CHAPTER 25

Brushed Permanent Magnet DC Motors

Most electric motors operate on the principle that current flowing through a magnetic field creates a force. Because of this relationship between current and force, electric motors can be used to convert electrical power to mechanical power. They can also be used to convert mechanical power to electrical power; as with, for example, generators in hydroelectric dams or regenerative braking in electric and hybrid cars.

In this chapter we study perhaps the simplest, cheapest, most common, and arguably most useful electrical motor: the brushed permanent magnet direct current (DC) motor. For brevity, we refer to these simply as DC motors. A DC motor has two input terminals, and a voltage applied across these terminals causes the motor shaft to spin. For a constant load or resistance at the motor shaft, the motor shaft achieves a speed proportional to the input voltage. Positive voltage causes spinning in one direction, and negative voltage causes spinning in the other.

Depending on the specifications, DC motors cost anywhere from tens of cents up to thousands of dollars. For most small-scale or hobby applications, appropriate DC motors typically cost a few dollars. DC motors are often outfitted with a sensing device, most commonly an encoder, to track the position and speed of the motor, and a gearhead to reduce the output speed and increase the output torque.

25.1 Motor Physics

DC motors exploit the *Lorentz force law*,

$$\mathbf{F} = \ell \mathbf{I} \times \mathbf{B}, \tag{25.1}$$

where $\mathbf{F}$, $\mathbf{I}$, and $\mathbf{B}$ are three-vectors, $\mathbf{B}$ describes the magnetic field created by permanent magnets, $\mathbf{I}$ is the current vector (including the magnitude and direction of the current flow through the conductor), ℓ is the length of the conductor in the magnetic field, and $\mathbf{F}$ is the force on the conductor. For the case of a current perpendicular to the magnetic field, the force is easily understood using the right-hand rule for cross-products: with your right hand, point your index finger along the current direction and your middle finger along the magnetic field flux lines. Your thumb will then point in the direction of the force (see Figure 25.1).

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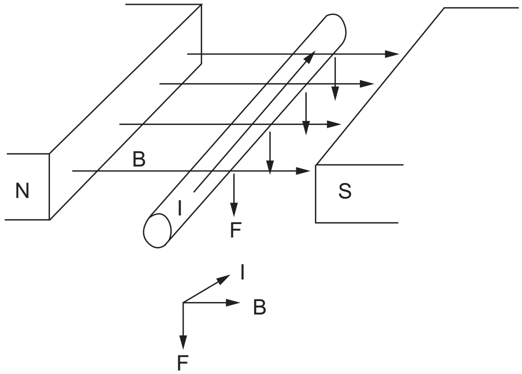


Figure 25.1
Two magnets create a magnetic field **B**, and a current **I** along the conductor causes a force **F** on the conductor.

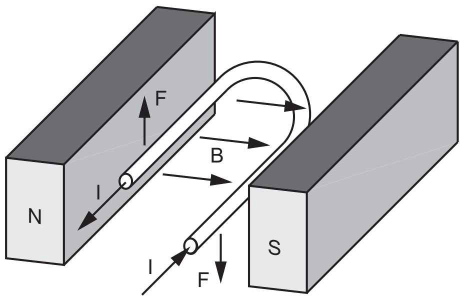


Figure 25.2
A current-carrying loop of wire in a magnetic field.

Now let us replace the conductor by a loop of wire, and constrain that loop to rotate about its center. See Figures 25.2 and 25.3. In one half of the loop, the current flows into the page, and in the other half of the loop the current flows out of the page. This creates forces of opposite directions on the loop. Referring to Figure 25.3, let the magnitude of the force acting on each half of the loop be f, and let d be the distance from the halves of the loop to the center of the loop. Then the total torque acting on the loop about its center can be written

$$\tau = 2df \cos\theta,$$

where θ is the angle of the loop. The torque changes as a function of θ. For $-90° < \theta < 90°$, the torque is positive, and it is maximum at $\theta = 0$. A plot of the torque on the loop as a function of θ is shown in Figure 25.4(a). The torque is zero at $\theta = -90°$ and $90°$, and of these two, $\theta = 90°$ is a stable equilibrium while $\theta = -90°$ is an unstable equilibrium. Therefore, if we send a constant current through the loop, it will likely come to rest at $\theta = 90°$.

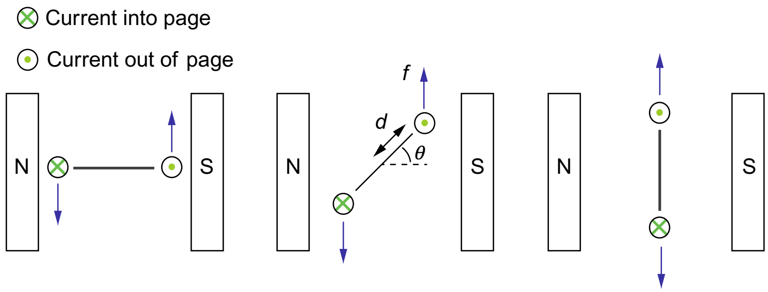


Figure 25.3
A loop of wire in a magnetic field, viewed end-on. Current flows into the page on one side of the loop and out of the page on the other, creating forces of opposite directions on the two halves of the loop. These opposite forces create torque on the loop about its center at most angles θ of the loop.

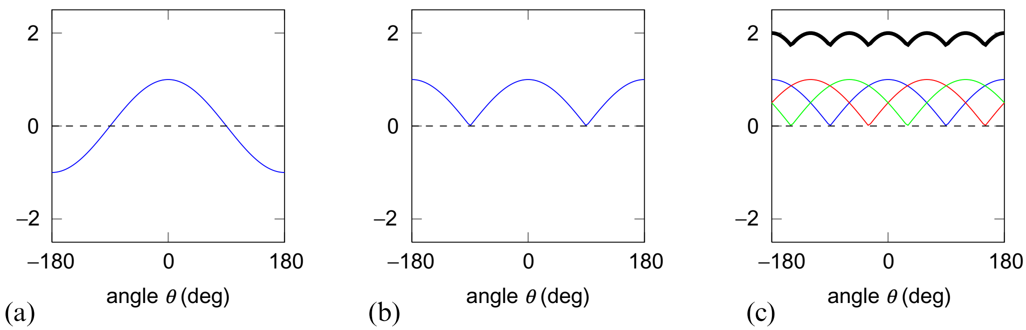


Figure 25.4
(a) The torque on the loop of Figure 25.3 as a function of its angle for a constant current. (b) If we reverse the current direction at the angles $\theta = -90°$ and $\theta = 90°$, we can make the torque nonnegative at all θ. (c) If we use several loops offset from each other, the sum of their torques (the thick curve) becomes more constant as a function of angle. The remaining variation contributes to torque ripple.

To make a more useful motor, we can reverse the direction of current at $\theta = -90°$ and $\theta = 90°$, which makes the torque nonnegative at all angles (Figure 25.4(b)). The torque is still zero at $\theta = -90°$ and $\theta = 90°$, however, and it undergoes a large variation as a function of θ. To make the torque more constant as a function of θ, we can introduce more loops of wire, each offset from the others in angle, and each reversing their current direction at appropriate angles. Figure 25.4(c) shows an example with three loops of wire offset from each other by 120°. Their component torques sum to give a more constant torque as a function of angle. The remaining variation in torque contributes to angle-dependent *torque ripple*.

Finally, to increase the torque generated, each loop of wire is replaced by a coil of wire (also called a winding) that loops back and forth through the magnetic field many times. If the coil consists of 100 loops, it generates 100 times the torque of the single loop for the same current. Wire used to create coils in motors, like magnet wire, is very thin, so there is resistance from one end of a coil to the other, typically from fractions of an ohm up to hundreds of ohms.

As stated previously, the current in the coils must switch direction at the appropriate angle to maintain non-negative torque. Figure 25.5 shows how brushed DC motors accomplish this current reversal. The two input terminals are connected to *brushes*, typically made of a soft conducting material like graphite, which are spring-loaded to press against the *commutator*, which is connected to the motor coils. As the motor rotates, the brushes slide over the commutator and switch between commutator *segments*, each of which is electrically connected to the end of one or more coils. This switching changes the direction of current through the coils. This process of switching the current through the coils as a function of the angle of the motor is called *commutation*. Figure 25.5 shows a schematic of a minimal motor design with three commutator segments and a coil between each pair of segments. Most high quality motors have more commutator segments and coils.

