CHAPTER 27

DC Motor Control

Driving a brushed DC motor with variable speed and torque requires variable high current. A microcontroller is capable of neither variable analog output nor high current. Both problems are solved through the use of digital PWM and an H-bridge. The H-bridge consists of a set of switches that are rapidly opened and closed by the microcontroller's PWM signal, alternately connecting and disconnecting high voltage to the motor. The effect is similar to the time-average of the voltage. Motion control of the motor is achieved using motor position feedback, typically from an encoder.

27.1 The H-Bridge and Pulse Width Modulation

Let us consider a series of improving ideas for driving a motor. By attempting to fix their problems, we arrive at the H-bridge.

Direct driving from a microcontroller pin (Figure 27.1(a))

The first idea is to simply connect a microcontroller digital output pin to one motor lead and connect the other motor lead to a positive voltage. The pin can alternate between low (ground) and high impedance (disconnected or "tristated"). This connection method is a bad idea, of course, since a typical digital output can only sink a few milliamps, and most motors must draw much more than that to do anything useful.

Current amplification with a switch (Figure 27.1(b))

To increase the current, we can use the digital pin to turn a switch on and off. The switch could be an electromechanical relay or a transistor, and it allows a much larger current to flow through the motor.

Consider, however, what happens when the switch has been closed for a while. A large current is flowing through the motor, which electrically behaves like a resistor and inductor in series. When the switch opens, the current must instantly drop to zero. The voltage across the inductor is governed by

$$V_L = L\frac{\mathrm{d}I}{\mathrm{d}t},$$

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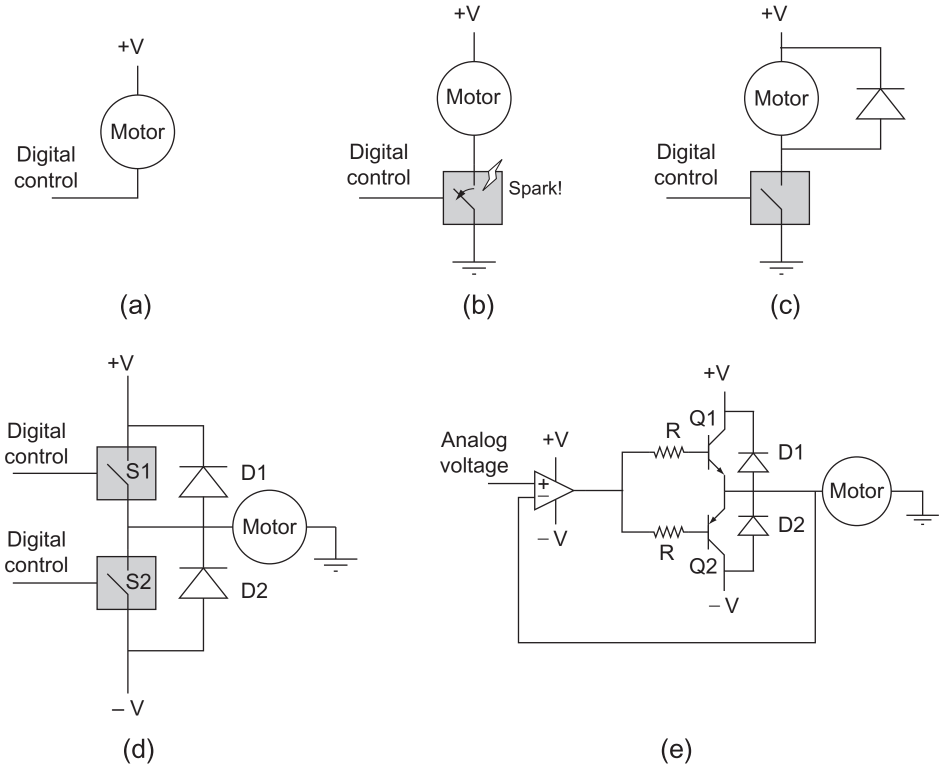

Figure 27.1

A progression of ideas to drive a motor, from worst to better. (a) Attempting to drive directly from a digital output. (b) Using a digital output to control a switch that allows more current to flow. (c) Adding a flyback diode to prevent sparking. (d) Using two switches and a bipolar supply to run the motor bidirectionally. (e) Using an analog control signal, an op-amp, and two transistors to make a linear push-pull amplifier.

so the instantaneous drop in current means that a large (theoretically infinite) voltage develops across the motor leads. The large voltage means that a spark will occur between the switch and the motor lead it was recently attached to. This sparking is certainly bad for the microcontroller.

Adding a flyback diode (Figure 27.1(c))

To prevent the instantaneous change in current and sparking, a *flyback diode* can be added to the circuit. Now when the closed switch is opened, the motor's current has a path to flow through—the diode in parallel with the motor. The voltage across the motor instantaneously changes from +V to the negative of the forward bias voltage of the diode, but that's okay, there is nothing in the resistor-inductor-diode circuit that tries to prevent that. With the switch open, the energy stored in the motor's inductance is dissipated by the current flowing through the motor's resistance, and the initial current I_0 will drop smoothly. Assuming the diode's forward bias voltage is zero, and treating the motor as a resistor and inductor in series, Kirchoff's voltage law tells us that the voltage around the closed loop satisfies

$$L\frac{\mathrm{d}I}{\mathrm{d}t}(t) + RI(t) = 0,$$

and the current through the motor after opening the switch can be solved as

$$I(t) = I_0 \mathrm{e}^{-\frac{R}{L}t},$$

a first-order exponential drop from the initial current I_0 to zero, with time constant equal to the electrical time constant of the motor, $T_e = L/R$.

When the switch is closed, the flyback diode has no effect on the circuit. Flyback diodes should be capable of carrying a lot of current, should be fast switching, and should have low forward bias voltage.

This circuit represents a viable approach to controlling a motor with variable speed: by opening and closing the switch rapidly, it is possible to create a variable average voltage across, and current through, the motor, depending on the duty cycle of the switching. The current always has the same sign, though, so the motor can only be driven in one direction.

