Dataflow Analysis

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Course IN4303, 2012/13
Compiler Construction

Overview

today's lecture

control flow graphs

Overview

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control flow graphs

data flow analyses

- liveness analysis
- reaching definitions
- available expressions

Overview

today's lecture

control flow graphs

data flow analyses

- liveness analysis
- reaching definitions
- available expressions

non-local optimisations

- dead code elimination
- constant & copy propagation
- common subexpression elimination

I

control-flow graphs

Intermediate language

quadruples

store

$a \leftarrow b \oplus c$

$a \leftarrow b$

memory access

$a \leftarrow M[b]$

$M[a] \leftarrow b$

functions

$f(a_1, \ldots, a_n)$

$b \leftarrow f(a_1, \ldots, a_n)$

jumps

L:

goto L

if $a \otimes b$

  goto L_1

else

  goto L_2

Control-flow graphs
example

```
    a ← 0
L1: b ← a + 1
    c ← c + b
    a ← 2 * b
    if a < N
       goto L1
    else
       goto L2
L2: return c
```

Control-flow graphs

example

```
    a ← 0
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    c ← c + b
    a ← 2 * b
    if a < N
       goto L1
    else
       goto L2
L2: return c
```
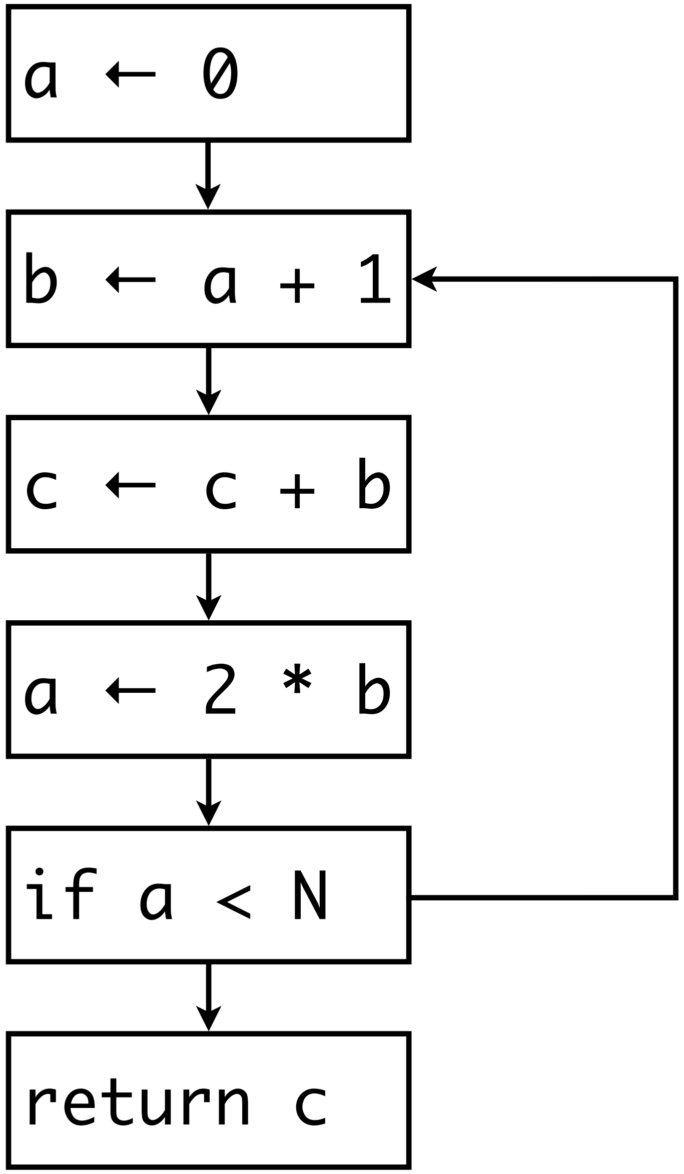
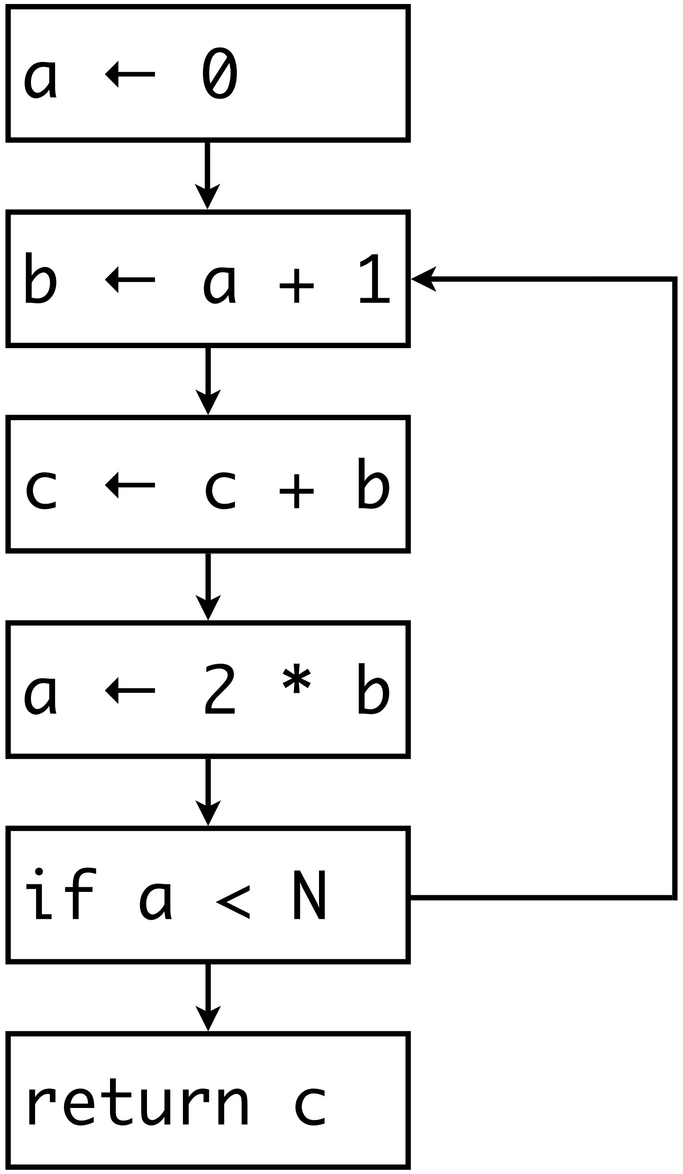



Control-flow graphs

terminology



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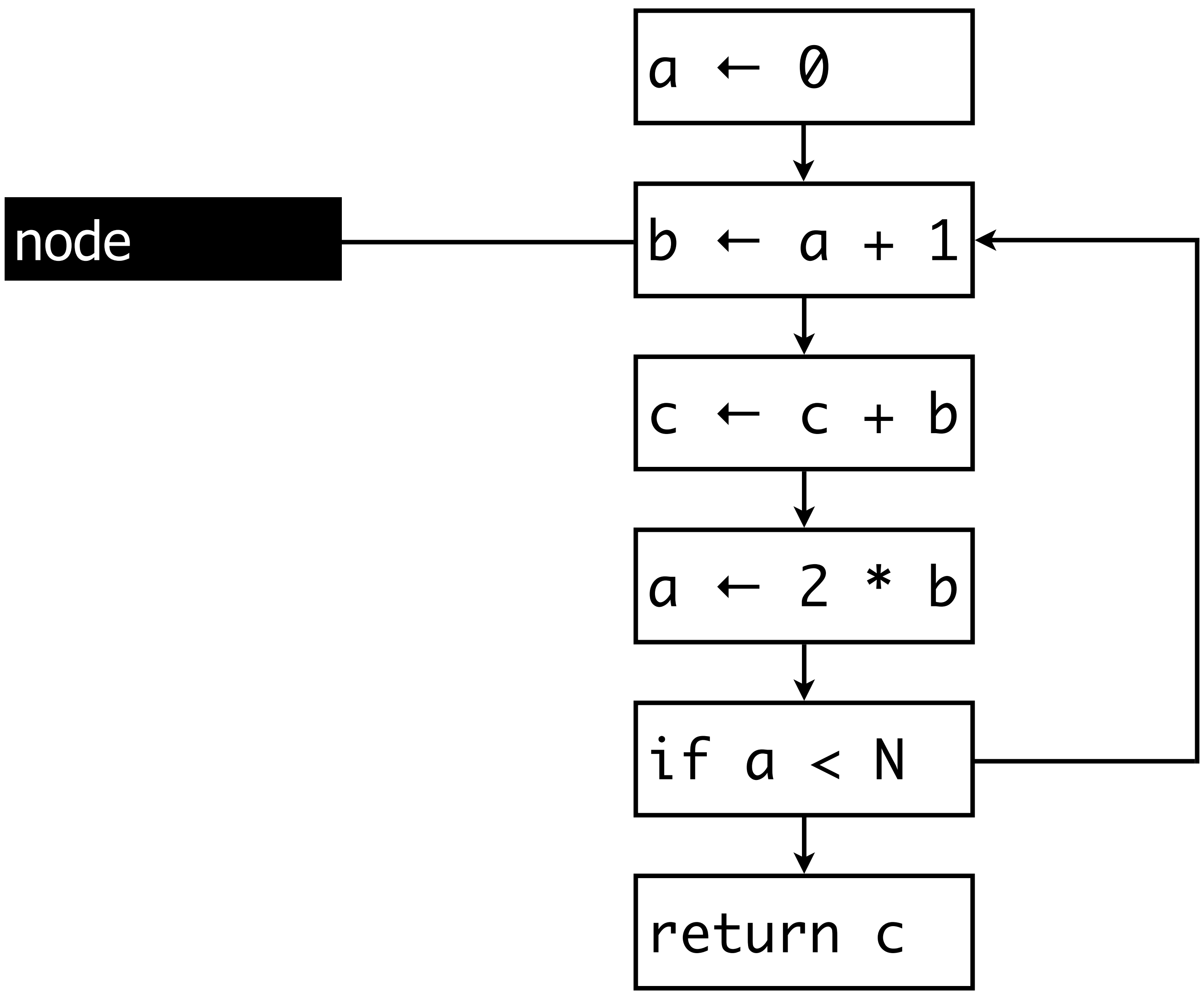

Control-flow graphs
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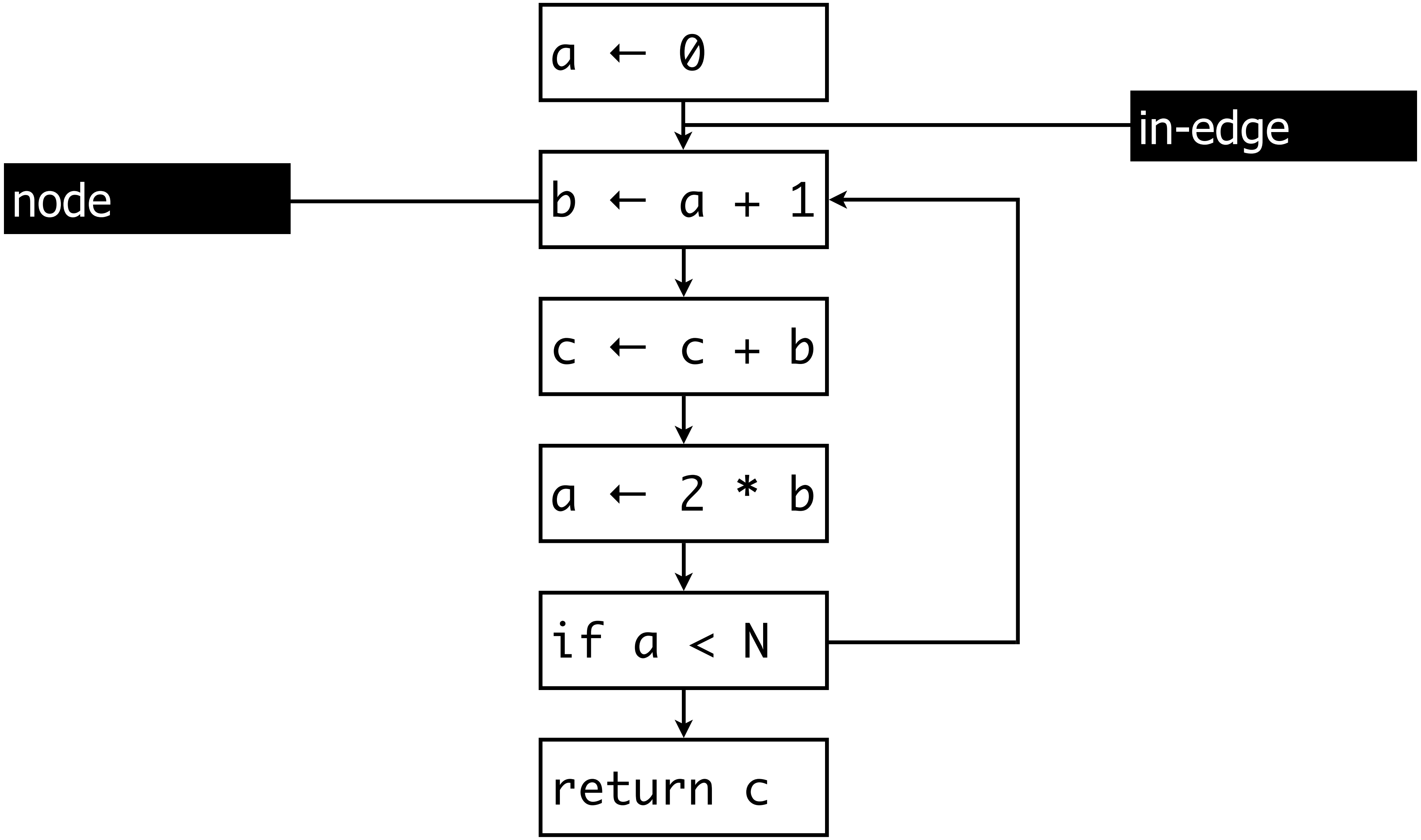

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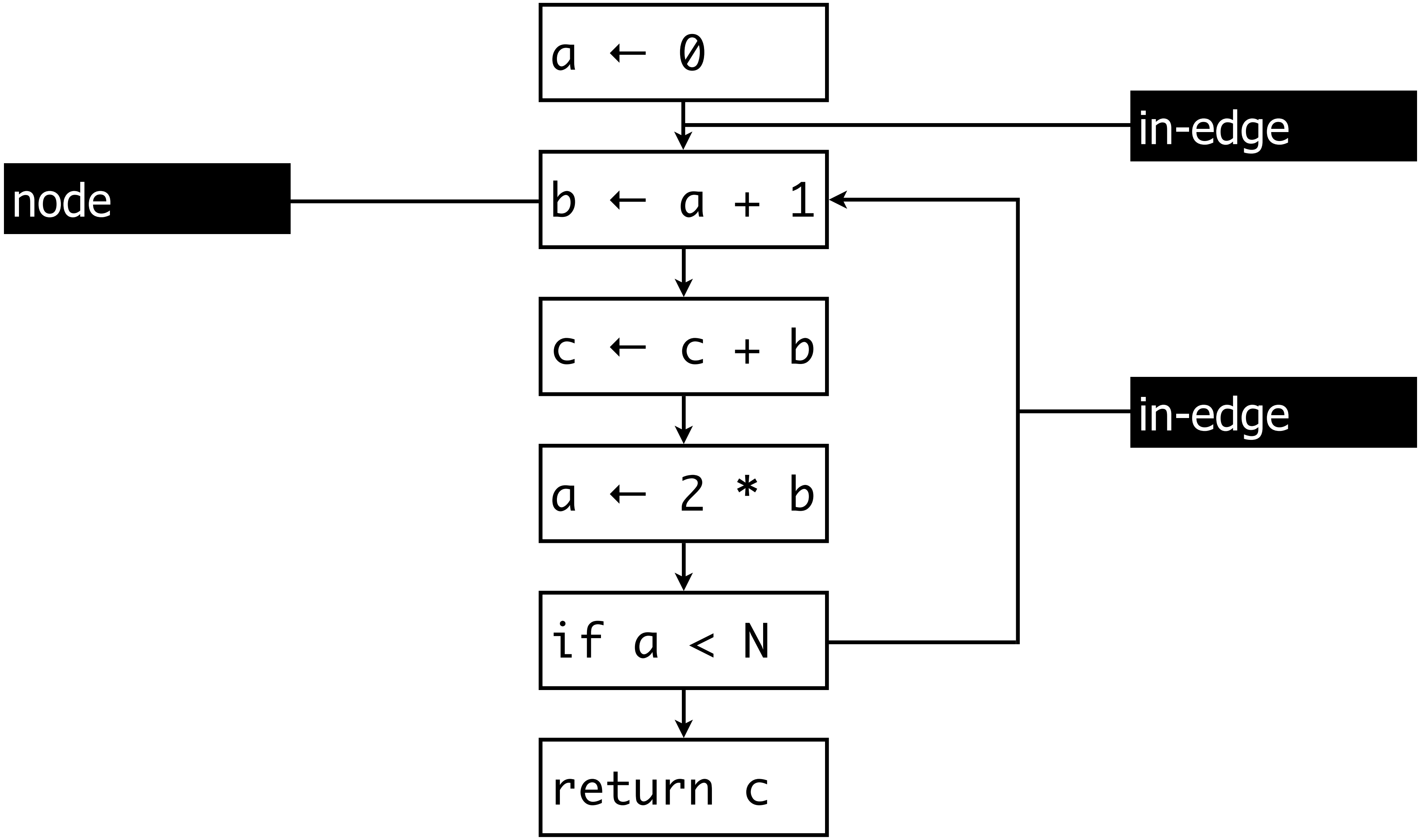

Control-flow graphs
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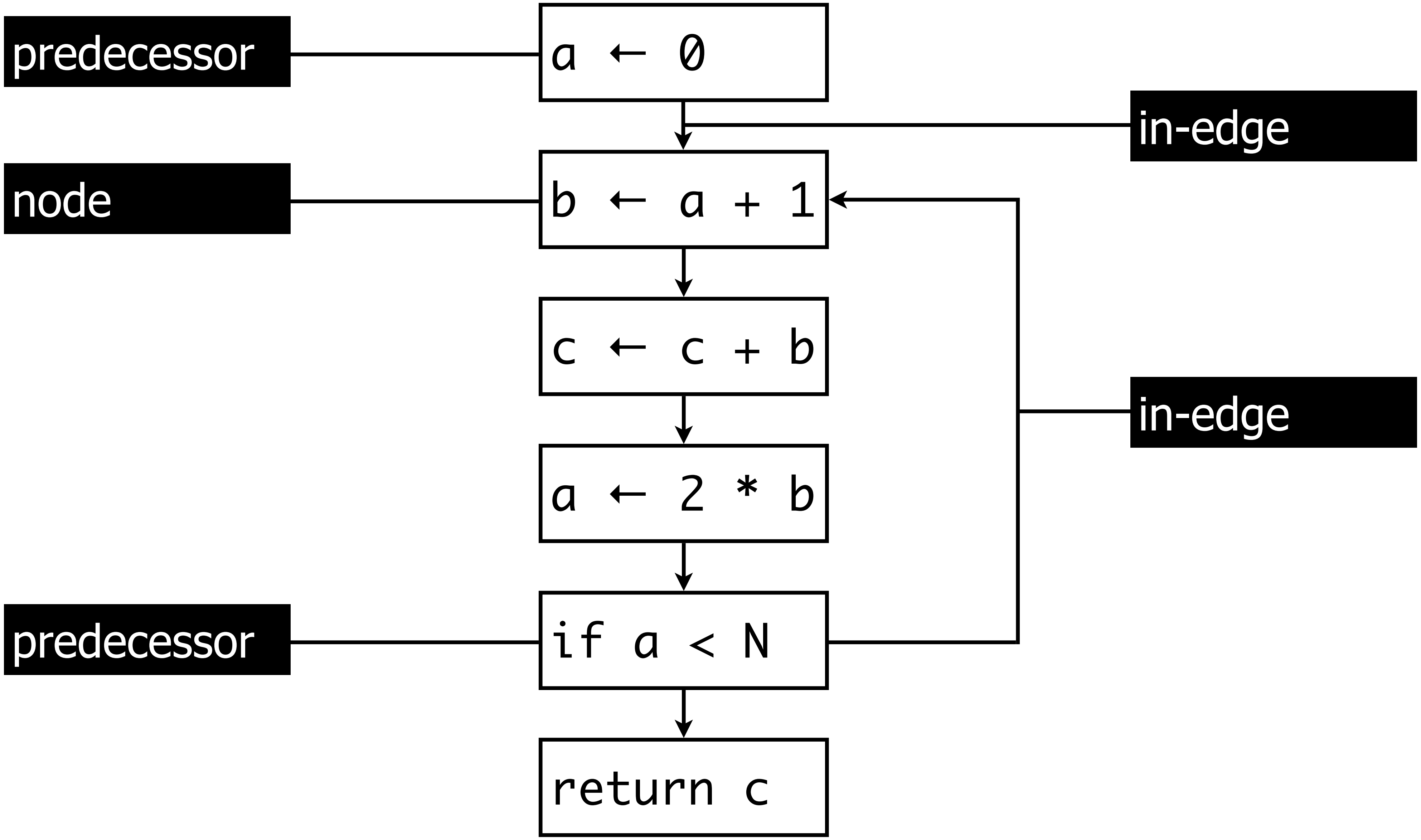
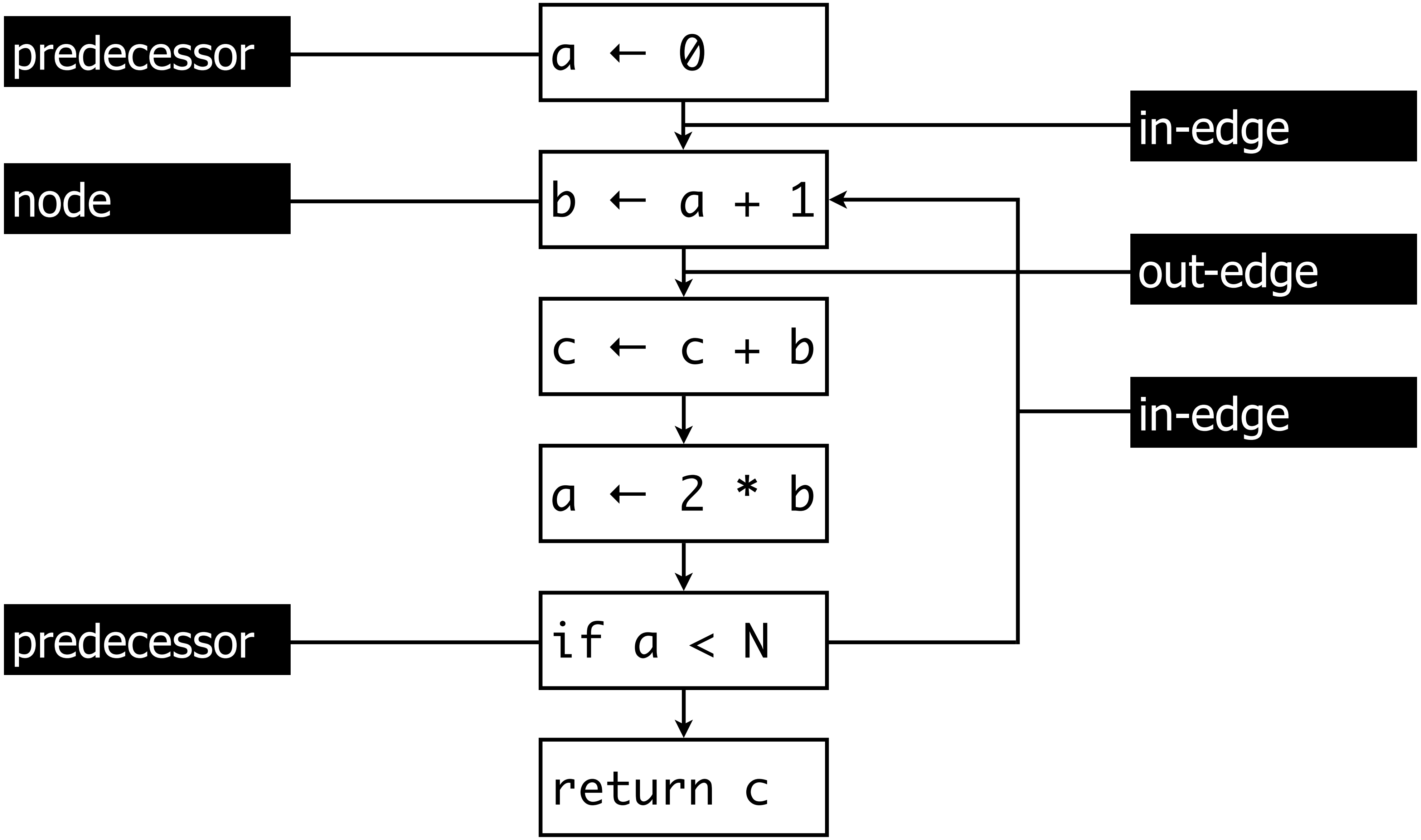