Unlike the simplified example in Figure 25.4, the brush-commutator geometry means that each coil in a real brushed motor is only energized at a subset of angles of the motor. Apart

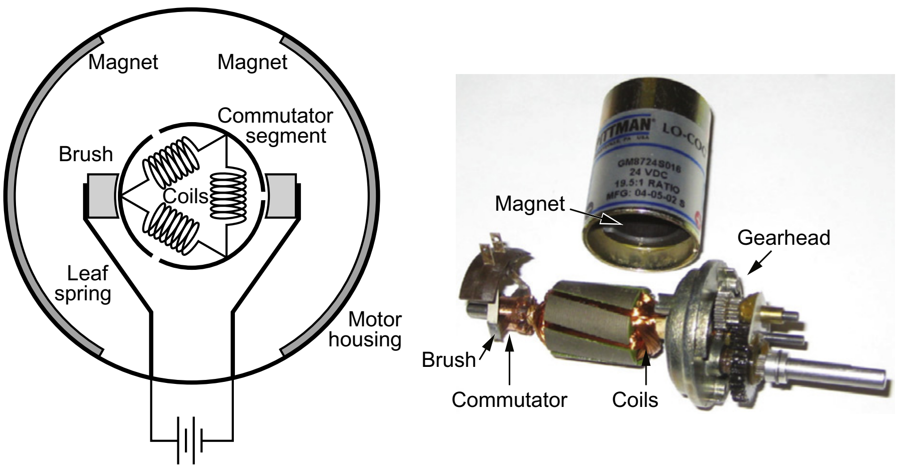


Figure 25.5
(Left) A schematic end-on view of a simple DC motor. The two brushes are held against the commutator by leaf springs which are electrically connected to the external motor terminals. This commutator has three segments and there are coils between each segment pair. The stator magnets are epoxied to the inside of the motor housing. (Right) This disassembled Pittman motor has seven commutator segments. The two brushes are attached to the motor housing, which has otherwise been removed. One of the two permanent magnets is visible inside the housing. The coils are wrapped around a ferromagnetic core to increase magnetic permeability. This motor has a gearhead on the output.

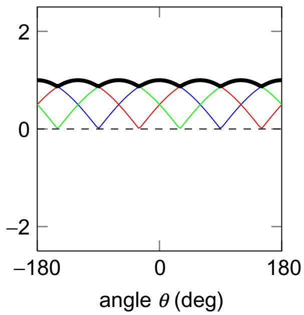


Figure 25.6

Figure 25.4(c) illustrates the sum of the torque of three coils offset by 120° if they are all energized at the same time. The geometry of the brushes and commutator ensure that not all coils are energized simultaneously, however. This figure shows the angle-dependent torque of a three-coil brushed motor that has only one coil energized at a time, which is approximately what happens if the brushes in Figure 25.5 are small. The energized coil is the one at the best angle to create a torque. The result is a motor torque as indicated by the thick curve; the thinner curves are the torques that would be provided by the other coils if they were energized. Comparing this figure to Figure 25.4(c) shows that this more realistic motor produces half the torque, but uses only one-third of the electrical power, since only one of the three coils is energized. Power is not wasted by putting current through coils that would generate little torque.

from being a consequence of the geometry, this has the added benefit of avoiding wasting power when current through a coil would provide little torque. Figure 25.6 is a more realistic version of Figure 25.4(c).

The stationary portion of the motor attached to the housing is called the stator, and the rotating portion of the motor is called the rotor.

Figure 25.7 shows a cutaway of a Maxon brushed motor, exposing the brushes, commutator, magnets, and windings. The figure also shows other elements of a typical motor application: an encoder attached to one end of the motor shaft to provide feedback on the angle and a gearhead attached to the other end of the motor shaft. The output shaft of the gearhead provides lower speed but higher torque than the output shaft of the motor.

Brushless motors are a variant that use electronic commutation as opposed to brushed commutation. For more on brushless DC motors, see Chapter 29.5.

25.2 Governing Equations

To derive an equation to model the motor's behavior, we ignore the details of the commutation and focus instead on electrical and mechanical power. The electrical power into the motor is IV, where I is the current through the motor and V is the voltage across the motor. We know that the motor converts some of this input power to mechanical power $\tau\omega$, where τ

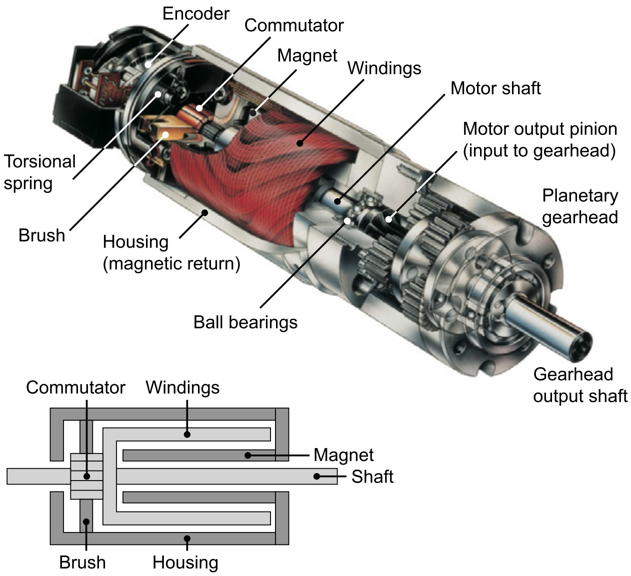


Figure 25.7

A cutaway of a Maxon brushed motor with an encoder and a planetary gearhead. The brushes are spring-loaded against the commutator. The bottom left schematic is a simplified cross-section showing the stationary parts of the motor (the stator) in dark gray and the rotating parts of the motor (the rotor) in light gray. In this "coreless" motor geometry, the windings spin in a gap between the permanent magnets and the housing. (Cutaway image courtesy of Maxon Precision Motors, Inc., maxonmotorusa.com.)

and ω are the torque and velocity of the output shaft, respectively. Electrically, the motor is described by a resistance R between the two terminals as well as an inductance L due to the coils. The resistance of the motor coils dissipates power I^2R as heat. The motor also stores energy $\frac{1}{2}LI^2$ in the inductor's magnetic field, and the time rate of change of this is $LI(\mathrm{d}I/\mathrm{d}t)$, the power into (charging) or out of (discharging) the inductor. Finally, power is dissipated as sound, heat due to friction at the brush-commutator interface and at the bearings between the motor shaft and the housing, etc. In SI units, all these power components are expressed in watts. Combining all of these factors provides a full accounting for the electrical power put into the motor:

$$IV = \tau\omega + I^2R + LI\frac{\mathrm{d}I}{\mathrm{d}t} + \text{ power dissipated due to friction, sound, etc.}$$

Ignoring the last term, we have our simple motor model, written in terms of power:

$$IV = \tau\omega + I^2R + LI\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.2}$$

From (25.2) we can derive all other relationships of interest. For example, dividing both sides of (25.2) by I yields

$$V = \frac{\tau}{I}\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.3}$$

The ratio τ/I is a constant, an expression of the Lorentz force law for the particular motor design. This constant, relating current to torque, is called the *torque constant* k_t. The torque constant is one of the most important properties of the motor:

$$k_t = \frac{\tau}{I} \quad \text{or} \quad \tau = k_t I. \tag{25.4}$$

The SI units of k_t are Nm/A. (In this chapter, we only use SI units, but you should be aware that many different units are used by different manufacturers, as on the speed-torque curve and data sheet in Figure 25.16 in the Exercises.) Equation (25.3) also shows that the SI units for k_t can be written equivalently as Vs/rad, or simply Vs. When using these units, we sometimes call the motor constant the *electrical constant* k_e. The inverse is sometimes called the *speed constant*. You should recognize that these terms all refer to the same property of the motor. For consistency, we usually refer to the torque constant k_t.