Bidirectional operation with two switches and a bipolar supply (Figure 27.1(d))

By using a bipolar power supply (+V, −V, and GND) and two switches, each controlled by a separate digital input, it is possible to achieve bidirectional motion. With the switch S1 closed and S2 open, current flows from +V, through the motor, to GND. With S2 closed and S1 open, current flows through the motor in the opposite direction, from GND, through the motor, to −V. Two flyback diodes are used to provide current paths when switches transition from closed to open. For example, if S2 is open and S1 switches from closed to open, the motor current that was formerly provided by S1 now comes from −V through the diode D2.

To prevent a short circuit, S1 and S2 should never be closed simultaneously.

Bidirectional average voltages between +V and −V are determined by the duty cycle of the rapidly opening and closing switches.

A drawback of this approach is that a bipolar power supply is needed. This issue is solved by the H-bridge. But before discussing the H-bridge, let us consider a commonly used variation of the circuit in Figure 27.1(d).

Linear push-pull amplifier (Figure 27.1(e))

Figure 27.1(e) shows a linear push-pull amplifier. The control signal is a low-current analog voltage to an op-amp configured with negative feedback. Because of the negative feedback and the effectively infinite gain of the op-amp, the op-amp output does whatever it can to make sure that the signals at the inverting and noninverting inputs are equal. Since the inverting input is connected to one of the motor leads, the voltage across the motor is equal to the control voltage at the noninverting input, except with higher current available due to the output transistors. Only one of the two transistors is active at a time: either the NPN bipolar junction transistor Q1 “pushes” current from +V, through the motor, to GND, or the PNP BJT Q2 “pulls” current from GND, through the motor, to −V. Thus the op-amp provides a high impedance input and voltage following of the low-current control voltage, while the transistors provide current amplification.

For example, if +V = 10 V, and the control signal is at 6 V, then the op-amp ensures 6 V across the motor. To double-check that our circuit works as we expect, we calculate the current that would flow through the motor when it is stalled. If the motor’s resistance is 6 Ω, then the current $I_e = 6$ V/6 Ω $= 1$ A must be provided by the emitter of Q1. If the transistor is capable of providing that much current, we then check if the op-amp is capable of providing the base current $I_b = I_e/(\beta + 1)$ required to activate the transistor, where β is the transistor gain. If so, we are in good shape. The voltage at the base of Q1 is a PN diode drop higher than the voltage across the motor, and the voltage at the op-amp output is that base voltage plus I_bR. Note that Q1 is dissipating power approximately equal to the 4 V between the collector and the emitter times the 1 A emitter current, or approximately 4 W. This power goes to heating the transistor, so the transistor must be heat-sinked to allow it to dissipate the heat without burning up.

An example application of a linear push-pull amplifier would be using a rotary knob to control a motor’s bidirectional speed. The ends of a potentiometer in the knob would be connected to +V and −V, with the wiper voltage serving as the control signal.

If the op-amp by itself can provide enough current, the op-amp output can be connected directly to the motor and flyback diodes, eliminating the resistors and transistors. Power op-amps are available, but they tend to be expensive relative to using output transistors to boost current.

We could instead eliminate the op-amp by connecting the control signal directly to the base resistors of the transistors. The drawback is that neither transistor would be activated for control signals between approximately −0.7 and 0.7 V, or whatever the base-emitter voltage is when the transistors are activated. We have a “deadband” from the control signal to the motor voltage.

Some issues with the linear push-pull amp, addressed by the H-bridge, include:

- A bipolar power supply is required.
- The control signal is an analog voltage, which is generally not available from a microcontroller.
- The output transistors operate in the linear regime, with significant voltage between the collectors and emitters. A transistor in the linear mode dissipates power approximately equal to the current through the transistor multiplied by the voltage across it. This heats the transistor and wastes power.

Linear push-pull amps are sometimes used when power dissipation and heat are not a concern. They are also common in speaker amplifiers. (A speaker is a current-carrying coil moving in a magnetic field, essentially a linear motor.) There are many improvements to, and variations on, the basic circuit in Figure 27.1(e), and audio applications have raised amplifier circuit design to an art form. You can use a commercial audio amplifier to drive a DC motor, but you would have to remove the high-pass filter on the amplifier input. The high-pass filter is there because we cannot hear sound below 20 Hz, and low-frequency currents simply heat up the speaker coil without producing audible sound.

27.1.1 The H-Bridge

For most motor applications, the preferred amplifier is an H-bridge (Figure 27.2). An H-bridge uses a unipolar power supply (Vm and GND), is controlled by digital pulse width modulation pulse trains that can be created by a microcontroller, and has output transistors (switches) operating in the saturated mode, therefore with little voltage drop across them and relatively little power wasted as heat.

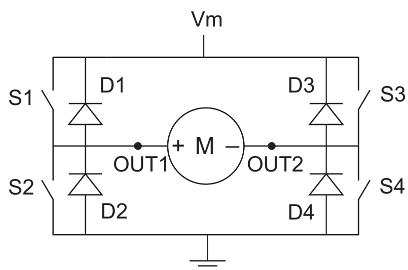

Figure 27.2
An H-bridge constructed of four switches and four flyback diodes. OUT1 and OUT2 are the H-bridge outputs, attached to the two terminals of the DC motor.

An H-bridge consists of four switches, S1–S4, typically implemented by MOSFETs, and four flyback diodes D1–D4.[1] An H-bridge can be used to run a DC motor bidirectionally, depending on which switches are closed:

Closed Switches	Voltage Across Motor
S1, S4	Positive (forward rotation)
S2, S3	Negative (reverse rotation)
S1, S3	Zero (short-circuit braking)
S2, S4	Zero (short-circuit braking)
None or one	Open circuit (coasting)

Switch settings not covered in the table (S1 and S2 closed, or S3 and S4 closed, or any set of three or four switches closed) result in a short circuit and should obviously be avoided!