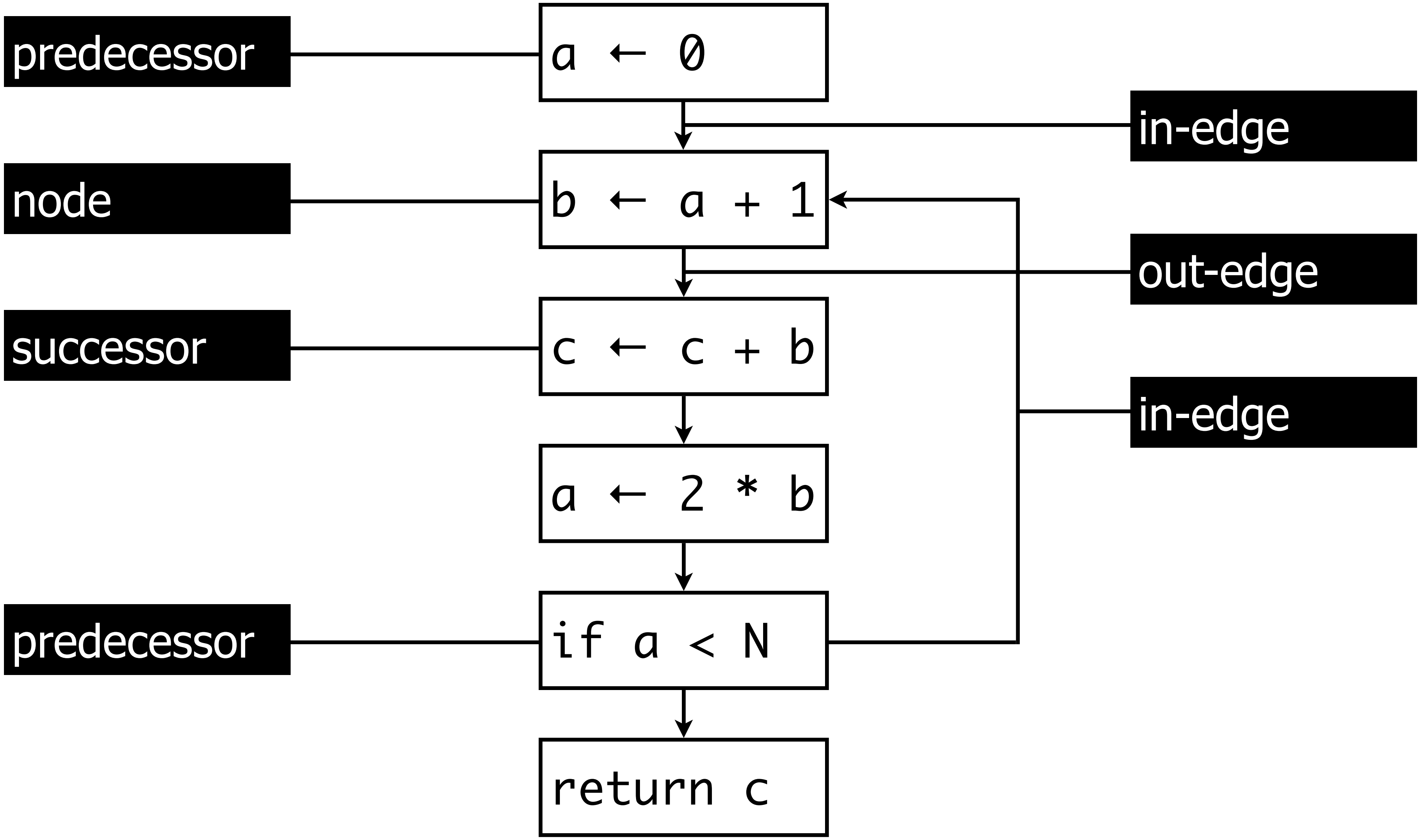
Control-flow graphs

terminology



Control-flow graphs

terminology

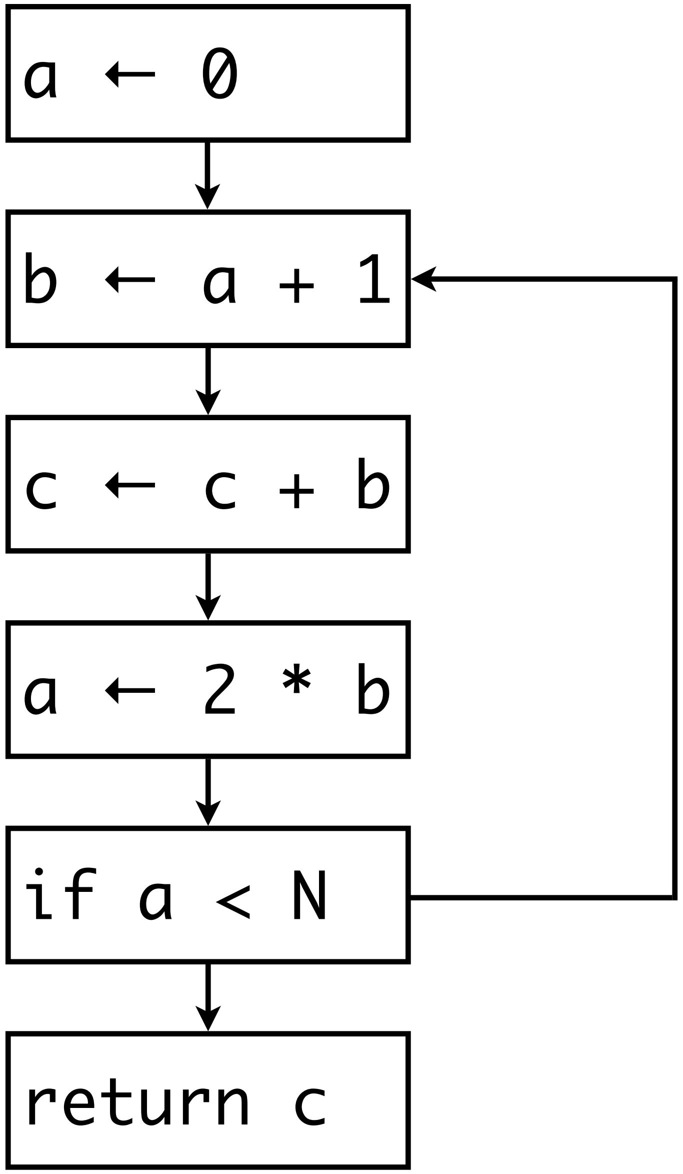


II

liveness analysis

Liveness analysis

terminology



Liveness analysis

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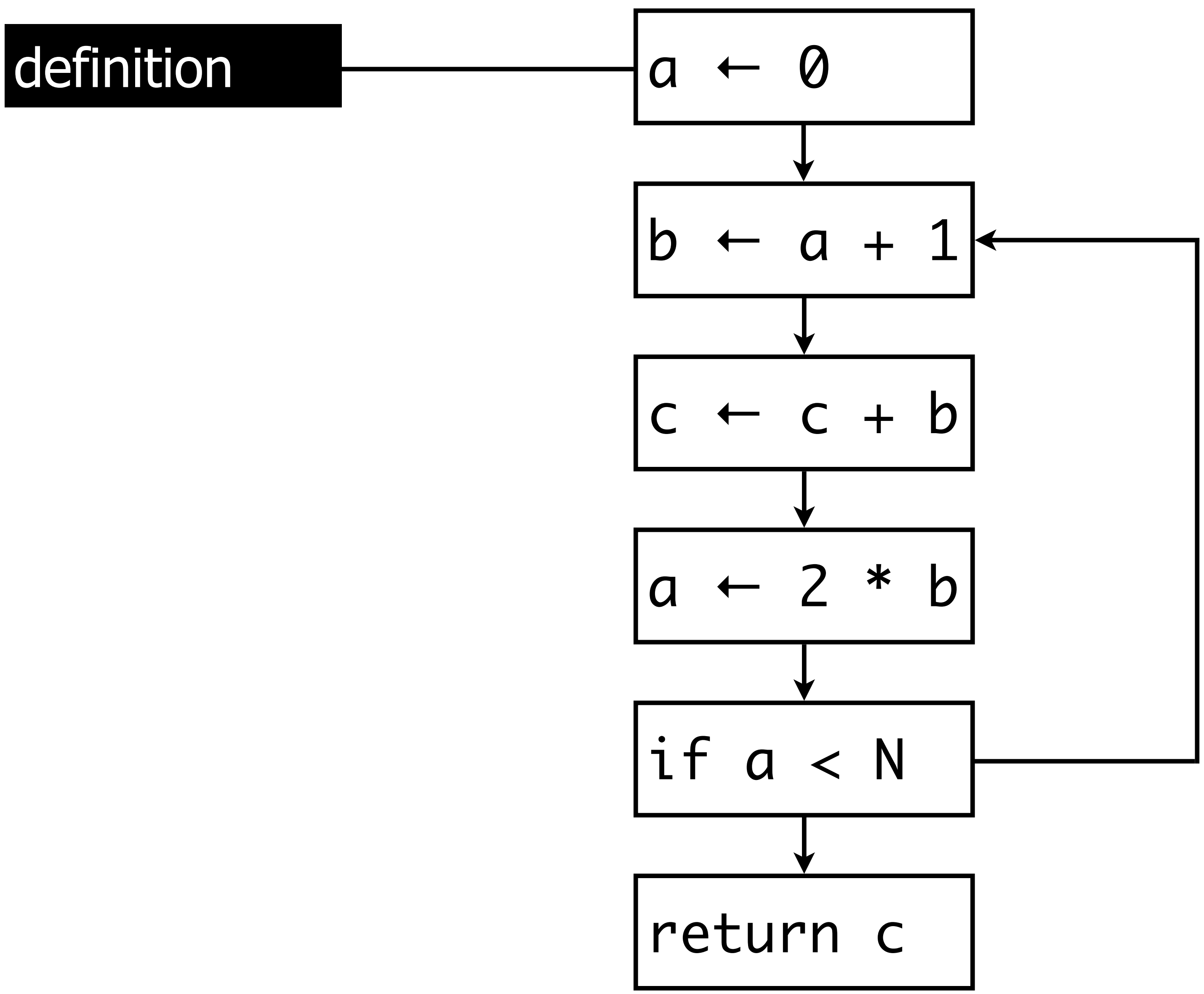

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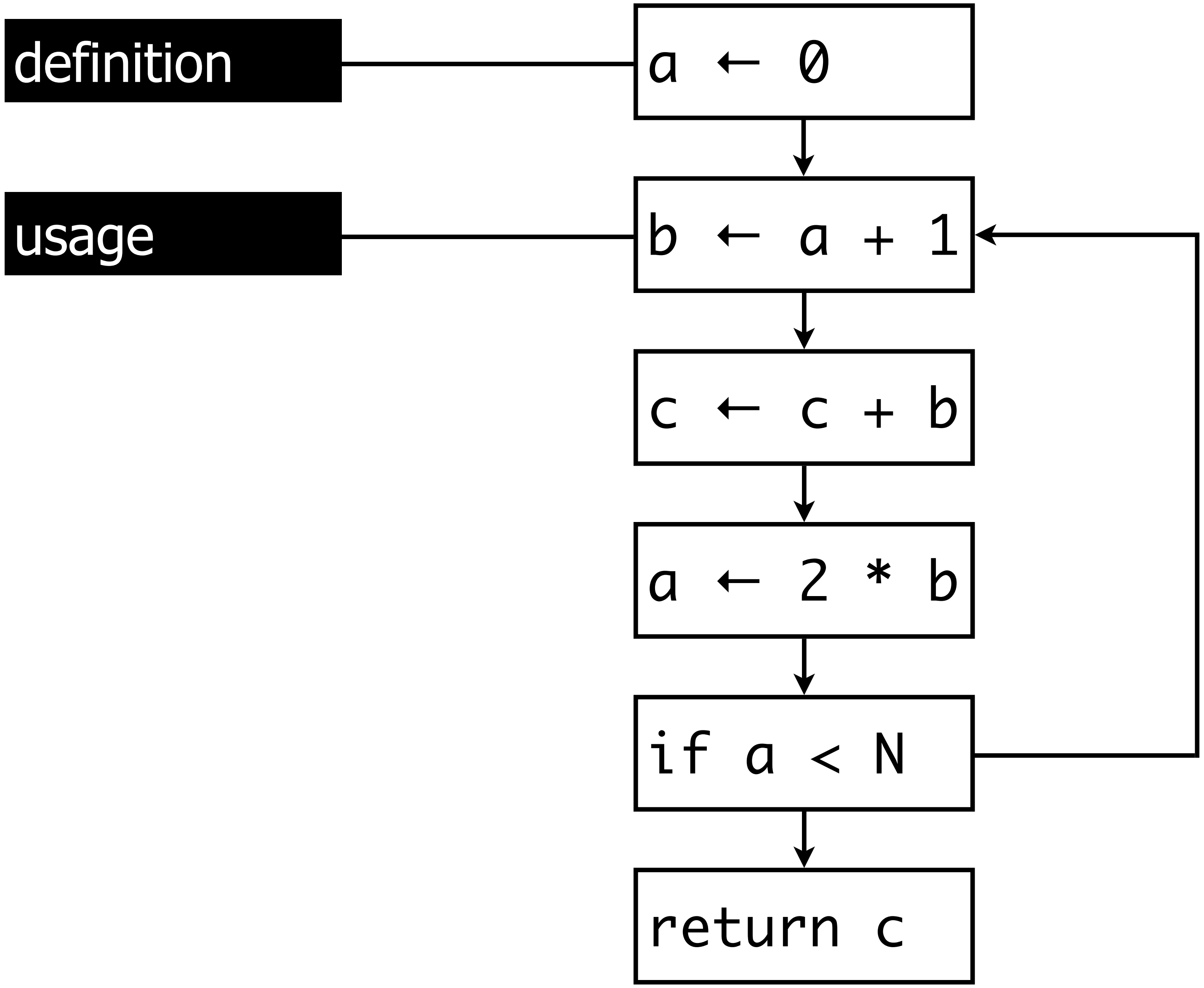

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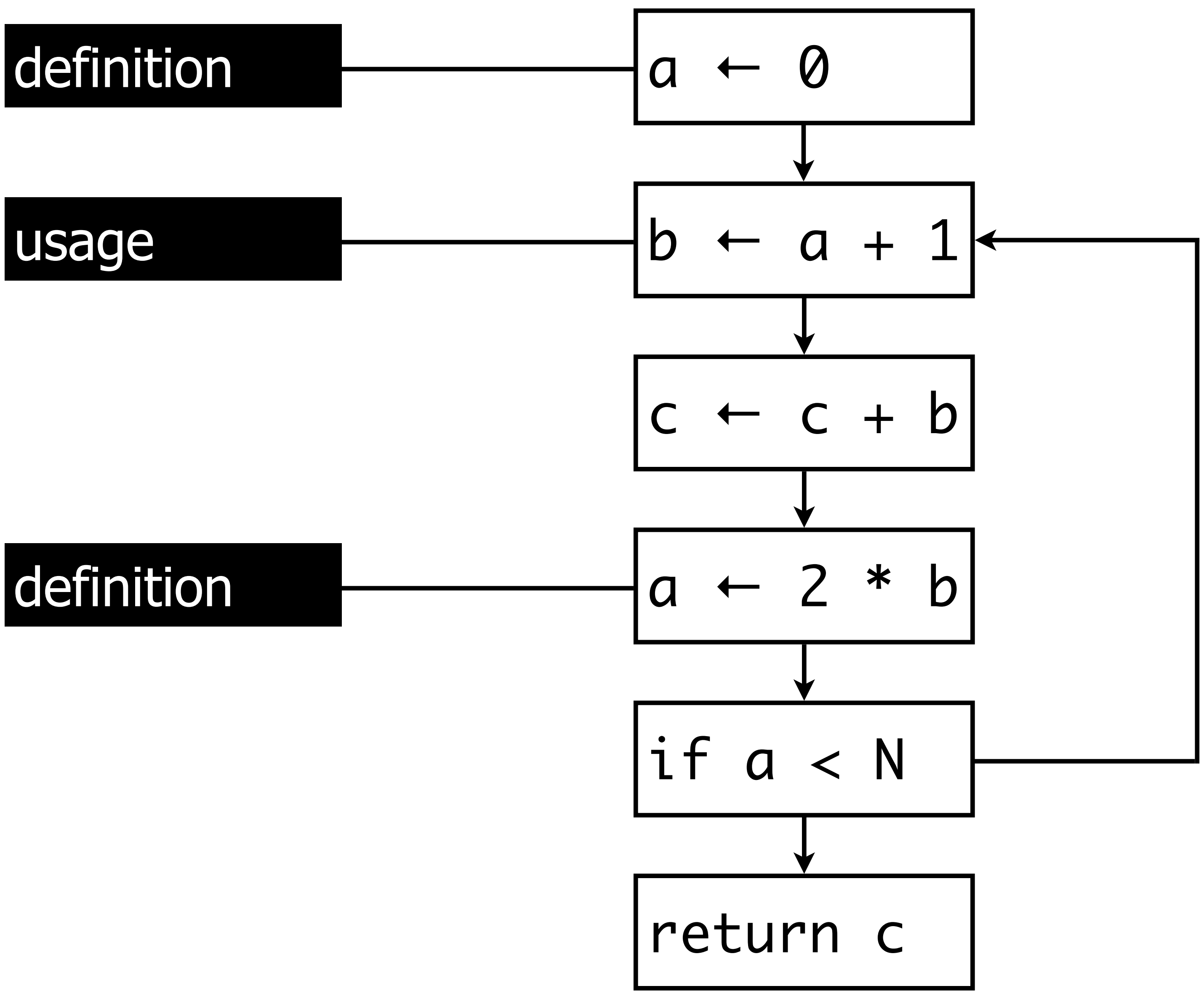

