Question 1 (8 points) There is provided the dataset "SwedenEcon.csv" which has been worked with in class. You will work with that dataset for this question:

- 1. Apply LOESS to the columns of "NominalExport" and "ExportGrowth" separately. Each plot should have the original data and the smoothing that LOESS provides (or LOWESS is also an approach to use).
- 2. Redraw those plots with a logarithmic scale for the y-axis values
- 3. Build a simple linear regression model and a polynomial regression model with powers up to the value of 3 to fit the prediction where the 'dependent is GDPbyActivityBasicPrices' and 'independent is ExportGrowth'. Plot the fits and the original data (3 lines).
- 4. Compare the 2 models using the F-test (anova(model1,model2,test="F")) and report on the decision for the choice of model.
- 5. Use the AIC function for the 2 models and report which model you choose on the outputs it provides.

```
In [108]: install.packages("ggpubr")
           install.packages("datarium")
          install.packages("glmnet")
           install.packages("readxl")
           install.packages("caret")
          install.packages("mlbench")
           install.packages("psych")
           install.packages("ggplot2")
           install.packages("DAAG")
          install.packages("MASS")
           install.packages("relaimpo")
           install.packages("TTR")
           install.packages("tseries")
          install.packages("data.table")
           install.packages("MTS")
           install.packages("ggfortify")
          Warning message:
           "package 'ggpubr' is in use and will not be installed"
          package 'datarium' successfully unpacked and MD5 sums checked
          The downloaded binary packages are in
                  C:\Users\Andrew\AppData\Local\Temp\RtmpI5MZBx\downloaded_packages
          Warning message:
           "package 'glmnet' is in use and will not be installed "Warning message:
           "package 'readxl' is in use and will not be installed"Warning message:
          "package 'caret' is in use and will not be installed"Warning message:
          "package 'mlbench' is in use and will not be installed "Warning message:
           "package 'psych' is in use and will not be installed "Warning message:
           "package 'ggplot2' is in use and will not be installed"Warning message:
          "package 'DAAG' is in use and will not be installed"Warning message:
           "package 'MASS' is in use and will not be installed Warning message:
           "package 'relaimpo' is in use and will not be installed"Warning message:
           "package 'TTR' is in use and will not be installed"Warning message:
          "package 'tseries' is in use and will not be installed"Warning message:
           "package 'data.table' is in use and will not be installed "Warning message:
           "package 'MTS' is in use and will not be installed"
          package 'ggfortify' successfully unpacked and MD5 sums checked
          The downloaded binary packages are in
                  {\tt C:\Users\Andrew\AppData\Local\Temp\RtmpI5MZBx\downloaded\_packages}
```

```
In [109]: library(ggpubr)
           #Library(tidyverse)
           library(readx1)
           library(glmnet)
           library(caret)
           library(mlbench)
           library(psych)
           library(ggplot2)
           library(DAAG)
           library(MASS)
           library(relaimpo)
           library(TTR)
           library(tseries)
           library(data.table)
           library(MTS)
           library(ggfortify)
          Warning message:
           "package 'ggfortify' was built under R version 3.6.3"
          Error: package or namespace load failed for 'ggfortify' in loadNamespace(j <- i[[11]], c(lib.loc, .libPaths()), ver
           sionCheck = vI[[j]]):
           namespace 'dplyr' 0.8.0.1 is already loaded, but >= 0.8.2 is required
          Traceback:

    library(ggfortify)

           2. tryCatch({
                  attr(package, "LibPath") <- which.lib.loc</pre>
                  ns <- loadNamespace(package, lib.loc)</pre>
                  env <- attachNamespace(ns, pos = pos, deps, exclude, include.only)</pre>
            . }, error = function(e) {
                  P <- if (!is.null(cc <- conditionCall(e)))
                      paste(" in", deparse(cc)[1L])
                  else "
                  msg <- gettextf("package or namespace load failed for %s%s:\n %s",</pre>
                      sQuote(package), P, conditionMessage(e))
                  if (logical.return)
                      message(paste("Error:", msg), domain = NA)
                  else stop(msg, call. = FALSE, domain = NA)
            . })
          3. tryCatchList(expr, classes, parentenv, handlers)
          4. tryCatchOne(expr, names, parentenv, handlers[[1L]])
          5. value[[3L]](cond)
          6. stop(msg, call. = FALSE, domain = NA)
```

Since you mention on question one, you want the original data, i did not take off NA

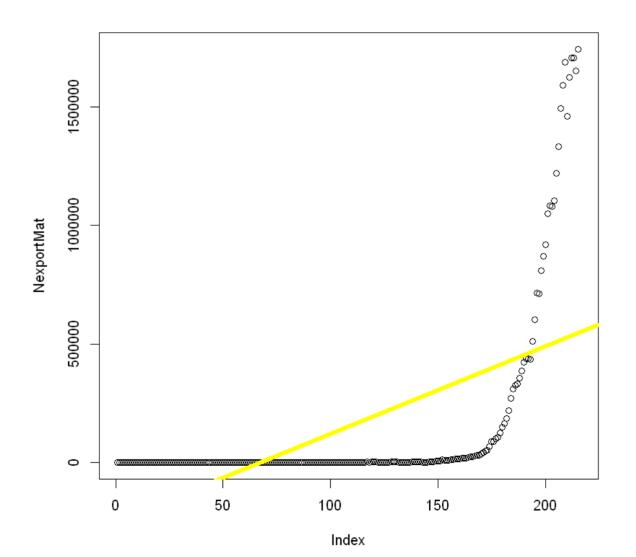
1. Apply LOESS to the columns of "NominalExport" and "ExportGrowth" separately. Each plot should have the original data and the smoothing that LOESS provides (or LOWESS is also an approach to use).

```
In [110]:
         SwedenEcon<- read.csv('C:/Users/Andrew/Desktop/SwedenEcon-1.csv')</pre>
         str(SwedenEcon)
         'data.frame': 216 obs. of 15 variables:
                                         : int 1800 1801 1802 1803 1804 1805 1806 1807 1808 1809 ...
          $ Year
          $ PrivateGrowthConsumptionVolume
                                            : num NA -2.66 3.98 1.09 11.92 ...
          $ PrivateGrowthConsumptionVolumeNominal: num 142 138 136 132 149 ...
          : num 15.8 20.9 22.3 20.8 19.2 ...
          $ NominalExport
                                            : num NA 32.54 8.5 -9.73 -9.71 ...
          $ ExportGrowth
                                            : num 14.8 19 14.5 13.8 23.5 ...
          $ NominalImport
                                            : num NA 24.49 -15.43 -5.26 66.81 ...
          $ ImportGrowth
          $ GDPbyActivityBasicPrices
                                           : num 110 113 116 110 109 ...
          $ GDPbyActivityCorrectedForChanges
                                            : num -4.449 3.812 0.545 -0.736 4.485 ...
          $ DividendIndex
                                             : num NA NA NA NA NA NA NA NA NA ...
          $ GovShortYield
                                             : num NA NA NA NA NA NA NA NA NA ...
          $ AgricultureEmployeeNumber
                                            : num NA NA NA NA NA NA NA NA NA ...
          $ GovernmentServiceEmployeeNumber
                                            : num NA ...
```

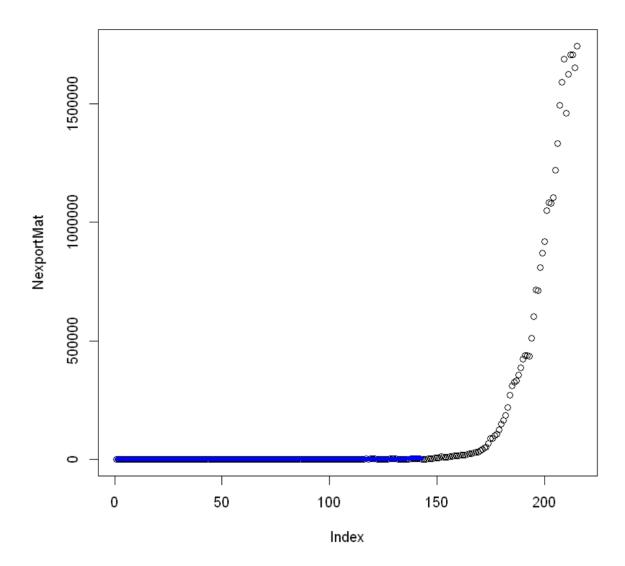
In [111]: head(SwedenEcon, 10) summary(SwedenEcon)

Year	${\bf Private Growth Consumption Volume}$	${\bf Private Growth Consumption Volume Nominal}$	${\bf Government Consumption Gowth Volume}$	GovernmentConsumpti
1800	NA	141.7600	NA	
1801	-2.6552163	138.3895	-1.3924178	
1802	3.9828373	135.7893	4.4855156	
1803	1.0860552	132.0632	8.0936523	
1804	11.9233158	148.9401	-6.1537242	
1805	-4.2713947	144.6281	3.0147266	
1806	-1.2301890	165.0820	-7.7087453	
1807	-5.2216500	160.7495	-0.5106037	
1808	4.0913070	203.2462	-0.9651354	
1809	-0.5753731	206.9595	-15.3344311	
4				>

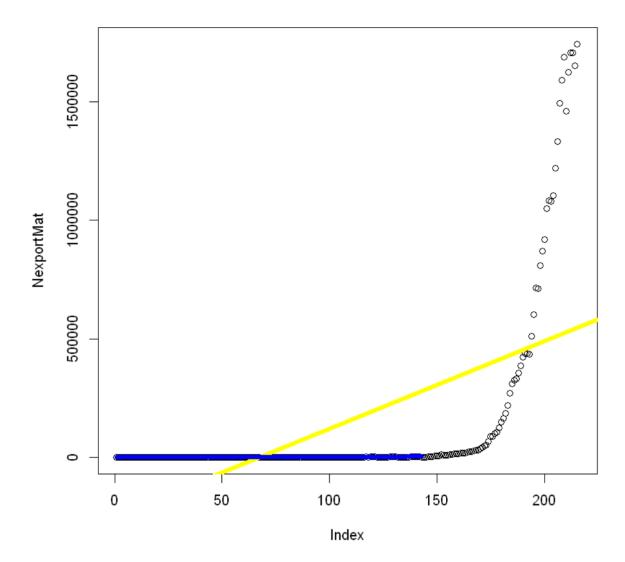
```
PrivateGrowthConsumptionVolume
    Year
Min. :1800 Min. :-10.2622
1st Ou.:1854
             1st Ou.: -0.7811
Median :1908
             Median : 2.2500
Mean :1908 Mean : 2.1255
3rd Qu.:1961 3rd Qu.: 4.4713
            Max. : 26.1188
NA's :2
Max. :2015
PrivateGrowthConsumptionVolumeNominal GovernmentConsumptionGowthVolume
Min. : 132.1
                                  Min. :-33.64338
1st Qu.:
           585.1
                                  1st Qu.: 0.03575
Median : 2595.0
                                  Median : 2.11766
Mean : 177589.1
                                  Mean : 2.35570
3rd Qu.: 44404.1
                                  3rd Qu.: 4.83320
Max. :1811947.0
NA's :1
                                  Max. : 29.33966
NA's :2
GovernmentConsumptionGowthNominal NominalExport
                                                  ExportGrowth
           12.6
                     Min. : 12.7
Min. :
                                                Min. :-44.718
1st Qu.:
           40.2
                               1st Qu.:
                                          81.6
                                                1st Qu.: -1.267
                              Median: 646.8 Median: 4.688
          246.3
Median :
Mean : 94434.9
                              Mean : 150340.1 Mean : 4.470
3rd Ou.: 12538.6
                               3rd Ou.: 18100.5
                                                3rd Ou.: 9.385
Max. :1030997.0
NA's :1
                              Max. :1743745.0
NA's :1
                                                Max. : 76.547
                                                      :2
                                                 NA's
NominalImport
                  ImportGrowth
                                  GDPbyActivityBasicPrices
Min. : 13.8
                  Min. :-45.090
                                  Min. : 108.7
           69.6
                  1st Qu.: -2.325
1st Qu.:
                                  1st Qu.:
                                             438.8
Median :
        712.6
                  Median : 5.200
                                  Median : 2077.1
Mean : 132527.5
                  Mean : 5.311 Mean : 140083.9
                  3rd Qu.: 10.659
                                  3rd Qu.: 29060.9
3rd Qu.: 17538.4
                  Max. :225.572
NA's :2
                                  Max. :1813900.4
Max. :1600463.0
NA's :1
                                  NA's :15
GDPbyActivityCorrectedForChanges DividendIndex
                                              GovShortYield
Min. :-10.5558
                             Min. : 0.0180
                                              Min. : 0.200
                              1st Qu.: 0.0460 1st Qu.: 3.750
1st Ou.: 0.5156
Median : 2.9639
                              Median : 0.0615
                                              Median : 5.000
                              Mean : 0.9897
Mean : 2.3776
                                              Mean : 5.173
3rd Qu.: 4.4715
                              3rd Qu.: 0.2152
                                              3rd Qu.: 6.000
Max. : 13.0124
                              Max. :15.9690
                                              Max. :14.150
NA's :16
                              NA's :74
                                              NA's :59
AgricultureEmployeeNumber GovernmentServiceEmployeeNumber
Min. : 129211
                       Min. : 49276
1st Qu.: 519458
                       1st Qu.: 89943
Median :1115254
                       Median : 141695
Mean : 899337
                       Mean : 397322
                       3rd Qu.: 510837
3rd Qu.:1233428
Max. :1334343
                       Max. :1436600
NA's :65
                       NA's :65
```



