# STAT UN1201 – Chapter 1

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#### Waitlist

- 1. The waitlist moves in order as places open up.
- 2. Course materials are available here during change of program period: 1201.info
- 3. It is strongly advised to keep up with the material if you are trying to get in the class.

EVERYONE: Once you've made a decision not to take the class, please be considerate and drop it from your schedule.

## Very brief course outline

- Descriptive statistics (Chapter 1)
- Probability (Chapters 2 5)
- Inferential statistics (Chapters 6 9, 12 13)

## Descriptive statistics

- numerical summaries: mean, median, etc.
- graphical summaries

## Anscombe's Quartet

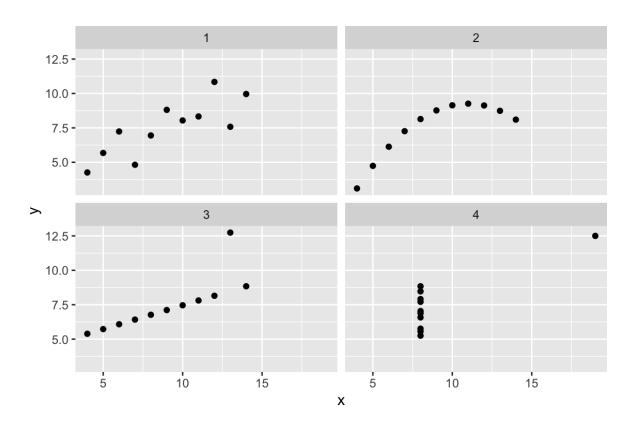
<b>x1</b>	<b>x2</b>	х3	<b>x4</b>	<b>y1</b>	<b>y2</b>	у3	у4
10	10	10	8	8.04	9.14	7.46	6.58
8	8	8	8	6.95	8.14	6.77	5.76
13	13	13	8	7.58	8.74	12.74	7.71
9	9	9	8	8.81	8.77	7.11	8.84
11	11	11	8	8.33	9.26	7.81	8.47
14	14	14	8	9.96	8.10	8.84	7.04
6	6	6	8	7.24	6.13	6.08	5.25
4	4	4	19	4.26	3.10	5.39	12.50
12	12	12	8	10.84	9.13	8.15	5.56
7	7	7	8	4.82	7.26	6.42	7.91
5	5	5	8	5.68	4.74	5.73	6.89

### Anscombe's Quartet, numerical summaries

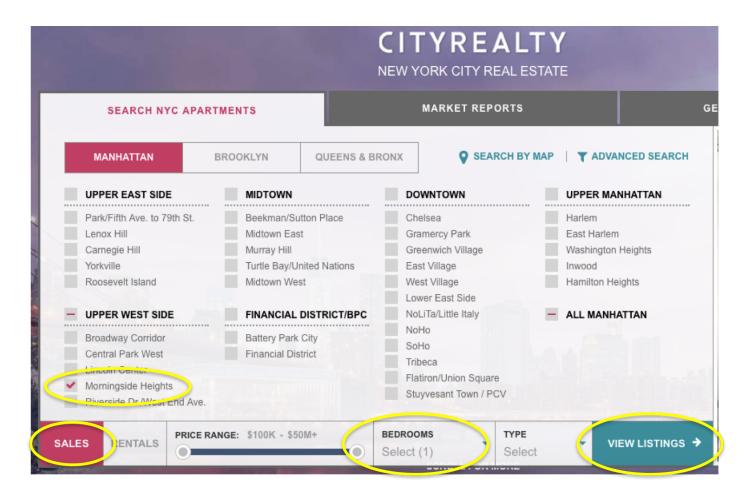
For each of the four data sets:

- Number of observations (n) = 11
- Mean of the *x*'s  $(\bar{x}) = 9.0$
- Mean of the *y*'s  $(\bar{y}) = 7.6$
- Equation of the regression line: y = 3 + 0.5x

