Exercise 3

for a set of real numbers $\{x_1, x_2, \ldots - x_n\}$
Greedy Solution:

1) Sort set in ascending order $x_1 \leq x_2 \leq \ldots \leq x_n$
2) Allow x_{min} be the smallest number in the set.
3) Place the closed interval at $[x, x+1]$
4) Remove all numbers that are not w/in $[x, x+1]$ from the sorted set.
5) Repeat steps 2-4 until the set is empty.

Running Time Analysis:

Step	Run Time
(1)	$O(n \ln n)$
(2) & (3)	$O(1)$ // returning a number in the set.
(4)	$O(n)$ // list is sorted so it will take $O(n)$ as a whole and $O(1)$ for each element.
Total	$O(n \lg n) + O(n) \Rightarrow O(n \lg n)$

Proof:
  Greedy Choice: One of the optimal solutions will have the
                 first unit-interval that starts at x_{min}
                 in the set.
  Proof: x_{min} // smallest number
         x_{min} should be covered by at least one of
         the unit intervals. For example x_{min} can be
         covered by $[y, y+1]$ in which $y \leq x_{min}$. Since
         there are clearly no numbers less than x_{min}, the
         interval $[y, y+1]$ can be moved to $[x_{min}, x_{min}+1]$
         without increasing the number of intervals.