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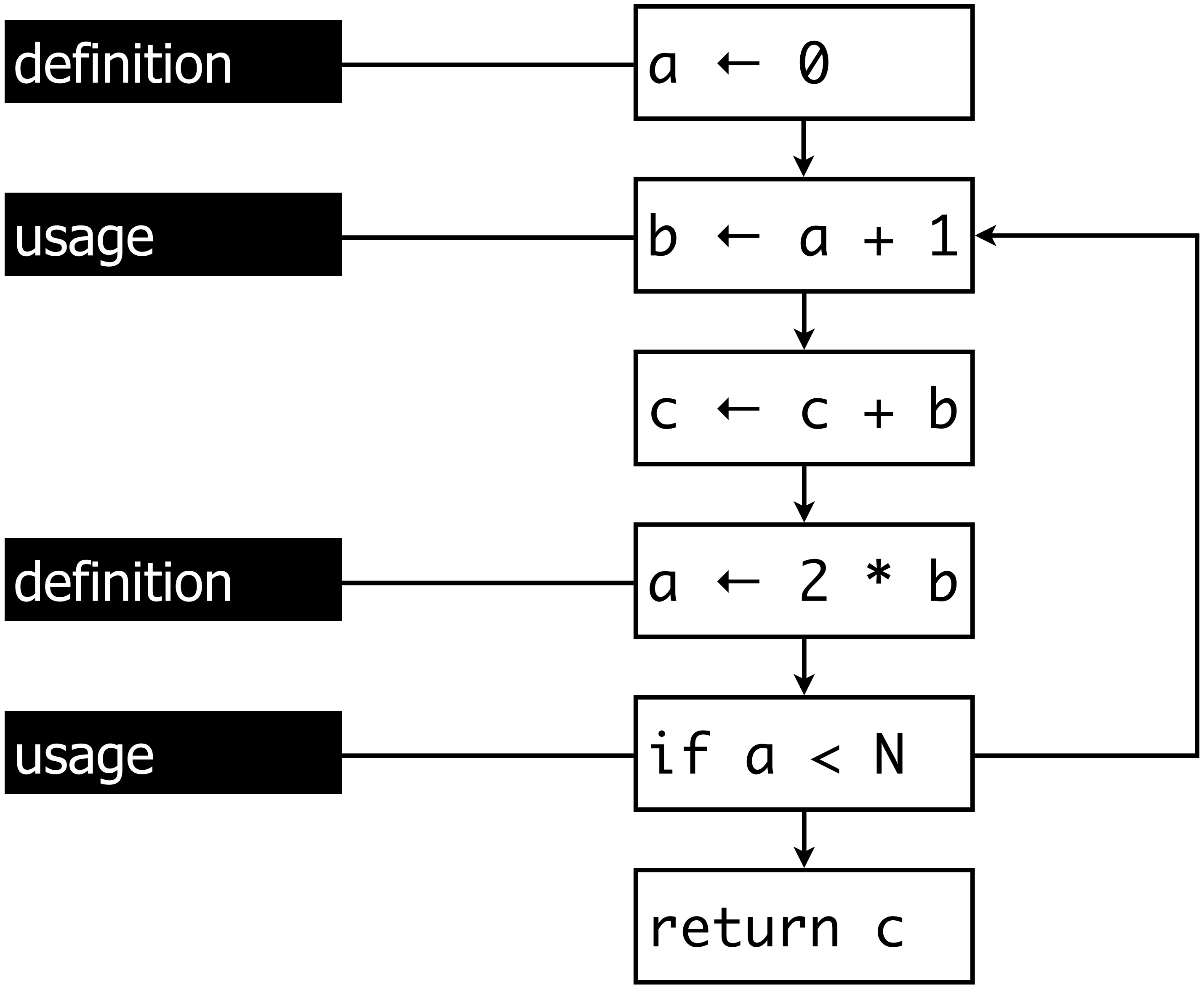

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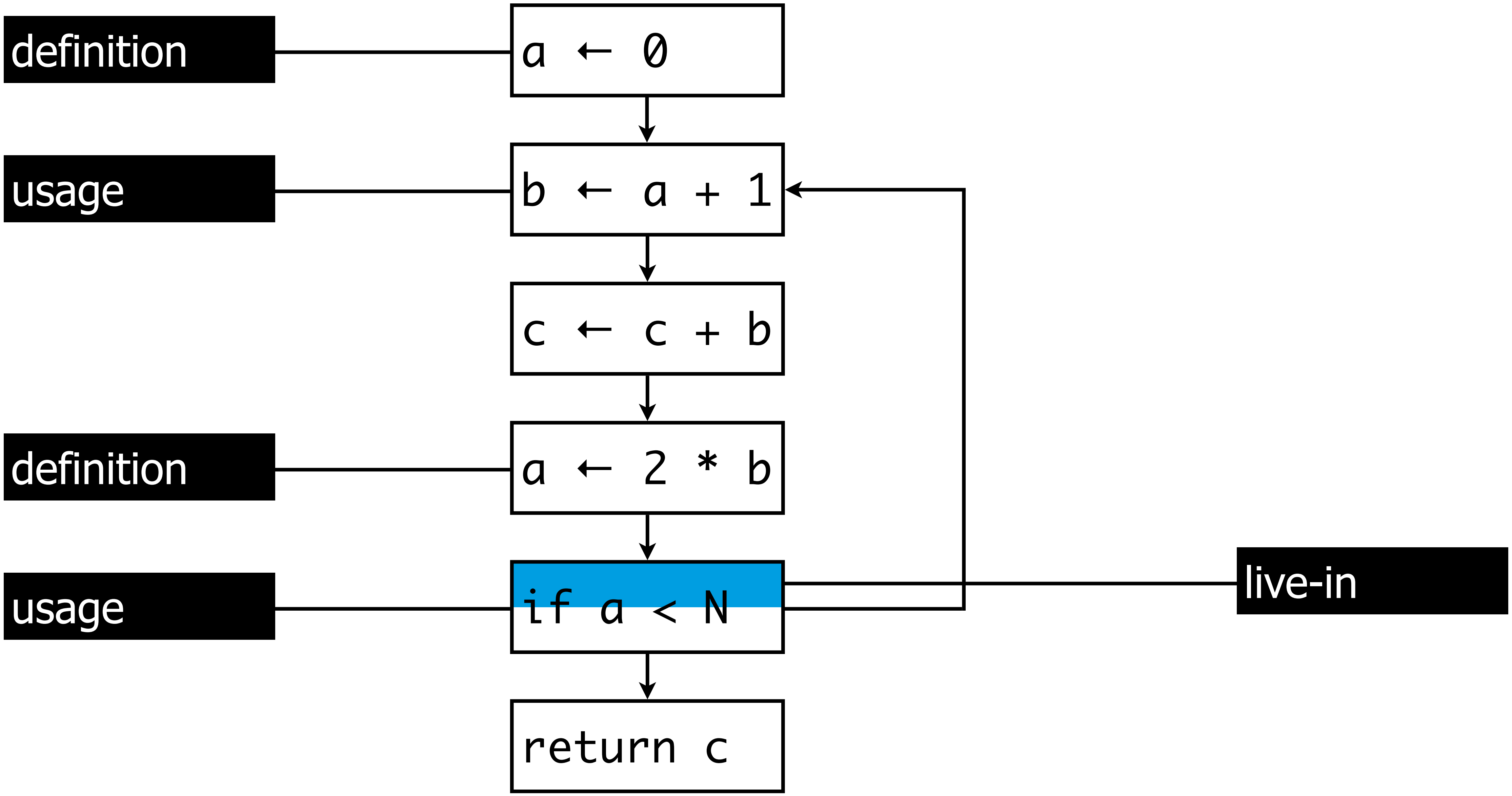

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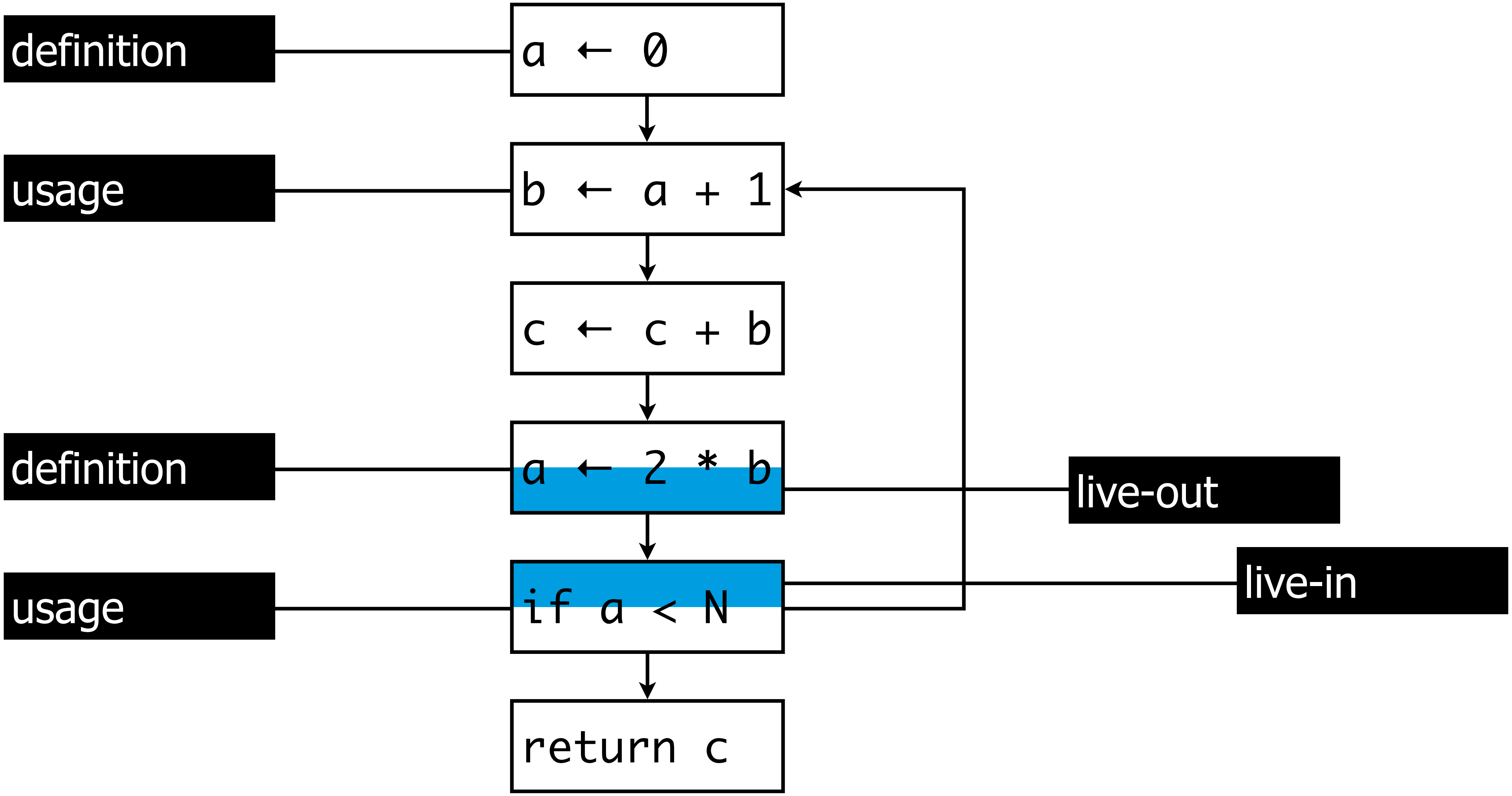

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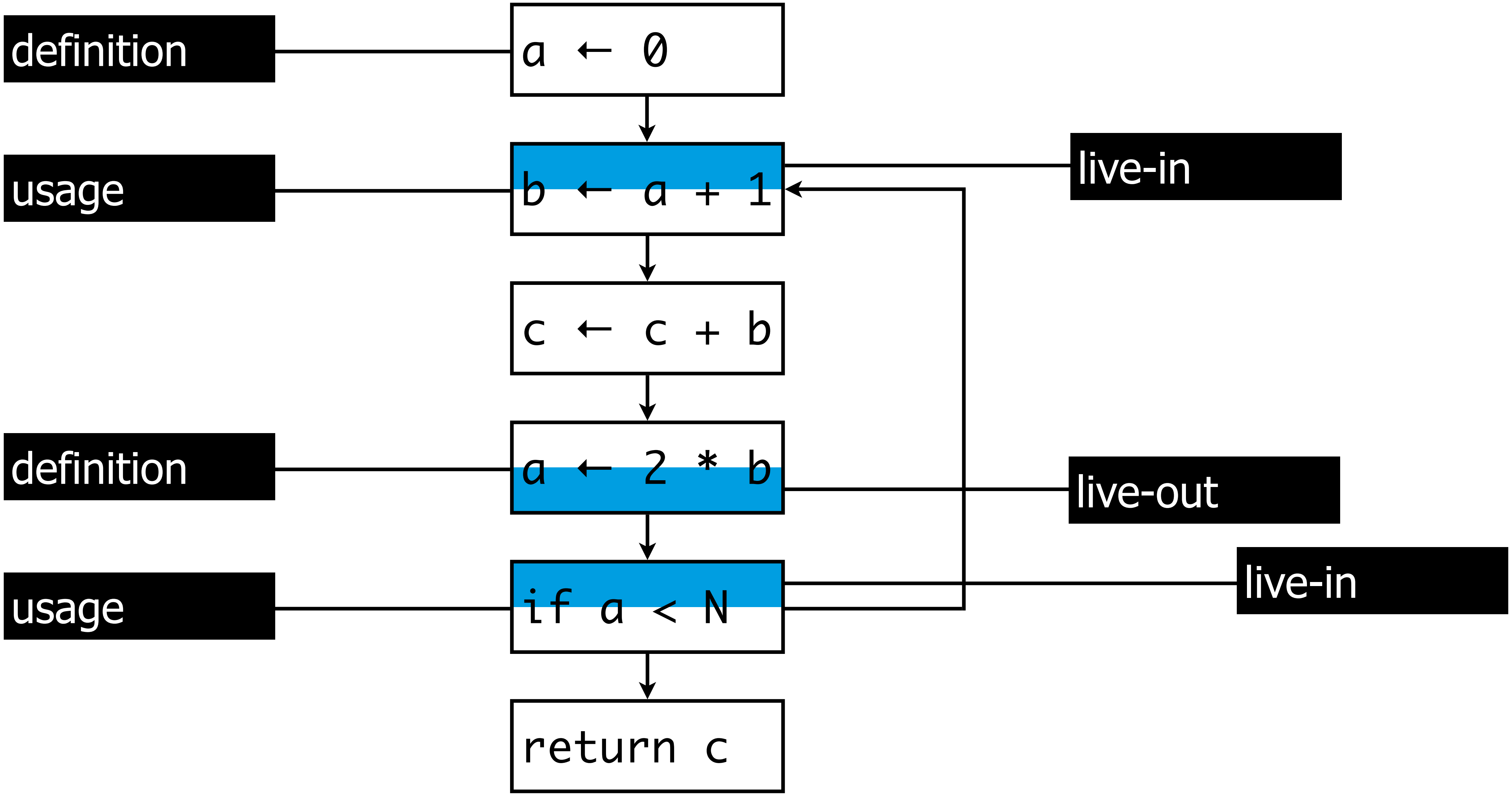

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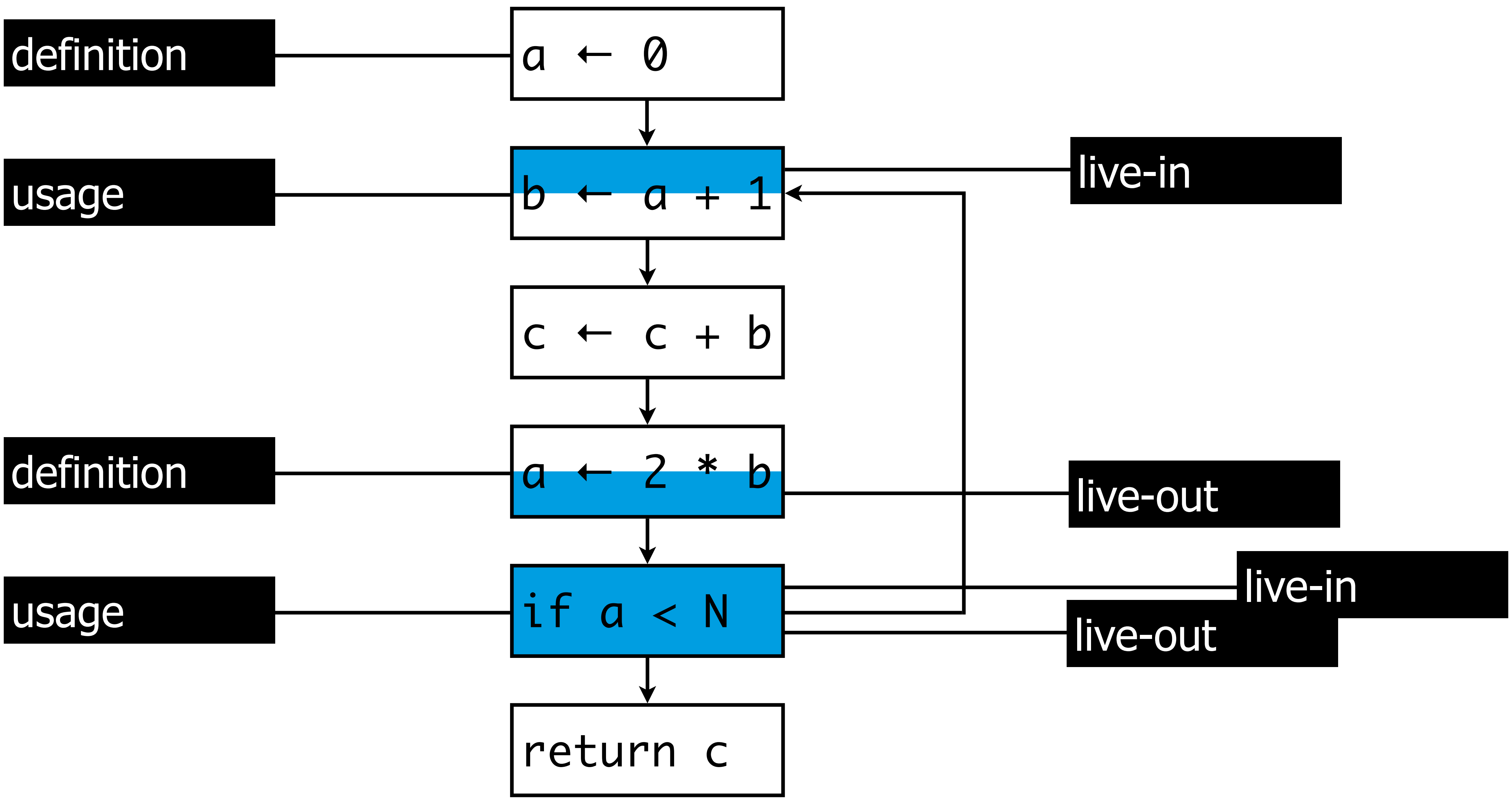

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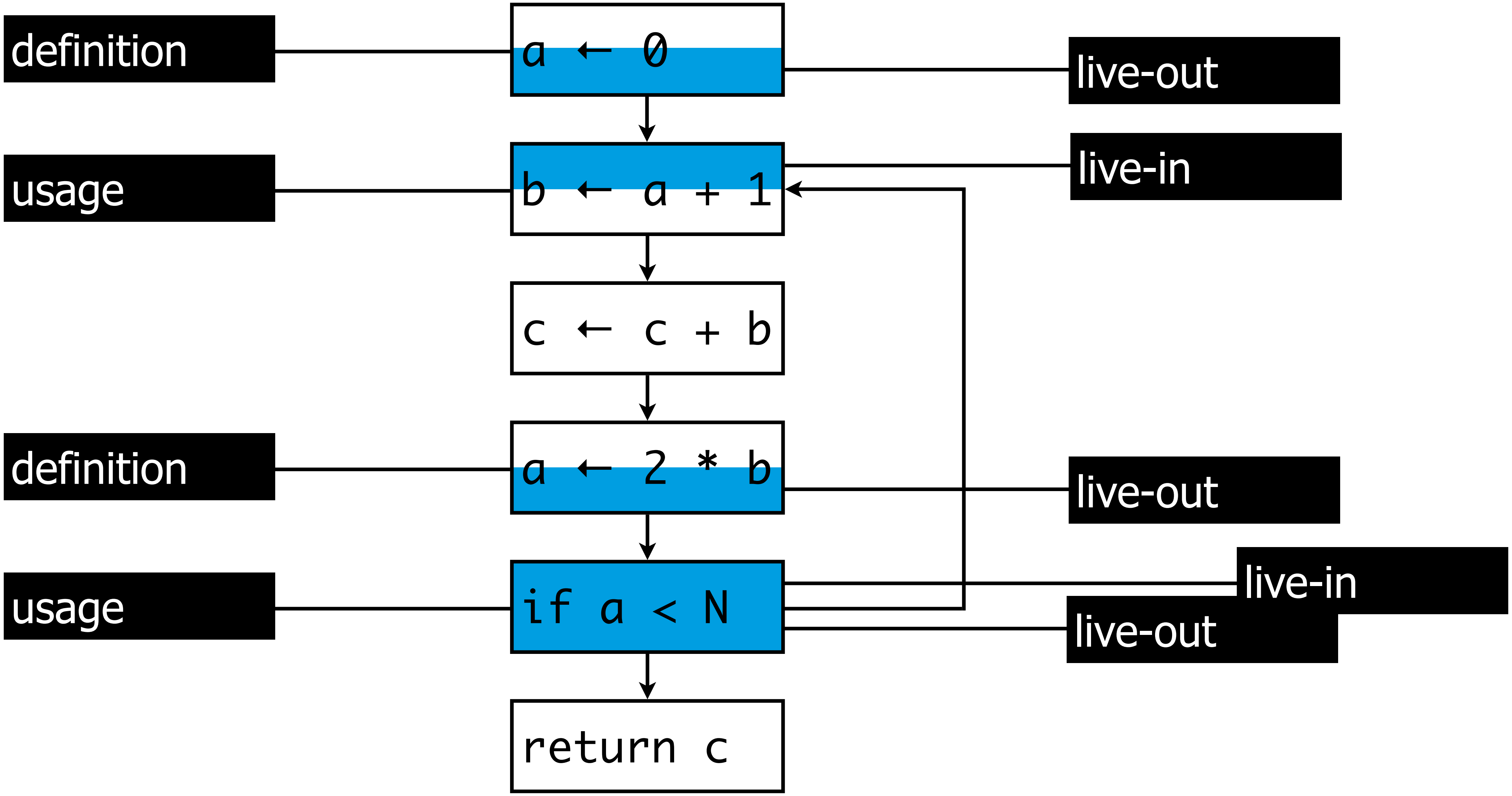

