

Algorithms Lab HS24
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cadmo.ethz.ch/education/lectures/HS24/algolab

## Exercise - Casterly Rock

"Interesting thing about my father... he built our house up from near ruin. He built our army, he built Casterly Rock as we know it, but he didn't build the sewers. That was beneath him, so he gave the job to the lowest person he could find... me."

Tyrion Lannister, A Song of Ice and Fire by George R. R. Martin

Lord Tywin Lannister has given his youngest son Tyrion charge of all the cisterns and drains at Casterly Rock, in an effort to make him a more responsible family member. After a close inspection of the existing antiquated facilities, Tyrion decides to rebuild the system from scratch.

We imagine the following situation in the standard x-y-plane, with horizontal x-axis pointing right (east), and vertical y-axis pointing up (north). That is, we ignore the issue of depth, meaning that all canals and pipes (as defined below) are allowed to cross and even overlap.

There is a number of houses at Casterly Rock that all need access to fresh water and a possibility to dispose of their sewage. Tyrion's grand plan is to build two massive underground canals, one for fresh water and the other for the sewage. Each canal is to be made in the form of a straight line so as to maximize the flow capacity and minimize the risk of congestion. To simplify construction, the two canals are to meet and cross at a right angle (90 degrees). Once they are in place, every house is to be connected via two separate pipes, where each pipe starts at the corresponding house and ends at either one of the two canals. To minimize the chance of confusing pipes of different types during installation, each fresh water pipe is to be built as a vertical straight line segment, and each sewage pipe as a horizontal one. Note that this further requires the fresh water canal to be non-vertical, and the sewer canal to be non-horizontal.

Every house at Casterly Rock belongs either to a noble family or to the common folk. Tyrion's sister Cersei is quite concerned with having to endure the bad smells of the commoners. She thus insists on having the sewer canal built in such a way that all noble houses are on its left and all common houses are on its right side. Being precisely on the sewer canal is also fine.

Lord Tywin is more concerned with the immense cost of his son's project. He identifies the sewage pipes as the most expensive component; after all, they are to be coated with Valyrian steel to make them really durable. He thus requires the sewer canal to be laid out in such a way that the sum of the lengths of all sewage pipes does not exceed a given threshold s. Note that by definition of pipe, overlapping sewage pipes are counted with multiplicity in our model.

Finally, Tyrion's brother Jaime, by some considered the greatest swordsman in the kingdom, worries most about his personal and his people's health. In particular, he wants to ensure good water quality. In order to keep the water from getting stale, Tyrion thus agrees to minimize the length of the longest fresh water pipe, subject to all the constraints mentioned before.

**Input** The first line of the input contains the number  $t \leq 30$  of test cases. Each of the t test cases is described as follows.

• It starts with a line containing three integers n m s, separated by a space. They denote

- n, the number of noble houses  $(0 \le n \le 500)$ ;
- m, the number of common houses (0  $\leq$  m  $\leq$  500);
- s, the maximum allowable sum of lengths of all sewage pipes ( $-1 \le s < 2^{50}$ , where the special value -1 is to be interpreted as  $s = \infty$ ).
- The following n lines define the positions of the noble houses.
- The following m lines define the positions of the common houses.

Each position is described by two integers x y, separated by a space and such that  $|x|, |y| < 2^{24}$ . You may assume that these  $n + m \ge 3$  positions are distinct and not all on a common line.

Output The output for each test case consists of a separate line. If a sewer canal cannot be built according to Cersei's constraint, the output is "Y" (for "Yuck!"). Otherwise, if Cersei's constraint can be satisfied but not Tywin's at the same time, the output is "B" for ("Bankrupt"). Otherwise, if both Cersei's and Tywin's constraint can be satisfied, the output consists of the minimum attainable largest length over all fresh water pipes, rounded up to the next integer.

You may assume that the rounded output number can be represented exactly as a double.

Points There are four groups of test sets, worth 100 points in total.

- 1. For the first group of test sets, worth 30 points, you may assume that s = 0. In effect, this means that you only have to distinguish the outputs "Y" and "B".
- 2. For the second group of test sets, worth 30 points, you may assume that m=0 and  $s=\infty$  (recall that the actual input is -1). In effect, this means that the output is never "Y" or "B".
- 3. For the third group of test sets, worth 20 points, you may assume that the two canals need not necessarily meet at a right angle. More precisely, the optimal solution without this constraint has the same (rounded) value as the optimal solution with this constraint.
- 4. For the fourth group of test sets, worth 20 points, there are no additional assumptions.

Corresponding sample test sets are contained in testi.in/out, for  $i \in \{1, 2, 3, 4\}$ .

Sample Input		Sample Output
4	2 2 4	Υ
2 2 3	-1 1	В
1 1	-1 -1	1
1 -1	1 1	1
-1 1	1 -1	
-1 -1	3 2 2	
2 2 3	0 1	
-1 1	-1 0	
-1 -1	0 -1	
1 1	0 0	
1 -1	1 0	