We now express the motor model in terms of voltage as

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.5}$$

You should remember, or be able to quickly derive, the power equation (25.2), the torque constant (25.4), and the voltage equation (25.5).

The term $k_t\omega$, with units of voltage, is called the *back-emf*, where emf is short for *electromotive force*. We could also call this "back-voltage." Back-emf is the voltage generated by a spinning motor to "oppose" the input voltage generating the motion. For example, assume that the motor's terminals are not connected to anything (open circuit). Then $I = 0$ and $\frac{\mathrm{d}I}{\mathrm{d}t} = 0$, so (25.5) reduces to

$$V = k_t\omega.$$

This equation indicates that back-driving the motor (e.g., spinning it by hand) will generate a voltage at the terminals. If we were to connect a capacitor across the motor terminals, then spinning the motor by hand would charge the capacitor, storing some of the mechanical energy we put in as electrical energy in the capacitor. In this situation, the motor acts as a generator, converting mechanical energy to electrical energy.

The existence of this back-emf term also means that if we put a constant voltage V across a free-spinning frictionless motor (i.e., the motor shaft is not connected to anything), after some time it will reach a constant speed V/k_t. At this speed, by (25.5), the current I drops to zero, meaning there is no more torque τ to accelerate the motor. This happens because as the motor accelerates, the back-emf increases, countering the applied voltage until no current flows (and hence there is no torque or acceleration).

25.3 The Speed-Torque Curve

Consider a motor spinning a boat's propeller at constant velocity. The torque τ provided by the motor can be written

$$\tau = \tau_{\text{fric}} + \tau_{\text{pushing water}},$$

where τ_{fric} is the torque the motor has to generate to overcome friction and begin to spin, while $\tau_{\text{pushing water}}$ is the torque needed for the propeller to displace water when the motor is spinning at velocity ω. In this section we assume $\tau_{\text{fric}} = 0$, so $\tau = \tau_{\text{pushing water}}$ in this example. In Section 25.4 we consider nonzero friction.

For a motor spinning at constant speed ω and providing constant torque τ (as in the propeller example above), the current I is constant and therefore $\mathrm{d}I/\mathrm{d}t = 0$. Under these assumptions, (25.5) reduces to

$$V = k_t\omega + IR. \tag{25.6}$$

Using the definition of the torque constant, we get the equivalent form

$$\omega = \frac{1}{k_t}V - \frac{R}{k_t^2}\tau. \tag{25.7}$$

Equation (25.7) gives ω as a linear function of τ for a given constant V. This line, of slope $-R/k_t^2$, is called the *speed-torque curve* for the voltage V.

The speed-torque curve plots all the possible constant-current operating conditions with voltage V across the motor. Assuming friction torque is zero, the line intercepts the $\tau = 0$ axis at

$$\omega_0 = V/k_t = \text{no load speed}.$$

The line intercepts the $\omega = 0$ axis at

$$\tau_{\text{stall}} = \frac{k_t V}{R} = \text{Stall torque}.$$

At the no-load condition, $\tau = I = 0$; the motor rotates at maximum speed with no current or torque. At the stall condition, the shaft is blocked from rotating, and the current ($I_{\text{stall}} = \tau_{\text{stall}}/k_t = V/R$) and output torque are maximized due to the lack of back-emf. Which point along the speed-torque curve the motor actually operates at is determined by the load attached to the motor shaft.

An example speed-torque curve is shown in Figure 25.8. This motor has $\omega_0 = 500$ rad/s and $\tau_{\text{stall}} = 0.1067$ Nm for a nominal voltage of $V_{\text{nom}} = 12$ V. The *operating region* is any point below the speed-torque curve, corresponding to voltages less than or equal to 12 V. If the motor is operated at a different voltage cV_{nom}, the intercepts of the speed-torque curve are linearly scaled to $c\omega_0$ and $c\tau_{\text{stall}}$.

Imagine squeezing the shaft of a motor powered by a voltage V and spinning at a constant velocity. Your hand is applying a small torque to the shaft. Since the motor is not accelerating and we are neglecting friction in the motor, the torque created by the motor's coils must be equal and opposite the torque applied by your hand. Thus the motor operates at a specific point on the speed-torque curve. If you slowly squeeze the shaft harder, increasing the torque you apply to the rotor, the motor will slow down and increase the torque it applies, to balance your hand's torque. Assuming the motor's current changes slowly (i.e., $L\mathrm{d}I/\mathrm{d}t$ is negligible), then the operating point of the motor moves down and to the right on the speed-torque curve as you increase your squeeze force. When you squeeze hard enough that the motor can no longer move, the operating point is at the stall condition, the bottom-right point on the speed-torque curve.

The speed-torque curve corresponds to constant V, but not to constant input power $P_{\text{in}} = IV$. The current I is linear with τ, so the input electrical power increases linearly with τ. The output mechanical power is $P_{\text{out}} = \tau\omega$, and the *efficiency* in converting electrical to mechanical power is $\eta = P_{\text{out}}/P_{\text{in}} = \tau\omega/IV$. We return to efficiency in Section 25.4.

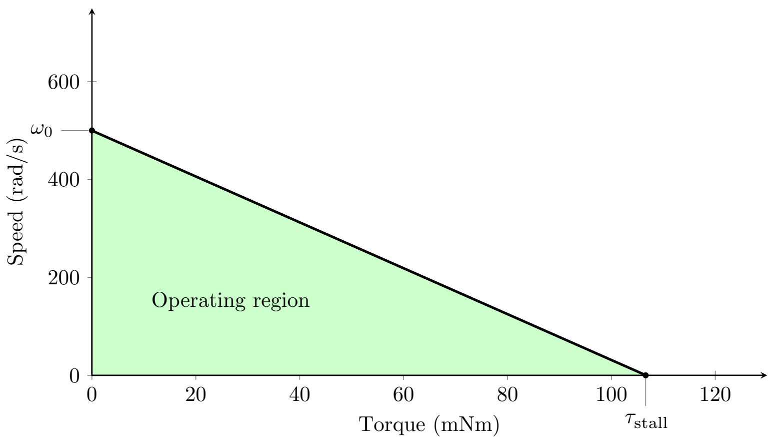


Figure 25.8
A speed-torque curve. Many speed-torque curves use rpm for speed, but we prefer SI units.

To find the point on the speed-torque curve that maximizes the mechanical output power, we can write points on the curve as $(\tau, \omega) = (c\tau_{\text{stall}}, (1-c)\omega_0)$ for $0 \leq c \leq 1$, so the output power is expressed as

$$P_{\text{out}} = \tau\omega = (c - c^2)\tau_{\text{stall}}\omega_0,$$

and the value of c that maximizes the power output is found by solving

$$\frac{\mathrm{d}}{\mathrm{d}c}\left((c-c^2)\tau_{\text{stall}}\omega_0\right) = (1-2c)\tau_{\text{stall}}\omega_0 = 0 \quad \rightarrow \quad c = \frac{1}{2}.$$

Thus the mechanical output power is maximized at $\tau = \tau_{\text{stall}}/2$ and $\omega = \omega_0/2$. This maximum output power is

$$P_{\max} = \left(\frac{1}{2}\tau_{\text{stall}}\right)\left(\frac{1}{2}\omega_0\right) = \frac{1}{4}\tau_{\text{stall}}\omega_0.$$

See Figure 25.9.

Motor current is proportional to motor torque, so operating at high torques means large coil heating power loss I^2R, sometimes called *ohmic heating*. For that reason, motor manufacturers specify a *maximum continuous current* I_{cont}, the largest continuous current such that the coils' steady-state temperature remains below a critical point.[1] The maximum continuous current has a corresponding *maximum continuous torque* τ_{cont}. Points to the left of this torque and under the speed-torque curve are called the *continuous operating region*. The motor can be

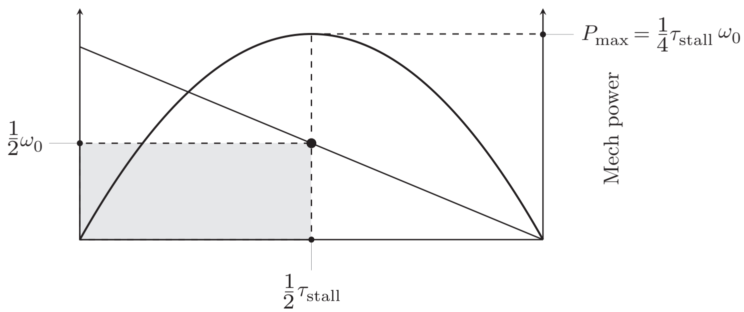


Figure 25.9
The quadratic mechanical power plot $P = \tau\omega$ plotted alongside the speed-torque curve. The area of the speed-torque rectangle below and to the left of the operating point is the mechanical power.