While you can build your own H-bridge out of discrete components, it is often easier to buy one packaged in an integrated circuit, particularly for low-power applications. Apart from reducing your component count, these ICs also make it impossible for you to accidentally cause a short circuit. An example H-bridge IC is the Texas Instruments DRV8835.

The DRV8835 has two full H-bridges, labeled A and B, each capable of providing up to 1.5 A continuously to power two separate motors. The two H-bridges can be used in parallel to provide up to 3 A to drive a single motor. The DRV8835 works with motor supply voltages (to power the motors) of up to 11 V and logic supply voltages (for interfacing with the microcontroller) between 2 and 7 V. It offers two modes of operation: the IN/IN mode, where the two inputs for each H-bridge control whether the H-bridge is in the forward, reverse, braking, or coasting mode, and the PHASE/ENABLE mode, where one input controls whether the H-bridge is enabled or braking and the other input controls forward vs. reverse if the H-bridge is enabled. We will focus on the PHASE/ENABLE mode.

In the PHASE/ENABLE mode, chosen by setting the MODE pin to logic high, the following truth table determines how the logic inputs (0 and 1) of one H-bridge controls its two outputs:

MODE	PHASE	ENABLE	OUT1	OUT2	Function
1	x	0	L	L	Short-circuit braking (S2, S4 closed)
1	0	1	H	L	Forward (S1, S4 closed)
1	1	1	L	H	Reverse (S2, S3 closed)

[1] MOSFETs themselves allow reverse currents, acting somewhat as flyback diodes, but typically dedicated flyback diodes are incorporated for better performance.

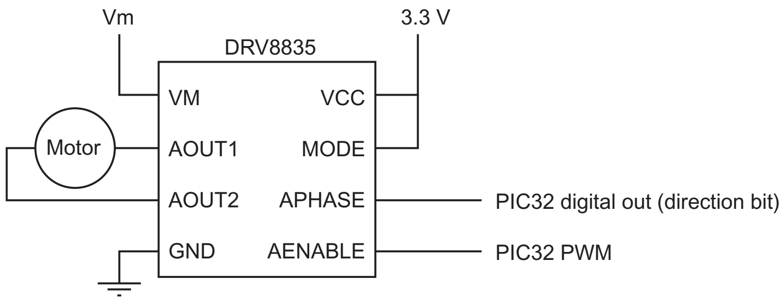

Figure 27.3
Wiring the DRV8835 to use H-bridge A. Not shown is a recommended 10 μF capacitor from VM to GND and a recommended 0.1 μF capacitor from VCC to GND, which may be included already on a DRV8835 breakout board.

When the ENABLE pin is low, OUT1 and OUT2 are held at the same (low) voltage, causing the motor to brake by its own short-circuit damping. When ENABLE is high, then if PHASE is low, switches S1 and S4 are closed, putting a positive voltage across the motor trying to drive it in the forward direction. When ENABLE is high and PHASE is high, switches S2 and S3 are closed, putting a negative voltage across the motor trying to drive it in the reverse direction. PHASE sets the sign of the voltage across the motor, and the duty cycle of the PWM on ENABLE determines the average magnitude of the voltage across the motor, by rapidly switching between approximately +Vm (or $-$Vm) and zero.

Figure 27.3 shows the wiring of the DRV8835 to use H-bridge A, where Vm is the voltage to power the motor. The logic high voltage VCC is 3.3 V. If the two H-bridges of the DRV8835 are used in parallel for more current, the following pins should be connected to each other: APHASE and BPHASE; AENABLE and BENABLE; AOUT1 and BOUT1; and AOUT2 and BOUT2.

27.1.2 Control with PWM

Rapidly switching ENABLE from high to low can effectively create an average voltage V_{ave} across the motor. Assuming PHASE = 0 (forward), then if DC is the duty cycle of ENABLE, where $0 \leq \mathrm{DC} \leq 1$, and if we ignore voltage drops due to flyback diodes and resistance at the MOSFETs, then the average voltage across the motor is

$$V_{\mathrm{ave}} \approx \mathrm{DC} * \mathrm{Vm}.$$

Ignoring the details of the motor's inductance charging and discharging, this yields an approximate average current through the motor of

$$I_{\mathrm{ave}} \approx (V_{\mathrm{ave}} - V_{\mathrm{emf}})/R,$$

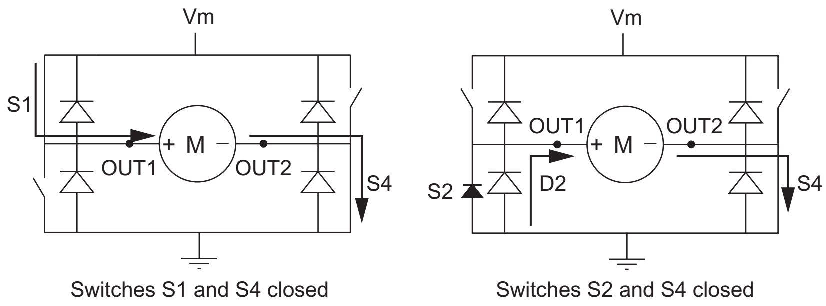

Figure 27.4

Left: The closed switches S1 and S4 provide current to the motor, from Vm to GND. Right: When S1 is opened and S2 is closed, the motor's need for a continuous current is satisfied by the flyback diode D2 and the switch S4. Current could also flow through the MOSFET S2, which acts like a diode to reverse current, but the flyback diode is designed to carry the current.

where $V_{\mathrm{emf}} = k_t\omega$ is the back-emf. Since the period of a PWM cycle is typically much shorter than the motor's mechanical time constant T_m, the motor's speed ω (and therefore V_{emf}) is approximately constant during a PWM cycle.