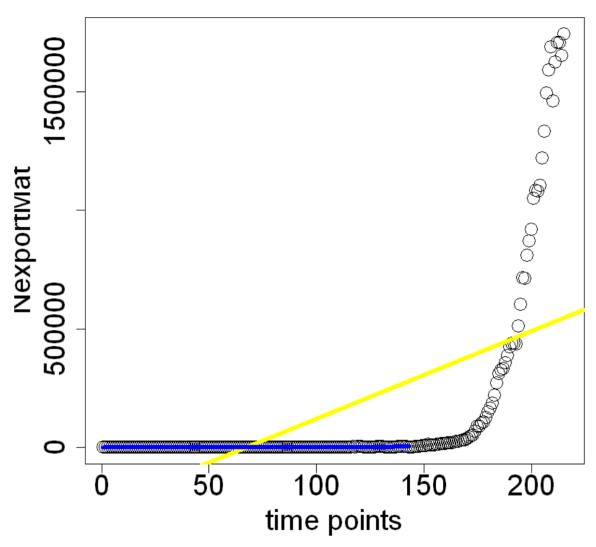
```
In [114]: #view the LOWESS
plot(NexportMat)
x=seq(1,nrow(NexportMat))
loessM1=lines(lowess(NexportMat), col="blue", lwd=5)
```



```
In [115]: #view both of them together on the same plot
plot(NexportMat)
lmM1=abline(lm(NexportMat ~x), col="yellow", lwd=5)
loessM1=lines(lowess(NexportMat), col="blue", lwd=5)
```



Nexport data with Im and LOWESS



```
In [117]: #now move to ExportGrowth variable
exportGrowth=SwedenEcon[,7]
exportGrowthMat=as.matrix(exportGrowth)
dim(exportGrowthMat)
is.matrix(exportGrowthMat)
head(exportGrowthMat)

216  1

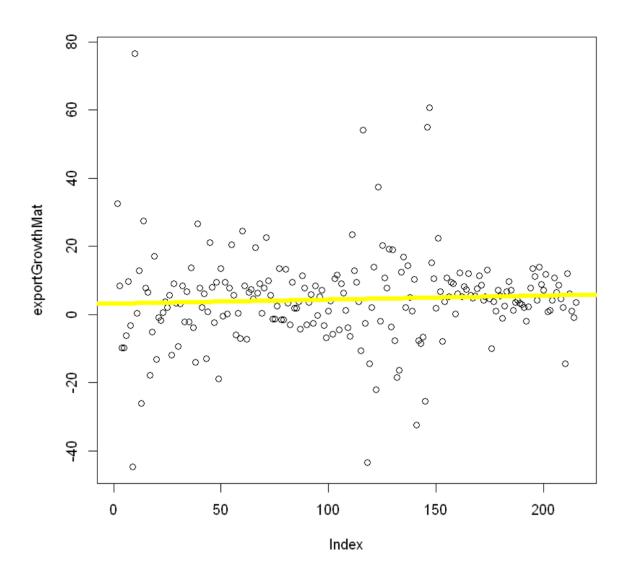
TRUE

NA
```

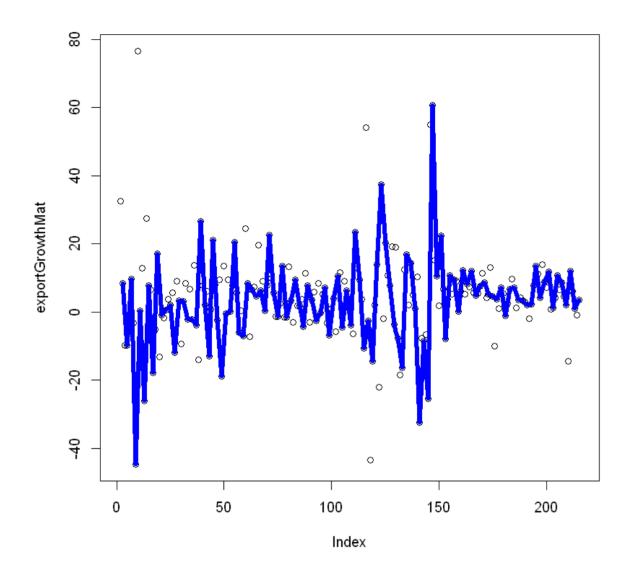
8.500821 -9.728642 -9.706756 -6.110455

32.542291

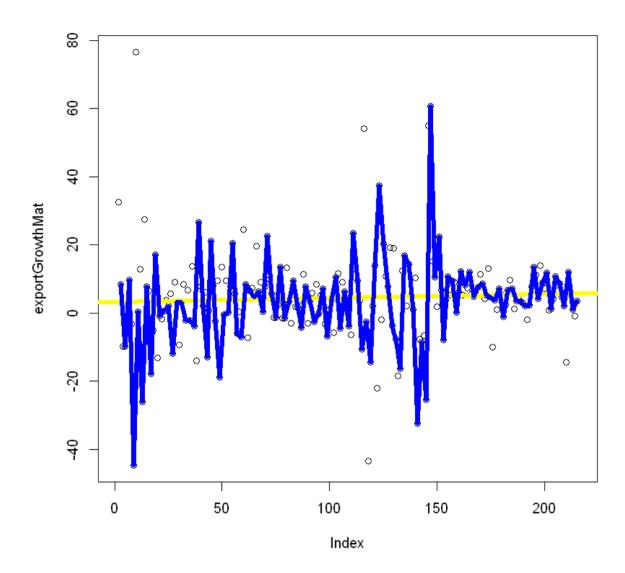
```
In [118]: plot(exportGrowthMat)
    x=seq(1,nrow(exportGrowthMat))
    lmM1=abline(lm(exportGrowthMat ~x), col="yellow", lwd=5)
```



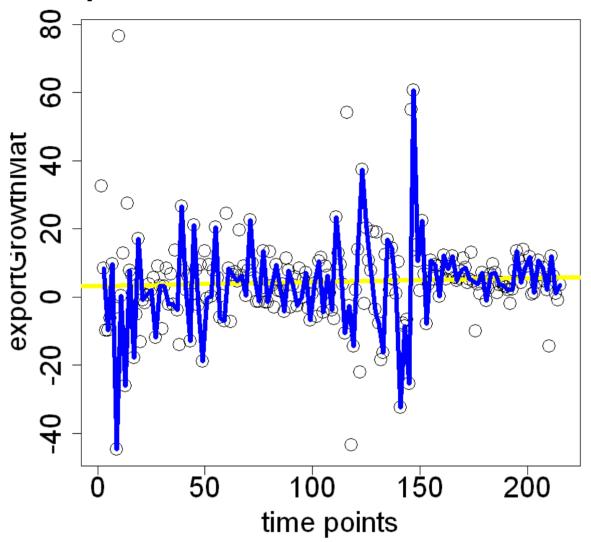
```
In [119]: #view the LOWESS
plot(exportGrowthMat)
x=seq(1,nrow(exportGrowthMat))
loessM1=lines(lowess(exportGrowthMat), col="blue", lwd=5)
```



```
In [120]: #view both of them together on the same plot
    plot(exportGrowthMat)
    lmM1=abline(lm(exportGrowthMat ~x), col="yellow", lwd=5)
    loessM1=lines(lowess(exportGrowthMat), col="blue", lwd=5)
```



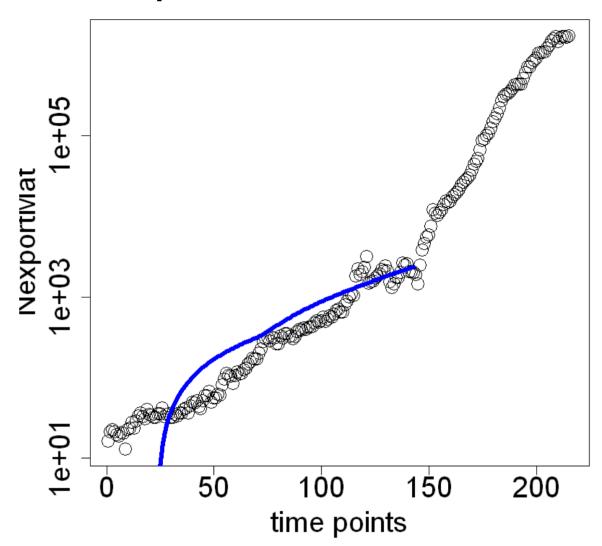
exportGrowth data with Im and LOWESS



In []:

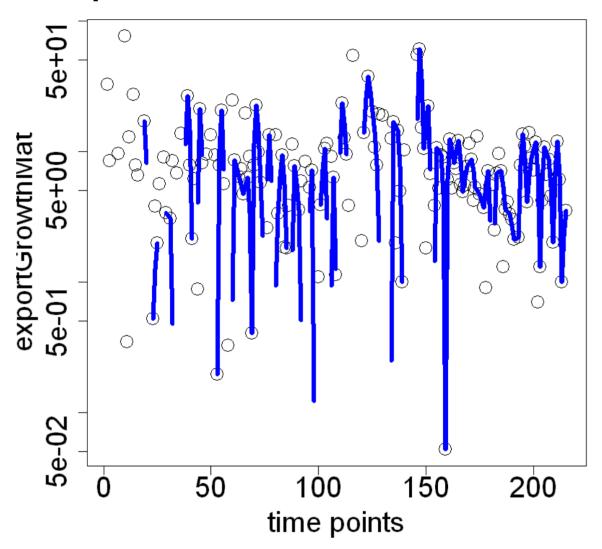
1. Redraw those plots with a logarithmic scale for the y-axis values

Nexport data with Im and LOWESS



Warning message in xy.coords(x, y, xlabel, ylabel, log): "60 y values <= 0 omitted from logarithmic plot"