# Anscombe's Quartet, graphical analysis



## Collecting data



# Collecting data

TIDE ALL PROTOSY SHOW ALL P	CE ALL PROTOS* OROW ALL PROTOS							
ADDRESS	TYPE	NEIGHBORHOOD	BEDS / BATH / SIZE	PRICE	PRICE/FT <sup>2</sup>	LISTED		
395 Riverside Drive, #4E 📜	CO-OP	Morningside Heights	1 bd / 1 ba	\$799K	-	Feb 22, 2016		
390 Riverside Drive, #4G	CO-OP	Morningside Heights	1 bd / 1 ba / 800 ft <sup>2</sup>	\$745K	\$931	Jul 19, 2016		
380 Riverside Drive, #8L	CO-OP	Morningside Heights	1 bd / 1 ba / 700 ft <sup>2</sup>	\$725K	\$1,036	Sep 06, 2016		
535 West 110th Street, #6B	CO-OP	Morningside Heights	1 bd / 1 ba / 750 ft <sup>2</sup>	\$725K	\$967	Sep 07, 2016		
535 West 110th Street, #6A 🗯	CO-OP	Morningside Heights	1 bd / 1 ba / 750 ft <sup>2</sup>	\$699K(-4%~)	\$933	Apr 26, 2016		
395 Riverside Drive, #2D 📜	CO-OP	Morningside Heights	1 bd / 1 ba / 900 ft <sup>2</sup>	\$699K	\$777	Mar 03, 2016		
528 West 111th Street, #24 📜	CO-OP	Morningside Heights	1 bd / 1 ba	\$675K(-6.9%~)	-	Apr 15, 2015		
545 West 111th Street, #9N 🗯	CO-OP	Morningside Heights	1 bd / 1 ba / 650 ft <sup>2</sup>	\$665K(-9.5%~)	\$1,023	Mar 30, 2016		
380 Riverside Drive, #6B 📂	CO-OP	Morningside Heights	1 bd / 1 ba	\$599K	-	Oct 07, 2015		
80 La Salle Street, #20C 📂	CO-OP	Morningside Heights	1 bd / 1 ba	\$545K	-	May 19, 2016		
609 West 114th Street, #36 📜	CO-OP	Morningside Heights	1 bd / 1 ba / 600 ft <sup>2</sup>	\$535K	\$892	Jun 22, 2016		
80 La Salle Street, #21B 📂	CO-OP	Morningside Heights	1 bd / 1 ba / 725 ft <sup>2</sup>	\$535K	\$738	May 04, 2016		
80 La Salle Street, #14D 📂	CO-OP	Morningside Heights	1 bd / 1 ba	\$529K	-	Apr 14, 2016		
175 Claremont Avenue, #3	CO-OP	Morningside Heights	1 bd / 1 ba	\$499K(-9.3%~)	-	Mar 30, 2016		
3117 Broadway, #56	CO-OP	Morningside Heights	1 bd / 1 ba / 500 ft <sup>2</sup>	\$450K	\$900	Jun 28, 2016		

#### TURED NEW DEVELOPMENTS





510 West 123rd Street, #38 📜	CO-OP	Morningside Heights	1 bd / 1 ba / 700 ft <sup>2</sup>	\$450K	\$643	Jul 02, 2016
3115 Broadway, #43 🏴	CO-OP	Morningside Heights	1 bd / 1 ba / 650 ft <sup>2</sup>	\$425K	\$654	Apr 18, 2016
3115 Broadway, #42	CO-OP	Morningside Heights	1 bd / 1 ba / 650 ft <sup>2</sup>	\$379K(-5%~)	\$583	Mar 28, 2016