[1] The maximum continuous current depends on thermal properties governing how fast coil heat can be transferred to the environment. This depends on the environment temperature, typically considered to be room temperature. The maximum continuous current can be increased by cooling the motor.

operated intermittently outside of the continuous operating region, in the *intermittent operating region*, provided the motor is allowed to cool sufficiently between uses in this region. Motors are commonly rated with a nominal voltage that places the maximum mechanical power operating point (at $\tau_{\text{stall}}/2$) outside the continuous operating region.

Given thermal characteristics of the motor of Figure 25.8, the speed-torque curve can be refined to Figure 25.10, showing the continuous and intermittent operating regions of the motor. The point on the speed-torque curve at τ_{cont} is the *rated* or *nominal operating point*, and the mechanical power output at this point is called the motor's *power rating*. For the motor of Figure 25.10, $\tau_{\text{cont}} = 26.67$ mNm, which occurs at $\omega = 375$ rad/s, for a power rating of

$$0.02667 \text{ Nm} \times 375 \text{ rad/s} = 10.0 \text{ W}.$$

Figure 25.10 also shows the constant output power hyperbola $\tau\omega = 10$ W passing through the nominal operating point.

The speed-torque curve for a motor is drawn based on a nominal voltage. This is a "safe" voltage that the manufacturer recommends. It is possible to overvolt the motor, however, provided it is not continuously operated beyond the maximum continuous current. A motor also may have a specified *maximum permissible speed* ω_{max}, which creates a horizontal line constraint on the permissible operating range. This speed is determined by allowable brush wear, or possibly properties of the shaft bearings, and it is typically larger than the no-load speed ω_0. The shaft and bearings may also have a maximum torque rating $\tau_{\text{max}} > \tau_{\text{stall}}$. These

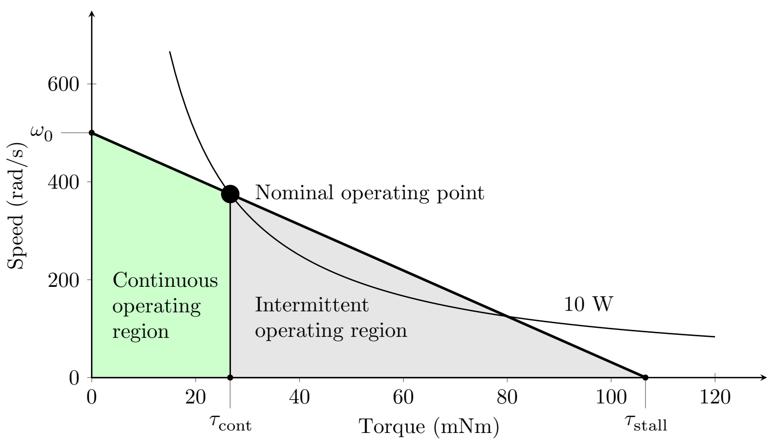


Figure 25.10
The continuous operating region (under the speed-torque curve and left of τ_{cont}) and the intermittent operating region (the rest of the area under the speed-torque curve). The 10 W mechanical power hyperbola is indicated, including the nominal operating point at τ_{cont}.

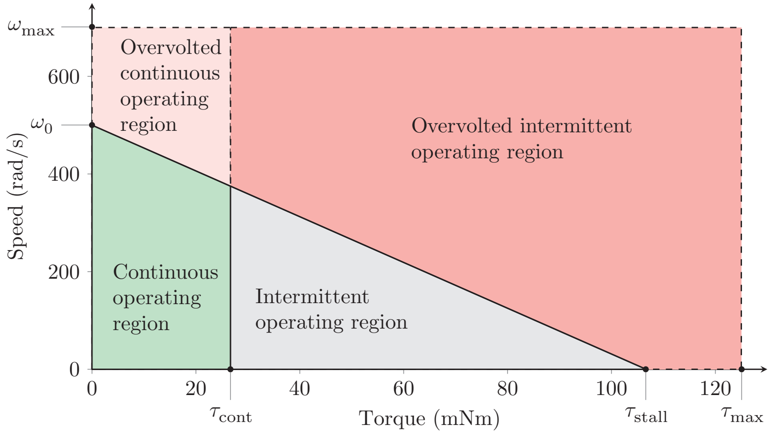


Figure 25.11
It is possible to exceed the nominal operating voltage, provided the constraints $\omega < \omega_{\max}$ and $\tau < \tau_{\max}$ are respected and τ_{cont} is only intermittently exceeded.

limits allow the definition of overvolted continuous and intermittent operating regions, as shown in Figure 25.11.

25.4 Friction and Motor Efficiency

Until now we have been assuming that the full torque $\tau = k_t I$ generated by the windings is available at the output shaft. In practice, some torque is lost due to friction at the brushes and the shaft bearings. Let us use a simple model of friction: assume a torque $\tau \geq \tau_{\mathrm{fric}} > 0$ must be generated to overcome friction and initiate motion, and any torque beyond τ_{fric} is available at the output shaft regardless of the motor speed (e.g., no friction that depends on speed magnitude). When the motor is spinning, the torque available at the output shaft is

$$\tau_{\mathrm{out}} = \tau - \tau_{\mathrm{fric}}.$$

Nonzero friction results in a nonzero *no-load current* $I_0 = \tau_{\mathrm{fric}}/k_t$ and a no-load speed ω_0 less than V/k_t. The speed-torque curve of Figure 25.11 is modified to show a small friction torque in Figure 25.12. The torque actually delivered to the load is reduced by τ_{fric}.

Taking friction into account, the motor's efficiency in converting electrical to mechanical power is

$$\eta = \frac{\tau_{\mathrm{out}}\omega}{IV}. \tag{25.8}$$

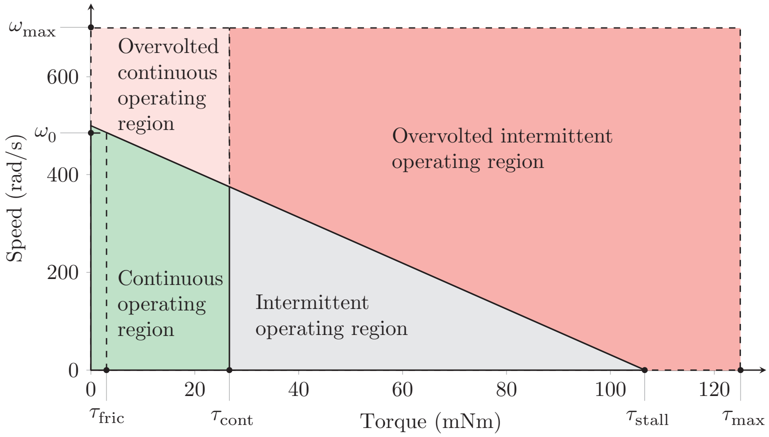


Figure 25.12
The speed-torque curve of Figure 25.11 modified to show a nonzero friction torque τ_{fric} and the resulting reduced no-load speed ω_0.

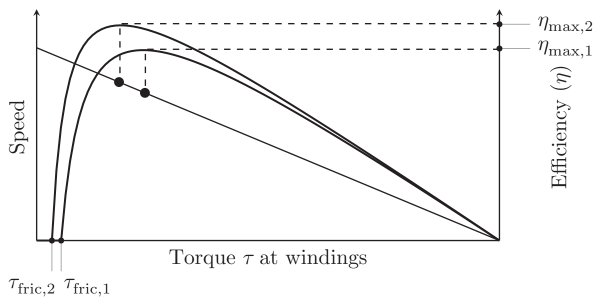


Figure 25.13
The speed-torque curve for a motor and two efficiency plots, one for high friction torque (case 1) and one for low friction torque (case 2). For each case, efficiency is zero for all τ below the level needed to overcome friction. The low friction version of the motor (case 2) achieves a higher maximum efficiency, at a higher speed and lower torque, than the high friction version (case 1).