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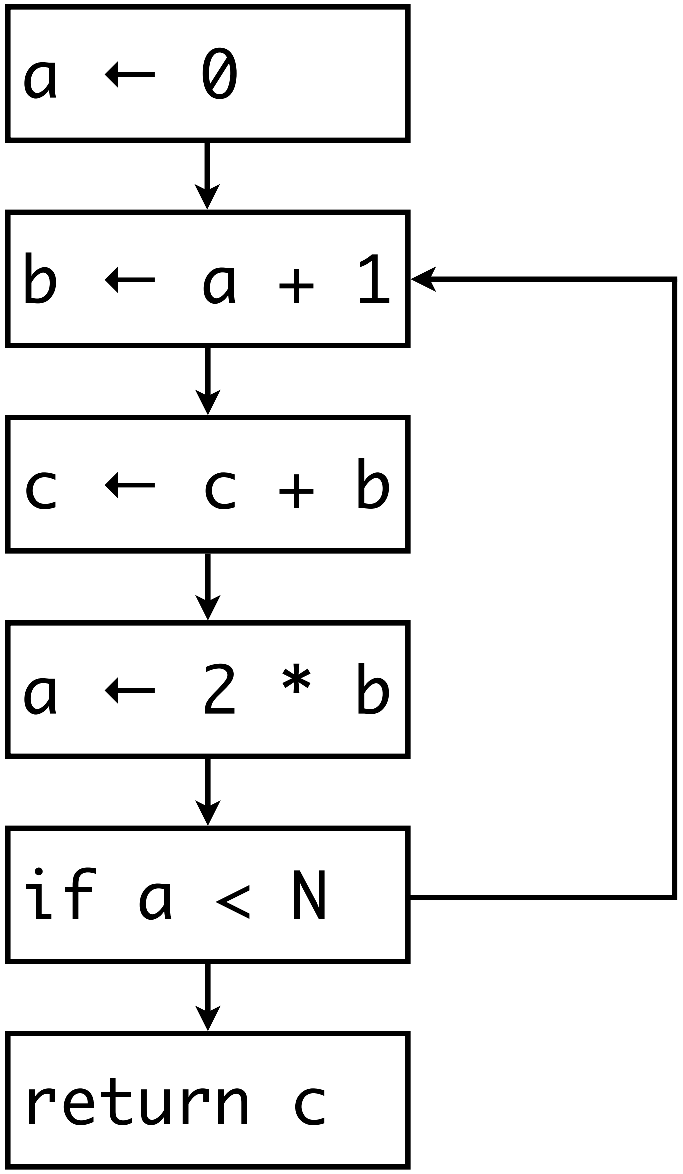
example



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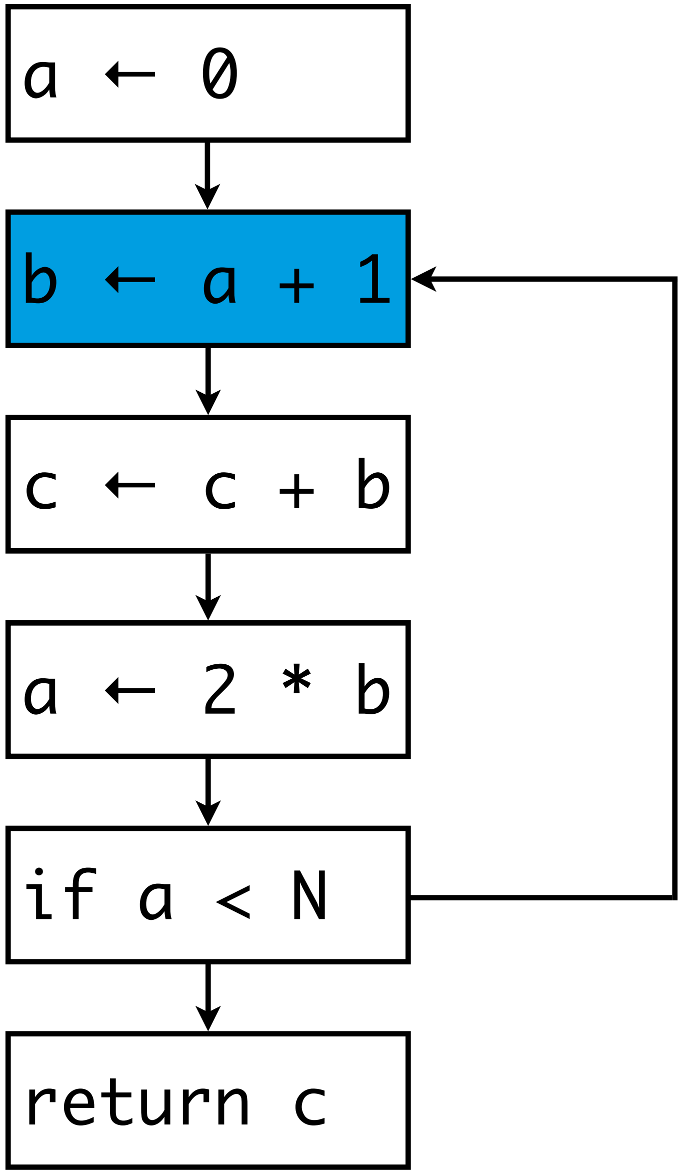

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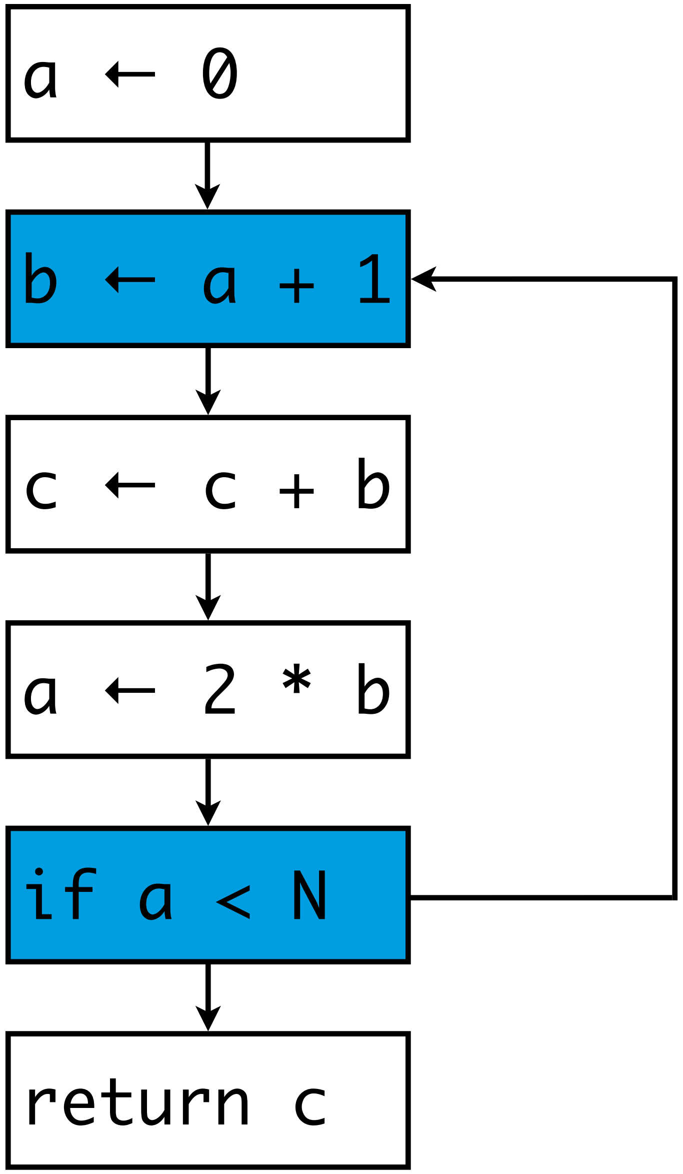

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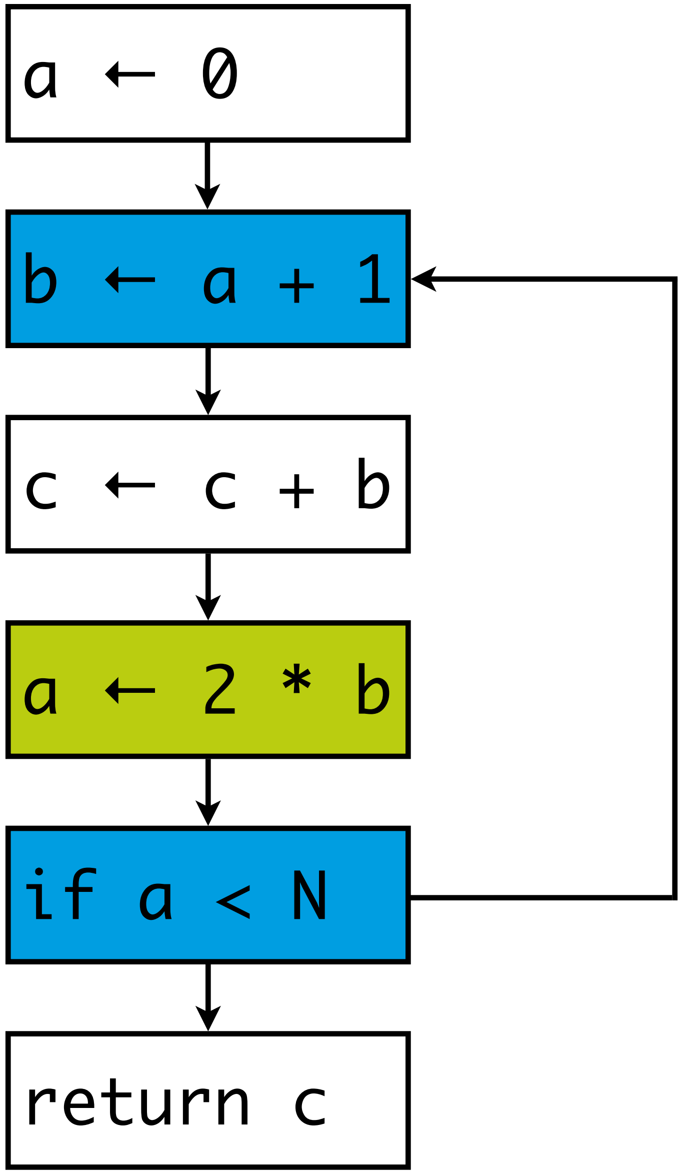

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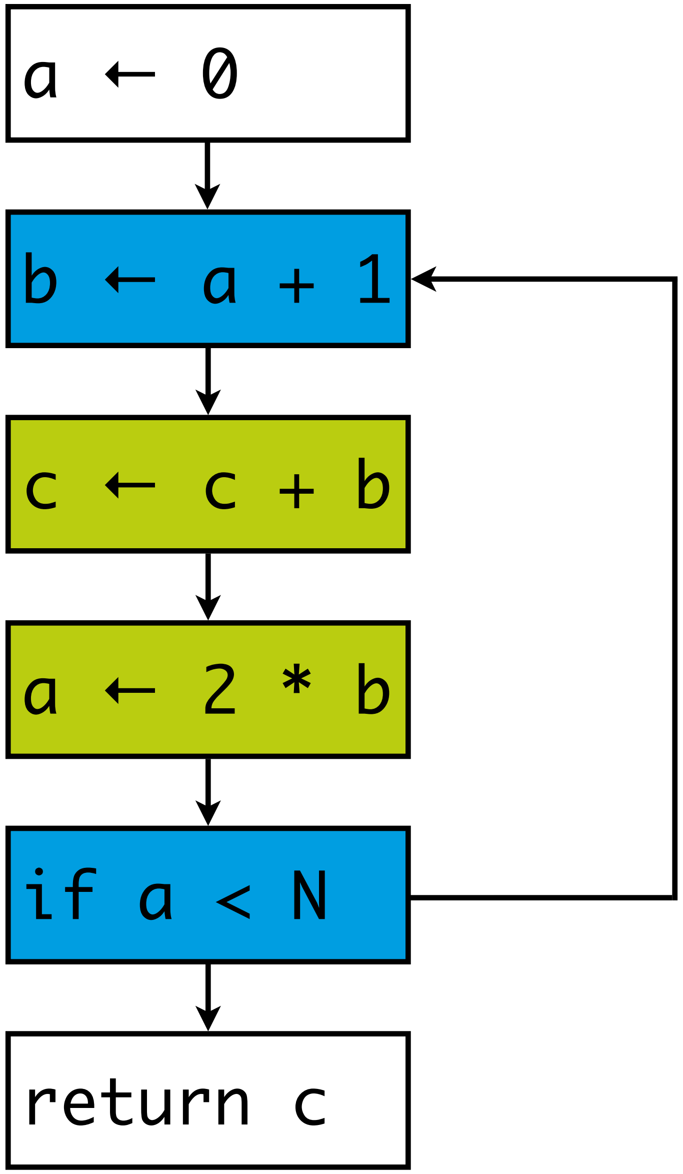

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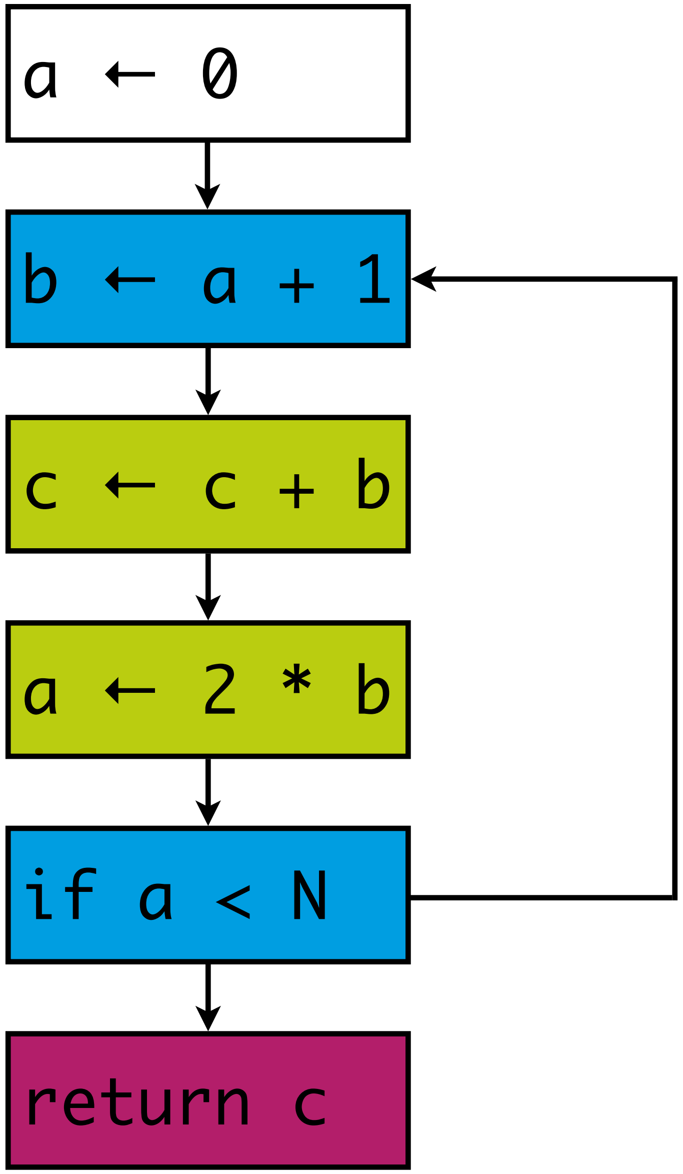

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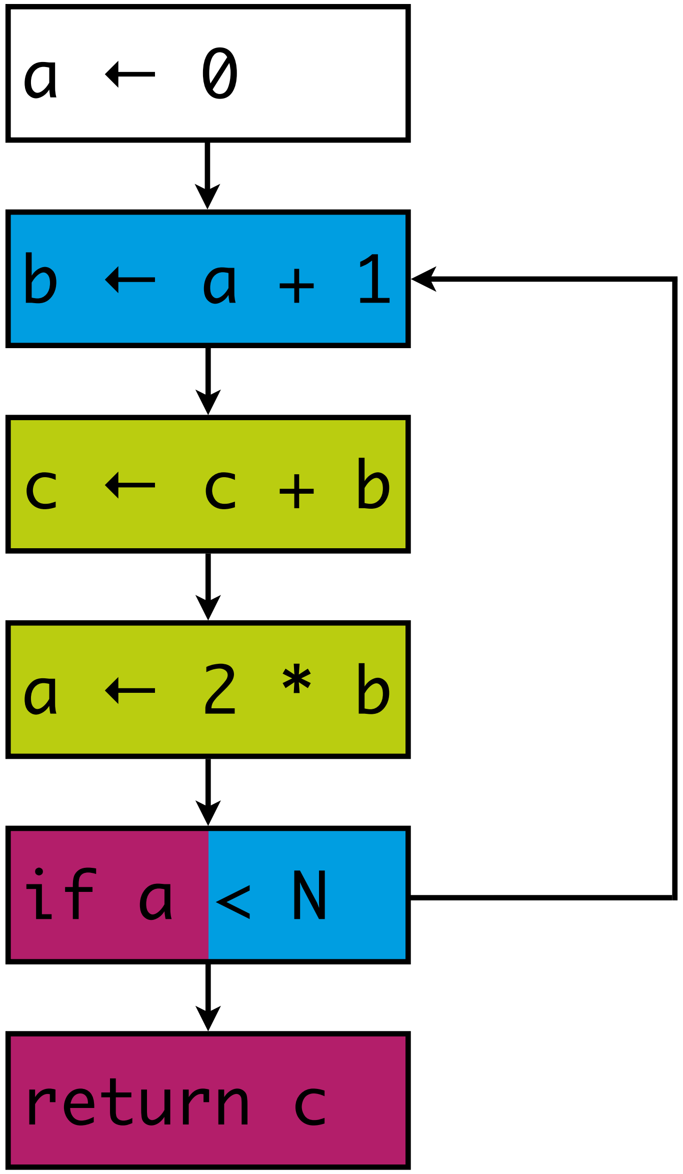
example



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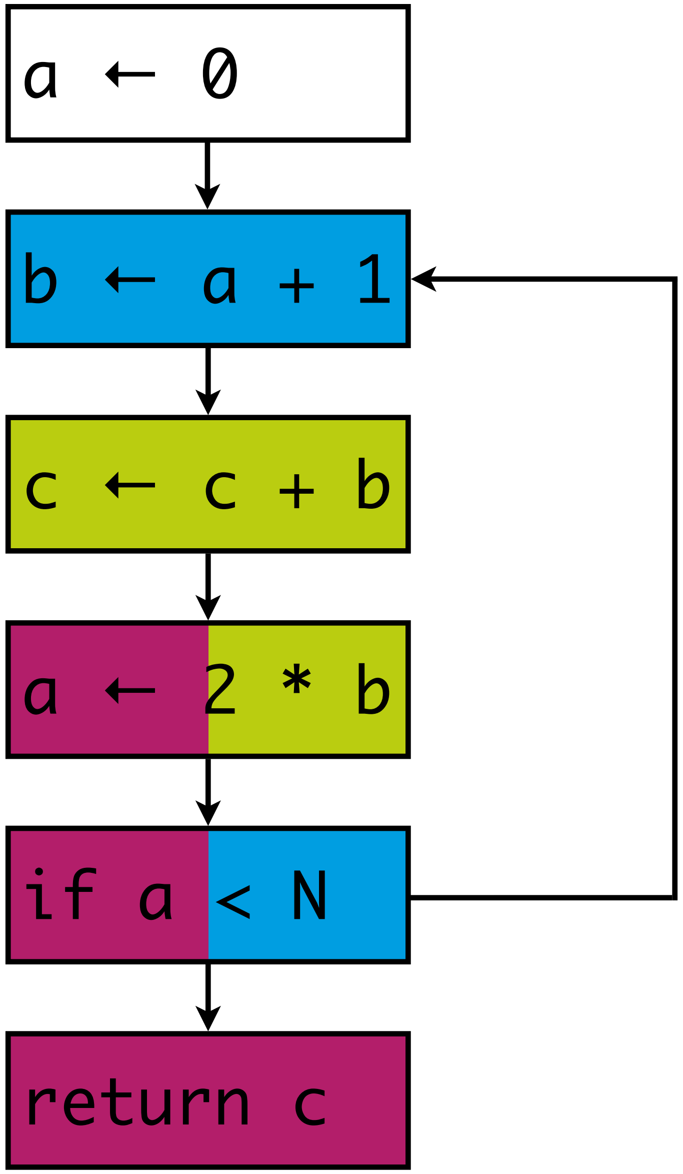