# Cleaning / organizing data

	Α	В	С	D	E	F	G	Н	
1		ADDRESS	TYPE	NEIGHBORH	BEDS / BATH	PRICE	PRICE/FT2	LISTED	
2	+								
3	3115 Broadw	CO-OP							
4	Morningside	1 bd / 1 ba /	\$379K	\$583	28-Mar-16				
5	+								
6	3115 Broadw	CO-OP							
7	Morningside	1 bd / 1 ba /	\$425K	\$654	18-Apr-16				
8	+								
9	510 West 12	CO-OP							
10	Morningside	1 bd / 1 ba /	\$450K	\$643	2-Jul-16				
11	+								
12	3117 Broadw	CO-OP						1	
13	Morningside	1 bd / 1 ba /	\$450K	\$900	28-Jun-16				
14	+								
15	175 Claremo	CO-OP							
16	Morningside	1 bd / 1 ba	\$499K	-	30-Mar-16				
17	+								
18	80 La Salle St	CO-OP							
19	Morningside	1 bd / 1 ba	\$529K	-	14-Apr-16				

# Cleaning / organizing data

	A	В	С	D
1	Address		Price	IntPrice
2	3115 Broadway, #42	1 bd / 1 ba / 650 ft2	\$379K	379
3	3115 Broadway, #43	1 bd / 1 ba / 650 ft2	\$425K	425
4	3117 Broadway, #56	1 bd / 1 ba / 500 ft2	\$450K	450
5	510 West 123rd Street, #38	1 bd / 1 ba / 700 ft2	\$450K	450
6	175 Claremont Avenue, #3	1 bd / 1 ba	\$499K	499
7	80 La Salle Street, #14D	1 bd / 1 ba	\$529K	529
8	609 West 114th Street, #36	1 bd / 1 ba / 600 ft2	\$535K	535
9	80 La Salle Street, #21B	1 bd / 1 ba / 725 ft2	\$535K	535
0	80 La Salle Street, #20C	1 bd / 1 ba	\$545K	545
1	380 Riverside Drive, #6B	1 bd / 1 ba	\$599K	599
2	545 West 111th Street, #9N	1 bd / 1 ba / 650 ft2	\$665K	665
3	528 West 111th Street, #24	1 bd / 1 ba	\$675K	675
4	535 West 110th Street, #6A	1 bd / 1 ba / 750 ft2	\$699K	699
15	395 Riverside Drive, #2D	1 bd / 1 ba / 900 ft2	\$699K	699
16	380 Riverside Drive, #8L	1 bd / 1 ba / 700 ft2	\$725K	725
17	535 West 110th Street, #6B	1 bd / 1 ba / 750 ft2	\$725K	725
18	390 Riverside Drive, #4G	1 bd / 1 ba / 800 ft2	\$745K	745
9	395 Riverside Drive, #4E	1 bd / 1 ba	\$799K	799
20				
1				

## Types of data

- Quantitative
  - Discrete (finite or can be listed in an infinite sequence)
  - Continuous (entire continuum of possible values)
- Qualitative (categorical)

## Visualizing one-dimensional continuous data

- stem-and-leaf plot
- histogram
  - frequency
  - relative frequency
  - density
- boxplot

## Stem-and-leaf plot

```
## [1] 10 32 33 37 41 43 44 44 52 55 55 56 59 61 62

##

## The decimal point is 1 digit(s) to the right of the |

##

## 1 | 0

## 2 |

## 3 | 237

## 4 | 1344

## 5 | 25569

## 6 | 12
```

## Stem-and-leaf plot

#### Prices:

```
## [1] 379 425 450 450 499 529 535 535 545 599 665
## [12] 675 699 699 725 725 745 799
```

Prices, in 1,000s, rounded to nearest 10,000:

```
## [1] 380 430 450 450 500 530 540 540 550 600 670
## [12] 680 700 700 730 730 750 800
```

## Stem-and-leaf plot

Prices:

```
## [1] 379 425 450 450 499 529 535 535 545 599 665
## [12] 675 699 699 725 725 745 799
```

Prices, in 1,000s, rounded to nearest 10,000:

```
## [1] 380 430 450 450 500 530 540 540 550 600 670
## [12] 680 700 700 730 730 750 800
```

```
##
## The decimal point is 2 digit(s) to the right of the |
##
## 3 | 8
## 4 | 355
## 5 | 03445
## 6 | 078
## 7 | 00335
## 8 | 0
```

## Stem and leaf plot

Total Fertility Rate (average births per woman)