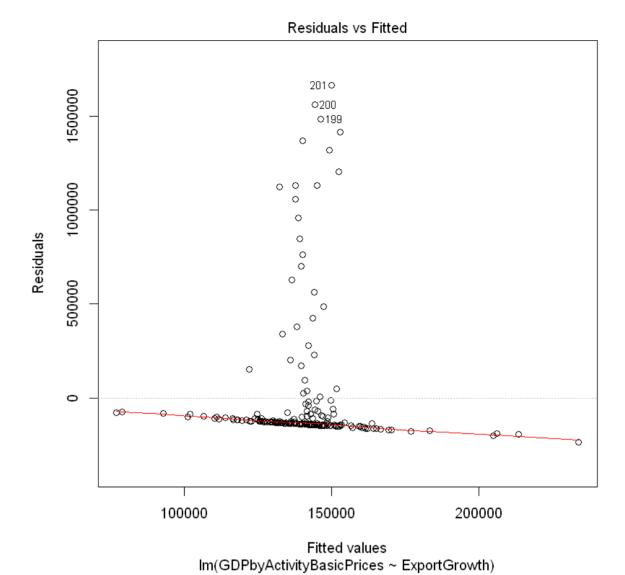
exportGrowth data with Im and LOWESS

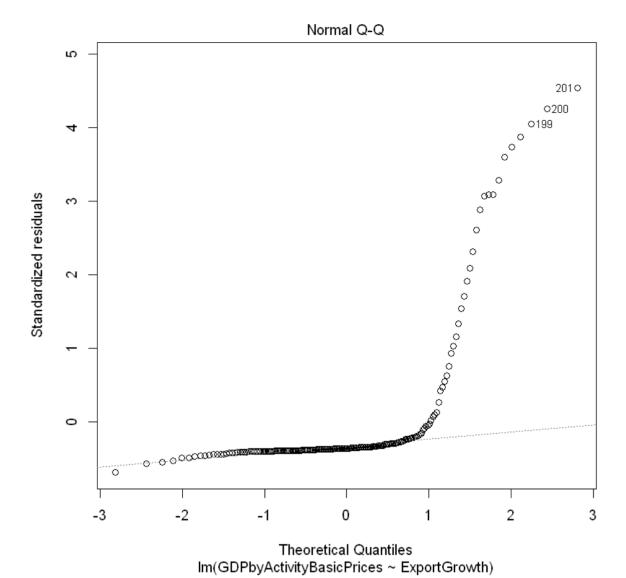


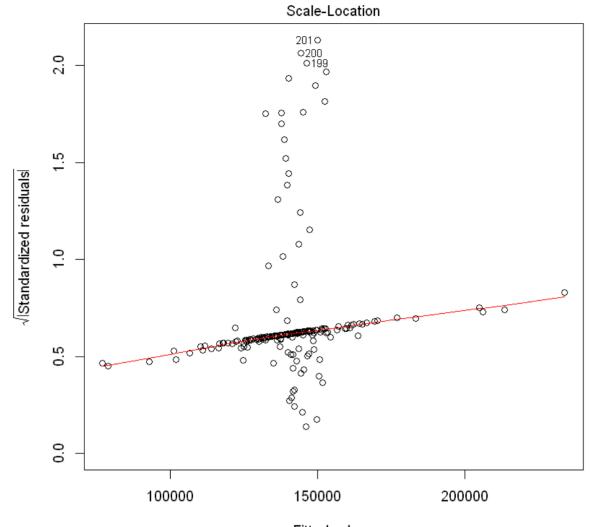
```
In [ ]:
```

1. Build a simple linear regression model and a polynomial regression model with powers up to the value of 3 to fit the prediction where the 'dependent is GDPbyActivityBasicPrices' and 'independent is ExportGrowth'. Plot the fits and the original data (3 lines).

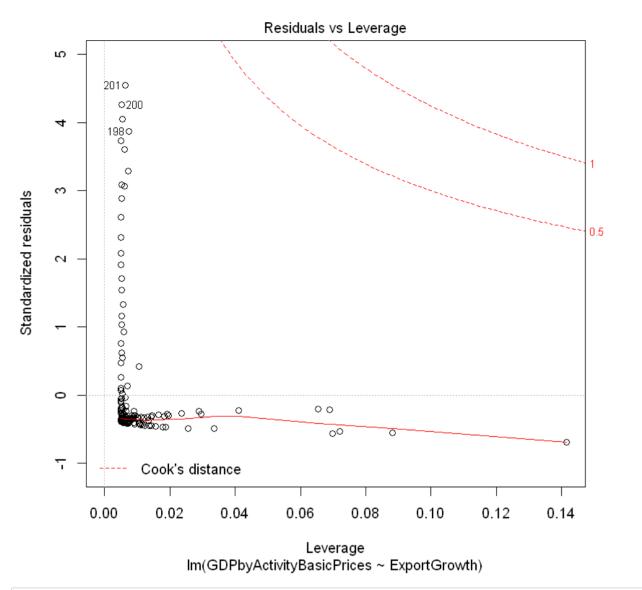
In [125]: plot(model)







Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth)



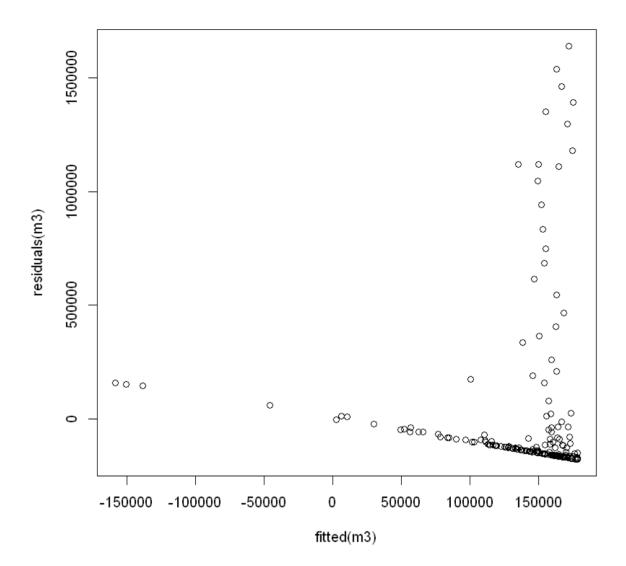
```
In [126]: summary(model)
          Call:
          lm(formula = GDPbyActivityBasicPrices ~ ExportGrowth, data = SwedenEcon)
          Residuals:
              Min
                       1Q Median
                                       3Q
                                              Max
          -233838 -143299 -131413 -96448 1663864
          Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
          (Intercept)
                         134888
                                     27366
                                            4.929 1.74e-06 ***
                                      1886
                                            0.686
                                                     0.493
          ExportGrowth
                           1295
          Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
          Residual standard error: 367400 on 198 degrees of freedom
            (16 observations deleted due to missingness)
          Multiple R-squared: 0.002374, Adjusted R-squared: -0.002665
          F-statistic: 0.4711 on 1 and 198 DF, p-value: 0.4933
```

my poly function doesn't work, i have to use alternative way

```
In [127]: #my poly function doesn't work, i have to use alternative way
          #polynomial regression model with powers up to the value of 3 to fit the prediction
          #Plot the fits and the original data (3 lines).
          m1<-lm(SwedenEcon$GDPbyActivityBasicPrices ~ SwedenEcon$ExportGrowth)</pre>
          m2<-lm(SwedenEcon$GDPbyActivityBasicPrices ~ SwedenEcon$ExportGrowth + I(SwedenEcon$ExportGrowth^2))</pre>
          m3<-lm(SwedenEcon$GDPbyActivityBasicPrices ~ SwedenEcon$ExportGrowth + I(SwedenEcon$ExportGrowth^2)+
                 I(SwedenEcon$ExportGrowth^3))
          Your code contains a unicode char which cannot be displayed in your
          current locale and R will silently convert it to an escaped form when the
          R kernel executes this code. This can lead to subtle errors if you use
          such chars to do comparisons. For more information, please see
          https://github.com/IRkernel/repr/wiki/Problems-with-unicode-on-windows
In [128]: #mean squred errors (MSE)
          print(mean(summary(m1$residuals^2)))
          print(mean(summary(m2$residuals^2)))
          print(mean(summary(m3$residuals^2)))
          [1] 493035291072
          [1] 482967339938
          [1] 481784155686
In [129]: summary(m3)
          Call:
          lm(formula = SwedenEcon$GDPbyActivityBasicPrices ~ SwedenEcon$ExportGrowth +
              I(SwedenEcon$ExportGrowth^2) + I(SwedenEcon$ExportGrowth^3))
          Residuals:
              Min
                       1Q Median
                                       3Q
          -178506 -161250 -134540 -65896 1641571
          Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)
                                        1.427e+05 3.062e+04 4.662 5.77e-06 ***
          (Intercept)
                                      3.487e+03 2.757e+03 1.265
          SwedenEcon$ExportGrowth
                                                                        0.207
          I(SwedenEcon$ExportGrowth^2) -7.922e+01 7.576e+01 -1.046
                                                                        0.297
          I(SwedenEcon$ExportGrowth^3) -2.314e-01 1.435e+00 -0.161
                                                                        0.872
          Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
          Residual standard error: 366200 on 196 degrees of freedom
            (16 observations deleted due to missingness)
          Multiple R-squared: 0.01912, Adjusted R-squared: 0.004108
          F-statistic: 1.274 on 3 and 196 DF, p-value: 0.2846
```

In [130]: confint(m3, level=0.95)

2.5 % 97.5 % (Intercept) 82365.950513 2.031314e+05 SwedenEcon\$ExportGrowth -1949.393345 8.923091e+03 I(SwedenEcon\$ExportGrowth^2) -228.633926 7.018710e+01 I(SwedenEcon\$ExportGrowth^3) -3.061038 2.598144e+00



In [132]: #use a few statement to fit the linear model and use the model to predict some values at points we have seen and no
 t seen
 m1<-lm(GDPbyActivityBasicPrices~ExportGrowth, data=SwedenEcon)
 new.df<-data.frame(ExportGrowth=c(10.5))
 predict(m1, new.df)</pre>

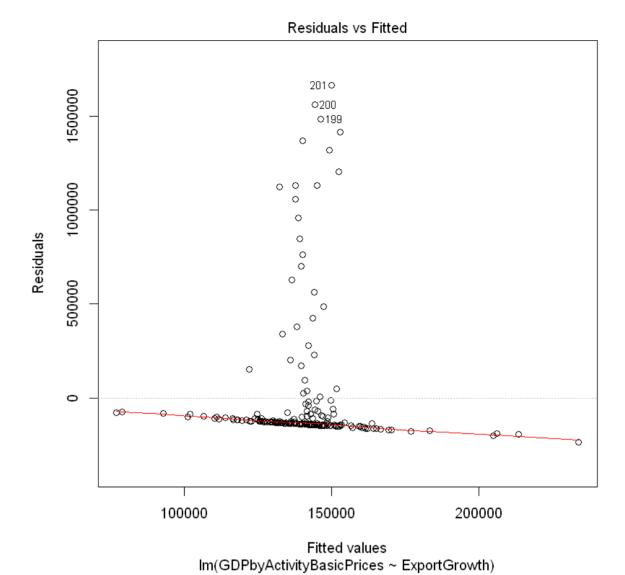
1: 148482.67500375

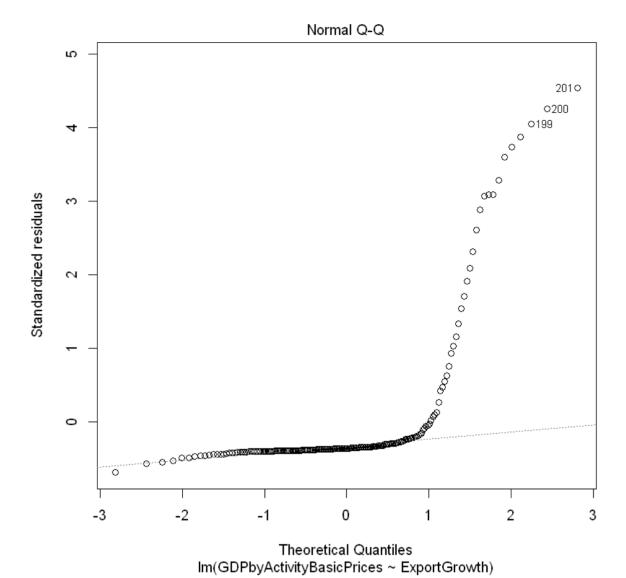
In [133]: #Let's Look at the prediction interval around a point that we specify
m1<-lm(GDPbyActivityBasicPrices~ExportGrowth, data=SwedenEcon)
new.df<-data.frame(ExportGrowth=c(10.5))
predict(m1, new.df, interval="prediction")</pre>

