

Exercise #5) Bipartite Graphs.

In One Direction:

If G is bipartite with vertex sets V_1 and V_2 , every step along the walk will take you from V_1 to V_2 or V_2 to V_1 . To end up where you started, you will have to take an even number of steps.

Conversely:

If every cycle of G is even, let v_0 be any vertex. For ~~every~~ each vertex v in the same component C_0 as v_0 , let $d(v)$ be the length of the shortest path from v_0 to v . We can color every vertex in C_0 whose distance from v_0 is even with red and color the other vertices of C_0 blue. We do this for each component of G . If G had any edge between two red vertices or between two blue vertices, it would have an odd cycle. Therefore, G is bipartite and the red and blue vertices are the two parts.