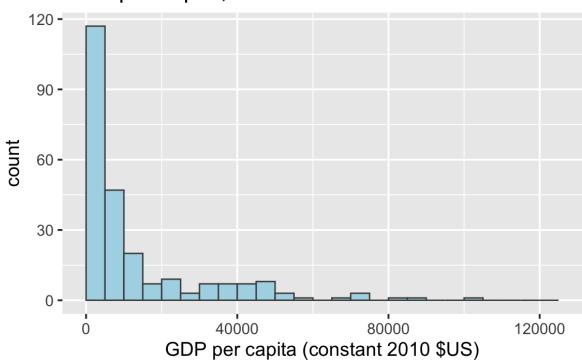
## Stem and leaf plot, change the length (scale)

#### **Total Fertility Rate**

```
##
##
    The decimal point is at the |
##
##
    1 | 33333333444444
       2 | 000000011112222233333334444444
    2 | 5555555666666679999
    3 | 01112233333344
    3 | 567889
##
    4 | 001111224
    4 | 5555667779999
    5 | 00012223333
    5 | 55788
    6 | 012344
##
    7 |
##
    7 | 6
```

## Frequency histogram

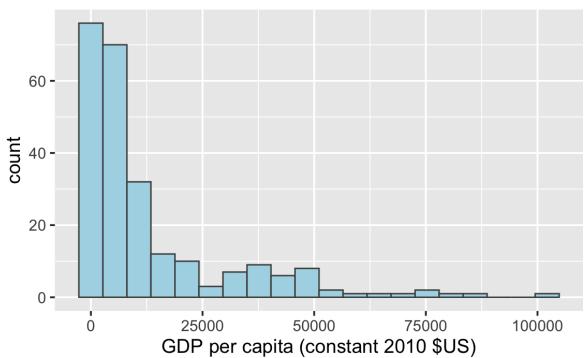




Source: World Bank

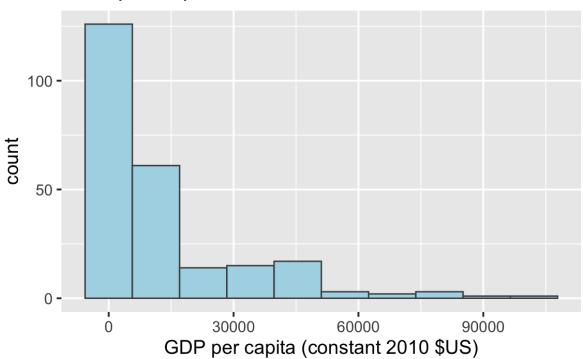
# Change number of bins to 20





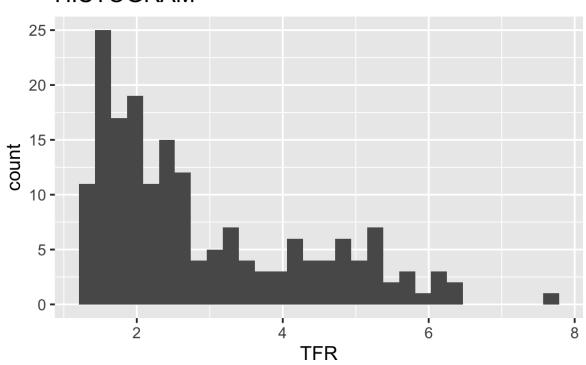
## Change number of bins to 10





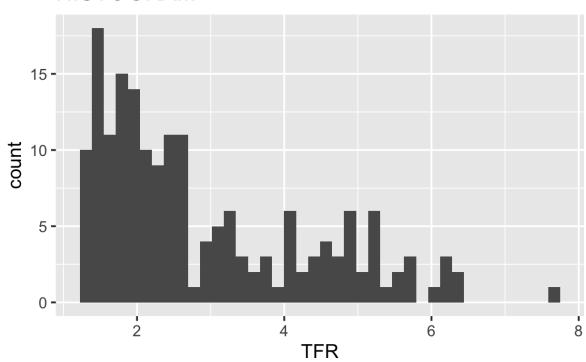
# Total fertility rate

#### HISTOGRAM



# Change the number of bins to 40

#### **HISTOGRAM**



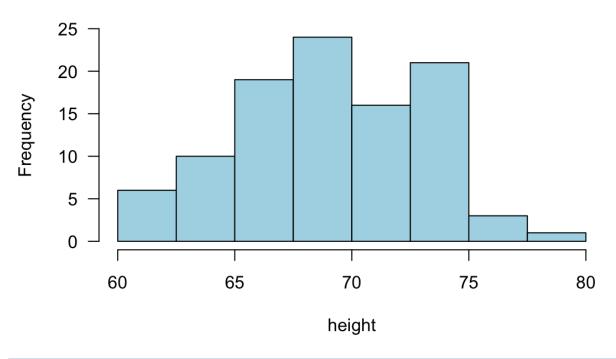
# Shapes of histograms

- Unimodal
- Bimodal
- Skewed left
- Skewed right