Figure 27.4 shows a motor with positive average current (from left to right). When switches S1 and S4 are closed and S2 and S3 are open (ENABLE is 1, OUT1 is high, and OUT2 is low), S1 and S4 carry current from Vm, through the motor, to ground. When the PWM on ENABLE becomes 0, S2 and S4 are closed and S1 and S3 are open (OUT1 and OUT2 are both low). Because the motor's inductance requires that the current not change instantaneously, the current must flow from ground, through the flyback diode D2, through the motor, then through switch S4 back to ground. (The "closed switch" S2 MOSFET is represented as a diode, since a MOSFET behaves similarly to a diode when current tries to flow in the reverse direction. But the flyback diode is designed to carry this current, so most current flows through D2.)

During the period when OUT1 and OUT2 are low, the voltage across the motor is approximately zero, and the motor current $I(t)$ satisfies

$$0 = L\frac{\mathrm{d}I}{\mathrm{d}t}(t) + RI(t) + V_{\mathrm{emf}},$$

causing an exponential drop in $I(t)$ with time constant $T_e = L/R$, the electrical time constant of the motor. Figure 27.5 shows an example of the voltage across the motor during two cycles of PWM, and the resulting current for two different motors: one with a large T_e and one with a small T_e. A large T_e results in a nearly constant current during a PWM cycle, while a small T_e results in a fast-changing current. The nearly constant motor velocity during a PWM cycle gives a nearly constant V_{emf}.

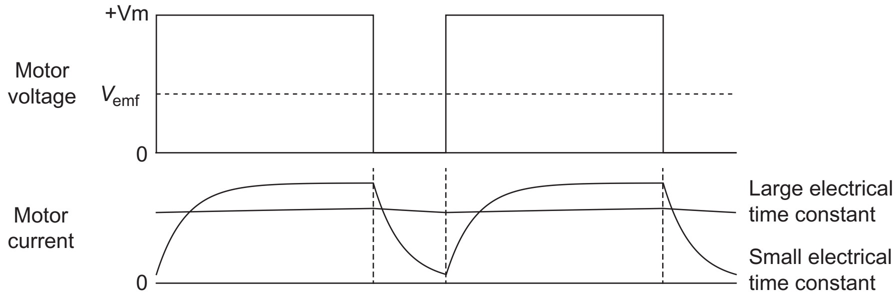

Figure 27.5
A PWM signal puts Vm across the motor for 75% of the PWM cycle and 0 V for 25% of the cycle. This causes a nearly constant positive current for a motor with large L/R, while the current for a motor with small L/R varies significantly.

To understand the behavior of the electromechanical system under PWM control, we need to consider three time scales: the PWM period T, the electrical time constant T_e, and the mechanical time constant T_m. The PWM frequency $1/T$ should be chosen to be much higher than $1/T_m$, to prevent significant speed variation during a PWM cycle. Ideally the PWM frequency would also be much higher than $1/T_e$, to minimize current variation during a cycle. One reason to want little current variation is that more-constant current results in less power wasted heating the motor coils. To see why, consider a motor with resistance $R = 1\ \Omega$ powered by a constant current of 2 A vs. a current alternating with 50% duty cycle between 4 and 0 A, for a time-average of 2 A. Both provide the same average torque, but the average power to heating the coils in the first case is $I^2R = 4$ W while it is $0.5(4\text{ A})^2(1\ \Omega) = 8$ W in the second.

Another consideration is audible noise: to make sure the switching is inaudible, the PWM frequency $1/T$ should be chosen at 20 kHz or larger.

On the other hand, the PWM frequency should not be chosen too high, as it takes time for the H-bridge MOSFETs to switch. During switching, when larger voltages are across the active MOSFETs, more power is wasted to heating. If switching occurs too often, the H-bridge may even overheat. The DRV8835 takes approximately 200 ns to switch, and its maximum recommended PWM frequency is 250 kHz.

Trading off the considerations above, common PWM frequencies are in the range 20-40 kHz.

27.1.3 Regeneration

When the voltage across the motor and the current through the motor have the same sign, the motor is consuming electrical power ($IV > 0$). When the voltage across the motor and the current through the motor have opposite signs ($IV < 0$), then the motor is acting as a generator and is actually producing electrical power. This phenomenon is called *regeneration*.

Regeneration may occur when braking a motor, for example. Regenerative braking is used in hybrid and electric cars to convert some of the car's kinetic energy into battery energy, instead of just wasting it heating the brake pads.

For concreteness, consider the H-bridge of Figure 27.2 powered by 10 V, flyback diodes with a forward bias voltage of 0.7 V, and a motor with a resistance of 1 Ω and a torque constant of 0.01 Nm/A (0.01 Vs/rad). Consider these two examples of regeneration.

1. **Forced motion of the motor output.** Assume all H-bridge switches are open and an external power source spins the motor shaft. The external source could be water falling over the blades of a turbine in a hydroelectric dam, for example. If the motor shaft spins at a constant $\omega = 2000$ rad/s, then the back-emf is $k_t\omega = (0.01 \text{ Vs/rad})(2000 \text{ rad/s}) = 20$ V. The flyback diodes cap the voltage across the motor to the range [−11.4 V, 11.4 V], however, so current must be flowing through the motor. Assuming the flyback diodes D1 and D4 are conducting, we have

$$11.4 \text{ V} = k_t\omega + IR = 20 \text{ V} + I(1 \ \Omega).$$

   Solving for I, we get a current of $I = -8.6$ A, flowing from ground through D4, the motor, and D1 to the 10 V supply (Figure 27.6). The power consumed by the motor is (−8.6 A)(11.4 V) = −98.04 W, i.e., the motor is generating 98.04 W.
   (If we had assumed the flyback diodes D2 and D3 were conducting instead, putting −11.4 V across the motor, and solved for I, we would have seen that the required negative current, from right to left, cannot be provided by D2 and D3. Therefore, D2 and D3 are not conducting.)
2. **Motor braking.** Assume the motor has a positive current of 2 A through it (left to right, carried by switches S1 and S4), then all switches are opened. Immediately after the