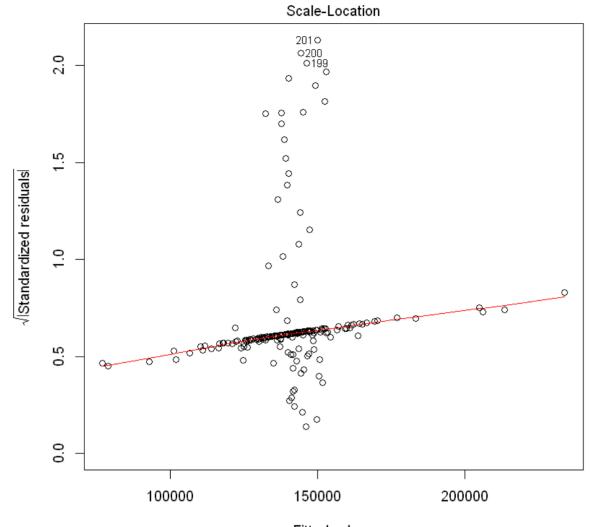
 fit
 lwr
 upr

 148482.7
 -578273.7
 875239

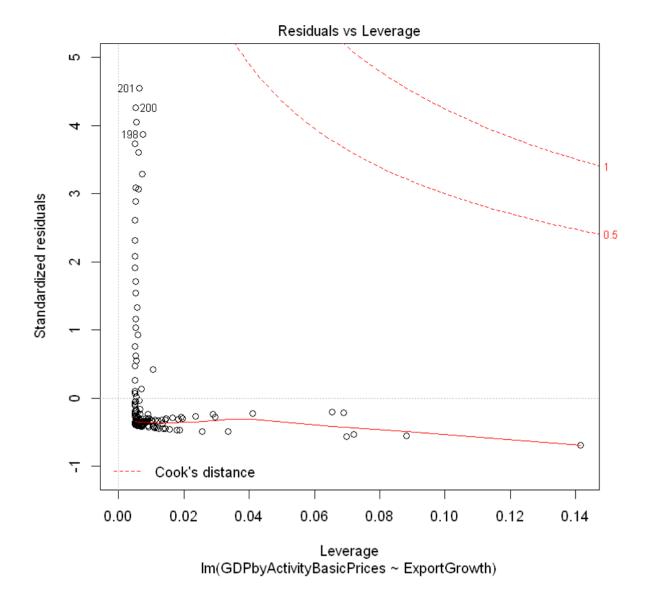
In [134]: plot(m1)







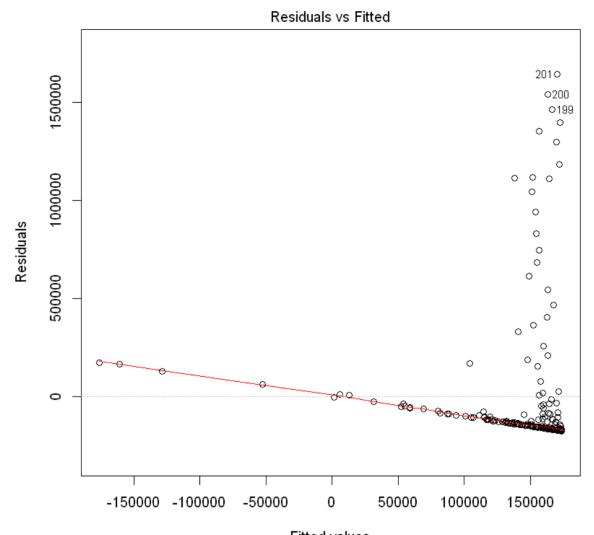
Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth)



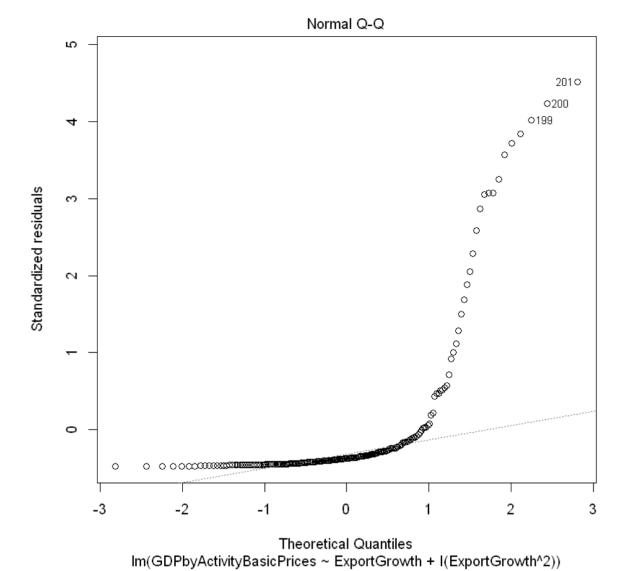
these code do not work for me plot(x=1:20, y=GDPbyActivityBasicPrices, col="blue", cex=2, cex.axis=1.5, cex.lab=2) new.df<-data.frame(ExportGrowth=c(1:20)) ynew<-predict(m1, new.df) points(ynew, col="red", cex=2)

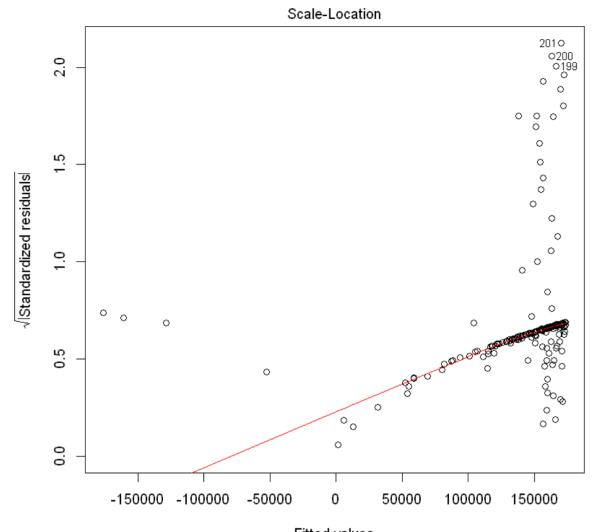
these code do not work for me: plot(x=1:20, y=GDPbyActivityBasicPrices, col="blue", cex=2, cex.axis=1.5, cex.lab=2) new.df<-data.frame(ExportGrowth=c(1:20)) ynew<-predict(m2, new.df) points(ynew, col="red", cex=4)

In [136]: plot(m2)

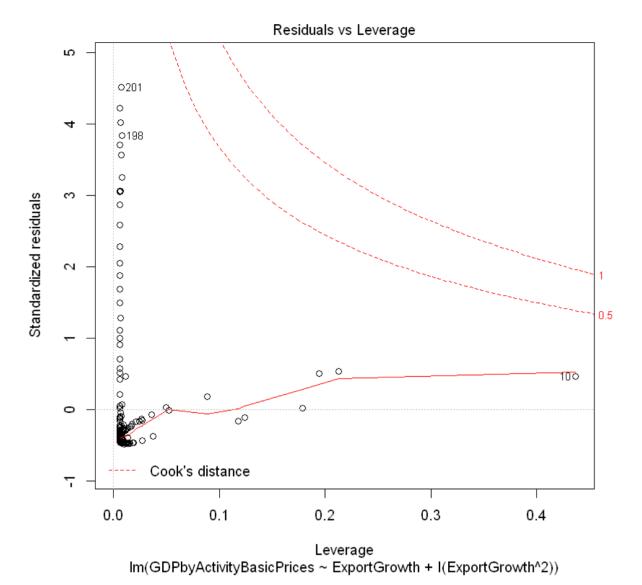


Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth + I(ExportGrowth^2))





Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth + I(ExportGrowth^2))

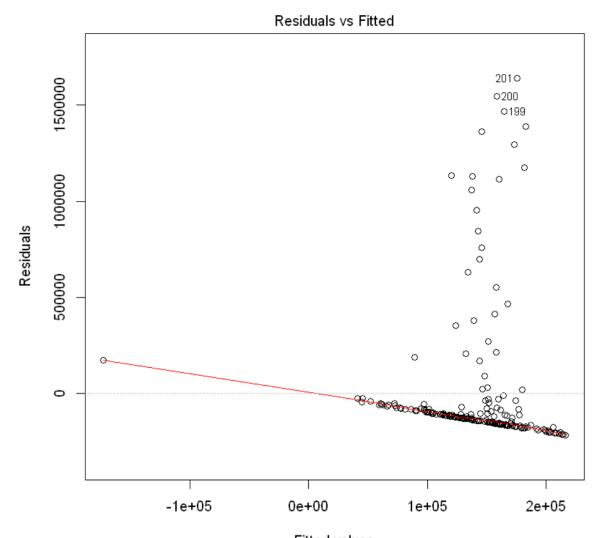


In [137]: #Let's Look at the prediction interval around a point that we specify
 m3<-lm(GDPbyActivityBasicPrices~ExportGrowth+I(ExportGrowth^3), data=SwedenEcon)
 new.df<-data.frame(ExportGrowth=c(10, 10.5))
 predict(m3, new.df, interval="prediction")</pre>

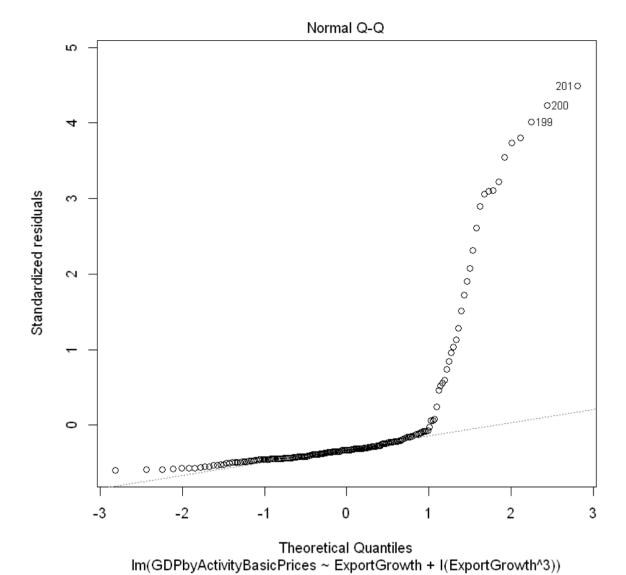
fit	lwr	upr
169057.0	-555917.8	894031.7
170919.9	-554172.1	896011.9