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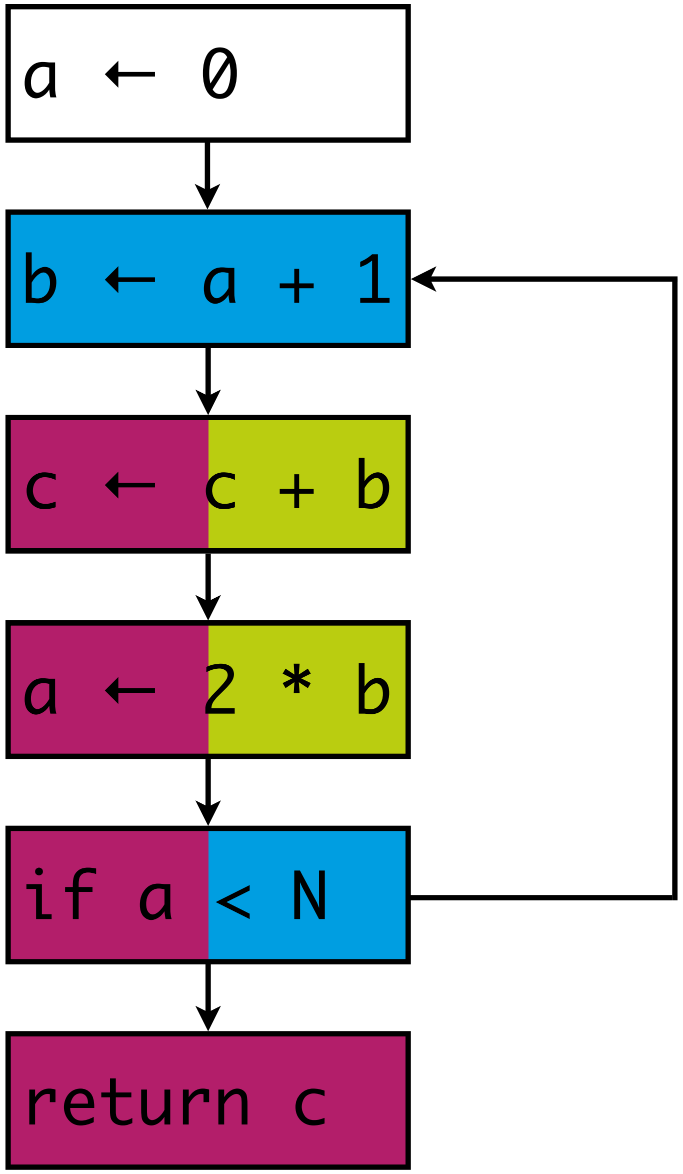

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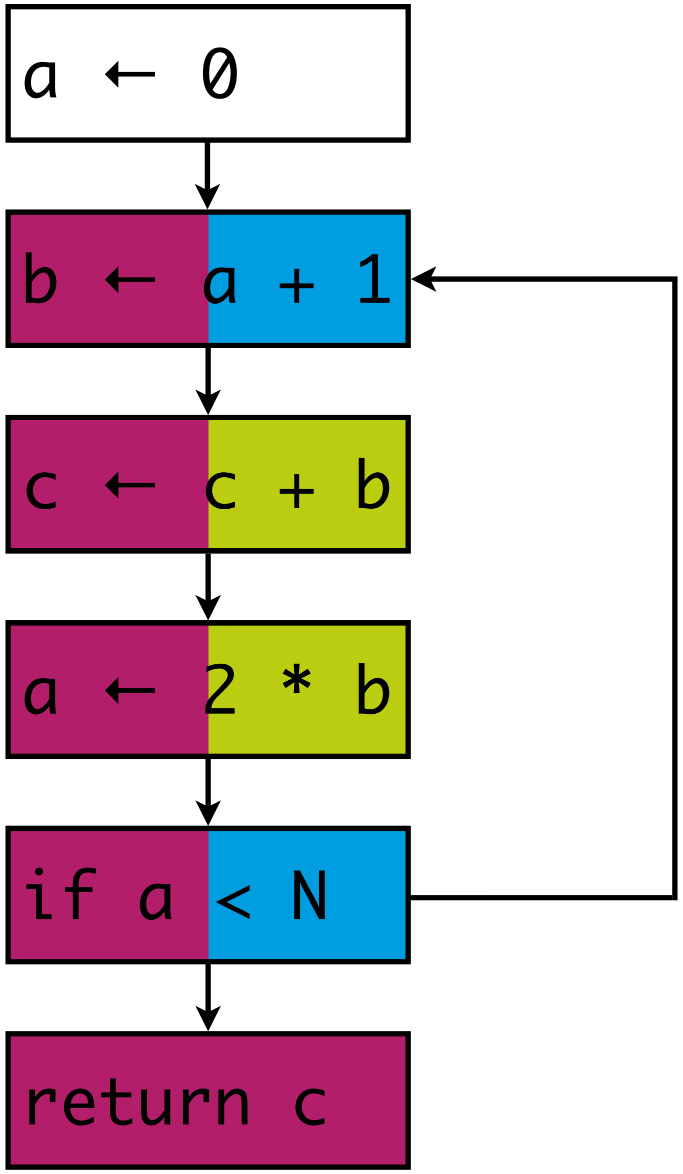

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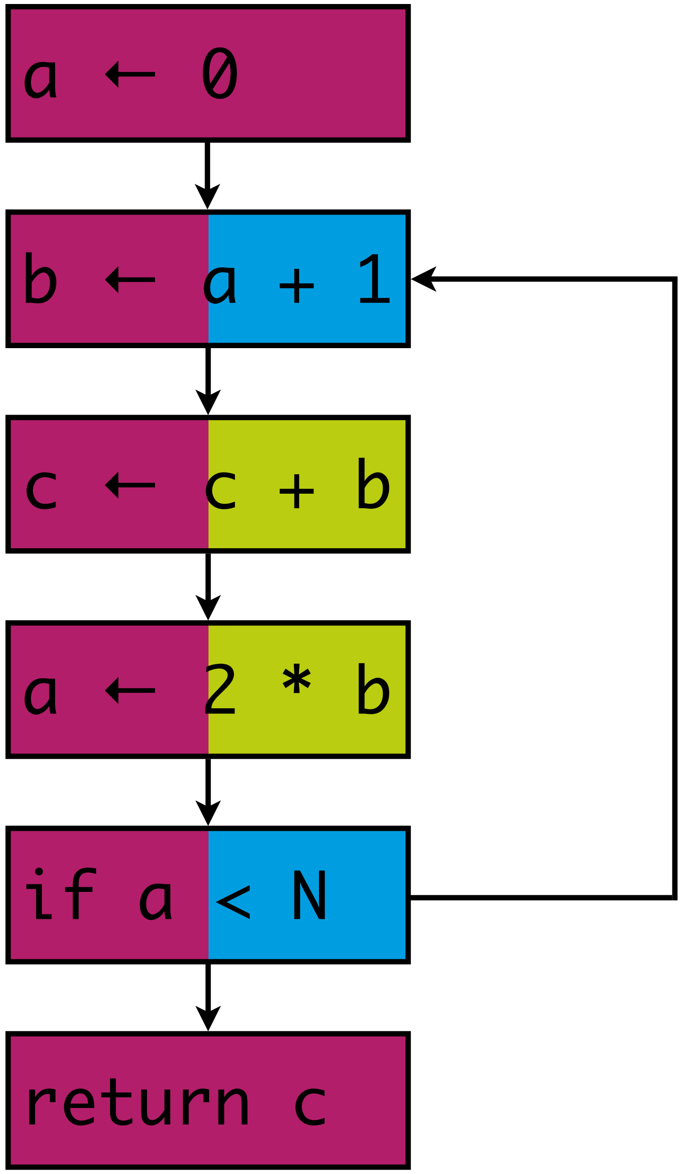


Liveness analysis
formalisation

$$in[n] = use[n] \cup \left(out[n] - def[n]\right)$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

1. If a variable is used at node n, then it is live-in at n.
2. If a variable is live-out but not defined at node n, it is live-in at n.
3. If a variable is live-in at node n, then it is live-out at its predecessors.

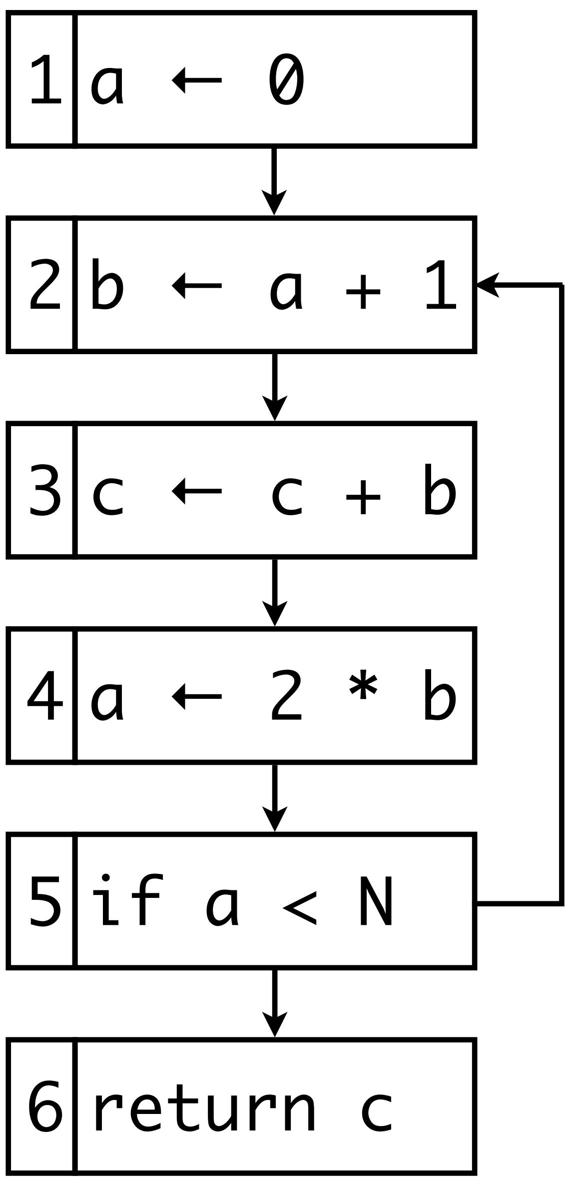
Liveness analysis

algorithm

```
for each n
    in[n] ← {} ; out[n] ← {}
repeat
    for each n
        in'[n] = in[n] ; out'[n] = out[n]
        in[n] = use[n] ∪ (out[n] - def[n])
        for each s in succ[n]
            out[n] = out[n] ∪ in[s]
until
    for all n: in[n] = in'[n] and out[n] = out'[n]
```

Liveness analysis

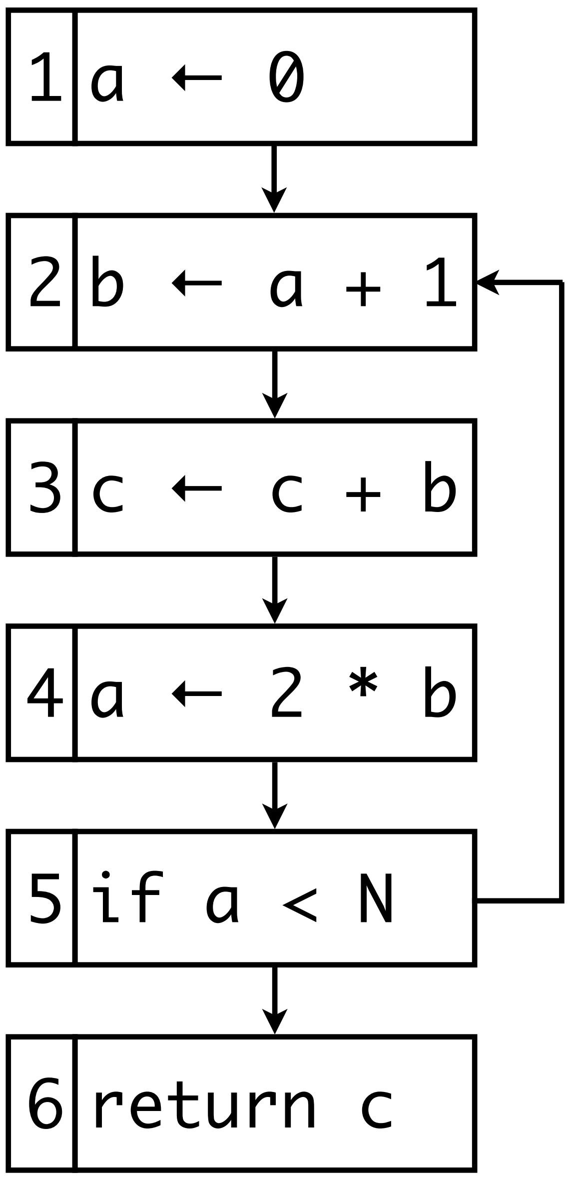
example



	u
1	
2	a
3	bc
4	b
5	a
6	c

Liveness analysis

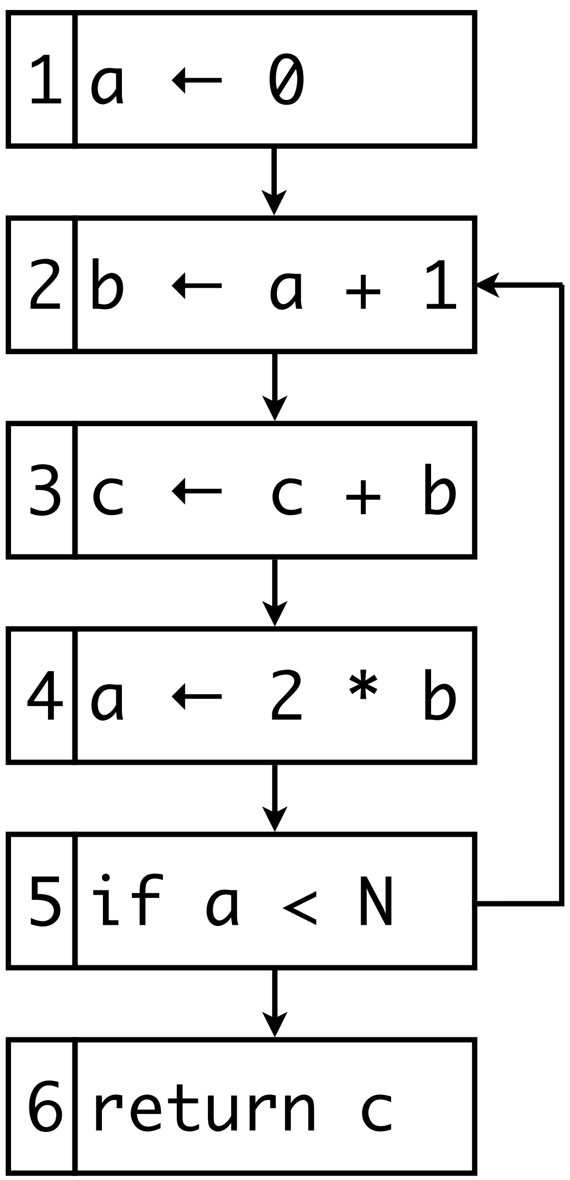
example



	u	d
1		a
2	a	b
3	bc	c
4	b	a
5	a	
6	c	

Liveness analysis

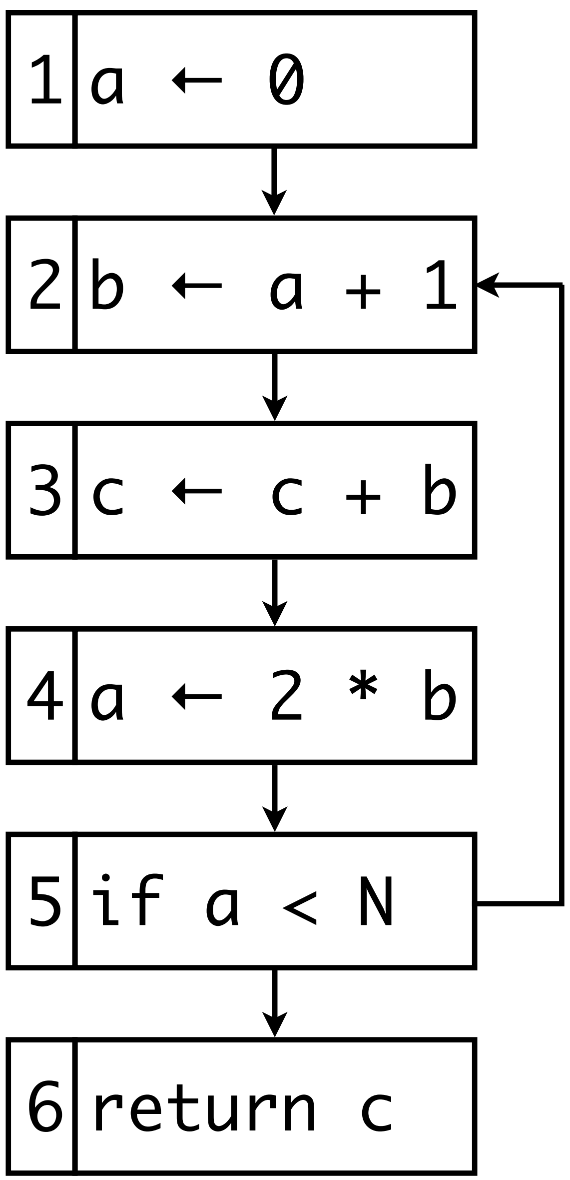
example



			1	
	u	d	i	o
1		a		
2	a	b	a	
3	bc	c	bc	
4	b	a	b	
5	a		a	a
6	c		c	

Liveness analysis

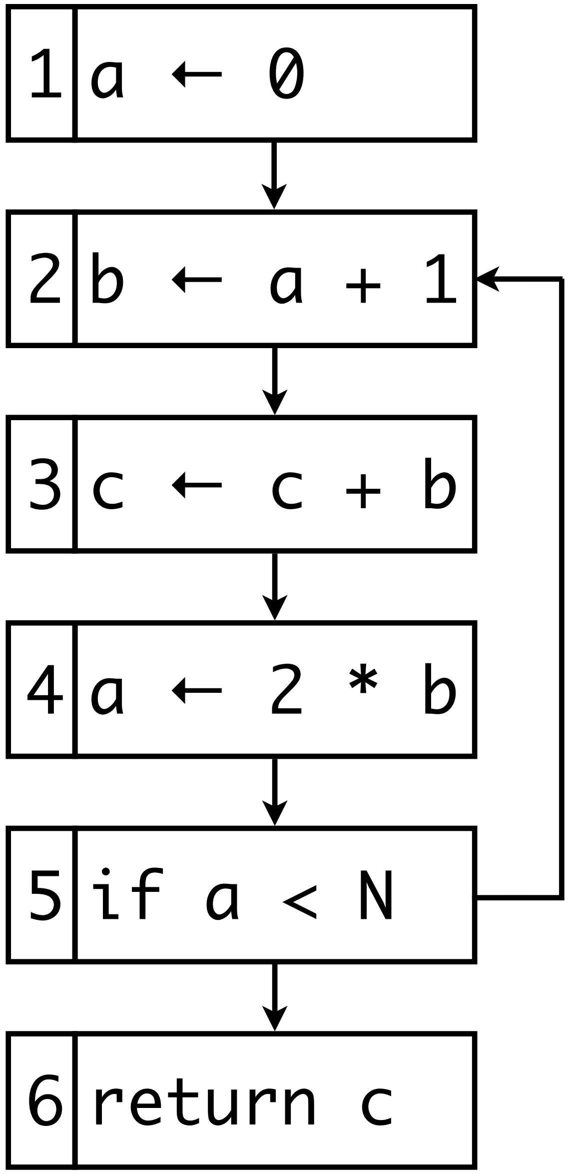
example



			1		2	
	u	d	i	o	i	o
1		a				a
2	a	b	a		a	bc
3	bc	c	bc		bc	b
4	b	a	b		b	a
5	a		a	a	a	ac
6	c		c		c	

Liveness analysis

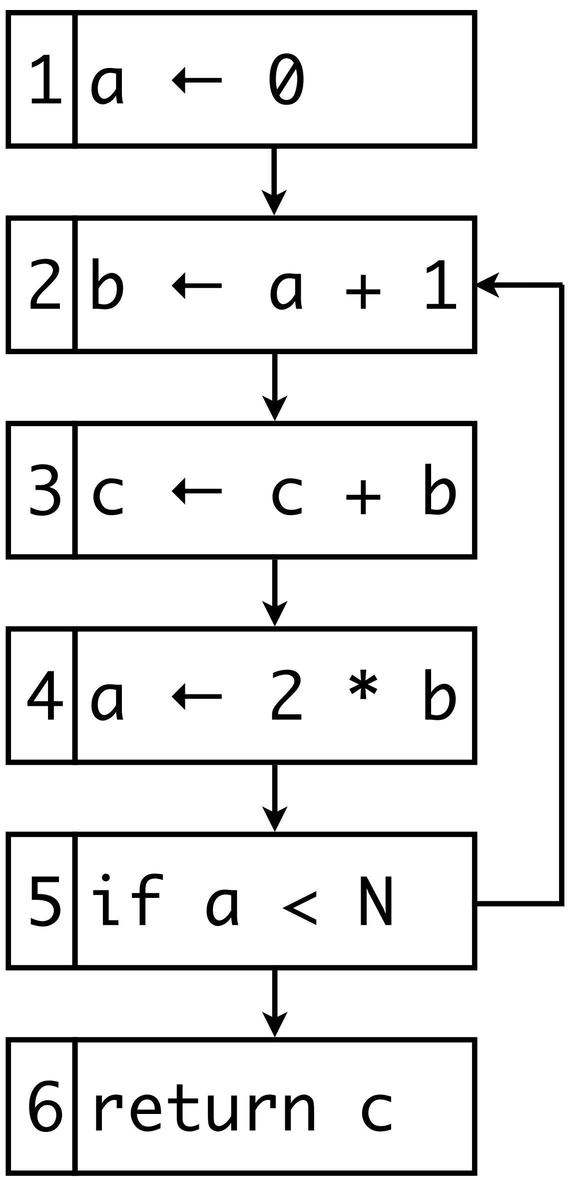
example



			1		2		3									
	u	d	i	o	i	o	i	o								
1		a				a		a								
2	a	b	a		a	bc	ac	bc								
3	bc	c	bc		bc	b	bc	b								
4	b	a	b		b	a	b	a								
5	a		a	a	a	ac	ac	ac								
6	c		c		c		c									