The efficiency depends on the operating point on the speed-torque curve, and it is zero when either τ_{out} or ω is zero, as there is no mechanical power output. Maximum efficiency generally occurs at high speed and low torque, approaching the limit of 100% efficiency at $\tau = \tau_{out} = 0$ and $\omega = \omega_0$ as τ_{fric} approaches zero. As an example, Figure 25.13 plots efficiency vs. torque for the same motor with two different values of τ_{fric}. Lower friction results in a higher maximum efficiency η_{max}, occurring at a higher speed and lower torque.

To derive the maximally efficient operating point and the maximum efficiency $\eta_{\max}$ for a given motor, we can express the motor current as

$$I = I_0 + I_a,$$

where I_0 is the no-load current necessary to overcome friction and I_a is the added current to create torque to drive the load. Recognizing that $\tau_{\text{out}} = k_t I_a$, $V = I_{\text{stall}} R$, and $\omega = R(I_{\text{stall}} - I_a - I_0)/k_t$ by the linearity of the speed-torque curve, we can rewrite the efficiency (25.8) as

$$\eta = \frac{I_a(I_{\text{stall}} - I_0 - I_a)}{(I_0 + I_a)I_{\text{stall}}}. \tag{25.9}$$

To find the operating point I_a^* maximizing η, we solve $\mathrm{d}\eta/\mathrm{d}I_a = 0$ for I_a^*, and recognizing that I_0 and I_{stall} are nonnegative, the solution is

$$I_a^* = \sqrt{I_{\text{stall}} I_0} - I_0.$$

In other words, as the no-load current I_0 goes to zero, the maximally efficient current (and therefore τ) goes to zero.

Plugging I_a^* into (25.9), we find

$$\eta_{\max} = \left(1 - \sqrt{\frac{I_0}{I_{\text{stall}}}}\right)^2.$$

This answer has the form we would expect: maximum efficiency approaches 100% as the friction torque approaches zero, and maximum efficiency approaches 0% as the friction torque approaches the stall torque.

Choosing an operating point that maximizes motor efficiency can be important when trying to maximize battery life in mobile applications. For the majority of analysis and motor selection problems, however, ignoring friction is a good first approximation.

25.5 Motor Windings and the Motor Constant

It is possible to build two different versions of the same motor by simply changing the windings while keeping everything else the same. For example, imagine a coil of resistance R with N loops of wire of cross-sectional area A. The coil carries a current I and therefore has a voltage drop IR. Now we replace that coil with a new coil with N/c loops of wire with cross-sectional area cA. This preserves the volume occupied by the coil, fitting in the same form factor with similar thermal properties. Without loss of generality, let us assume that the new coil has fewer loops and uses thicker wire ($c > 1$).

The resistance of the new coil is reduced to R/c^2 (a factor of c due to the shorter coil and another factor of c due to the thicker wire). To keep the torque of the motor the same, the new coil would have to carry a larger current cI to make up for the fewer loops, so that the current times the pathlength through the magnetic field is unchanged. The voltage drop across the new coil is $(cI)(R/c^2) = IR/c$.

Replacing the coils allows us to create two versions of the motor: a many-loop, thin wire version that operates at low current and high voltage, and a fewer-loop, thick wire version that operates at high current and low voltage. Since the two motors create the same torque with different currents, they have different torque constants. Each motor has the same *motor constant* k_m, however, where

$$k_m = \frac{\tau}{\sqrt{I^2 R}} = \frac{k_t}{\sqrt{R}}$$

with units of Nm/$\sqrt{\text{W}}$. The motor constant defines the torque generated per square root of the power dissipated by coil resistance. In the example above, the new coil dissipates $(cI)^2(R/c^2) = I^2R$ power as heat, just as the original coil does, while generating the same torque.

Figure 25.16 shows the data sheet for a motor that comes in several different versions, each identical in every way except for the winding. Each version of the motor has a similar stall torque and motor constant but different nominal voltage, resistance, and torque constant.

25.6 Other Motor Characteristics

Electrical time constant

When the motor is subject to a step in the voltage across it, the *electrical time constant* T_e measures the time it takes for the unloaded current to reach 63% of its final value. The motor's voltage equation is

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}.$$

Ignoring back-emf (because the motor speed does not change significantly over one electrical time constant), assuming an initial current through the motor of I_0, and an instantaneous drop in the motor voltage to 0, we get the differential equation

$$0 = I_0 R + L\frac{\mathrm{d}I}{\mathrm{d}t}$$

or

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\frac{R}{L}I_0$$

with solution

$$I(t) = I_0\ \mathrm{e}^{-tR/L} = I_0\ \mathrm{e}^{-t/T_e}.$$

The time constant of this first-order decay of current is the motor's electrical time constant, $T_e = L/R$.

Mechanical time constant

When the motor is subject to a step voltage across it, the *mechanical time constant* T_m measures the time it takes for the unloaded motor speed to reach 63% of its final value. Beginning from the voltage equation

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t},$$

ignoring the inductive term, and assuming an initial speed ω_0 at the moment the voltage drops to zero, we get the differential equation

$$0 = IR + k_t\omega_0 = \frac{R}{k_t}\tau + k_t\omega_0 = \frac{JR}{k_t}\frac{\mathrm{d}\omega}{\mathrm{d}t} + k_t\omega_0,$$

where we used $\tau = Jd\omega/dt$, where J is the inertia of the motor. We can rewrite this equation as

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{k_t^2}{JR}\omega_0$$

with solution

$$\omega(t) = \omega_0\ \mathrm{e}^{-t/T_m},$$

with a time constant of $T_m = JR/k_t^2$. If the motor is attached to a load that increases the inertia J, the mechanical time constant increases.

Short-circuit damping

When the terminals of the motor are shorted together, the voltage equation (ignoring inductance) becomes

$$0 = k_t\omega + IR = k_t\omega + \frac{\tau}{k_t}R$$

or

$$\tau = -B\omega = -\frac{k_t^2}{R}\omega,$$

where $B = k_t^2/R$ is the short-circuit damping. A spinning motor is slowed more quickly by shorting its terminals together, compared to leaving the terminals open circuit, due to this damping.

25.7 Motor Data Sheet

Motor manufacturers summarize motor properties described above in a speed-torque curve and in a data sheet similar to the one in Figure 25.14. When you buy a motor second-hand or surplus, you may need to measure these properties yourself. We will use all SI units, which is not the case on most motor data sheets.

Many of these properties have been introduced already. Below we describe some methods for estimating them.

Experimentally Characterizing a Brushed DC Motor

Given a mystery motor with an encoder, you can use a function generator, oscilloscope, multimeter and perhaps some resistors and capacitors to estimate most of the important properties of the motor. Below are some suggested methods; you may be able to devise others.

Terminal resistance R

You can measure R with a multimeter. The resistance may change as you rotate the shaft by hand, as the brushes move to new positions on the commutator. You should record the minimum resistance you can reliably find. A better choice, however, may be to measure the current when the motor is stalled.