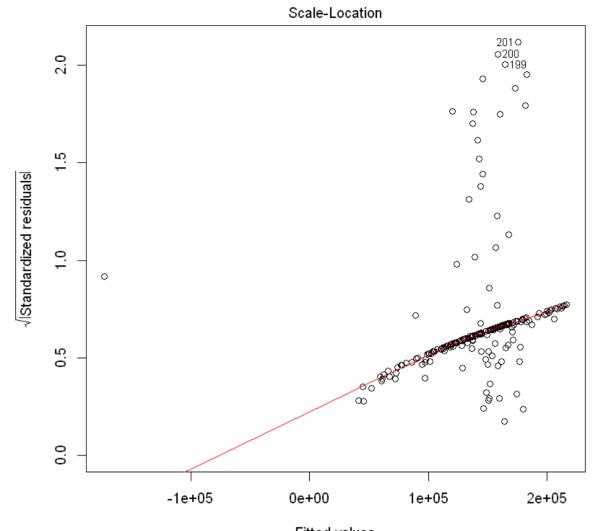
these code do not work for me: plot(x=1:20, y=GDPbyActivityBasicPrices, col="blue", cex=2, cex.axis=1.5, cex.lab=2) new.df<-data.frame(ExportGrowth=c(1:20)) ynew<-predict(m3, new.df) points(ynew, col="red", cex=4)

In [138]: plot(m3)

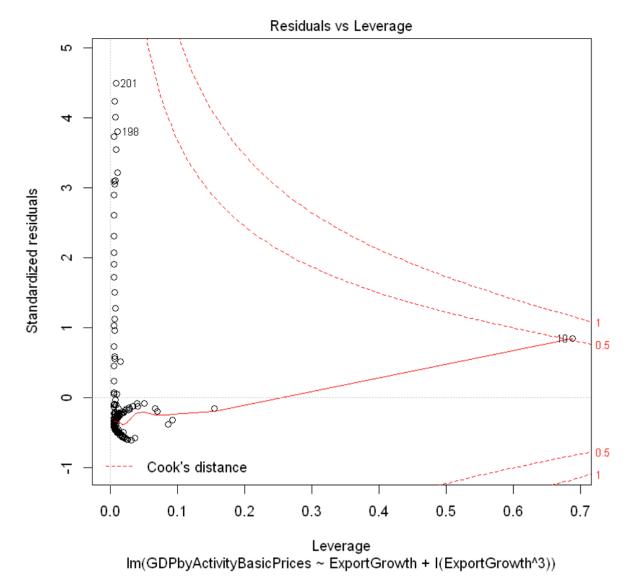


Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth + I(ExportGrowth^3))





Fitted values Im(GDPbyActivityBasicPrices ~ ExportGrowth + I(ExportGrowth^3))



In []:

1. Compare the 2 models using the F-test (anova(model1,model2,test="F")) and report on the decision for the choice of model.

we can see that the 'Df' is that there is a single additional parameter and the p value is bigger than 0.05 since there was no change in a perfect fit

In []:

1. Use the AIC function for the 2 models and report which model you choose on the outputs it provides.

```
In [140]: AIC(m1, m2)

to df AIC
m1 3 5697.298
m2 4 5695.939
```

I will choose m2 because the model has smaller AIC

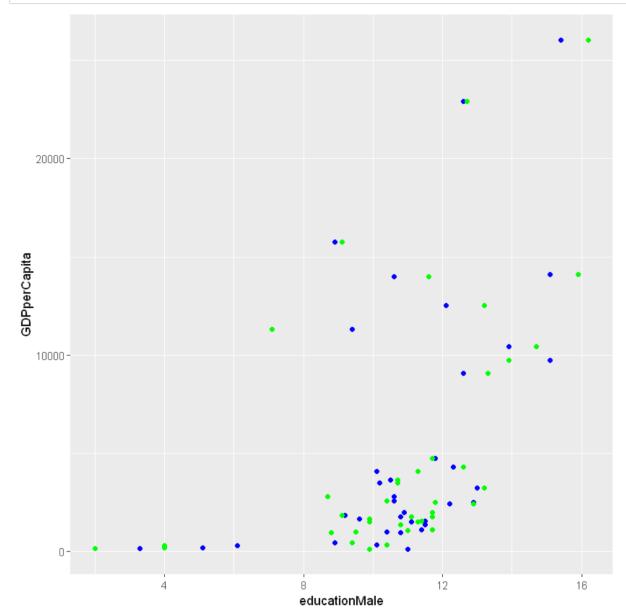
\$ illiteracyFemale

Question 2 (10 points) You will use the dataset "UN98.csv" for this question:

- 1. In a 2D scatterplot display the points for (educationMale,GDPperCapita) in the color blue, and add to that same plot the points for (educationFemale,GDPperCapita) in green.
- 2. In the dataset check (validate) that the column 'region' is a Factor and if it is not a factor make it so.
- 3. Use the library, 'nnet', (library(nnet), install.packages("nnet")) to use the function 'multinom' to do multinomial logistic regression where the dependent is the 'region', and the independents are 'infantMortality' and 'GDPperCapita'. What can you conclude from the model production?
- 4. Can you use the 'predict' function with your model on a new data point where 'infantMortality = 10' and 'GDPperCapita = 20000' and report on the output? This same function was not shown for multinom's use but it was shown for binary logitic regression and for decision trees etc. If it cannot be used say why or else apply it as well to report on the results.
- 5. Build a decision tree where the dependent is the 'region', and the independents are 'infantMortality' and 'GDPperCapita'. Display the tree.
- 6. Can you use the 'predict' function with your decision tree model on a new data point where 'infantMortality = 10' and 'GDPperCapita = 20000' and report on the output?
- 1. In a 2D scatterplot display the points for (educationMale,GDPperCapita) in the color blue, and add to that same plot the points for (educationFemale,GDPperCapita) in green.

```
In [142]: UN98<- read.csv('C:/Users/Andrew/Desktop/UN98-1.csv')</pre>
          str(UN98)
          'data.frame': 207 obs. of 14 variables:
           $ X
                                   : Factor w/ 207 levels "Afghanistan",..: 1 2 3 4 5 6 7 8 9 10 ...
           $ region
                                   : Factor w/ 5 levels "Africa", "America", ...: 3 4 1 3 4 1 2 2 4 5 ...
           $ tfr
                                   : num 6.9 2.6 3.81 NA NA 6.69 NA 2.62 1.7 1.89 ...
           $ contraception
                                  : int NA NA 52 NA NA NA 53 NA 22 76 ...
           $ educationMale
                                 : num NA NA 11.1 NA NA NA NA NA NA 16.3 ...
           $ educationFemale
                                  : num NA NA 9.9 NA NA NA NA NA NA 16.1 ...
           $ lifeMale
                                   : num 45 68 67.5 68 NA 44.9 NA 69.6 67.2 75.4 ...
           $ lifeFemale
                                   : num 46 74 70.3 73 NA 48.1 NA 76.8 74 81.2 ...
           $ infantMortality
                                   : int 154 32 44 11 NA 124 24 22 25 6 ..
           $ GDPperCapita
                                   : int
                                         2848 863 1531 NA NA 355 6966 8055 354 20046 ...
           $ economicActivityMale : num 87.5 NA 76.4 58.8 NA NA 74.4 76.2 65 74 ...
           $ economicActivityFemale: num 7.2 NA 7.8 42.4 NA NA 56.2 41.3 52 53.8 ...
           $ illiteracyMale
                                  : num 52.8 NA 26.1 0.264 NA NA NA 3.8 0.3 NA ...
```

: num 85 NA 51 0.36 NA NA NA 3.8 0.5 NA ...



```
In [ ]:
```

1. In the dataset check (validate) that the column 'region' is a Factor and if it is not a factor make it so.

```
In [144]: is.factor(UN98NoNA$region)

TRUE
```

column 'region' is a Factor

```
In [ ]:
```

1. Use the library, 'nnet', (library(nnet), install.packages("nnet")) to use the function 'multinom' to do multinomial logistic regression where the dependent is the 'region', and the independents are 'infantMortality' and 'GDPperCapita'. What can you conclude from the model production?

```
library(nnet)
          Warning message:
           "package 'nnet' is in use and will not be installed"
In [146]: levels(UN98NoNA$region)
           contrasts(UN98NoNA$region)
          'Africa' 'America' 'Asia' 'Europe' 'Oceania'
                    America Asia Europe Oceania
             Africa
                                              0
           America
                              0
                                      0
                                              0
                         1
              Asia
                         0
                                      0
                                              0
            Europe
                         0
                              0
                                      1
                                              0
            Oceania
                              0
                                      0
                         0
                                              1
In [147]: #built multi nomial regression
           model<-multinom(region ~ infantMortality + GDPperCapita, family=binomial(link='logit'), data=UN98NoNA)</pre>
           summary(model)
           coef(model)
          # weights: 20 (12 variable)
          initial value 62.768079
          iter 10 value 43.455051
          iter 20 value 35.327942
           iter 30 value 35.102365
          iter 40 value 35.081895
          final value 35.081616
          converged
          Call:
          multinom(formula = region ~ infantMortality + GDPperCapita, data = UN98NoNA,
               family = binomial(link = "logit"))
          Coefficients:
                   (Intercept) infantMortality GDPperCapita
           America
                     8.4005450
                                    -0.2114168
                                                0.0007464656
          Asia
                     5.7103749
                                     -0.1589191 0.0008987694
          Europe
                    10.5592752
                                    -0.3494420 0.0006553723
                                     -0.0331512 -0.0004236457
          Oceania 0.5501728
          Std. Errors:
                    (Intercept) infantMortality GDPperCapita
          America 0.0006918875
                                      0.02409275 0.0003169448
                  0.0006992323
                                      0.02523785 0.0003164302
          Asia
          Europe 0.0019344713
                                      0.04442970 0.0003227298
          Oceania 0.0002508197
                                      0.02256019 0.0010608704
          Residual Deviance: 70.16323
          AIC: 94.16323
                    (Intercept) infantMortality
                                          GDPperCapita
           America
                     8.4005450
                                 -0.2114168
                                           0.0007464656
                    5.7103749
                                 -0.1589191
                                           0.0008987694
              Asia
            Europe
                   10.5592752
                                 -0.3494420
                                           0.0006553723
                                 -0.0331512 -0.0004236457
            Oceania
                    0.5501728
```

From the summary of the model, From the coeficient result, we can see infantMortality and region is negative relationship, increase on region and infantMortality decrease. Africa is 0 and other region is 1, it means other regions has lower infant Mortality.

In []:

In [145]: | install.packages("nnet")

^{1.} Can you use the 'predict' function with your model on a new data point where 'infantMortality = 10' and 'GDPperCapita = 20000' and report on the output? This same function was not shown for multinom's use but it was shown for binary logitic regression and for decision trees etc. If it cannot be used say why or else apply it as well to report on the results.