Liveness analysis

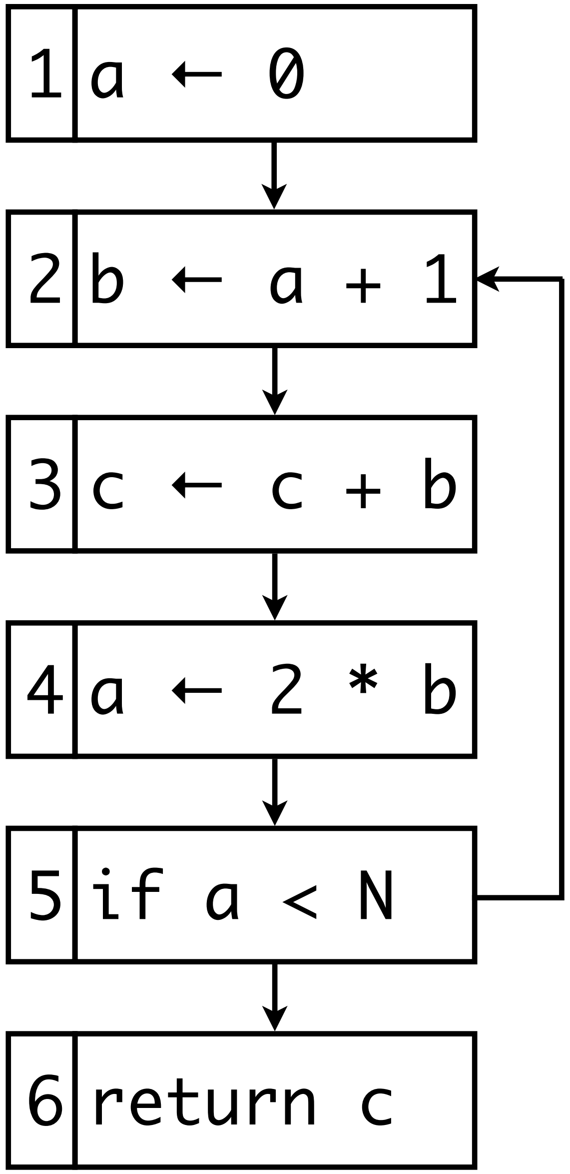
example



			1		2		3		4							
	u	d	i	o	i	o	i	o	i	o						
1		a				a		a		ac						
2	a	b	a		a	bc	ac	bc	ac	bc						
3	bc	c	bc		bc	b	bc	b	bc	b						
4	b	a	b		b	a	b	a	b	ac						
5	a		a	a	a	ac	ac	ac	ac	ac						
6	c		c		c		c		c							

Liveness analysis

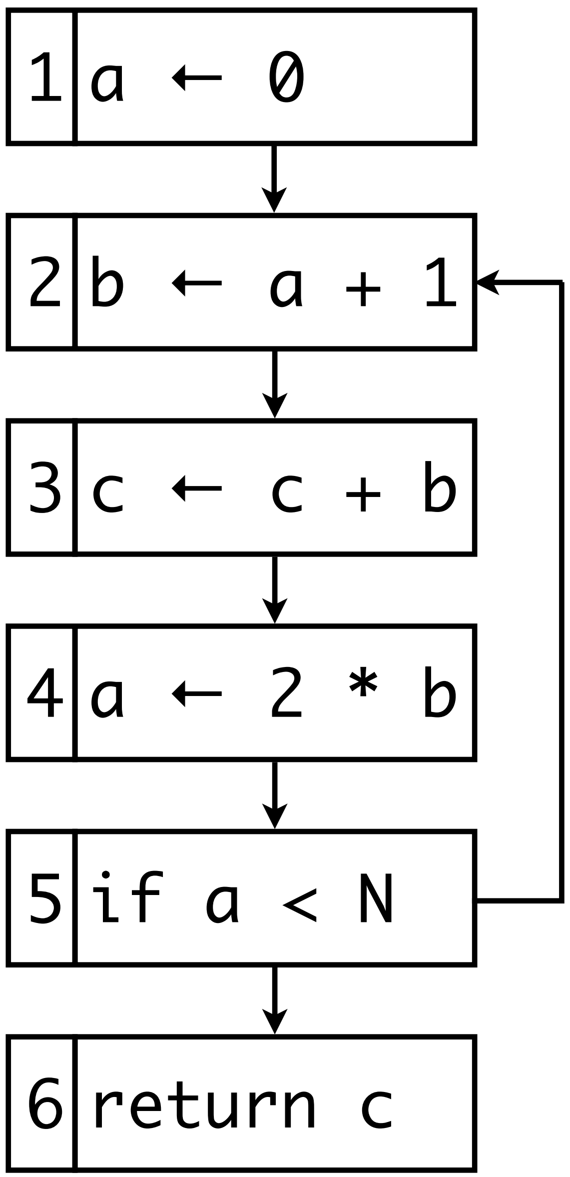
example



			1		2		3		4		5	
	u	d	i	o	i	o	i	o	i	o	i	o
1		a				a		a		ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b
4	b	a	b		b	a	b	a	b	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c	

Liveness analysis

example



			1		2		3		4		5		6	
	u	d	i	o	i	o	i	o	i	o	i	o	i	o
1		a				a		a		ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c	

Liveness analysis

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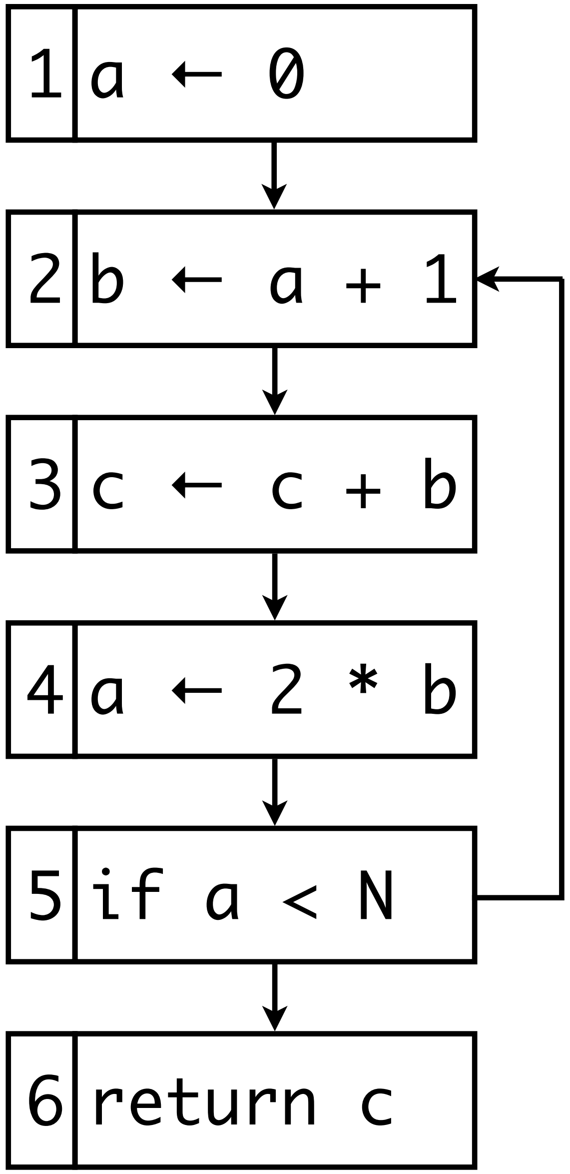
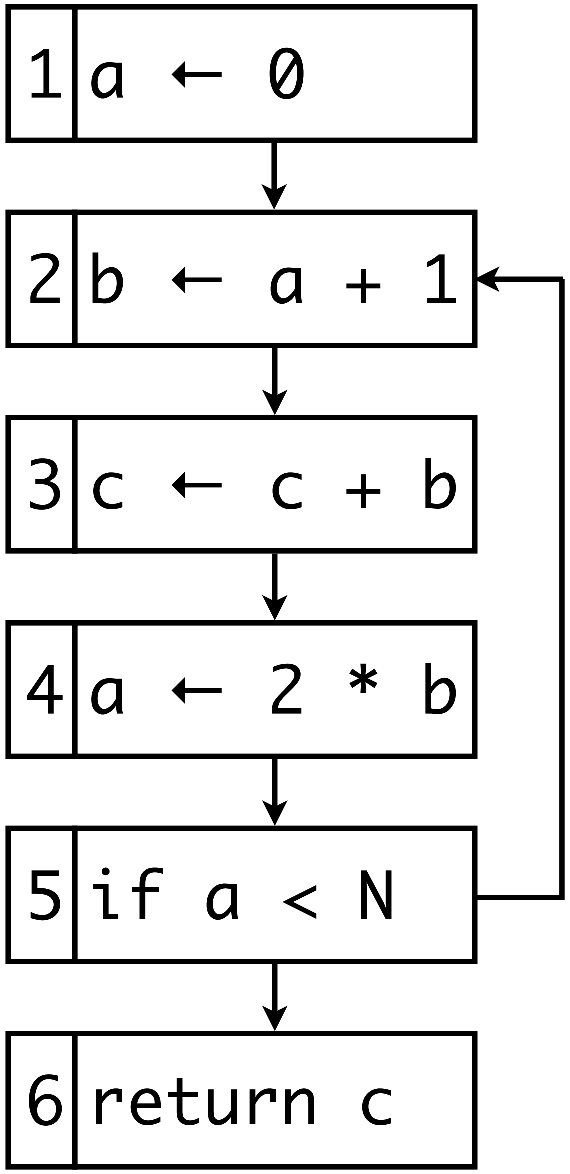


			1		2		3		4		5		6		7	
	u	d	i	o	i	o	i	o	i	o	i	o	i	o	i	o
1		a				a		a		ac	c	ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c		c	

Liveness analysis

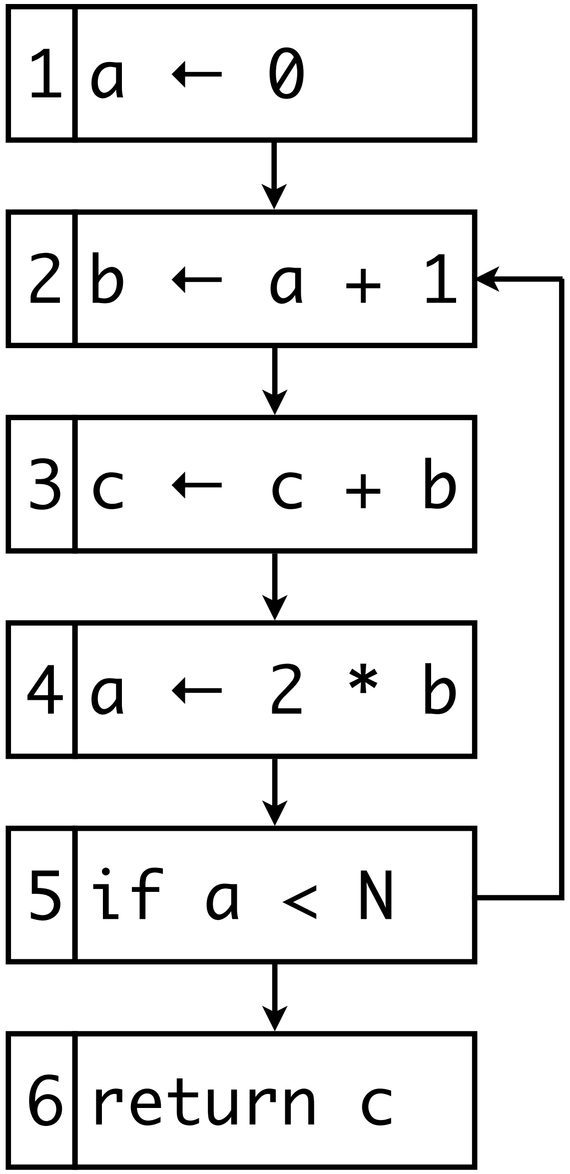
optimisation



	u							
6	c							
5	a							
4	b							
3	bc							
2	a							
1								

Liveness analysis

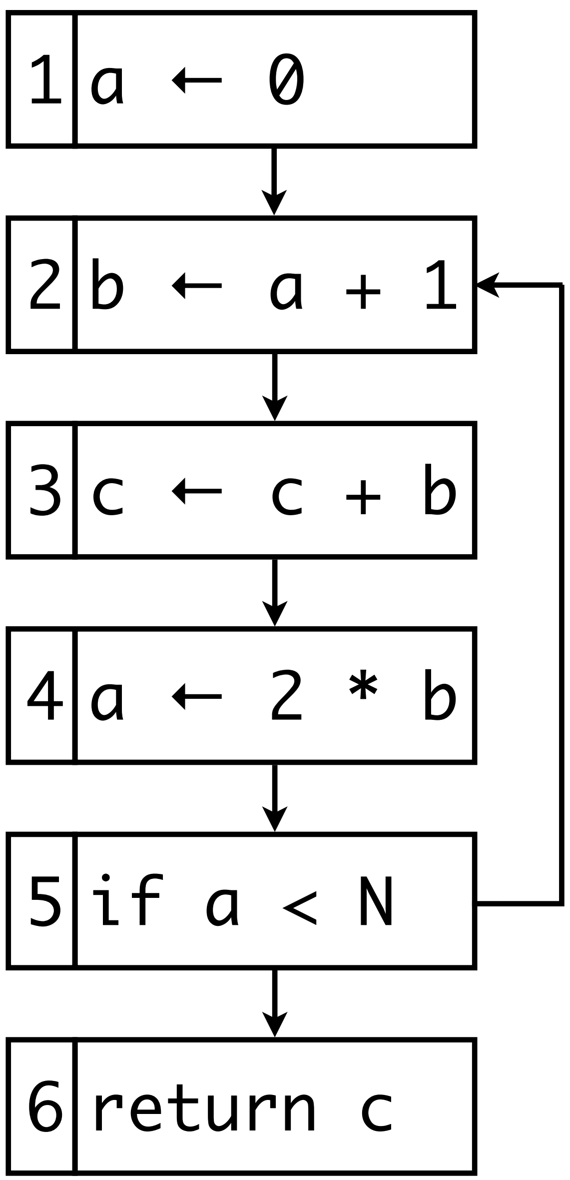
optimisation



	u	d						
6	c							
5	a							
4	b	a						
3	bc	c						
2	a	b						
1		a						

Liveness analysis

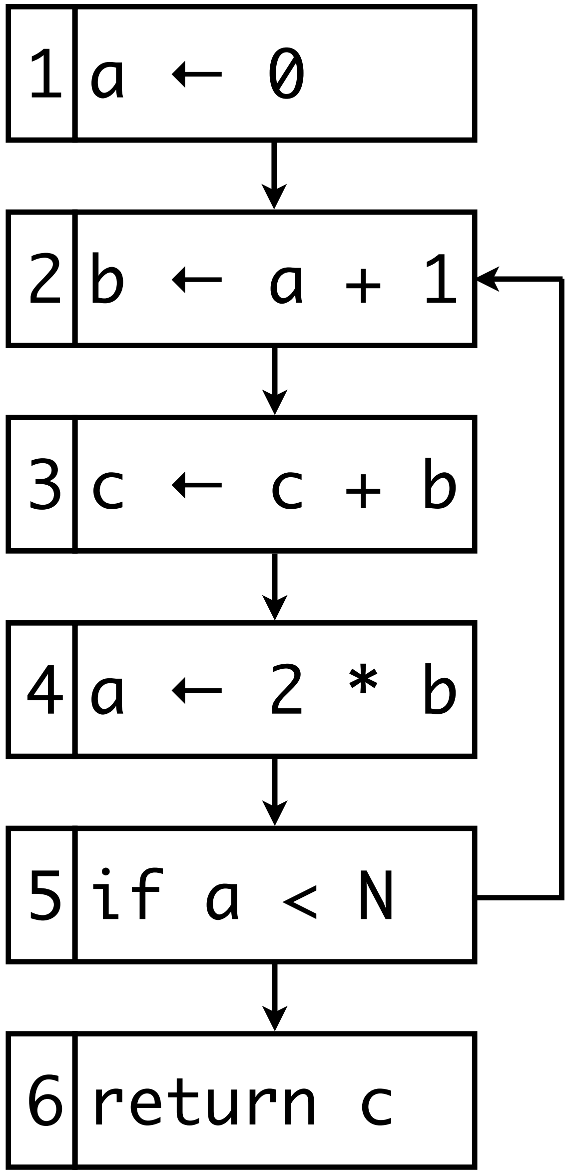
optimisation



			1					
	u	d	o	i				
6	c			c				
5	a		c	ac				
4	b	a	ac	bc				
3	bc	c	bc	bc				
2	a	b	bc	ac				
1		a	ac	a				

Liveness analysis

optimisation



			1		2	
	u	d	o	i	o	i
6	c			c		c
5	a		c	ac	ac	ac
4	b	a	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac
1		a	ac	a	ac	c

Liveness analysis

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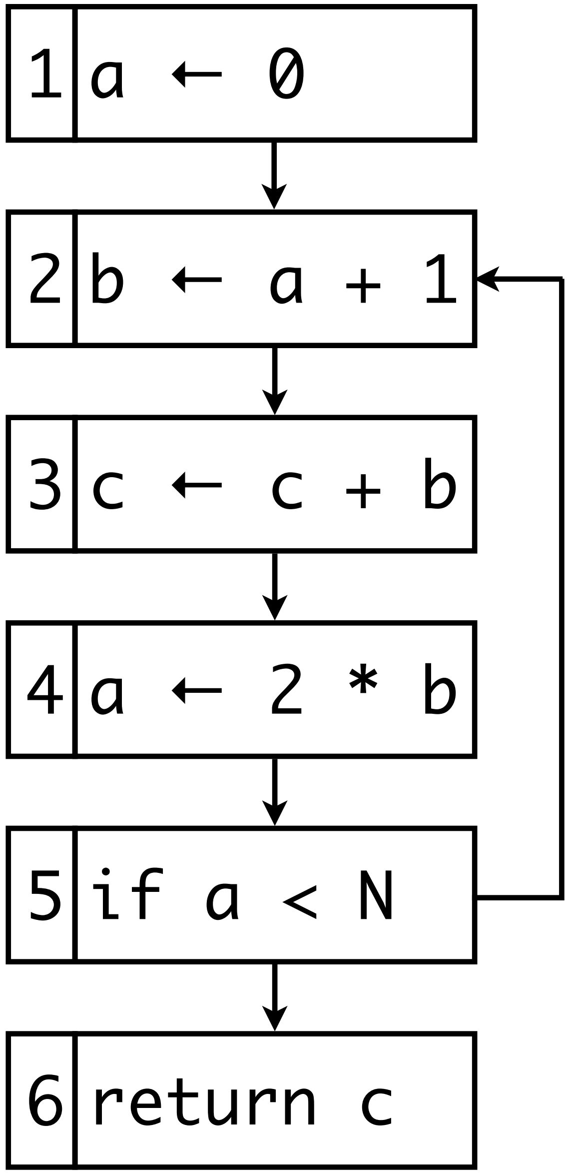
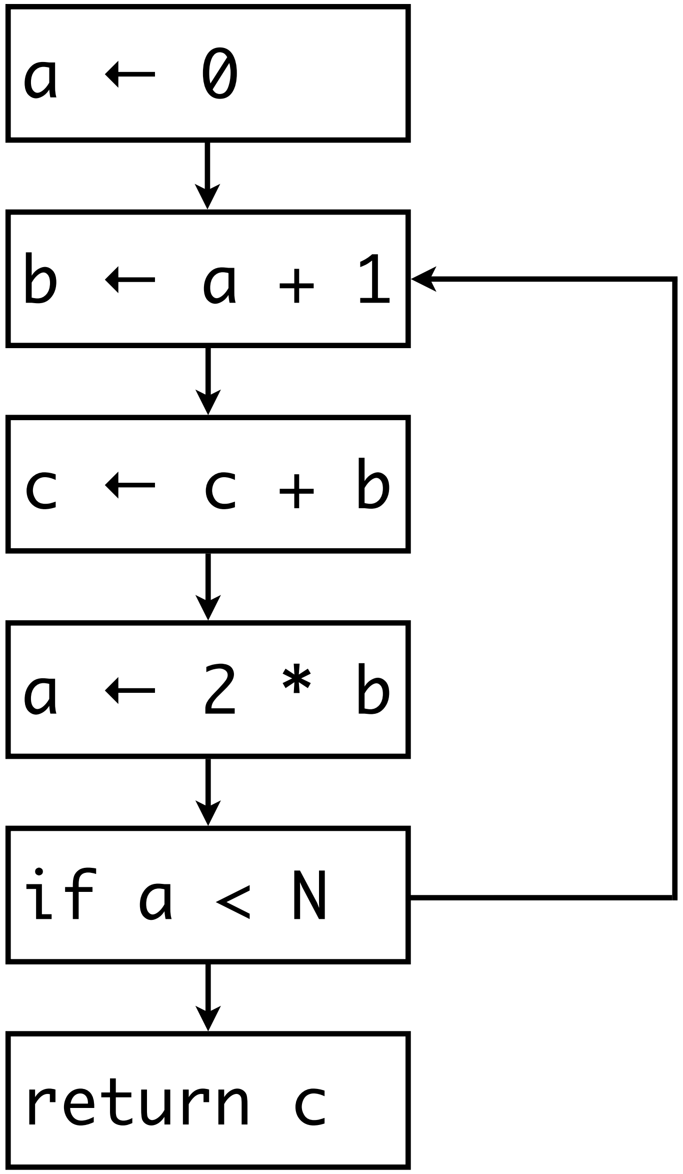