Torque constant k_t

You can measure this by spinning the shaft of the motor, measuring the back-emf at the motor terminals, and measuring the rotation rate ω using the encoder. Or, if friction losses are

Motor Characteristic	Symbol	Value	Units	Comments
Terminal resistance	R		Ω	Resistance of the motor windings. May change as brushes slide over commutator segments. Increases with heat.
Torque constant	k_t		Nm/A	The constant ratio of torque produced to current through the motor.
Electrical constant	k_e		Vs/rad	Same numerical value as the torque constant (in SI units). Also called voltage or back-emf constant.
Speed constant	k_s		rad/(Vs)	Inverse of electrical constant.
Motor constant	k_m		Nm/$\sqrt{\text{W}}$	Torque produced per square root of power dissipated by the coils.
Max continuous current	I_{cont}		A	Max continuous current without overheating.
Max continuous torque	τ_{cont}		Nm	Max continuous torque without overheating.
Short-circuit damping	B		Nms/rad	Not included in most data sheets, but useful for motor braking (and haptics).
Terminal inductance	L		H	Inductance due to the coils.
Electrical time constant	Te		s	The time for the motor current to reach 63% of its final value. Equal to L/R.
Rotor inertia	J		kgm^2	Often given in units gcm^2.
Mechanical time constant	Tm		s	The time for the motor to go from rest to 63% of its final speed under constant voltage and no load. Equal to JR/kt^2.
Friction				Not included in most data sheets. See explanation.
Values at Nominal Voltage				
Nominal voltage	V_{nom}		V	Should be chosen so the no-load speed is safe for brushes, commutator, and bearings.
Power rating	P		W	Output power at the nominal operating point (max continuous torque).
No-load speed	ω_0		rad/s	Speed when no load and powered by $V\text{nom}$. Usually given in rpm (revs/min, sometimes m^{-1}).
No-load current	I_0		A	The current required to spin the motor at the no-load condition. Nonzero because of friction torque.
Stall current	I		A	Same as starting current, $V\text{nom}/R$.
Stall torque	τ_{stall}		Nm	The torque achieved at the nominal voltage when the motor is stalled.
Max mechanical power	P_{max}		W	The max mechanical power output at the nominal voltage (including short-term operation).
Max efficiency	η		%	The maximum efficiency achievable in converting electrical to mechanical power.

Figure 25.14
A sample motor data sheet, with values to be filled in.

negligible, a good approximation is V_{nom}/ω_0. This eliminates the need to spin the motor externally.

Electrical constant k_e

Identical to the torque constant in SI units. The torque constant k_t is often expressed in units of Nm/A or mNm/A or English units like oz-in/A, and often k_e is given in V/rpm, but k_t and k_e have identical numerical values when expressed in Nm/A and Vs/rad, respectively.

Speed constant k_s

Just the inverse of the electrical constant.

Motor constant k_m

The motor constant is calculated as $k_m = k_t/\sqrt{R}$.

Max continuous current I_{cont}

This is determined by thermal considerations, which are not easy to measure. It is typically less than half the stall current.

Max continuous torque τ_{cont}

This is determined by thermal considerations, which are not easy to measure. It is typically less than half the stall torque.

Short-circuit damping B

This is most easily calculated from estimates of R and k_t: $B = k_t^2/R$.

Terminal inductance L

There are several ways to measure inductance. One approach is to add a capacitor in parallel with the motor and measure the oscillation frequency of the resulting RLC circuit. For example, you could build the circuit shown in Figure 25.15, where a good choice for C may be 0.01 or 0.1 µF. The motor acts as a resistor and inductor in series; back-emf will not be an issue, because the motor will be powered by tiny currents at high frequency and therefore will not move.

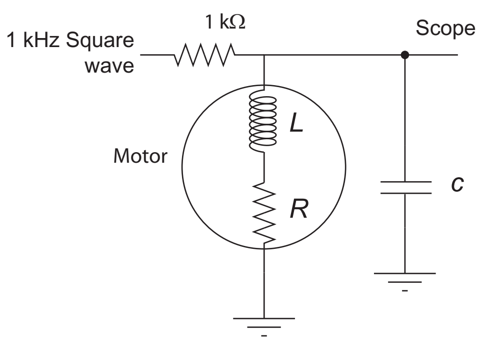


Figure 25.15
Using a capacitor to create an RLC circuit to measure motor inductance.

Use a function generator to put a 1 kHz square wave between 0 and 5 V at the point indicated. The 1 kΩ resistor limits the current from the function generator. Measure the voltage with an oscilloscope where indicated. You should be able to see a decaying oscillatory response to the square wave input when you choose the right scales on your scope. Measure the frequency of the oscillatory response. Knowing C and that the natural frequency of an RLC circuit is $\omega_n = 1/\sqrt{LC}$ in rad/s, estimate L.

Let us think about why we see this response. Say the input to the circuit has been at 0 V for a long time. Then your scope will also read 0 V. Now the input steps up to 5 V. After some time, in steady state, the capacitor will be an open circuit and the inductor will be a closed circuit (wire), so the voltage on the scope will settle to 5 V $\times$ $(R/(1000 + R))$—the two resistors in the circuit set the final voltage. Right after the voltage step, however, all current goes to charge the capacitor (as the zero current through the inductor cannot change discontinuously). If the inductor continued to enforce zero current, the capacitor would charge to 5 V. As the voltage across the capacitor grows, however, so does voltage across the inductor, and therefore so does the rate of change of current that must flow through the inductor (by the relation $V_L + V_R = V_C$ and the constitutive law $V_L = L\,\mathrm{d}I/\mathrm{d}t$). Eventually the integral of this rate of change dictates that all current is redirected to the inductor, and in fact the capacitor will have to provide current to the inductor, discharging itself. As the voltage across the capacitor drops, though, the voltage across the inductor will eventually become negative, and therefore the rate of change of current through the inductor will become negative. And so on, to create the oscillation. If R were large, i.e., if the circuit were heavily damped, the oscillation would die quickly, but you should be able to see it.

Note that you are seeing a damped oscillation, so you are actually measuring a damped natural frequency. But the damping is low if you are seeing at least a couple of cycles of oscillation, so the damped natural frequency is nearly indistinguishable from the undamped natural frequency.

Electrical time constant T_e

The electrical time constant can be calculated from L and R as $T_e = L/R$.

Rotor inertia J

The rotor inertia can be estimated from measurements of the mechanical time constant T_m, the torque constant k_t, and the resistance R. Alternatively, a ballpark estimate can be made based on the mass of the motor, a guess at the portion of the mass that belongs to the spinning rotor, a guess at the radius of the rotor, and a formula for the inertia of a uniform density cylinder. Or, more simply, consult a data sheet for a motor of similar size and mass.

Mechanical time constant T_m

The time constant can be measured by applying a constant voltage to the motor, measuring the velocity, and determining the time it takes to reach 63% of final speed. Alternatively, you could make a reasonable estimate of the rotor inertia J and calculate $T_m = JR/k_t^2$.

Friction

Friction torque arises from the brushes sliding on the commutator and the motor shaft spinning in its bearings, and it may depend on external loads. A typical model of friction includes both Coulomb friction and viscous friction, written

$$\tau_{\text{fric}} = b_0 \operatorname{sgn}(\omega) + b_1 \omega,$$

where b_0 is the Coulomb friction torque ($\operatorname{sgn}(\omega)$ just returns the sign of ω) and b_1 is a viscous friction coefficient. At no load, $\tau_{\text{fric}} = k_t I_0$. An estimate of each of b_0 and b_1 can be made by running the motor at two different voltages with no load.

Nominal voltage V_{nom}

This is the specification you are most likely to know for an otherwise unknown motor. It is sometimes printed right on the motor itself. This voltage is just a recommendation; the real issue is to avoid overheating the motor or spinning it at speeds beyond the recommended value for the brushes or bearings. Nominal voltage cannot be measured, but a typical no-load speed for a brushed DC motor is between 3000 and 10,000 rpm, so the nominal voltage will often give a no-load speed in this range.

Power rating P

The power rating is the mechanical power output at the max continuous torque.

No-load speed ω_0

You can determine ω_0 by measuring the unloaded motor speed when powered with the nominal voltage. The amount that this is less than V_{nom}/k_t can be attributed to friction torque.

No-load current I_0

You can determine I_0 by using a multimeter in current measurement mode.

Stall current I_{stall}

Stall current is sometimes called starting current. You can estimate this using your estimate of R. Since R may be difficult to measure with a multimeter, you can instead stall the motor shaft and use your multimeter in current sensing mode, provided the multimeter can handle the current.

Stall torque τ_{stall}

This can be obtained from k_t and I_{stall}.

Max mechanical power P_{max}

The max mechanical power occurs at $\frac{1}{2}\tau_{\text{stall}}$ and $\frac{1}{2}\omega_0$. For most motor data sheets, the max mechanical power occurs outside the continuous operation region.

Max efficiency η_{max}

Efficiency is defined as the power out divided by the power in, $\tau_{\text{out}}\omega/(IV)$. The wasted power is due to coil heating and friction losses. Maximum efficiency can be estimated using the no-load current I_0 and the stall current I_{stall}, as discussed in Section 25.4.