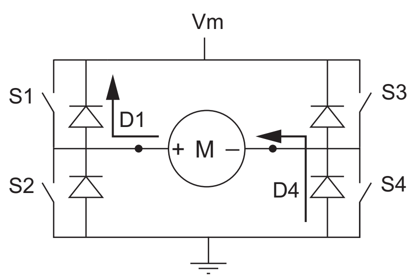

Figure 27.6
One example of regeneration, where the current through and voltage across a motor have opposite signs. The motor is forced to spin forward, by an external source, at a speed ω such that the back-emf $k_t\omega$ is larger than Vm. This forces a negative current to flow through the motor, carried by the flyback diodes D1 and D4. Electrical power is dumped into the power supply.

switches are opened, the only option to continue providing 2 A to the motor is from ground, through D2, the motor, and D3 to the 10 V supply. The voltage across the motor must therefore be −11.4 V: two diode drops and the 10 V supply voltage. The motor consumes (2 A)(−11.4 V) = −22.8 W, i.e., it is acting as a generator, providing 22.8 W of power just after the switches are opened.

As these examples show, regeneration dumps current back into the power supply, charging it up, whether it wants to be charged or not. Some batteries can directly accept the regeneration current. For a wall-powered supply, however, a high-capacitance, high-voltage, typically polarized electrolytic capacitor at the power supply outputs can act as storage for energy dumped back into the power supply. While such a capacitor may be present in a linear power supply, a switched-mode power supply is unlikely to have one, so an external capacitor would have to be added. If the power supply capacitor voltage gets too high, a voltage-activated switch can allow the back-current to be redirected to a "regen" power resistor, which is designed to dissipate electrical energy as heat.

27.1.4 Other Practical Considerations

Motors are electrically noisy devices, creating both electromagnetic interference (EMI), e.g., induced currents on sensitive electronics due to changing magnetic fields induced by large changing motor currents, as well as voltage spikes, due to brush switching and changing PWM current and voltage. These effects can disrupt the functioning of your microcontroller, cause erroneous readings on your sensor inputs, etc. Dealing with EMI is beyond our scope, but it can be minimized by keeping the motor leads short and far from sensitive circuitry, and by using shielded cable or twisted wires for motor and sensor leads.

Optoisolators can be used to separate noisy motor power supplies from clean logic voltage supplies. An optoisolator consists of an LED and a phototransistor. When the LED turns on, the phototransistor is activated, allowing current to flow from its collector to its emitter. Thus a digital on/off signal can be passed between the logic circuit and the power circuit using only an optical connection, eliminating an electrical connection. In our case, the PIC32's H-bridge control signals would be applied to the LEDs and converted by the phototransistors to high and low signals to be passed to the inputs of the H-bridge.

Optoisolators can be bought in packages with multiple optoisolators. Each LED-phototransistor pair uses four pins: two for the internal LED and two for the collector and emitter of the phototransistor. Thus you can get a 16-pin DIP chip with four optoisolators, for example.

It is also common to directly solder a nonpolarized capacitor across the motor terminal leads, effectively turning the motor into a capacitor in parallel with the resistance and inductance of the motor. This capacitor helps to smooth out voltage spikes due to brushed commutation.

Finally, the H-bridge chip should be heat-sinked to prevent overheating. The heat sink dissipates heat due to MOSFET switching and MOSFET output resistance (on the order of hundreds of mΩ for the DRV8835).

27.2 Motion Control of a DC Motor

An example block diagram for control of a DC motor is shown in Figure 27.7.[2] A trajectory generator creates a reference position as a function of time. To drive the motor to follow this reference trajectory, we use two nested control loops: an outer motion control loop and an inner current control loop. These two loops are roughly motivated by the two time scales of the system: the mechanical time constant of the motor and load and the electrical time constant of the motor.

- **Outer motion control loop.** This outer loop runs at a lower frequency, typically a few hundred Hz to a few kHz. The motion controller takes as input the desired position and/or velocity, as well as the motor's current position, as measured by an encoder or potentiometer, and possibly the motor's current velocity, as measured by a tachometer. The output of the controller is a commanded current I_c. The current is directly proportional to the torque. Thus the motion control loop treats the mechanical system as if it has direct control of motor torque.
- **Inner current control loop.** This inner loop typically runs at a higher frequency, from a few kHz to tens of kHz, but no higher than the PWM frequency. The purpose of the current controller is to deliver the current requested by the motion controller. To do this, it

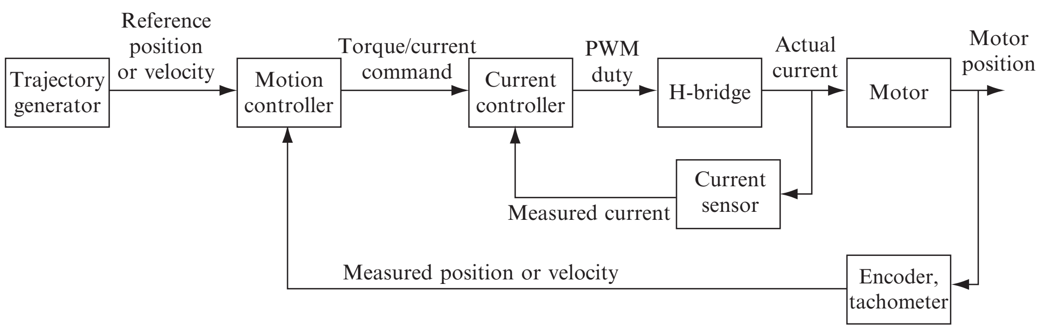

Figure 27.7
A block diagram for motion control.

[2] A simpler block diagram would have the motion controller block directly output a PWM duty cycle to an H-bridge, with no inner-loop control of the actual motor current, which would be sufficient for many applications. However, the block diagram in Figure 27.7 is more typical of industrial implementations.

monitors the actual current flowing through the motor and outputs a commanded average voltage V_c (expressed as a PWM duty cycle).

Traditionally a mechanical engineer might design the motion control loop, and an electrical engineer might design the current control loop. But you are a mechatronics engineer, so you will do both.