Yes I can predict region under the condition of 'infantMortality = 10' and 'GDPperCapita = 20000'. It predict in Asia.

```
In [ ]:
```

1. Build a decision tree where the dependent is the 'region', and the independents are 'infantMortality' and 'GDPperCapita'. Display the tree.

```
In [149]: #build decision tree
   install.packages("rpart")
   install.packages("rpart.plot")
   library(rpart)
   library(rpart.plot)
```

Warning message:
"package 'rpart' is in use and will not be installed"Warning message:
"package 'rpart.plot' is in use and will not be installed"

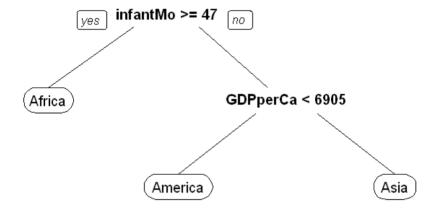
```
#draw the tree model
prp(regionTree)
regionTree

n= 39

node), split, n, loss, yval, (yprob)
    * denotes terminal node

1) root 39 26 America (0.23 0.33 0.26 0.15 0.026)
    2) infantMortality>=46.5 11 3 Africa (0.73 0 0.18 0 0.091) *
    3) infantMortality< 46.5 28 15 America (0.036 0.46 0.29 0.21 0)
    6) GDPperCapita< 6904.5 19 8 America (0.053 0.58 0.16 0.21 0) *
    7) GDPperCapita>=6904.5 9 4 Asia (0 0.22 0.56 0.22 0) *
```

In [150]: regionTree=rpart(region ~ infantMortality + GDPperCapita, data=UN98NoNA)



```
In [151]: summary(regionTree)
         rpart(formula = region ~ infantMortality + GDPperCapita, data = UN98NoNA)
           n= 39
                  CP nsplit rel error
                                       xerror
         1 0.3076923
                         0 1.0000000 1.0000000 0.1132277
         2 0.1153846
                         1 0.6923077 0.7307692 0.1200566
         3 0.0100000
                         2 0.5769231 0.7307692 0.1200566
         Variable importance
         infantMortality
                         GDPperCapita
                      60
         Node number 1: 39 observations,
                                          complexity param=0.3076923
           predicted class=America expected loss=0.6666667 P(node) =1
             class counts: 9 13 10 6 1
            probabilities: 0.231 0.333 0.256 0.154 0.026
           left son=2 (11 obs) right son=3 (28 obs)
           Primary splits:
               infantMortality < 46.5 to the right, improve=5.992507, (0 missing)
               GDPperCapita < 1550.5 to the left, improve=3.000000, (0 missing)
           Surrogate splits:
               GDPperCapita < 1448.5 to the left, agree=0.846, adj=0.455, (0 split)
         Node number 2: 11 observations
           predicted class=Africa expected loss=0.2727273 P(node) =0.2820513
                                 0
                                        2
             class counts: 8
            probabilities: 0.727 0.000 0.182 0.000 0.091
         Node number 3: 28 observations,
                                         complexity param=0.1153846
           predicted class=America expected loss=0.5357143 P(node) =0.7179487
             class counts: 1 13 8 6 0
            probabilities: 0.036 0.464 0.286 0.214 0.000
           left son=6 (19 obs) right son=7 (9 obs)
           Primary splits:
               GDPperCapita
                             < 6904.5 to the left, improve=1.760652, (0 missing)
               infantMortality < 27      to the right, improve=1.281119, (0 missing)</pre>
           Surrogate splits:
               infantMortality < 8.5 to the right, agree=0.821, adj=0.444, (0 split)
         Node number 6: 19 observations
           predicted class=America expected loss=0.4210526 P(node) =0.4871795
             class counts: 1 11
                                       3 4
            probabilities: 0.053 0.579 0.158 0.211 0.000
         Node number 7: 9 observations
           predicted class=Asia expected loss=0.4444444 P(node) =0.2307692
             class counts: 0 2 5 2
            probabilities: 0.000 0.222 0.556 0.222 0.000
 In [ ]:
```

1. Can you use the 'predict' function with your decision tree model on a new data point where 'infantMortality = 10' and 'GDPperCapita = 20000' and report on the output?

Yes, I can predict region under the condition of 'infantMortality = 10' and 'GDPperCapita = 20000', the region 56% chance is in Asia and equal 22% chance are in either America or Europe.

Question 3 (8 points) There is another dataset 'books.csv' from Kaggle on the data collected from GoodReads.com which registers activity on a website where people read and share opinions, rating etc.

- 1. Produce a box plot for the 'average rating', 'num pages', 'ratings count', and 'text reviews count'.
- 2. Produce the same type of plot as before but make it a 'violin box plot' as shown in class.
- 3. Is the average rating correlated with the number of pages?
- 4. Make a model to predict the average rating from the number of pages and the ratings count (dependent is the 'avarage rating' and the independents 'num pages' and 'ratings count'). Is the model significant?
- 1. Produce a box plot for the 'average rating', 'num pages', 'ratings count', and 'text reviews count'.

```
In [154]: b<- read.csv('C:/Users/Andrew/Desktop/books.csv')</pre>
In [155]: install.packages("microbenchmark")
           install.packages("magrittr")
           install.packages("tidyr")
           install.packages("dplyr")
           library(microbenchmark)
           library(tidyr)
           library(dplyr)
           library(magrittr)
          Warning message:
           "package 'microbenchmark' is in use and will not be installed"Warning message:
           "package 'magrittr' is in use and will not be installed"also installing the dependency 'dplyr'
          Warning message:
           "package 'dplyr' is in use and will not be installed"
          package 'tidyr' successfully unpacked and MD5 sums checked
          The downloaded binary packages are in
                   {\tt C:\Users\Andrew\AppData\Local\Temp\RtmpI5MZBx\downloaded\_packages}
          Warning message:
           "package 'dplyr' is in use and will not be installed "Warning message:
           "package 'tidyr' was built under R version 3.6.3"
          Error: package or namespace load failed for 'tidyr' in loadNamespace(j <- i[[1L]], c(lib.loc, .libPaths()), version
          Check = vI[[j]]):
           namespace 'dplyr' 0.8.0.1 is already loaded, but >= 0.8.2 is required
          Traceback:

    library(tidyr)

          2. tryCatch({
                 attr(package, "LibPath") <- which.lib.loc</pre>
                 ns <- loadNamespace(package, lib.loc)</pre>
                 env <- attachNamespace(ns, pos = pos, deps, exclude, include.only)</pre>
            . }, error = function(e) {
                 P <- if (!is.null(cc <- conditionCall(e)))
                     paste(" in", deparse(cc)[1L])
                 msg <- gettextf("package or namespace load failed for %s%s:\n %s",</pre>
                      sQuote(package), P, conditionMessage(e))
                 if (logical.return)
                     message(paste("Error:", msg), domain = NA)
                 else stop(msg, call. = FALSE, domain = NA)
           . })
          3. tryCatchList(expr, classes, parentenv, handlers)
          4. tryCatchOne(expr, names, parentenv, handlers[[1L]])
          5. value[[3L]](cond)
          6. stop(msg, call. = FALSE, domain = NA)
```

```
In [156]: #take off NA
    bNoNA=na.omit(b)
    #change factor variabes to numeric
    average_rating_int=as.integer(bNoNA$average_rating)
    X..num_pages_int = as.integer(bNoNA$X..num_pages)

is.factor(average_rating_int)
    class(average_rating_int)

is.factor(X..num_pages_int)

class(X..num_pages_int)
```

FALSE

'integer'

FALSE

'integer'

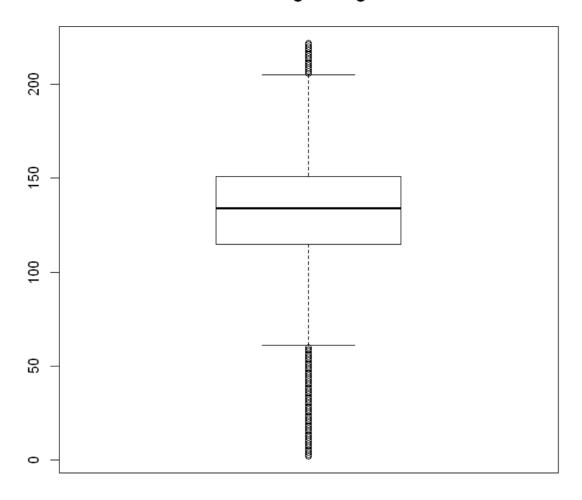
In [157]: summary(bNoNA) str(bNoNA) head(bNoNA)

```
bookID
                                         title
                                                                authors
Min. : 1
                'Salem's Lot
                                           : 11 Agatha Christie : 69
 1st Qu.:10621
                One Hundred Years of Solitude: 11
                                                     Stephen King :
 Median :21322
               The Brothers Karamazov
                                         : 10
                                                     Orson Scott Card: 48
                                            : 10 Rumiko Takahashi: 46
: 9 P.G. Wodehouse : 42
: 9 Terry Brooks : 40
 Mean :22161
                The Iliad
                A Midsummer Night's Dream
 3rd Ou.:33322
 Max. :47709
                                                    Terry Brooks : 40
                A Tale of Two Cities
                                                     (Other)
                (Other)
                                            :13659
                                                                   :13408
 average_rating
                       isbn
                                            isbn13
                                                        language_code
4.00 : 261
                        : 1 0008987059752: 1
                                                        eng :10594
                0.00
                                                        en-US : 1699
 3.96 : 231
                000100039X: 1 0020049130001: 1
      : 222
                0001713191: 1 0023755004321: 1 spa
0002005883: 1 0034406054602: 1 en-GB
 3.95
                                                              : 419
 3.89
       : 219
                                                       en-GB : 341
                0002259834: 1 0049086007763: 1 ger : 238
 3.98 : 219
 3.99 : 217
                0002261987: 1 0073999140774: 1 fre : 209
 (Other):12350
                (Other) :13713 (Other) :13713
                                                       (Other): 219
 X..num_pages
                ratings_count
                                 text_reviews_count
                Min. : 0 Min. : 0.0
 288 : 252
                1st Qu.:
 320 : 248
                           83 1st Qu.:
                                            7.0
 192
       : 244
                Median :
                           630
                                 Median :
                                           40.0
                                 Mean : 533.6
 256
       : 236
                Mean : 17759
                                 3rd Qu.: 222.0
                3rd Qu.: 4737
      : 230
224
      : 227 Max. :5629932 Max. :93619.0
 (Other):12282
'data.frame': 13719 obs. of 10 variables:
$ bookID
                : int 1 2 3 4 5 8 9 10 12 13 ...
 $ title
                    : Factor w/ 12428 levels "","'Salem's Lot",...: 3808 3809 3812 3806 3811 3813 11750 3815 11082
11081 ...
                   : Factor w/ 7606 levels "","A.B. Yehoshua-Hillel Halkin",..: 3044 3044 3044 3036 3044 3044 724
$ authors
8 3036 1696 1696 ...
$ average_rating : Factor w/ 222 levels ""," Jr.-C.S. Lewis-P.G. Wodehouse-Michael Moorcock-L. Sprague de Camp-
Fletcher Pratt-Eric Knight-Mervyn Peake-Pier"| __truncated__,..: 194 187 185 179 193 215 107 211 176 176 ... $ isbn : Factor w/ 13720 levels "","0.00","000100039X",..: 4973 4890 4932 4931 4953 4958 10416 4980 6
001 3356 ...
                   : Factor w/ 13720 levels "","0008987059752",..: 4996 4913 4955 4954 4976 4981 10428 5003 6024
$ isbn13
3379 ...
                   : Factor w/ 36 levels "","9780674842113",..: 14 14 14 14 14 14 13 14 14 14 ...
$ language_code
                   : Factor w/ 1093 levels "","0","1","10",...: 794 1007 426 463 555 361 204 442 956 956 ...
$ X..num_pages
                 : int 1944099 1996446 5629932 6267 2149872 38872 18 27410 3602 240189 ...
 $ ratings count
 $ text reviews count: int 26249 27613 70390 272 33964 154 1 820 258 3954 ...
 - attr(*, "na.action")= 'omit' Named int 4012 5689 7058 10604 10672
 ..- attr(*, "names")= chr "4012" "5689" "7058" "10604" ...
```