### Discrete data

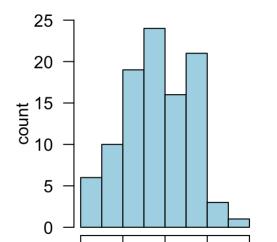
#### Discrete data

#### Heights of 100 college students, in inches



### Discrete data histogram

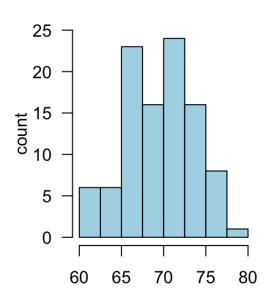
#### Histogram of height



65

60

#### Histogram of height



**RIGHT CLOSED, LEFT OPEN** 

70

75

80

**RIGHT OPEN, LEFT CLOSED** 

#### **EXERCISE**

Draw a histogram of the asking prices for one-bedroom apartments in Morningside Heights (prices in thousands of \$)

Data source: cityrealty.com, 9/13/2016

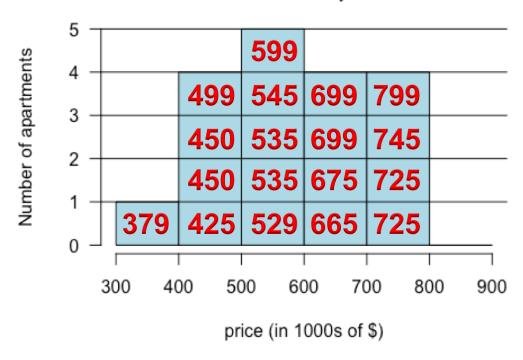
379, 425, 450, 450, 499, 529, 535, 535, 545, 599, 665, 675, 699, 699, 725, 725, 745, 799

### Drawing histograms

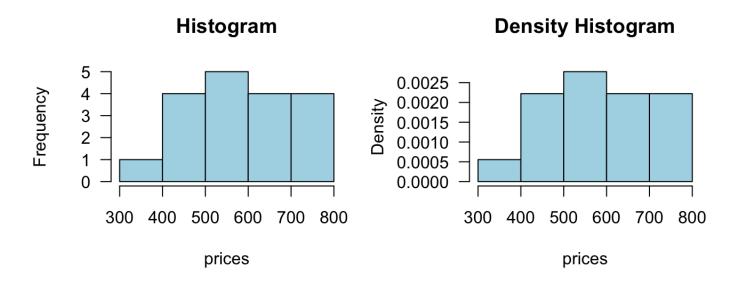
Asking prices for one-bedroom apartments in Morningside Heights, in thousands Data source: cityrealty.com, 9/13/2016

379, 425, 450, 450, 499, 529, 535, 535, 545, 599, 665, 675, 699, 699, 725, 725, 745, 799