			1		2		3	
	u	d	o	i	o	i	o	i
6	c			c		c		c
5	a		c	ac	ac	ac	ac	ac
4	b	a	ac	bc	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac	bc	ac
1		a	ac	a	ac	c	ac	c

Liveness analysis
generalisation



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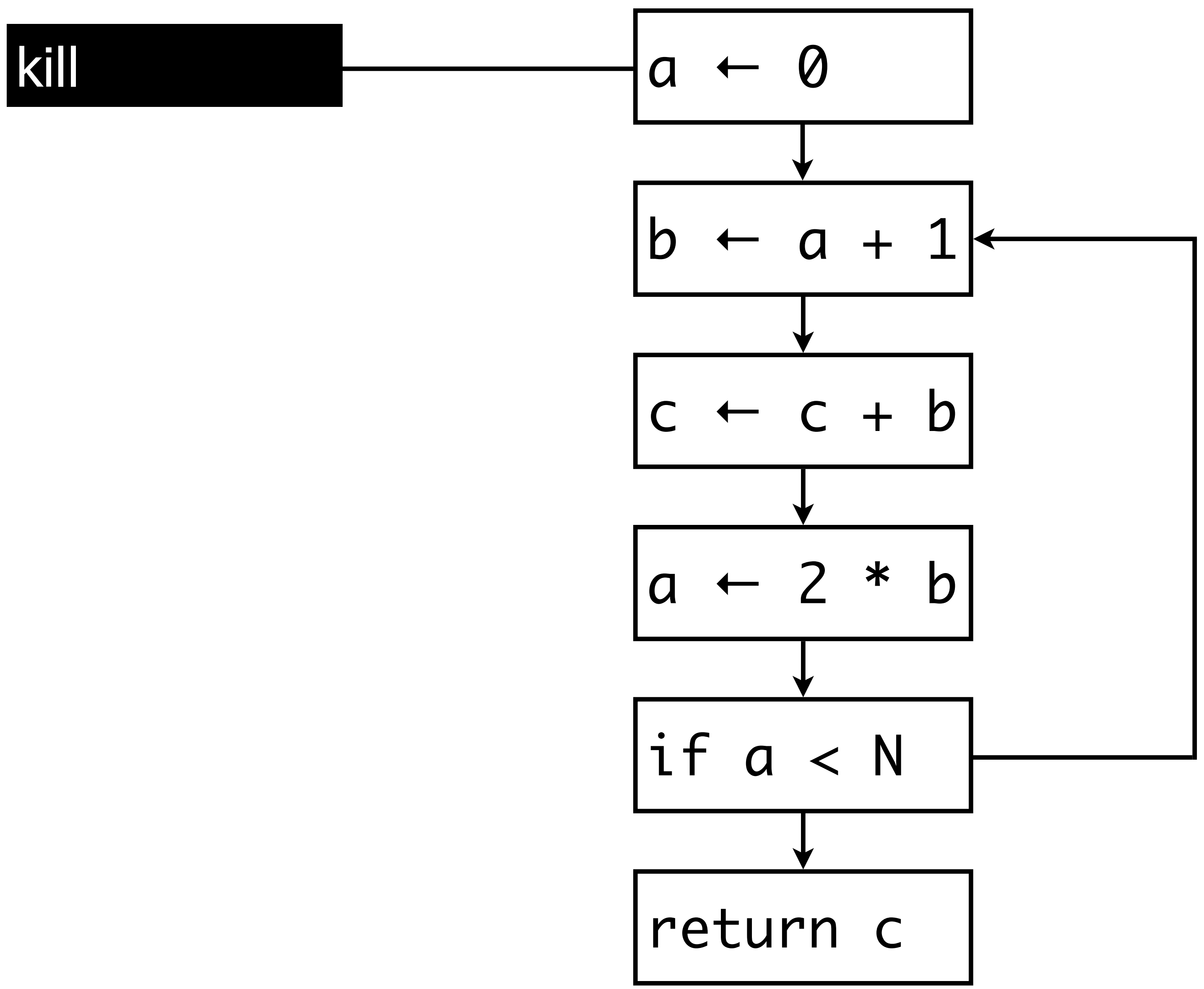

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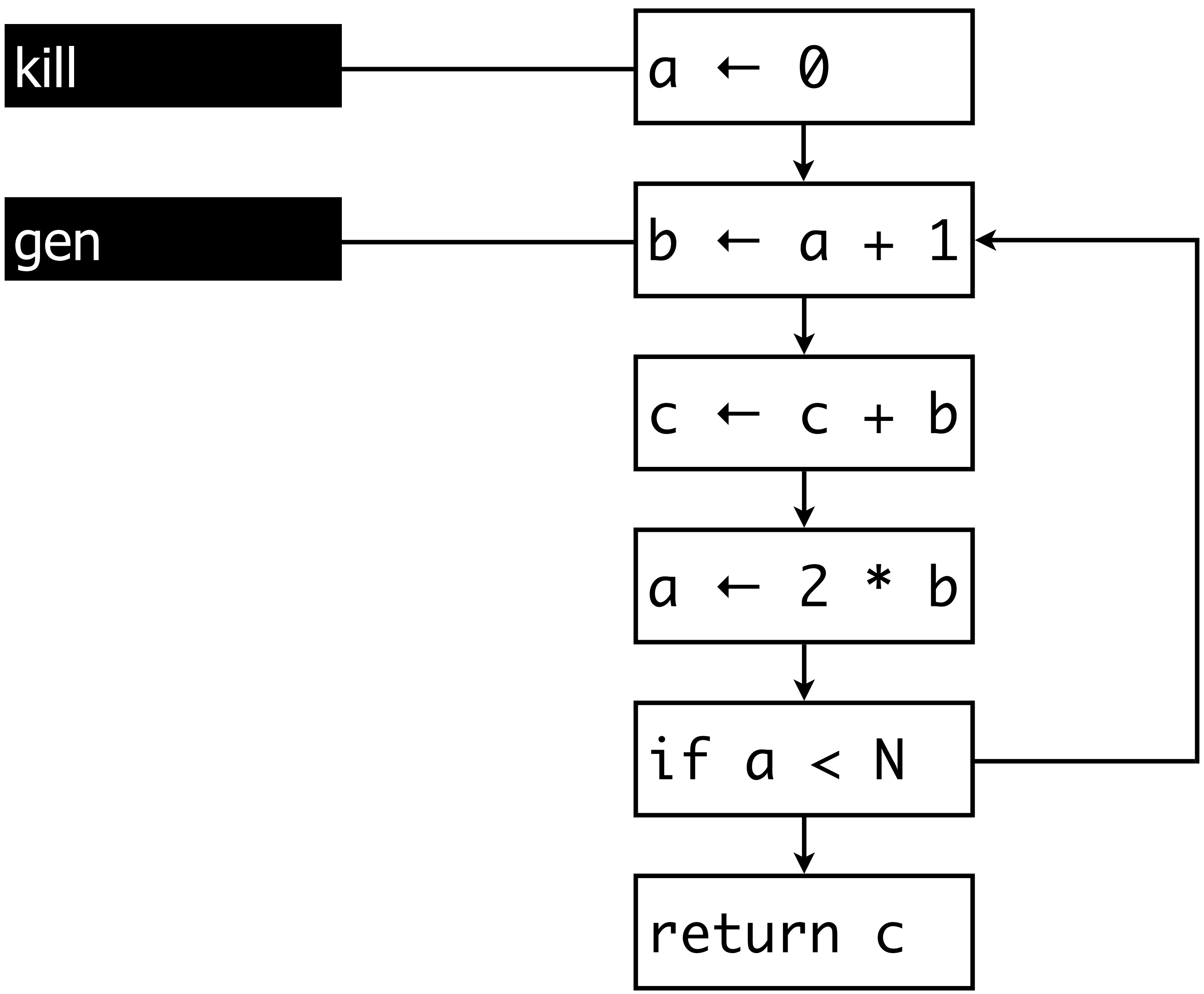

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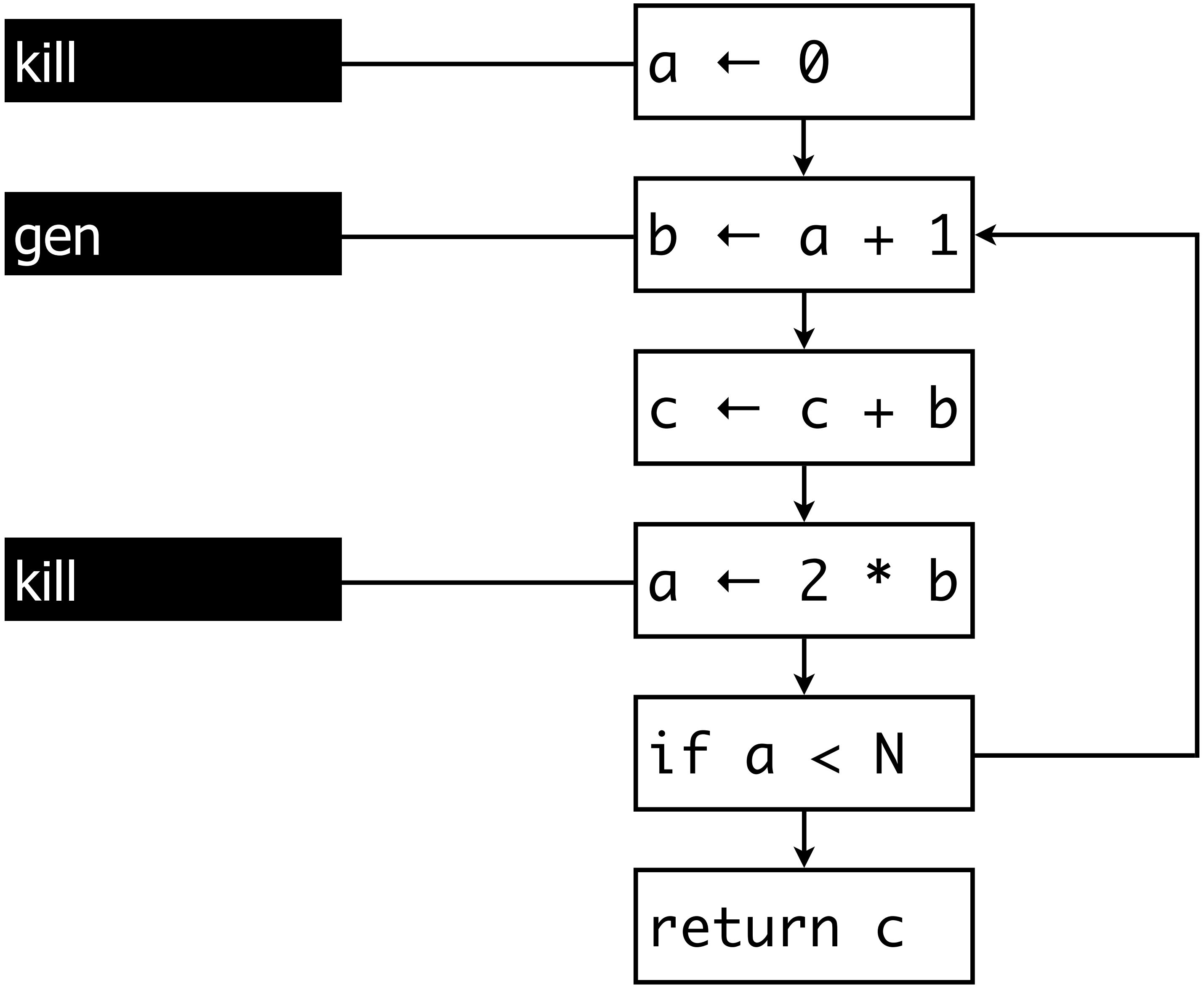

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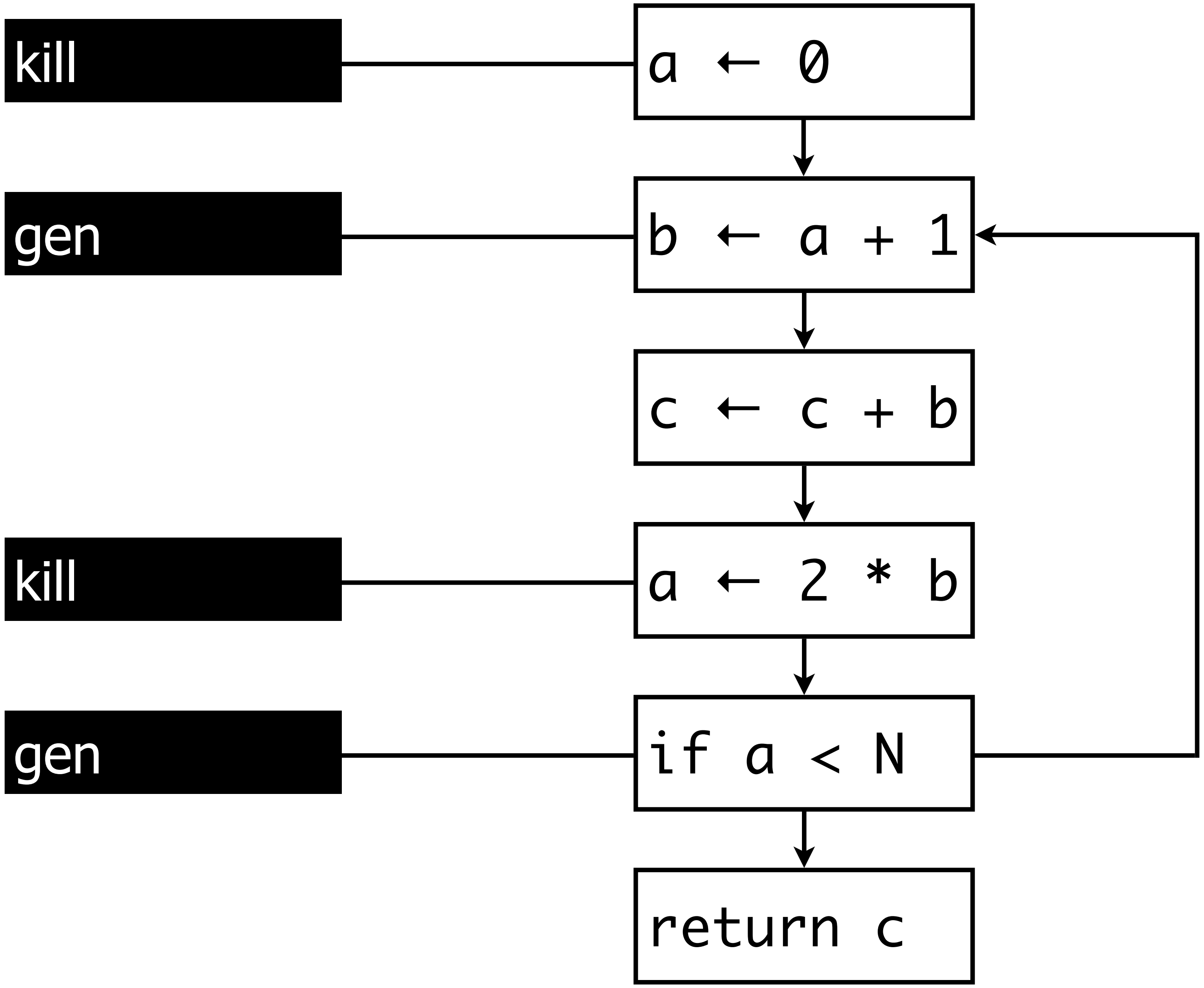

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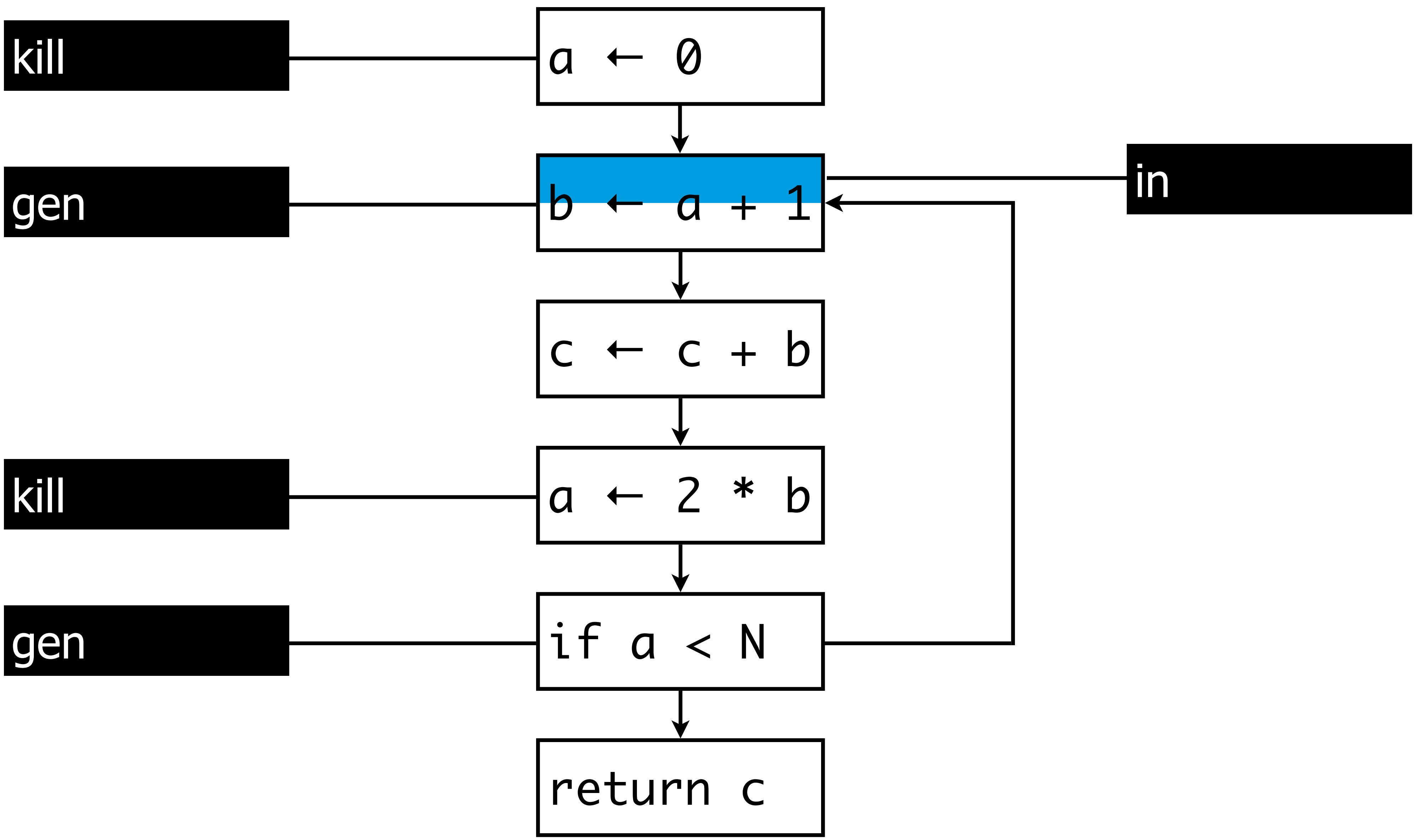