25.8 Chapter Summary

- The Lorentz force law says that a current-carrying conductor in a constant magnetic field feels a net force according to

$$\mathbf{F} = \ell \mathbf{I} \times \mathbf{B},$$

  where ℓ is the length of the conductor in the field, $\mathbf{I}$ is the current vector, and $\mathbf{B}$ is the (constant) magnetic field vector.
- A brushed DC motor consists of multiple current-carrying coils attached to a rotor, and magnets on the stator to create a magnetic field. Current is transmitted to the coils by two brushes connected to the stator sliding over a commutator ring attached to the rotor. Each coil attaches to two different commutator segments.
- The voltage across a motor's terminals can be expressed as

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t},$$

  where k_t is the torque constant and $k_t\omega$ is the back-emf.
- The speed-torque curve is obtained by plotting the steady-state speed as a function of torque for a given motor voltage V,

$$\omega = \frac{1}{k_t}V - \frac{R}{k_t^2}\tau.$$

The maximum speed (at $\tau = 0$) is called the no-load speed ω_0 and the maximum torque (at $\omega = 0$) is called the stall torque τ_{stall}.

- The continuous operating region of a motor is defined by the maximum current I the motor coils can conduct continuously without overheating due to I^2R power dissipation.
- The mechanical power $\tau\omega$ delivered by a motor is maximized at half the stall torque and half the no-load speed, $P_{\max} = \frac{1}{4}\tau_{\text{stall}}\omega_0$.
- A motor's electrical time constant $T_e = L/R$ is the time needed for current to reach 63% of its final value in response to a step input in voltage.
- A motor's mechanical time constant $T_m = JR/k_t^2$ is the time needed for the motor speed to reach 63% of its final value in response to a step change in voltage.

25.9 Exercises

1. Assume a DC motor with a five-segment commutator. Each segment covers 70° of the circumference of the commutator circle. The two brushes are positioned at opposite ends of the commutator circle, and each makes contact with 10° of the commutator circle.
   a. How many separate coils does this motor likely have? Explain.
   b. Choose one of the motor coils. As the rotor rotates 360°, what is the total angle over which that coil is energized? (For example, an answer of 360° means that the coil is energized at all angles; an answer of 180° means that the coil is energized at half of the motor positions.)
2. Figure 25.16 gives the data sheet for the 10 W Maxon RE 25 motor. The columns correspond to different windings.
   a. Draw the speed-torque curve for the 12 V version of the motor, indicating the no-load speed (in rad/s), the stall torque, the nominal operating point, and the rated power of the motor.
   b. Explain why the torque constant is different for the different versions of the motor.
   c. Using other entries in the table, calculate the maximum efficiency $\eta_{\max}$ of the 12 V motor and compare to the value listed.
   d. Calculate the electrical time constant T_e of the 12 V motor. What is the ratio to the mechanical time constant T_m?
   e. Calculate the short-circuit damping B for the 12 V motor.
   f. Calculate the motor constant k_m for the 12 V motor.
   g. How many commutator segments do these motors have?
   h. Which versions of these motors are likely to be in stock?

Stock program
Standard program
Special program (on request)

Article Numbers

Motor Data		118740	118741	118742	118743	118744	118745	118746	118747	118748
Values at nominal voltage										
1 Nominal voltage	V	4.5	8	9	12	15	18	24	32	48
2 No load speed	rpm	5360	5320	5230	4850	4980	4790	5190	5510	5070
3 No load current	mA	79.7	44.4	38.7	26.3	21.8	9.88	14.4	11.7	6.96
4 Nominal speed	rpm	4980	4520	4220	3800	3920	3710	4130	4450	4000
5 Nominal torque (max. continuous torque)	mNm	11.4	20.9	23.9	28.6	28.2	28.7	28	27.9	27.9
6 Nominal current (max. continuous current)	A	1.5	1.5	1.5	1.24	1.01	0.811	0.652	0.516	0.317
7 Stall torque	mNm	131	132	119	129	131	126	136	144	132
8 Starting current	A	16.5	9.23	7.31	5.5	4.57	3.52	3.1	2.61	1.47
9 Max. efficiency	%	87	87	86	87	87	90	87	87	87
Characteristics										
10 Terminal resistance	Ω	0.273	0.867	1.23	2.18	3.28	5.11	7.73	12.3	32.6
11 Terminal inductance	mH	0.0275	0.0882	0.115	0.238	0.353	0.551	0.832	1.31	3.48
12 Torque constant	mNm/A	7.99	14.3	16.3	23.5	28.6	35.8	43.9	55.2	89.9
13 Speed constant	rpm/V	1200	668	584	406	334	267	217	173	106
14 Speed / torque gradient	rpm/mNm	40.9	40.5	44	37.7	38.3	38.2	38.3	38.5	38.6
15 Mechanical time constant	ms	4.99	4.4	4.37	4.25	4.23	4.22	4.22	4.22	4.23
16 Rotor inertia	gcm^2	11.7	10.4	9.49	10.8	10.6	10.6	10.5	10.5	10.5

Specifications

Thermal data

17	Thermal resistance housing-ambient	14 K/W
18	Thermal resistance winding-housing	3.1 K/W
19	Thermal time constant winding	12.5 s
20	Thermal time constant motor	612 s
21	Ambient temperature	-20…+85°C
22	Max. permissible winding temperature	+100°C

Mechanical data (ball bearings)

23	Max. permissible speed	5500 rpm
24	Axial play	0.05 - 0.15 mm
25	Radial play	0.025 mm
26	Max. axial load (dynamic)	3.2 N
27	Max. force for press fits (static)	64 N
	(static, shaft supported)	800 N
28	Max. radial loading, 5 mm from flange	16 N

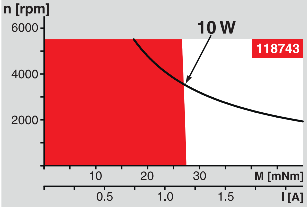
Other specifications

29	Number of pole pairs	1
30	Number of commutator segments	11
31	Weight of motor	130 g
	CLL = Capacitor Long Life	

Values listed in the table are nominal.
Explanation of the figures on page 49.

Option
Preloaded ball bearings

Operating Range



Comments

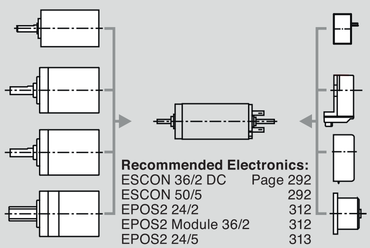
Continuous operation
In observation of above listed thermal resistance (lines 17 and 18) the maximum permissible winding temperature will be reached during continuous operation at 25°C ambient.
= Thermal limit.

Short term operation
The motor may be briefly overloaded (recurring).

Assigned power rating

maxon Modular System Overview on page 16 - 21

Planetary Gearhead
Ø26 mm
0.5 - 4.5 Nm
Page 231/232
Planetary Gearhead
Ø32 mm
0.75 - 6.0 Nm
Page 234/235/237
Koaxdrive
Ø32 mm
1.0 - 4.5 Nm
Page 240
Spindle Drive
Ø32 mm
Page 255/256/257



Recommended Electronics:

ESCON 36/2 DC	Page 292
ESCON 50/5	292
EPOS2 24/2	312
EPOS2 Module 36/2	312
EPOS2 24/5	313
EPOS2 50/5	313
EPOS2 P 24/5	316
EPOS3 70/10 EtherCAT	319
Notes	**18**

Encoder MR
128 - 1000 CPT,
3 channels
Page 272
Encoder Enc
22 mm
100 CPT, 2 channels
Page 274
Encoder HED_ 5540
500 CPT,
3 channels
Page 276/278
DC-Tacho DCT
Ø22 mm
0.52 V
Page 286

Figure 25.16
The data sheet for the Maxon RE 25 motor. The columns correspond to different windings for different nominal voltages. (Image courtesy of Maxon Precision Motors. Motor data is subject to change at any time; consult maxonmotorusa.com for the latest data sheets.)

i. (Optional) Motor manufacturers may specify slightly different continuous and intermittent operating regions than the ones described in this chapter. For example, the limit of the continuous operating region is not quite vertical in the speed-torque plot of Figure 25.16. Come up with a possible explanation, perhaps using online resources.