#### Histogram of Morningside Heights One-Bedroom Apt. Prices



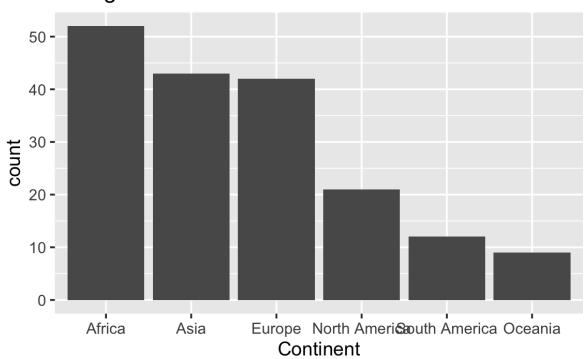
## Density histogram



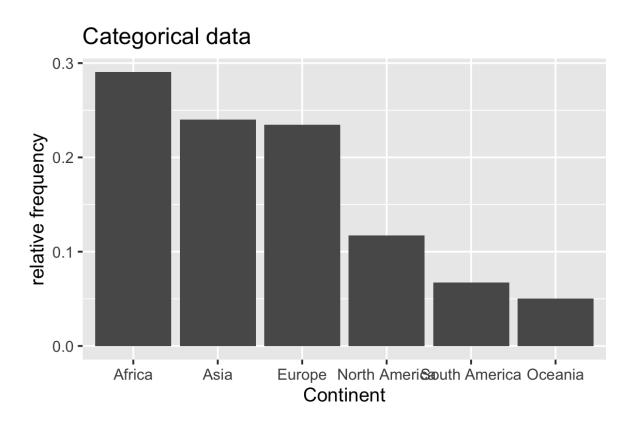
Class	Frequency	Rel. Frequency	Density	
(300, 400]	1	.056	.00056	
(400, 500]	4	.222	.00222	
(500, 600]	5	.278	.00278	
(600, 700]	4	.222	.00222	
(700, 800]	4	.222	.00222	

# Frequency bar chart

#### Categorical data



## Relative frequency bar chart



## Five number summary

- 1. min
- 2. lower fourth
- 3. median
- 4. upper fourth
- 5. max

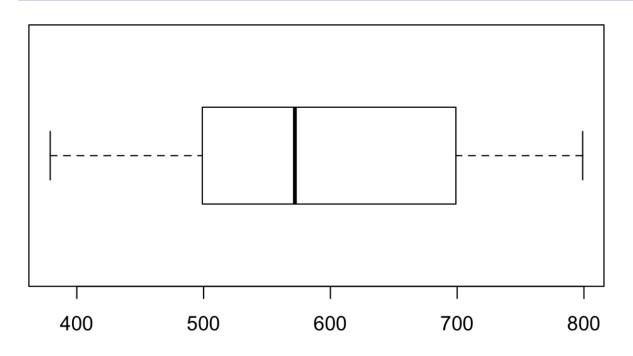
fivenum(prices)

## [1] 379 499 572 699 799

## Boxplot (no outliers)

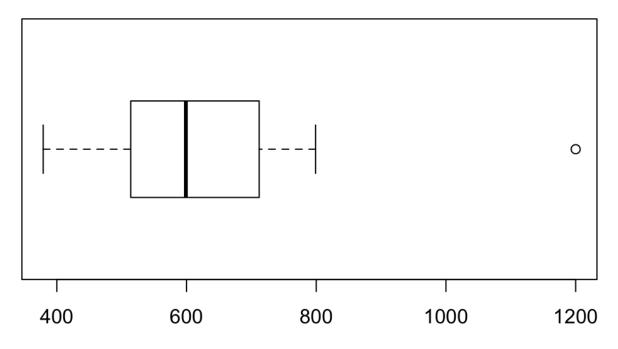
379, 425, 450, 450, 499, 529, 535, 535, 545, 599, 665, 675, 699, 699, 725, 725, 745, 799

## [1] 379 499 572 699 799



### Boxplot with outlier

```
## [1] 379 425 450 450 499 529 535 535 545
## [10] 599 665 675 699 699 725 725 745 799
## [19] 1200
```



