

Algorithms Lab HS24
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cadmo.ethz.ch/education/lectures/HS24/algolab

Exercise - Alastor "Mad-Eye" Moody

Constant vigilance!

Alastor Moody, Harry Potter and the Goblet of Fire by J. K. Rowling

There are only a couple of days before Harry Potter finally turns seventeen and the charm that protects him at his home from Lord Voldemort gets lifted. Since Dumbledore's tragic demise, Alastor "Mad-Eye" Moody has been in charge of the Order of the Phoenix and is now trying to devise a plan to safely transport Harry from his home at 4 Privet Drive to the Burrow.

On the night of the 27th of July, members of the Order will come to 4 Privet Drive (from now on 4PD) as previously agreed with Harry. At a precisely determined time, the whole party (members of the Order and Harry) will leave 4PD and fly towards a designated location at which a *Portkey* awaits. A Portkey is an ordinary looking, but enchanted, item which immediately transports anyone who touches it first (or multiple people if they touch it at the same time) to another location—in this case the Burrow. Everyone in the party will be equipped with the same model of a flying broomstick to ensure everyone flies at the same speed.

To make the party harder to track down, Mad-Eye wants them to only fly along specific flyways. Each flyway connects a pair of locations (4PD and the location of the Portkey are two of several such locations) and can be flown in either direction, which takes some time to do. The party does not stop at any location but uses them to cover their tracks and immediately proceed taking the next flyway. First and foremost, it is absolutely crucial for Mad-Eye that everyone in the party reaches the Burrow in the least amount of time possible. As the Portkey's teleportation enchantment wears off right after the first person touches it, they all need to reach the location at which the Portkey awaits at the same time.

To complicate things further, Lord Voldemort found out about parts of Mad-Eye's plan. He and his followers, the Death Eaters, are going to keep a close eye on the sky that night. To confuse them, Mad-Eye makes it possible for different people in the party to take different sequences of flyways from 4PD to the Portkey. Of course, multiple people can take some (or even all) flyways together, but each flyway can be flown by some maximum number of people, so as to not be noticed by the Death Eaters. Lastly, as things may escalate quickly, it would be nice to have as many members of the Order out there that night to protect Harry.

Mad-Eye's plan is considered to be *cautious* if all people in the party take the least amount of time possible to fly from 4PD to the Portkey and reach it at exactly the same time. He is now interested in the maximum size of the party (so, members of the Order including Harry) that he can have out there that night in a cautious plan.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line containing four integers n m s p, separated by a space. They denote
 - n, the total number of locations the party may visit along the way, including 4PD and the location at which the Portkey awaits; $(2 \le n \le 10^3)$;

- m, the total number of flyways $(1 \le m \le 5 \cdot 10^3)$;
- s, the location denoting 4 Privet Drive $(0 \le s \le n-1)$;
- p, the location at which the Portkey awaits $(0 \le p \le n-1 \text{ and } p \ne s)$.
- The following m lines describe the flyways. Each line consists of four integers u v c d, separated by a space, denoting that the flyway between locations u and v ($0 \le u, v \le n-1$) can be flown by at most c people ($1 \le c \le 10^4$) and that it takes d minutes to do so ($1 \le d \le 10^4$). There may be flyways between the same location (u = v) and there may be several flyways between the same pair of locations.

It is guaranteed that there is always a sequence of flyways that starts at 4PD and ends at the location at which the Portkey awaits.

Output For each test case output a line with a single integer that denotes the maximum number of the people in the party (so, members of the Order including Harry) that can travel from 4PD to the Portkey that night in a cautious plan.

Points There are four groups of test sets, each of which is worth 25 points. So, there are $4 \cdot 25 = 100$ points in total.

- 1. For the first group of test sets, you may assume that (i) for every flyway there is some cautious plan that uses it and (ii) for each flyway (u, v) in the input the time needed to reach u from 4PD is shorter than the time needed to reach the Portkey from v.
- 2. For the second group of test sets, you may assume that the sequence of flyways from 4PD to the Portkey which takes the least amount of time to fly is unique.
- 3. For the third group of test sets, you may assume that every location other than 4PD and the Portkey has exactly two flyways that connect to it.
- 4. For the fourth group of test sets there are no additional assumptions.

Corresponding sample test sets are contained in testi.in/out, for $i \in \{1, 2, 3, 4\}$.

Sample Input		Sample Output
5	4 5 0 3	6
3 3 0 2	0 2 4 2	4
0 2 4 2	0 1 3 1	3
0 1 3 1	1 2 1 1	5
1 2 2 1	2 3 2 1	2
3 3 0 2	1 3 4 2	
0 2 4 1	4 5 0 3	
0 1 3 1	0 2 4 1	
1 2 2 1	0 1 3 1	
4 5 0 3	1 2 1 1	
0 2 4 3	2 3 2 1	
0 1 3 1	1 3 4 2	
1 2 1 1		
2 3 2 1		
1 3 4 2		