27.2.1 Motion Control

Feedback control

Let θ and $\dot{\theta}$ be the actual position and velocity of the motor, and r and $\dot{r}$ be the desired position and velocity. Define the error $e = r - \theta$, error rate of change $\dot{e} = \dot{r} - \dot{\theta}$, and error sum (approximating an integral) $e_{\text{int}} = \sum_k e(k)$. Then a reasonable choice of controller would be a PID controller (Chapter 23),

$$I_{\text{c,fb}} = k_p e + k_i e_{\text{int}} + k_d \dot{e}, \tag{27.1}$$

where $I_{\text{c,fb}}$ is the commanded current by the feedback controller. The $k_p e$ term acts as a virtual spring that creates a force proportional to the error, pulling the motor to the desired angle. The $k_d \dot{e}$ term acts as a virtual damper that creates a force proportional to the "velocity" of the error, driving the error rate of change toward zero. The $k_i e_{\text{int}}$ term creates a force proportional to the time integral of error. See Chapter 23 for more on PID control.

In the absence of a model of the motor's dynamics, a reasonable commanded current I_c is simply $I_c = I_{\text{c,fb}}$.

An alternative form of the feedback controller (27.1) is

$$\ddot{\theta}_d = k_p e + k_i e_{\text{int}} + k_d \dot{e}, \tag{27.2}$$

where the feedback gains set a desired corrective acceleration of the motor $\ddot{\theta}_d$ instead of a current. This alternative form of the PID controller is used in conjunction with a system model in the next section.

Feedforward plus feedback control

If you have a decent model of the motor and its load, a model-based controller can be combined with a feedback controller to yield better performance. For example, for an unbalanced load as in Figure 27.8, you could choose a feedforward current command to be

$$I_{\text{c,ff}} = \frac{1}{k_t}(J\ddot{r} + mgd \sin\theta + b_0 \operatorname{sgn}(\dot{\theta}) + b_1\dot{\theta}),$$

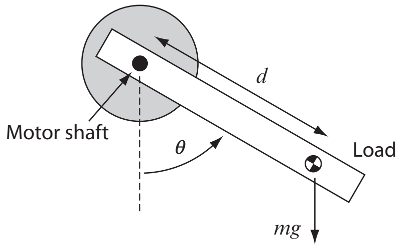

Figure 27.8
An unbalanced load in gravity.

where k_t is the torque constant, J is the motor and load inertia about the motor's axis, the planned motor acceleration $\ddot{r}$ can be obtained by finite differencing the desired trajectory, mg is the weight of the load, d is the distance of the load center of mass from the motor axis, θ is the angle of the load from vertical, b_0 is Coulomb friction torque, and b_1 is a viscous friction coefficient. To compensate for errors, the feedback current command $I_{\mathrm{c,fb}}$ can be combined with the feedforward command to yield

$$I_{\mathrm{c}} = I_{\mathrm{c,ff}} + I_{\mathrm{c,fb}}. \tag{27.3}$$

Alternatively, the motor acceleration feedback law (27.2) could be combined with the system model to yield the controller

$$I_{\mathrm{c}} = \frac{1}{k_t}(J(\ddot{r} + \ddot{\theta}_d) + mgd\sin\theta + b_0\,\mathrm{sgn}(\dot{\theta}) + b_1\dot{\theta}), \tag{27.4}$$

an implementation of a model-based controller from Chapter 23.4.

27.2.2 Current Control

To implement a current controller, a current sensor is required. In this chapter we assume a current-sense resistor with a current-sense amplifier, as described in Chapter 21.10.1 and Figure 21.22.

The output of the current controller is V_{c}, the commanded average voltage (to be converted to a PWM duty cycle). The simplest current controller would be

$$V_{\mathrm{c}} = k_V I_{\mathrm{c}}$$

for some gain k_V. This controller would be a good choice if your load were only a resistance R, in which case you would choose $k_V = R$. Even if not, if you do not have a good mechanical model of your system, achieving a particular current/torque may not matter anyway. You can just tune your motion control PID gains, use $k_V = 1$, and not worry about what the actual current is, eliminating the inner control loop.

On the other hand, if your battery pack voltage changes (due to discharging, or changing batteries, or changing from a 6 to a 12 V battery pack), the change in behavior of your overall controller will be significant if you do not measure the actual current in your current controller. More sophisticated current controller choices might be a mixed model-based and integral feedback controller

$$V_c = I_c R + k_t \dot{\theta} + k_{I,i}\, e_{I,\text{int}}$$

or, recognizing that the electrical system is a first-order system (using voltage to control a current through a resistor and inductor), a PI feedback controller

$$V_c = k_{I,p}\, e_I + k_{I,i}\, e_{I,\text{int}},$$

where e_I is the error between the commanded current I_c and the measured current, $e_{I,\text{int}}$ is the integral of current error, R is the motor resistance, k_t is the torque constant, $k_{I,p}$ is a proportional current control gain, and $k_{I,i}$ is an integral current control gain. A good current controller would closely track the commanded current.

27.2.3 An Industrial Example: The Copley Controls Accelus Amplifier

Copley Controls is a well known manufacturer of amplifiers for brushed and brushless motors for industrial applications and robotics. One of their models is the Accelus, pictured in Figure 27.9. The Accelus supports many different operating modes. Examples include control of motor current or velocity to be proportional to either an analog voltage input or the duty cycle of a PWM input. A microcontroller on the Accelus interprets the analog input or PWM duty cycle and implements a controller similar to that in Figure 27.7.

Figure 27.9
The Copley Controls Accelus amplifier. (Image courtesy of Copley Controls, copleycontrols.com.)

The mode most relevant to us is the Programmed Position mode. In this mode, the user specifies a few parameters to describe a desired rest-to-rest motion of the motor. The controller's job is to drive the motor to track this trajectory.

When the amplifier is first paired with a motor, some initialization steps must be performed. A GUI interface on the host computer, provided by Copley, communicates with the microcontroller on the Accelus using RS-232.

