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Exercise – *The Lernaean Hydra*

"Will you forget the head slicing thing?!"

Chiron yelling at Hercules to stop beheading the Lernaean Hydra

The second labor of Hercules is to slay the *Lernaean Hydra*. This many-headed creature lives in swamps close to Lake Lerna and has been raised by Hera[1] for the sole purpose of killing Hercules.

With a band of cloth across his nose and mouth to protect him from the poisonous fumes and armed with his famed sword, Hercules engages the Hydra in combat. He soon realizes that every time he cuts off one of the Hydra's heads, the head simply grows back, making this quest significantly harder than initially imagined.

With the help from Athena[2], Hercules gains a crucial insight as to how to slay the Hydra. He learns that there is a secret way to *eradicate* a head, that is to cut it off so that it cannot grow back. What Athena gives him is a list of *eradication patterns*, each consisting of exactly k (not necessarily distinct) Hydra's heads in a certain order. These patterns hold the key to slaying the Hydra once and for all.

Assume the Hydra's heads are labeled from left to right with $0, \ldots, n-1$. Hercules is to perform a sequence of cuts of heads which may contain some heads multiple times. Whenever he cuts some head i of the Hydra, that (and only that!) particular head is eradicated if and only if all heads $j < i$ have been eradicated *and* the last k cuts (including the current one) in the sequence form an eradication pattern; in other words, there exists an eradication pattern on Athena's list that consists of the last k heads of the sequence in the same order that they were cut. Otherwise, the head grows back. Once a head has been eradicated, it cannot be cut any more. Glancing at Athena's list, Hercules notices one last vital piece of information: each of Hydra's heads can be eradicated by only a small number of patterns.

Hercules, even being as strong as he is, wisely desires to save strength for future labors. Hence, in addition to the primary goal to slay the Hydra by any means, he wants to minimize the number of cuts it takes to do it. Does there exist a sequence of cuts which eradicates all of Hydra's heads? If so what is the length of the shortest such sequence Hercules can perform to complete this quest?

Input The first line of the input contains the number $t \leqslant 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that consists of two integers n m, separated by a space, denoting the number of Hydra's heads ($2 \leqslant n \leqslant 10^3$) and the number of eradication patterns on Athena's list ($1 \leqslant m \leqslant 2 \cdot 10^4$).

[1] In Greek mythology, Hera is the goddess of women, marriage, family, childbirth, one of the twelve Olympians, and the wife of Zeus.

[2] In Greek mythology, Athena is the goddess of wisdom, handicraft, and warfare, one of the twelve Olympians, and the daughter of Zeus.

- The following line consists of two integers `k d`, separated by a space, denoting the length of every eradication pattern ($2 \leqslant k \leqslant 10$) and the maximum number of times a head may appear as the last head in the list of eradication patterns ($1 \leqslant d \leqslant 20$).
- The following m lines define Athena's eradication patterns. The i-th such line consists of k integers $h_0 \ldots h_{k-1}$, separated by a space, and such that $0 \leqslant h_j < n$ for every $j \in \{0, \ldots, k-1\}$. It denotes a pattern of cuts that, if performed by Hercules, can be used to eradicate the head h_{k-1}.

Output For each test case output a single line with the minimum number of cuts Hercules needs to perform in order to eradicate all of Hydra's heads. If no such sequence exists, output "`Impossible!`" instead.

Points There are four groups of test sets, each worth 25 points. So, there are 100 points in total.

1. For the first group of test sets, you may assume that the number of patterns on Athena's list is at most $2 \cdot 10^3$ ($m \leqslant 2 \cdot 10^3$) and the length of each pattern is exactly two ($k = 2$).
2. For the second group of test sets, you may assume that the number of patterns on Athena's list is at most $2 \cdot 10^3$ ($m \leqslant 2 \cdot 10^3$) and the length of each pattern is at most three ($k \leqslant 3$).
3. For the third group of test sets, you may assume that the length of each pattern is at most three ($k \leqslant 3$).
4. For the fourth group of test sets, there are no additional assumptions.

Corresponding sample test sets are contained in `testi.in/out`, for $i \in \{1, 2, 3, 4\}$.

Sample Input

```
2
3 4
2 2
0 0
2 0
1 1
0 2
3 4
3 2
0 0 0
0 2 1
0 0 2
2 1 2
```

Sample Output

```
Impossible!
6
```