3. There are 21 entries on the motor data sheet from Section 25.7. Let us assume zero friction, so we ignore the last entry. To avoid thermal tests, you may also assume a maximum continuous power that the motor coils can dissipate as heat before overheating.

Of the 20 remaining entries, under the assumption of zero friction, how many independent entries are there? That is, what is the minimum number N of entries you need to be able to fill in the rest of the entries? Give a set of N independent entries from which you can derive the other $20 - N$ dependent entries. For each of the $20 - N$ dependent entries, give the equation in terms of the N independent entries. For example, V_{nom} and R will be two of the N independent entries, from which we can calculate the dependent entry $I_{\text{stall}} = V_{\text{nom}}/R$.

4. This exercise is an experimental characterization of a motor. For this exercise, you need a low-power motor (preferably without a gearhead to avoid high friction) with an encoder. You also need a multimeter, oscilloscope, and a power source for the encoder and motor. Make sure the power source for the motor can provide enough current when the motor is stalled. A low-voltage battery pack is a good choice.
   a. Spin the motor shaft by hand. Get a feel for the rotor inertia and friction. Try to spin the shaft fast enough that it continues spinning briefly after you let go of it.
   b. Now short the motor terminals by electrically connecting them. Spin again by hand, and try to spin the shaft fast enough that it continues spinning briefly after you let go of it. Do you notice the short-circuit damping?
   c. Try measuring your motor's resistance using your multimeter. It may vary with the angle of the shaft, and it may not be easy to get a steady reading. What is the minimum value you can get reliably? To double-check your answer, you can power your motor and use your multimeter to measure the current as you stall the motor's shaft by hand.
   d. Attach one of your motor's terminals to scope ground and the other to a scope input. Spin the motor shaft by hand and observe the motor's back-emf.
   e. Power the motor's encoder, attach the A and B encoder channels to your oscilloscope, and make sure the encoder ground and scope ground are connected together. Do *not* power the motor. (The motor inputs should be disconnected from anything.) Spin the motor shaft by hand and observe the encoder pulses, including their relative phase.
   f. Now power your motor with a low-voltage battery pack. Given the number of lines per revolution of the encoder, and the rate of the encoder pulses you observe on your scope, calculate the motor's no-load speed for the voltage you are using.
   g. Work with a partner. Couple your two motor shafts together by tape or flexible tubing. (This may only work if your motor has no gearhead.) Now plug one terminal of one of the motors (we shall call it the *passive* motor) into one channel of a scope, and plug the other terminal of the passive motor into GND of the same scope. Now power the other motor (the *driving* motor) with a battery pack so that both motors spin. Measure the speed of the passive motor by looking at its encoder count rate on your scope. Also measure its back-emf. With this information, calculate the passive motor's torque constant k_t.
5. Using techniques discussed in this chapter, or techniques you come up with on your own, create a data sheet with all 21 entries for your nominal voltage. Indicate how you

calculated the entry. (Did you do an experiment for it? Did you calculate it from other entries? Or did you estimate by more than one method to cross-check your answer?) For the friction entry, you can assume Coulomb friction only—the friction torque opposes the rotation direction ($b_0 \neq 0$), but is independent of the speed of rotation ($b_1 = 0$). For your measurement of inductance, turn in an image of the scope trace you used to estimate ω_n and L, and indicate the value of C that you used.

If there are any entries you are unable to estimate experimentally, approximate, or calculate from other values, simply say so and leave that entry blank.

6. Based on your data sheet from above, draw the speed-torque curves described below, and answer the associated questions. Do not do any experiments for this exercise; just extrapolate your previous results.
   a. Draw the speed-torque curve for your motor. Indicate the stall torque and no-load speed. Assume a maximum power the motor coils can dissipate continuously before overheating and indicate the continuous operating regime. Given this, what is the power rating P for this motor? What is the max mechanical power $P_{\max}$?
   b. Draw the speed-torque curve for your motor assuming a nominal voltage four times larger than in Exercise 6a. Indicate the stall torque and no-load speed. What is the max mechanical power $P_{\max}$?
7. You are choosing a motor for the last joint of a new direct-drive robot arm design. (A direct-drive robot does not use gearheads on the motors, creating high speeds with low friction.) Since it is the last joint of the robot, and it has to be carried by all the other joints, you want it to be as light as possible. From the line of motors you are considering from your favorite motor manufacturer, you know that the mass increases with the motor's power rating. Therefore you are looking for the lowest power motor that works for your specifications. Your specifications are that the motor should have a stall torque of at least 0.1 Nm, should be able to rotate at least 5 revolutions per second when providing 0.01 Nm, and the motor should be able to operate continuously while providing 0.02 Nm. Which motor do you choose from Table 25.1? Give a justification for your answer.
8. The speed-torque curve of Figure 25.8 is drawn for the positive speed and positive torque quadrant of the speed-torque plane. In this exercise, we will draw the motor's operating region for all four quadrants. The power supply used to drive the motor is 24 V, and assume the H-bridge motor controller (discussed in Chapter 27) can use that power supply to create any average voltage across the motor between −24 and 24 V. The motor's resistance is 1 Ω and the torque constant is 0.1 Nm/A. Assume the motor has zero friction.

   a. Draw the four-quadrant speed-torque operating region for the motor assuming the 24 V power supply (and the H-bridge driver) has no limit on current. Indicate the torque and speed values where the boundaries of the operating region intersect the $\omega = 0$ and $\tau = 0$ axes. Assume there are no other speed or torque constraints on the motor except for the one due to the 24 V limit of the power supply. (Hint: the operating region is unbounded in both speed and torque!)

Table 25.1: Motors to choose from

Assigned power rating	W	3	10	20	90
Nominal voltage	V	15	15	15	15
No load speed	rpm	13,400	4980	9660	7180
No load current	mA	36.8	21.8	60.8	247
Nominal speed	rpm	10,400	3920	8430	6500
Max continuous torque	mNm	2.31	28.2	20.5	73.1
Max continuous current	mA	259	1010	1500	4000
Stall torque	mNm	10.5	131	225	929
Stall current	mA	1030	4570	15,800	47,800
Max efficiency	%	65	87	82	83
Terminal resistance	Ohm	14.6	3.28	0.952	0.314
Terminal inductance	mH	0.486	0.353	0.088	0.085
Torque constant	mNm/A	10.2	28.6	14.3	19.4
Speed constant	rpm/V	932	334	670	491
Mechanical time constant	ms	7.51	4.23	4.87	5.65
Rotor inertia	gcm^2	0.541	10.6	10.4	68.1
Max permissible speed	rpm	16,000	5500	14,000	12,000
Cost	USD	88	228	236	239

Note that sometimes the "Assigned power rating" is different from the mechanical power output at the nominal operating point, for manufacturer-specific reasons. The meanings of the other terms in the table are unambiguous.

b. Update the operating region with the constraint that the power supply can provide a maximum current of 30 A. What is the maximum torque that can be generated using this power supply, and what are the maximum and minimum motor speeds possible at this maximum torque? What is the largest back-emf voltage that can be achieved?
c. Update the operating region with the constraint that the maximum recommended speed for the motor brushes and shaft bearings is 250 rad/s.
d. Update the operating region with the constraint that the maximum recommended torque at the motor shaft is 5 Nm.
e. Update the operating region to show the continuous operating region, assuming the maximum continuous current is 10 A.
f. We typically think of a motor as consuming electrical power ($IV > 0$, or "motoring") and converting it to mechanical power, but it can also convert mechanical power to electrical power ($IV < 0$, or "regenerating"). This occurs in electric car braking systems, for example. Update the operating region to show the portion where the motor is consuming electrical power and the portion where the motor is generating electrical power.

Further Reading

Hughes, A., & Drury, B. (2013). *Electric motors and drives: Fundamentals, types and applications* (4th ed.). Amsterdam: Elsevier.

Maxon DC motor RE 25, ø 25 mm, graphite brushes, 20 Watt. (2015). Maxon.