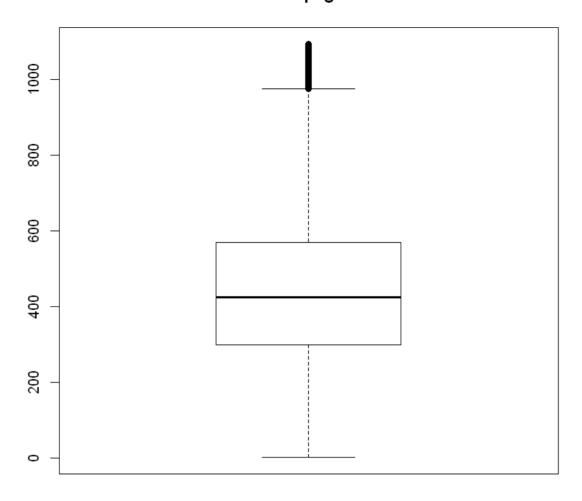
bookID	title	authors	average_rating	isbn	isbn13	language_code	Xnum_pages	ratings_count	text_reviews_count
1	Harry Potter and the Half- Blood Prince (Harry Potter #6)	J.K. Rowling- Mary GrandPré	4.56	0439785960	9780439785969	eng	652	1944099	26249
2	Harry Potter and the Order of the Phoenix (Harry Potter #5)	J.K. Rowling- Mary GrandPré	4.49	0439358078	9780439358071	eng	870	1996446	27613
3	Harry Potter and the Sorcerer's Stone (Harry Potter #1)	J.K. Rowling- Mary GrandPré	4.47	0439554934	9780439554930	eng	320	5629932	70390
4	Harry Potter and the Chamber of Secrets (Harry Potter #2)	J.K. Rowling	4.41	0439554896	9780439554893	eng	352	6267	272
5	Harry Potter and the Prisoner of Azkaban (Harry Potter #3)	J.K. Rowling- Mary GrandPré	4.55	043965548X	9780439655484	eng	435	2149872	33964
8	Harry Potter Boxed Set Books 1-5 (Harry Potter #1- 5)	J.K. Rowling- Mary GrandPré	4.78	0439682584	9780439682589	eng	2690	38872	154

```
In [158]: boxplot(average_rating_int, main="average rating")
boxplot(X..num_pages_int, main="num pages")
boxplot(b$ratings_count, main=" ratings count")
boxplot(b$text_reviews_count, main="text reviews count")
```

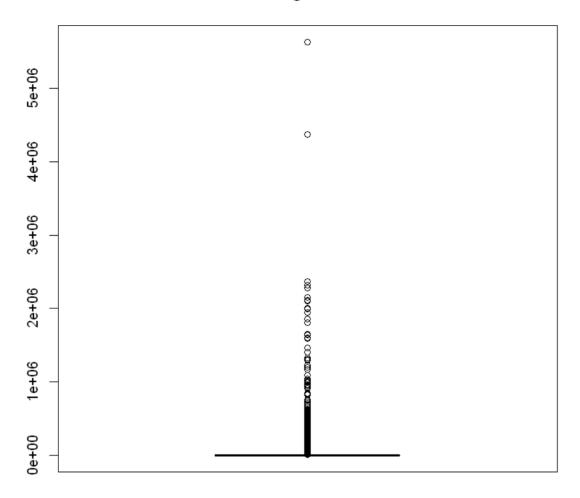
average rating



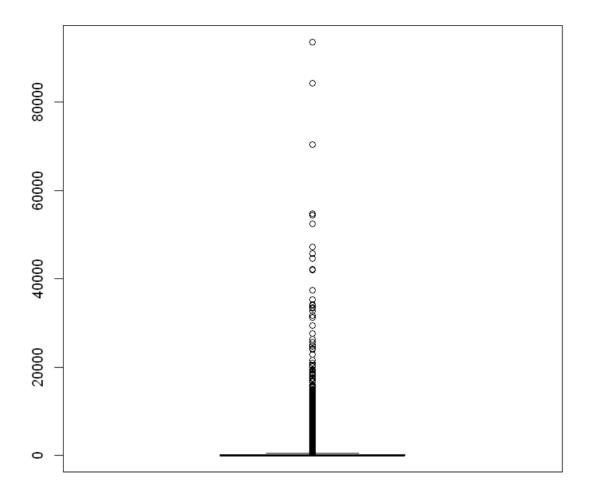
num pages



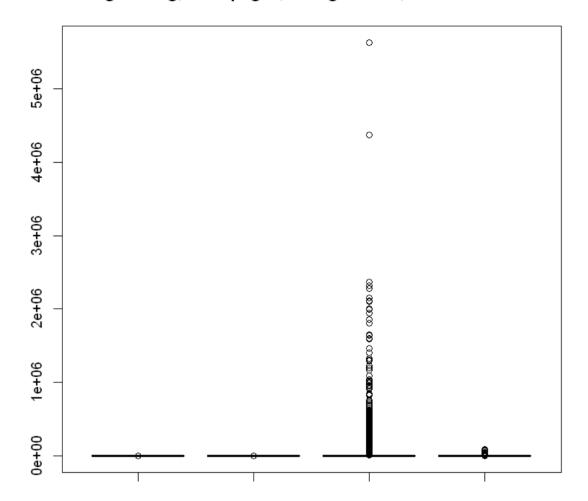
ratings count



text reviews count



average rating, num pages, ratings count, text reviews count





1. Produce the same type of plot as before but make it a 'violin box plot' as shown in class.

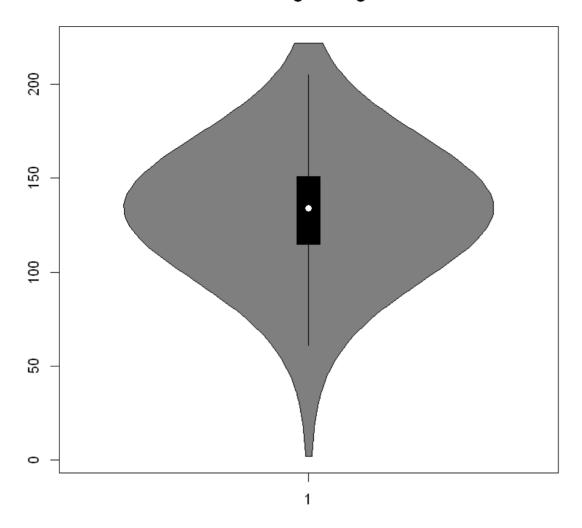
```
In [160]: install.packages("sm")
            install.packages("zoo")
            install.packages("vioplot")
install.packages("GGally")
            install.packages("htmlwidgets")
            install.packages("rpivotTable")
            install.packages("gplots")
install.packages("graphics")
            install.packages("corrplot")
            library(sm)
            library(vioplot)
            library(zoo)
            library(GGally)
            library(htmlwidgets)
            library(rpivotTable)
            library(gplots)
            library(graphics)
            library(corrplot)
```

Warning message:

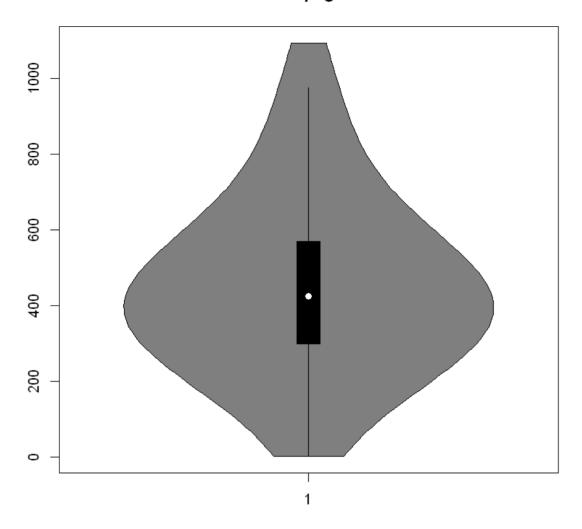
```
"package 'sm' is in use and will not be installed"Warning message:
"package 'zoo' is in use and will not be installed"Warning message:
"package 'vioplot' is in use and will not be installed"Warning message:
"package 'GGally' is in use and will not be installed"Warning message:
"package 'htmlwidgets' is in use and will not be installed"Warning message:
"package 'rpivotTable' is in use and will not be installed"Warning message:
"package 'gplots' is in use and will not be installed"Warning message:
"package 'graphics' is not available (for R version 3.6.1)"Warning message:
"package 'graphics' is a base package, and should not be updated"Warning message:
"package 'corrplot' is in use and will not be installed"
```

```
In [161]: vioplot(average_rating_int, main="average rating")
    vioplot(X..num_pages_int, main="num pages")
    vioplot(b$ratings_count, main=" ratings count")
    vioplot(b$text_reviews_count, main="text reviews count")
```

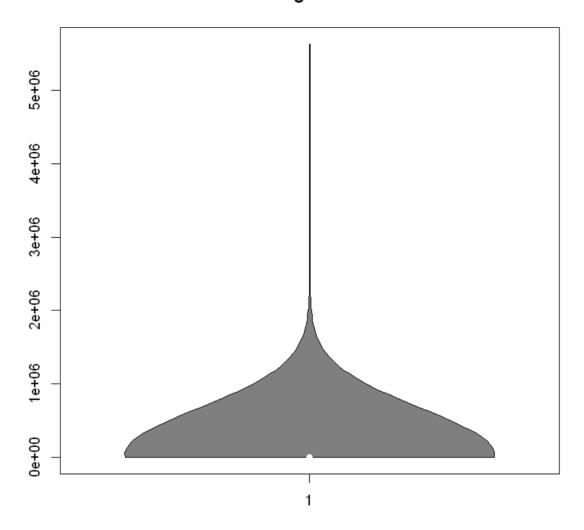
average rating



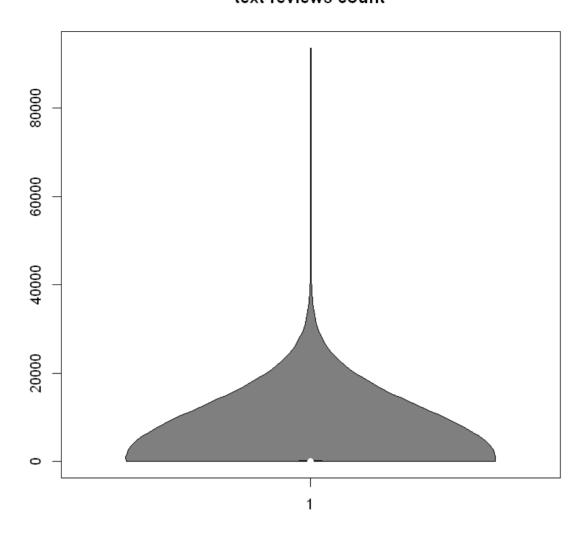
num pages



ratings count



text reviews count



```
In [ ]:
```

1. Is the average rating correlated with the number of pages?

From the chi square test, the p value is very small, states variable average rating correlated with the number of pages.