1. **Enter motor parameters.** From the motor's data sheet, enter the inertia, peak torque, continuous torque, maximum speed, torque constant, resistance, and inductance. These values are used for initial guesses at control gains for motion and current control. Also enter the number of lines per revolution of the encoder.
2. **Tune the inner current control loop.** Set a limit on the recent current to avoid overheating the motor. This limit is based on the integral $\int_{T_1}^{T_2} I^2(t)\,\mathrm{d}t$, which is a measure of how much energy the motor coils have dissipated recently. (When this limit is exceeded, the motor current is limited to the continuous operating current until the history of currents indicates that the motor has cooled.) Also, tune the values of P and I control gains for a PI current controller. This tuning is assisted by plots of reference and actual currents as a function of time. See Figure 27.10, which shows a square wave reference current of amplitude 1 A and frequency 100 Hz. The zero average current and high frequency of the reference waveform ensure that the motor does not move during current tuning, which focuses on the electrical properties of the motor.
   The current control loop executes at 20 kHz, which is also the PWM frequency (i.e., the PWM duty cycle is updated every cycle).

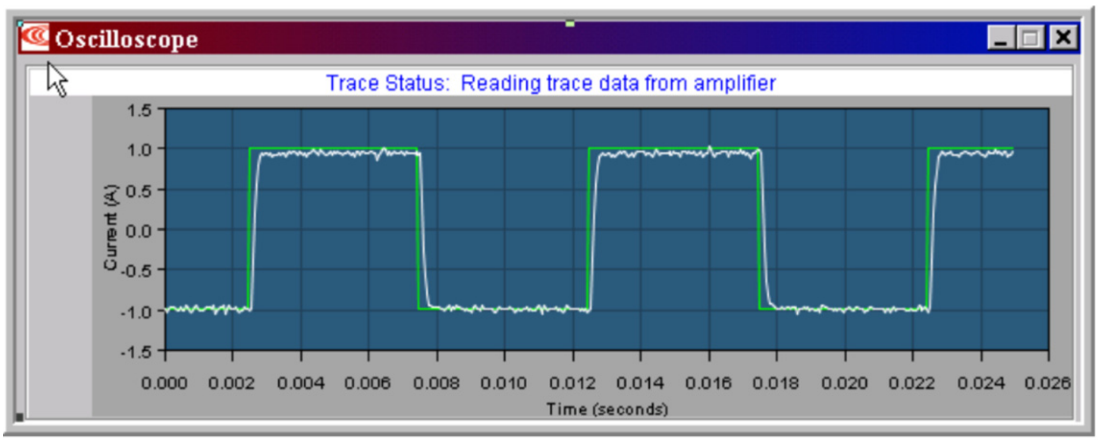

Figure 27.10
A plot of the reference square wave current and the actual measured current during PI current controller tuning.

3. **Tune the outer motion control loop with the load attached.** Attach the load to the motor and tune PID feedback control gains, a feedforward acceleration term, and a feedforward velocity term to achieve good tracking of sample reference trajectories. This process is assisted by plots of reference and actual positions and velocities as a function of time. The motion control loop executes at 4 kHz.

Once the initial setup procedures have been completed, the Accelus microcontroller saves all the motor parameters and control gains to nonvolatile flash memory. These tuned parameters then survive power cycling and are available the next time you power up the amplifier.

Now the amplifier is ready for use. The user specifies a desired trajectory using any of a number of interfaces (RS-232, CAN, etc.), and the amplifier uses the saved parameters to drive the motor to track the trajectory.

27.3 Chapter Summary

- An H-bridge amplifier allows bidirectional control of a DC motor based on a PWM control signal.
- Flyback diodes are used with an H-bridge to provide a current path for the inductive load (the motor) at all times as the H-bridge transistors switch on and off.
- A motor acts as an electrical generator, generating electrical power instead of consuming it, when the voltage across the motor and the current through it have opposite signs. An example is regenerative braking in electric and hybrid vehicles.
- A typical motor control system has a nested structure: an outer motion-control loop, which commands a torque (or current) from the inner current-control loop, which attempts to deliver the current requested by the outer-loop controller. Typically the inner-loop controller executes at a higher frequency than the outer-loop controller.

27.4 Exercises

1. The switch in Figure 27.1(b), with no flyback diode, has been closed for a long time, and then it is opened. The voltage supply is 10 V, the motor's resistance is $R = 2\ \Omega$, the motor's inductance is $L = 1$ mH, and the motor's torque constant is $k_t = 0.01$ Nm/A. Assume the motor is stalled.
   a. What is the current through the motor just before the switch is opened?
   b. What is the current through the motor just after the switch is opened?
   c. What is the torque being generated by the motor just before the switch is opened?
   d. What is the torque being generated by the motor just after the switch is opened?
   e. What is the voltage across the motor just before the switch is opened?
   f. What is the voltage across the motor just after the switch is opened?