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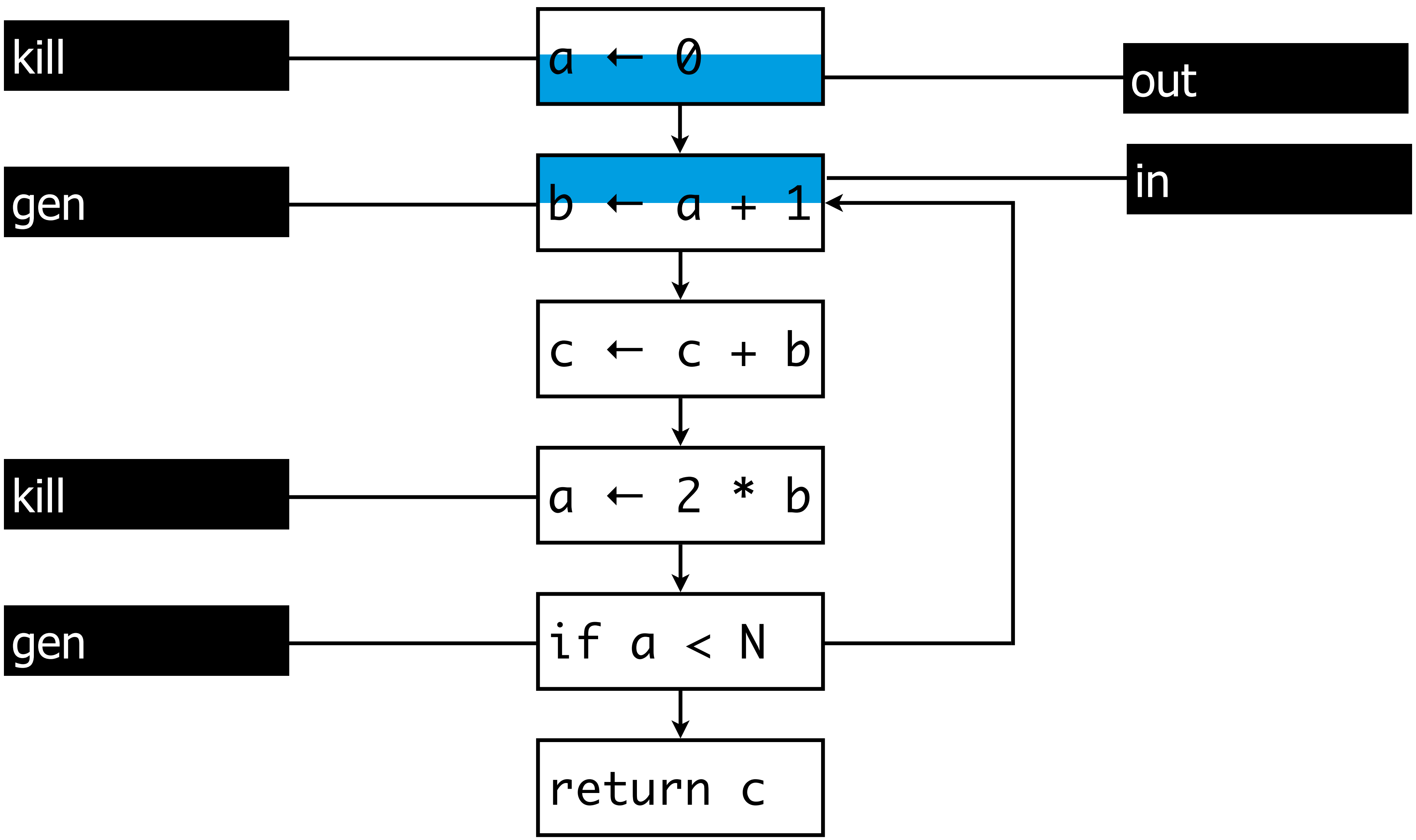

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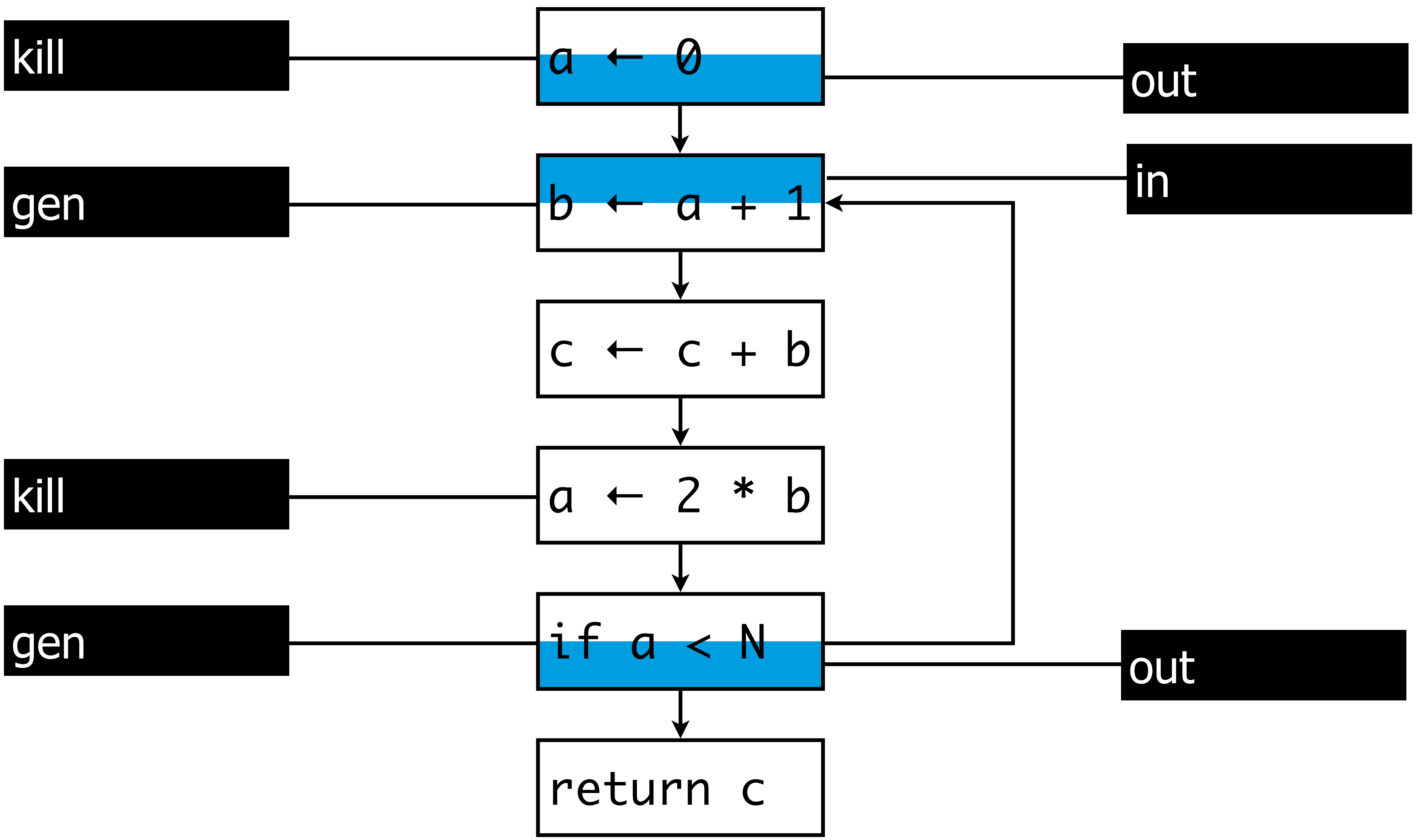
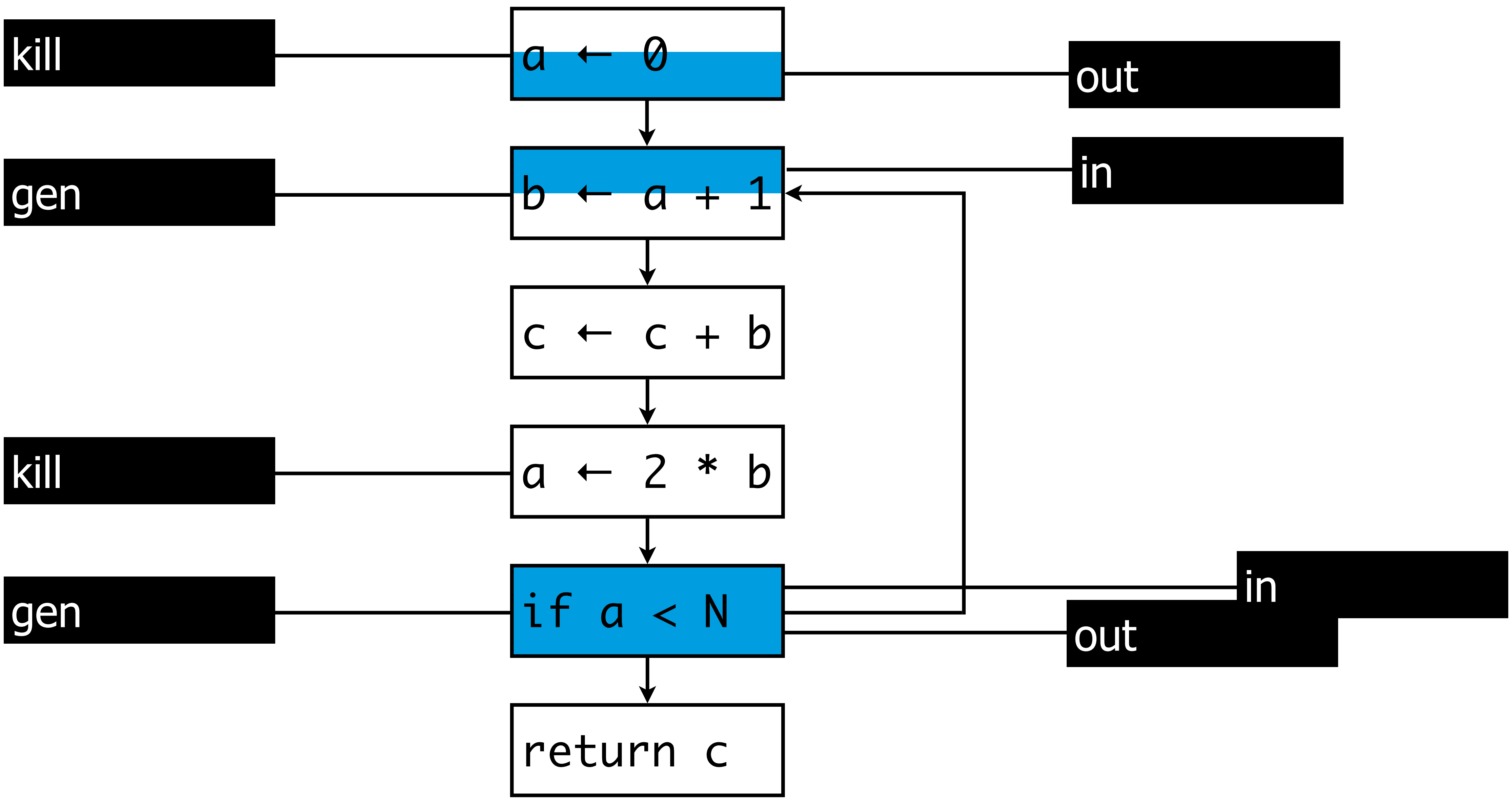
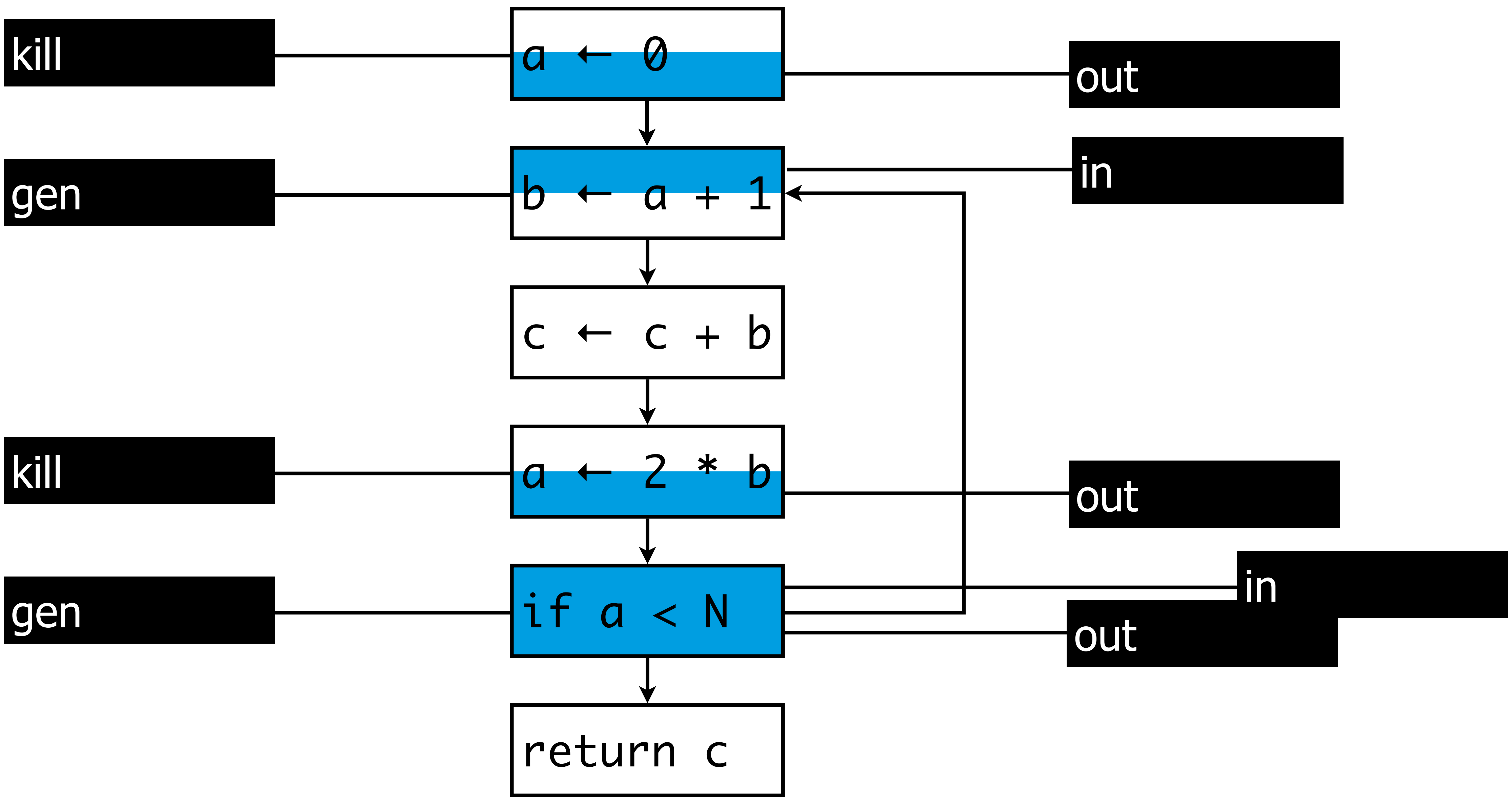


Liveness analysis
generalisation



Liveness analysis

generalisation



Liveness analysis

gen & kill

	gen	
$a \leftarrow b \oplus c$	{b,c}	
$a \leftarrow b$	{b}	
$a \leftarrow M[b]$	{b}	
$M[a] \leftarrow b$	{a,b}	
$f(a_1, \ldots, a_n)$	$\{a_1, \ldots, a_n\}$	
$a \leftarrow f(a_1, \ldots, a_n)$	$\{a_1, \ldots, a_n\}$	
goto L		
if $a \otimes b$	{a,b}	

Liveness analysis

gen & kill

	gen	kill
$a \leftarrow b \oplus c$	{b,c}	{a}
$a \leftarrow b$	{b}	{a}
$a \leftarrow M[b]$	{b}	{a}
$M[a] \leftarrow b$	{a,b}	
$f(a_1, \ldots, a_n)$	$\{a_1, \ldots, a_n\}$	
$a \leftarrow f(a_1, \ldots, a_n)$	$\{a_1, \ldots, a_n\}$	{a}
goto L		
if $a \otimes b$	{a,b}	

Liveness analysis
generalisation

$$in[n] = gen[n] \cup (out[n] - kill[n])$$

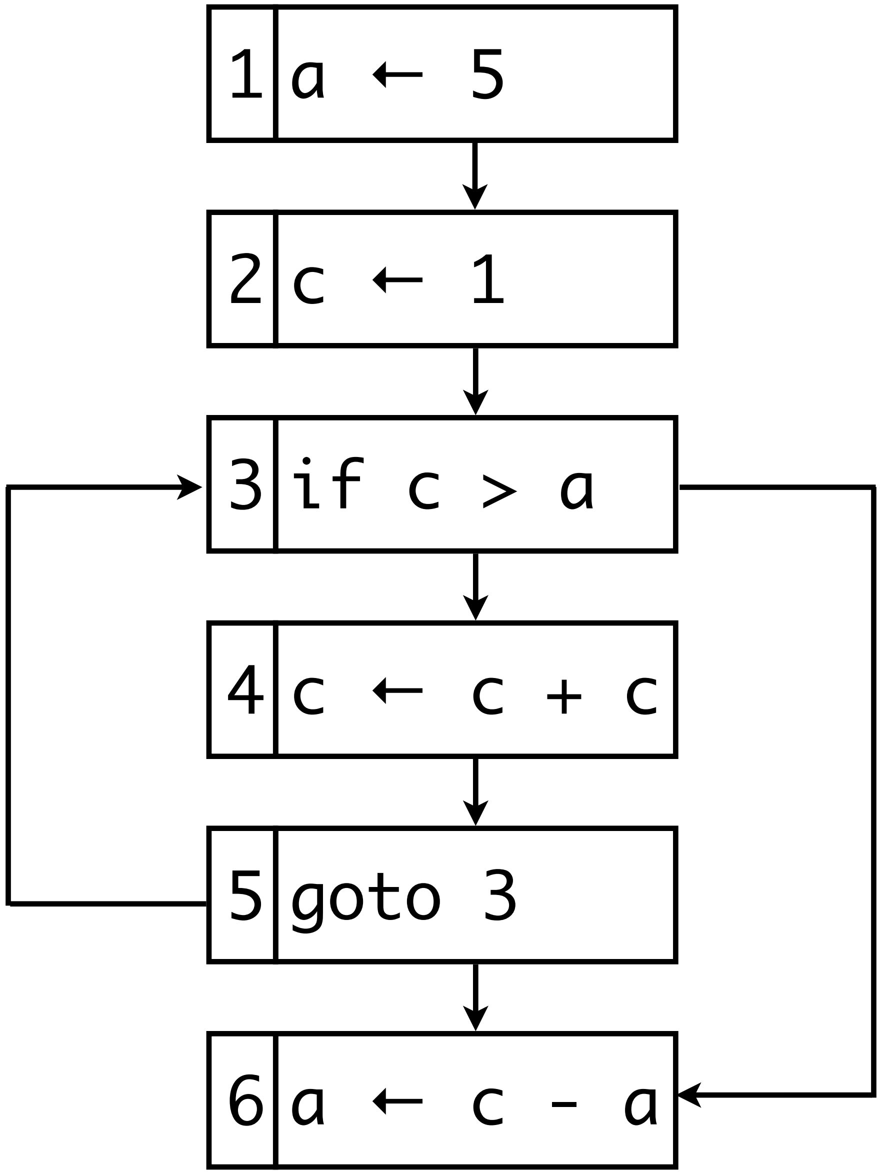
$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

coffee break

III

more analyses

More analyses
reaching definitions



Reaching definitions

gen & kill

	gen	
d: a ← b ⊕ c	{d}	
d: a ← b	{d}	
d: a ← M[b]	{d}	
M[a] ← b		
f(a_1, …, a_n)		
d: a ← f(a_1, …, a_n)	{d}	
goto L		
if a ⊗ b		

Reaching definitions

gen & kill

	gen	kill
d: a ← b ⊕ c	{d}	defs(a)-{d}
d: a ← b	{d}	defs(a)-{d}
d: a ← M[b]	{d}	defs(a)-{d}
M[a] ← b		
f(a_1, …, a_n)		
d: a ← f(a_1, …, a_n)	{d}	defs(a)-{d}
goto L		
if a ⊗ b		

Reaching definitions

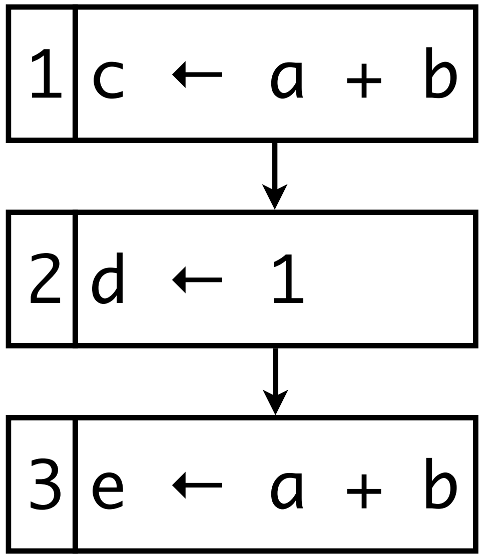
formalisation

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

More analyses

available expressions



Available expressions

gen & kill

	gen	
d: a ← b ⊕ c	{b⊕c}-kill	
d: a ← b		
d: a ← M[b]	{M[b]}-kill	
M[a] ← b		
f(a_1, …, a_n)		
d: a ← f(a_1, …, a_n)		
goto L		
if a ⊗ b		

Available expressions

gen & kill

	gen	kill
d: a ← b ⊕ c	{b⊕c}-kill	exps(a)
d: a ← b		
d: a ← M[b]	{M[b]}-kill	exps(a)
M[a] ← b		exps(M[_])
f(a_1, …, a_n)		exps(M[_])
d: a ← f(a_1, …, a_n)		exps(M[_]) ∪ exps(a)
goto L		
if a ⊗ b		

Available expressions

formalisation

$$in[n] = \bigcap_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

IV

optimisations

Dead code elimination

example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← 0
b ← a + 1
c ← c + b
return c
```

Constant propagation

example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← 0
b ← 0 + 1
c ← c + b
a ← 2 * b
return c
```

Copy propagation
example

```
a ← e
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← e
b ← e + 1
c ← c + b
a ← 2 * b
return c
```

Common subexpression elimination

example

$c \leftarrow a + b$

$d \leftarrow 1$

$e \leftarrow a + b$

$x \leftarrow a + b$

$c \leftarrow x$

$d \leftarrow 1$

$e \leftarrow x$

V

summary

Summary
lessons learned

liveness analysis

- intermediate language
- control-flow graphs
- definition & algorithm

more dataflow analyses

- reaching definitions
- available expressions

optimisations

Literature

learn more

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

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