From the Pearson's test, the p value is very small, states the numerica variable average rating correlated with the numerica variable number of pages.

```
In [ ]:
```

1. Make a model to predict the average rating from the number of pages and the ratings count (dependent is the 'avarage rating' and the independents 'num pages' and 'ratings count'). Is the model significant?

```
In [164]: install.packages("dplyr")
          library(dplyr)
          Warning message:
          "package 'dplyr' is in use and will not be installed"
In [165]: #build multi regression model using converted numberic variables
          modelglm<-glm(average_rating_int ~ X..num_pages_int + ratings_count, data=bNoNA)</pre>
          summary(modelglm)
          Call:
          glm(formula = average_rating_int ~ X..num_pages_int + ratings_count,
              data = bNoNA)
          Deviance Residuals:
              Min 10
                               Median
                                            30
                                                     Max
          -136.584 -16.671 1.684 19.465
                                                   94.340
          Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
                         1.266e+02 5.714e-01 221.570 < 2e-16 ***
          (Intercept)
          X..num_pages_int 1.096e-02 1.126e-03 9.732 < 2e-16 ***
          ratings_count 1.172e-05 2.247e-06 5.214 1.87e-07 ***
          Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
          (Dispersion parameter for gaussian family taken to be 883.5907)
             Null deviance: 12228453 on 13718 degrees of freedom
          Residual deviance: 12119330 on 13716 degrees of freedom
          AIC: 132007
          Number of Fisher Scoring iterations: 2
```

From the summary of the model, we can see the p value are very small, the model is statiscal significant.

```
In [166]: #make categories from average rating
bNoNA$average_ratingGroup=ntile(bNoNA$average_rating, 4)
head(bNoNA$average_ratingGroup, 10)
#make categories from number of page
bNoNA$X..num_pagesGroup=ntile(bNoNA$X..num_pages, 4)
head(bNoNA$X..num_pagesGroup, 10)
4 4 4 4 4 4 1 4 4 4
```

4 4 3 3 3 2 1 3 4 4

```
In [167]: install.packages("nnet")
          library(nnet)
          Warning message:
          "package 'nnet' is in use and will not be installed"
In [168]: modelmultiA<-multinom(average ratingGroup ~ X..num pagesGroup + ratings count, data=bNoNA)
          summary(modelmultiA)
          # weights: 16 (9 variable)
          initial value 19018.572340
          iter 10 value 18940.197564
          final value 18940.025065
          converged
          Call:
          multinom(formula = average_ratingGroup ~ X..num_pagesGroup +
              ratings_count, data = bNoNA)
          Coefficients:
           (Intercept) X..num_pagesGroup ratings_count
                              0.1688427 1.303554e-06
          2 -0.4261228
          3 -0.5240801
                               0.2044145 1.689709e-06
          4 -0.5270193
                               0.2048218 1.762504e-06
          Std. Errors:
             (Intercept) X..num_pagesGroup ratings_count
                            4.019918e-12 4.061006e-07
          2 1.606716e-12
          3 1.547282e-12
                              3.892790e-12 3.912453e-07
                             3.897852e-12 3.899449e-07
          4 1.551771e-12
          Residual Deviance: 37880.05
          AIC: 37898.05
In [169]: | modelmultiB<-multinom(average_ratingGroup ~ ratings_count, data=bNoNA)</pre>
          summary(modelmultiB)
          # weights: 12 (6 variable)
          initial value 19018.572340
          iter 10 value 19000.738745
          iter 10 value 19000.738744
          iter 10 value 19000.738744
          final value 19000.738744
          converged
          Call:
          multinom(formula = average_ratingGroup ~ ratings_count, data = bNoNA)
          Coefficients:
           (Intercept) ratings_count
          2 -0.01825108 1.416013e-06
          3 -0.02552116 1.808376e-06
          4 -0.02740774 1.881022e-06
          Std. Errors:
             (Intercept) ratings_count
          2 1.729308e-12 4.152665e-07
          3 1.678784e-12 4.008923e-07
```

4 1.684662e-12 3.996655e-07 Residual Deviance: 38001.48

AIC: 38013.48

```
In [170]: modelmultiC<-multinom(average_ratingGroup ~ X..num_pagesGroup, data=bNoNA)</pre>
          summary(modelmultiC)
          # weights: 12 (6 variable)
          initial value 19018.572340
          iter 10 value 18956.154320
          iter 10 value 18956.154297
          iter 10 value 18956.154297
          final value 18956.154297
          converged
          Call:
          multinom(formula = average_ratingGroup ~ X..num_pagesGroup, data = bNoNA)
          Coefficients:
            (Intercept) X..num_pagesGroup
          2 -0.4138021
                               0.1707184
          3 -0.5065104
                                0.2070364
          4 -0.5083656
                                0.2076432
          Std. Errors:
            (Intercept) X..num_pagesGroup
          2 0.05804869
                               0.02176307
          3 0.05858073
                               0.02179188
          4 0.05859458
                               0.02179406
          Residual Deviance: 37912.31
          AIC: 37924.31
In [171]: #tried A B C 3 models with different vaiables, AIC of model A is the smallest
          # i am going to use model A to predict
          #try a prediction use random data frame
          new<- data.frame(X..num_pagesGroup=c(4), ratings_count=c(2500))</pre>
          print(new)
          str(new)
          predict(modelmultiA, new)
            X..num_pagesGroup ratings_count
                                       2500
          1
                            4
          'data.frame': 1 obs. of 2 variables:
           $ X..num_pagesGroup: num 4
           $ ratings count : num 2500
          3
          ► Levels:
```

I created two groups factor variables to predict average rating group (1-4). For an example, scenario num page group 4 and ratings count 2500, the model prediction of average rating group is 3.

Question 4 (6 points) There are 2 datasets; 'hr.7257.csv' and 'hr.11839.csv'. These datasets are recordings of heart rates (heart beat data) from 2 subjects at the same period on the same conditions.

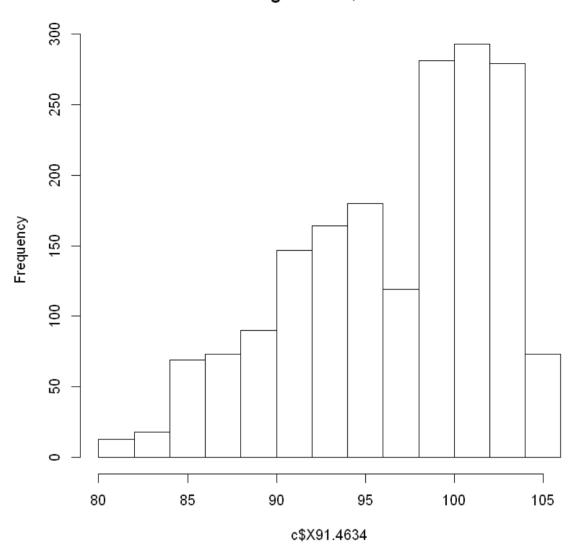
- 1. Produce a single 2 column dataset from the 2 datasets.
- 2. Produce 2 histograms from the values in each column.
- 3. Produce a single plot with the 2 histograms overlayed with a vertical line for each mean value of the distributions.
- 4. Use the T-test to see if there is significant difference between the distributions.
- 1. Produce a single 2 column dataset from the 2 datasets.

```
In [173]: h7257<- read.csv('C:/Users/Andrew/Desktop/hr.7257.csv')</pre>
          str(h7257)
          h11839<- read.csv('C:/Users/Andrew/Desktop/hr.11839.csv')
          str(h11839)
          'data.frame': 1799 obs. of 1 variable:
           $ X91.4634: num 91.5 91.2 91.9 91.2 89.8 ...
          'data.frame': 1799 obs. of 1 variable:
           $ X84.2697: num 84.3 84.1 85.7 87.2 87.1 ...
In [174]: summary(h7257)
          summary(h11839)
              X91.4634
           Min. : 80.21
           1st Qu.: 92.42
           Median : 98.25
           Mean : 96.64
           3rd Qu.:101.36
           Max. :104.89
              X84.2697
           Min. : 73.44
           1st Qu.: 88.86
           Median : 92.21
           Mean : 92.60
           3rd Qu.: 96.40
           Max. :106.76
In [175]: #colomun combine to one dataset called c
          c<-cbind(h7257,h11839)</pre>
          head(c)
          str(c)
           X91.4634 X84.2697
                    84.2697
            91.4634
            91.1834
                    84.0619
                    85.6542
            91.8788
            91.1772
                    87.2093
            89.7992
                    87.1246
            90.3571 86.8726
          'data.frame': 1799 obs. of 2 variables:
           $ X91.4634: num 91.5 91.2 91.9 91.2 89.8 ...
           $ X84.2697: num 84.3 84.1 85.7 87.2 87.1 ...
 In [ ]:
```

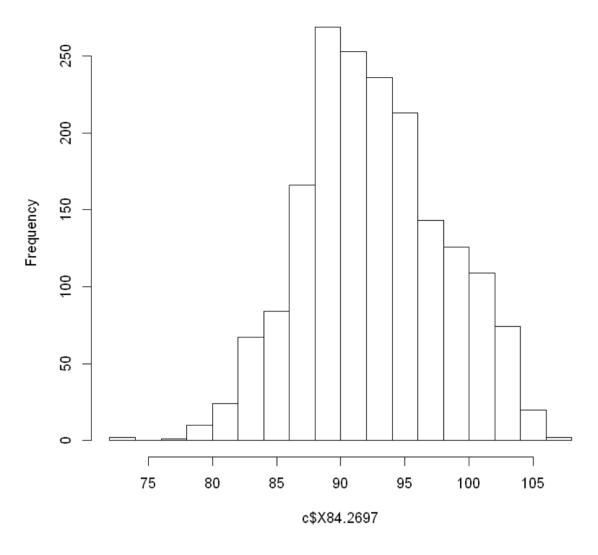
1. Produce 2 histograms from the values in each column.

In [176]: h1<-hist(c\$X91.4634)
h2<-hist(c\$X84.2697)</pre>

Histogram of c\$X91.4634



Histogram of c\$X84.2697



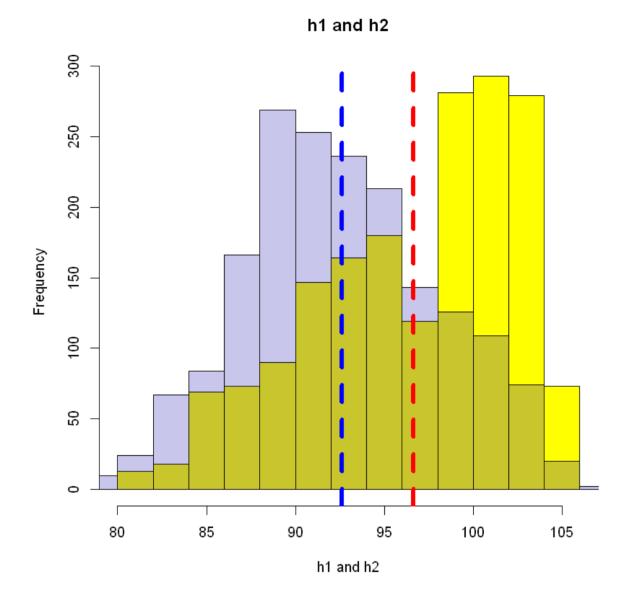
In []:

1. Produce a single plot with the 2 histograms overlayed with a vertical line for each mean value of the distributions.

In [177]: install.packages("plyr")
library(plyr)

Warning message:
"package 'plyr' is in use and will not be installed"

```
In [178]: plot(h1, col="yellow", main="h1 and h2", xlab="h1 and h2")
    plot(h2, col=rgb(0.15, .1, .7, 1/4), xlim=c(-5, 15), add=TRUE )
    abline(v=mean(c$X91.4634), col="red", lwd=5, lty=2)
    abline(v=mean(c$X84.2697), col="blue", lwd=5, lty=2)
```



```
In [ ]:
```

1. Use the T-test to see if there is significant difference between the distributions.

```
In [179]: t.test(c$X91.4634, c$X84.2697)

Welch Two Sample t-test

data: c$X91.4634 and c$X84.2697
    t = 21.68, df = 3591.4, p-value < 2.2e-16
    alternative hypothesis: true difference in means is not equal to 0
    95 percent confidence interval:
        3.674287 4.404927
    sample estimates:
    mean of x mean of y
    96.64035 92.60074</pre>
```

In []:	:	
In []:	:	