2. The switch in Figure 27.1(c), with the flyback diode, has been closed for a long time, and then it is opened. The voltage supply is 10 V, the motor's resistance is $R = 2\ \Omega$, the motor's inductance is $L = 1$ mH, and the motor's torque constant is $k_t = 0.01$ Nm/A. The flyback diode has a forward bias voltage drop of 0.7 V. Assume the motor is stalled.
   a. What is the current through the motor just before the switch is opened?
   b. What is the current through the motor just after the switch is opened?
   c. What is the torque being generated by the motor just before the switch is opened?
   d. What is the torque being generated by the motor just after the switch is opened?
   e. What is the voltage across the motor just before the switch is opened?
   f. What is the voltage across the motor just after the switch is opened?
   g. What is the rate of change of current through the motor $\mathrm{d}I/\mathrm{d}t$ just after the switch is opened? (Make sure to use a correct sign, relative to your current answers above.)
3. In Figure 27.1(d), the voltage supplies are ± 10 V, the motor's resistance is $R = 5\ \Omega$, the motor's inductance is $L = 1$ mH, and the motor's torque constant is $k_t = 0.01$ Nm/A. The flyback diodes have a forward bias voltage drop of 0.7 V. Switch S1 has been closed for a long time, with no voltage drop across it, and the motor is stalled. Switch S2 is open. Then switch S1 is opened while switch S2 remains open. Immediately after S1 opens, which flyback diode conducts current? What is the voltage across the motor? What is the current through the motor? What is the rate of change of the current through the motor?
4. Give some advantages of driving a DC motor using an H-bridge with PWM over a linear push-pull amplifier with an analog control input. Give at least one advantage of using a linear push-pull amplifier over an H-bridge. (Hint: consider the case of low PWM frequency or low motor inductance.)
5. Explain why an initially spinning motor comes to rest faster if the two motor leads are shorted to each other rather than left disconnected. Derive the result from the motor voltage equation.
6. Provide a circuit diagram showing the DRV8835 configured to drive a single motor with more than 2 A continuous.
7. Consider a motor connected to an H-bridge with all switches opened (motor is unpowered). The motor rotor is rotated by external forces (e.g., rushing water in a hydroelectric dam spinning the blades of a turbine). If the H-bridge is connected to a battery supply of voltage Vm, and the forward bias voltage of the flyback diodes is Vd, give a mathematical expression for the rotor speed at which the battery begins to charge. When this speed is exceeded, and assuming that the battery voltage Vm is constant (e.g., it acts somewhat like a very high capacitance capacitor, accepting current without changing voltage), give an expression for the current through the motor as a function of the rotor speed. Also give an expression for the power lost due to the heating the windings.

   How does the presence of the hydrogenerator affect the total energy of a bucket of water at the top of the dam compared to the total energy just before that water splashes into the river at the bottom of the dam? (The total energy is the potential energy plus the kinetic energy.)

8. To create an average current I through a motor, you could send I constantly, or you could alternate quickly between kI for $100\%/k$ of a cycle and zero current for the rest of the cycle. Provide an expression for the average power dissipated by the motor coils for each of these cases.
9. Imagine a motor with a 500 line incremental encoder on one end of the motor shaft and a gearhead with $G = 50$ on the other end. If the encoder is decoded in 4x mode, how many encoder counts are counted per revolution of the gearhead output shaft?
10. A simple outer-loop motion controller is to command a torque (current) calculated by a PID controller. A more sophisticated controller would attempt to use a model of the motor-load dynamics to get better trajectory tracking. One possibility is the control law (27.3). Another possibility is the control law (27.4). Describe any advantages of (27.4) over (27.3).
11. Choose R2/R1 for the current-sense amplifier in Figure 21.22 in Chapter 21 so the output voltage OUT swings full range (0-3.3 V) for a current range of ±1 A.
12. Choose an example R and C for the current-sense amplifier in Figure 21.22 in Chapter 21 to create a cutoff frequency of 200 Hz.
13. For the current-sense amplifier in Figure 21.22 in Chapter 21, the reference voltage at REFIN should be constant. We know that the currents into and out of the high-impedance op-amp inputs REFIN and FB are negligible, as is the current into the PIC32 analog input. But the currents through the external resistors R1 and R2 may not be small. As a result, REFIN actually varies as a function of the sensed input voltage and the output voltage OUT. We should choose the resistances R1, R2, and R3 so the voltage variation at REFIN is small.

    Pay attention only to the op-amp portion of the circuit in the bottom of Figure 21.22 in Chapter 21. Now assume that OUT is 3.3 V. What is the voltage at REFIN as a function of R1, R2, and R3? (Note that it is not exactly 1.65 V.) Use your equation to comment on how to choose the relative values of R1, R2, and R3 to make sure that REFIN is close to 1.65 V. Explain other practical constraints on the absolute values (minimum and maximum values) of R1, R2, and R3. (You may focus on slowly changing signals at the + input of the op amp. For high-frequency inputs, using large resistances R1 and R2 may combine with small parasitic capacitances in the circuit to create RC time delays that adversely affect the response.)
14. You decide on a current amplifier gain of $G = 101$. Using your result from the previous exercise, choose R1, R2, and R3 to achieve the gain G while ensuring that REFIN does not vary by more than 1 mV for any voltage at OUT in the range [0, 3.3 V]. For your choice of R3, indicate how much power is used in the R3-R3 voltage divider.
15. Due to natural variations in resistor values within their tolerance ranges, the gain G, and the voltage at REFIN, that you design for your current-sense circuit in Figure 21.22 in

Chapter 21 will not be exactly realized. Due to this and other variations, you need to calibrate your current sensor by running some experiments. Clearly explain what experiments you would do, and how you would use the results to interpret analog voltages read at the PIC32 as motor currents. Be specific; the interpretation should be easy to implement in software.

16. Clearly outline how you would implement the motor controller in Figure 27.7 in software on a PIC32. Indicate what peripherals, ISRs, NU32 functions, and global variables you might use. You may use concepts from the LED brightness control project, if it is helpful. In particular, indicate how you would implement:
    - **The trajectory generator.** Assume that desired trajectories are sent from MATLAB on the host computer, and trajectory tracking results are plotted in MATLAB.
    - **The outer-loop motion controller.** Assume that an external decoder/counter chip maintains the encoder count, and that communication with the chip occurs using SPI. How would you collect trajectory tracking data to be plotted in MATLAB?
    - **The inner-loop current controller**, using the current sensor described in this chapter. How would you collect data on tracking the desired current to be plotted in MATLAB?

Further Reading

Accelus panel: Digital servoamplifier for brushless/brush motors. (2011). Copley Controls.

DRV8835: Dual low voltage H-bridge IC. (2014). Texas Instruments.

DRV8835 dual motor driver carrier. (2015). https://www.pololu.com/product/2135 (Accessed: May 6, 2015).

MAX9918/MAX9919/MAX9920 −20V to +75V input range, precision uni-/bidirectional, current-sense amplifiers. (2015). Maxim Integrated.