STAT UN1201 – Chapter 3, pt 1

Prof. Joyce Robbins

Chapter 3

Discrete Random Variables and Probability Distributions

Sample Questions

- If your free throw percentage is .85, what is the probability that you will make 5 of your next 10 shots?

- If a coin is flipped 20 times, what's the probability of getting tails more than 15 times?

Random Variable

X = # of tails on three coin flips

X can be 0, 1, 2, or 3

Probability Distribution

assigns probabilities to every value of the random variable:

x	**p(x)**
0	.125
1	.375
2	.375
3	.125

$$\Sigma p(x) = 1$$

Bernoulli Random Variable

two values: 0 and 1

$$Y = \begin{cases} 0 & miss\, free\ throw \\ 1 & make\, free\ throw \end{cases}$$

parameter:

p = probability of one event 1 - p = probability of the other

family of Bernoulli random variables: different p's

Probability Mass Function (PMF)

$$p(x) = P(X = x)$$

$$p(0) = P(X = 0) = .1$$

$$p(1) = P(X = 1) = .15$$

$$p(2) = P(X = 2) = .35$$

$$p(3) = P(X = 3) = .4$$

Cumumulative Distribution Function (CDF)

$$F(x) = P(X \leq x)$$

$$F(0) = P(X \leq 0) = .1$$

$$F(1) = P(X \leq 1) = .25$$

$$F(2) = P(X \leq 2) = .6$$

$$F(3) = P(X \leq 3) = 1$$

X = # of telephone lines in use

x	**p(x)**
0	**0.10**
1	**0.15**
2	**0.20**
3	**0.25**
4	**0.20**
5	**0.06**
6	**0.04**

(based on p. 107, #13)

1. Determine the cumulative distribution function (cdf)

EXERCISE (cont.)

2. Calculate the probability of each: (Solve using the cdf)

   {a} at most 3 lines are in use

   {b} fewer than 3 lines are in use

3. What is the probability that between 2 and 5 lines (inclusive) are in use? (Solve using the cdf)

EXERCISE

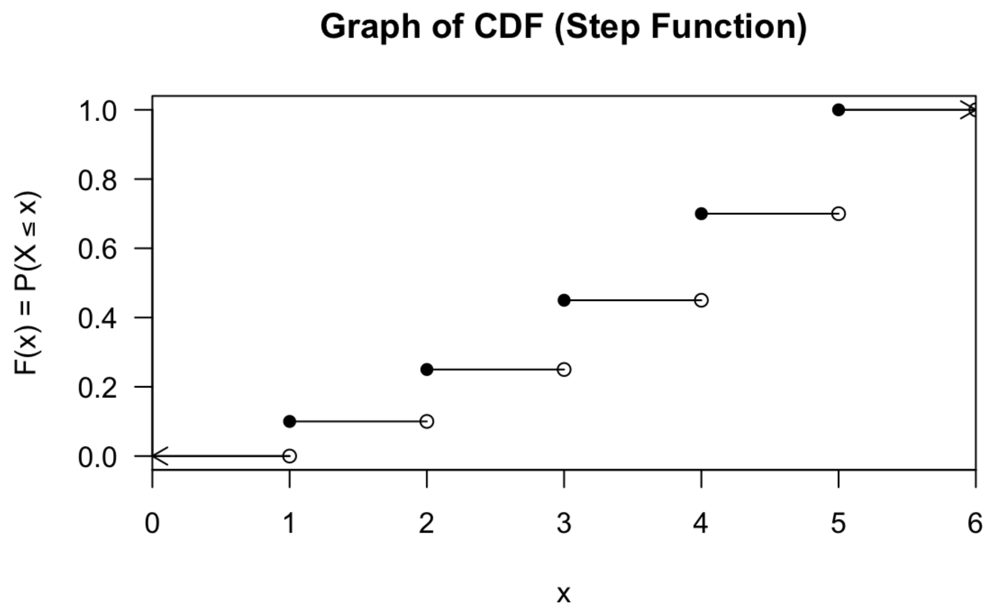
X is a discrete random variable with the probability mass function $p(x) = .05x + .05$ for integers between 1 and 5 inclusive, and 0 otherwise.

{a} Is $p(x)$ a legitimate PMF?

{b} Write inequalities to show the value of $F(x)$ for all values of x.

{c} Graph the step function $F(x)$.

Graph of F(x)



Expected value (3.3)

You play a game in which 70% of the time you win $5 and 30% of the time you win $10. How much would you expect to win on average per game?

X = number of tails on four coin flips

What is the *expected value* of X?

(or: What would we expect to happen?)

Formula for expected value

$$E(X) = \mu_X = \sum x \cdot p(x)$$

(for all possible values of x)

Expected value

Bernoulli RV

$$X = \begin{cases} 0 & \textit{something does not happen} \\ 1 & \textit{something happens} \end{cases}$$

$p(0) = 1 - p$ (or q)

$p(1) = p$

What is the expected value of X?

Expected value of a function

$$E[h(x)] = \sum h(x) \cdot p(h(X)) = \sum h(x) \cdot p(x)$$

Expected value of a linear function

$$h(X) = aX + b$$

$$E(aX + b) = a \cdot E(X) + b$$

Variance of X

Population variance = sum of squared deviations from mean / n

$V(random\ variable\ X) = \sum (X - \mu)^2 \cdot p(x)$

$E(X) = \mu$

EXERCISE: Find the variance of X

$$V(X) = \sum (X - \mu)^2 \cdot p(x)$$

x	p(x)	x * p(x)	(x - u)^2	(x - u)^2 * p(x)
1	0.10	0.1		
2	0.15	0.3		
3	0.20	0.6		
4	0.25	1.0		
5	0.30	1.5		

Step 1: $\mu = \sum x \cdot p(x) = 3.5$

EXERCISE: Find the variance of X

$$V(X) = \sum (X - \mu)^2 \cdot p(x)$$

x	p(x)	x * p(x)	(x - u)^2	(x - u)^2 * p(x)
1	0.10	0.1	6.25	0.6250
2	0.15	0.3	2.25	0.3375
3	0.20	0.6	0.25	0.0500
4	0.25	1.0	0.25	0.0625
5	0.30	1.5	2.25	0.6750

$$V(X) = \sum (x - \mu)^2 \cdot p(x) = \mathbf{1.75}$$

Another formula for variance

$V(X) = \sum (X - \mu)^2 \cdot p(x) \qquad E(X) = \sum x \cdot p(x)$

$V(X) = E(X - \mu)^2$

$= E(X^2 - 2X\mu +$

$mu^2)$

$= E(X^2) - 2E(X)\mu + \mu^2$

$= E(X^2) - 2\mu^2 + \mu^2$

$= E(X^2) - \mu^2 = E(X^2) - [E(X)]^2$

Using the alternate variance formula

$$= E(X^2) - \mu^2 = E(X^2) - [E(X)]^2$$

x	p(x)	x * p(x)	x^2	x^2 * p(x)
1	0.10	0.1		
2	0.15	0.3		
3	0.20	0.6		
4	0.25	1.0		
5	0.30	1.5		

Variance of a Bernoulli RV

$E(X) = ?$

$V(X) = ?$

Variance and sd of a linear function of X

$$V(aX + b) = a^2 \cdot V(X)$$

$$\sigma_{aX+b} = |a| \cdot \sigma_X$$

$$\sigma_{aX} = |a| \cdot \sigma_X$$

$$\sigma_{X+b} = \sigma_X$$