Box shows lower fourth, median, upper fourth

Whiskers show data within 1.5 times the fourth spread ( $f_s$ ) of the nearest fourth

$$f_s = 712 - 514 = 198$$

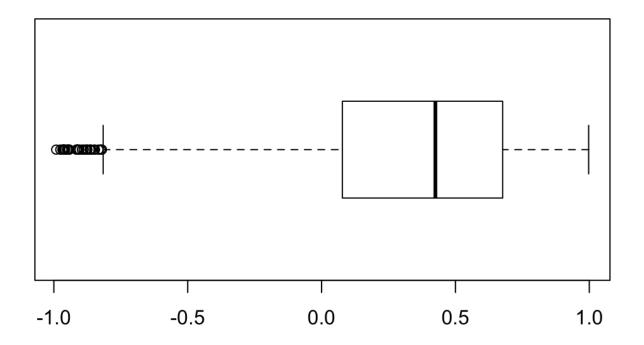
$$1.5f_s = 297$$

lower outlier boundary: lower fourth -  $1.5f_s$  \$6514 - 297 = 217

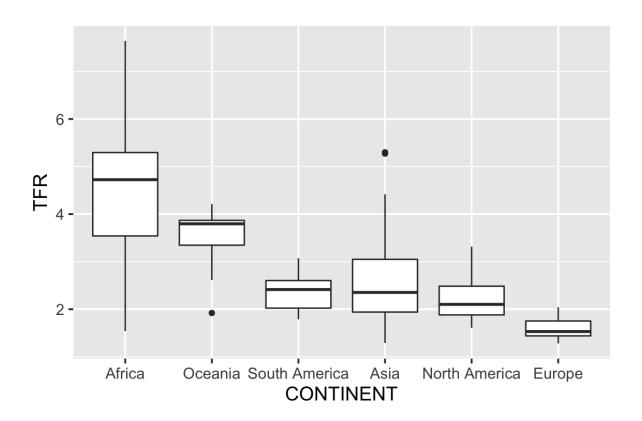
upper outlier boundary: upper fourth +  $1.5f_s$  = 712 + 297 = 1009

Add outliers and then draw whiskers to the lowest and highest data values that are *not* outliers.

# Boxplot with outliers



# Multiple box plots



### **EXERCISE**

(based on #72, p. 49)

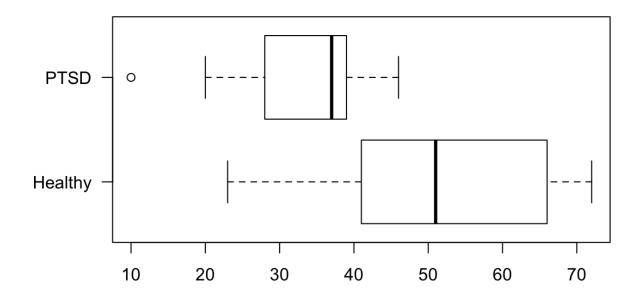
Data on a receptor binding measure:

PTSD: 10, 20, 25, 28, 31, 35, 37, 38, 38, 39, 39, 42, 46

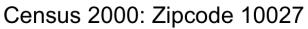
Healthy: 23, 39, 40, 41, 43, 47, 51, 58, 63, 66, 67, 69, 72

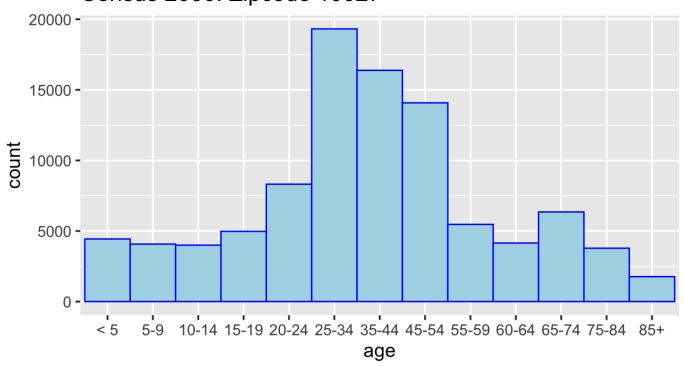
Draw a comparative boxplot.

# Comparative boxplot



## Histogram: what's wrong?



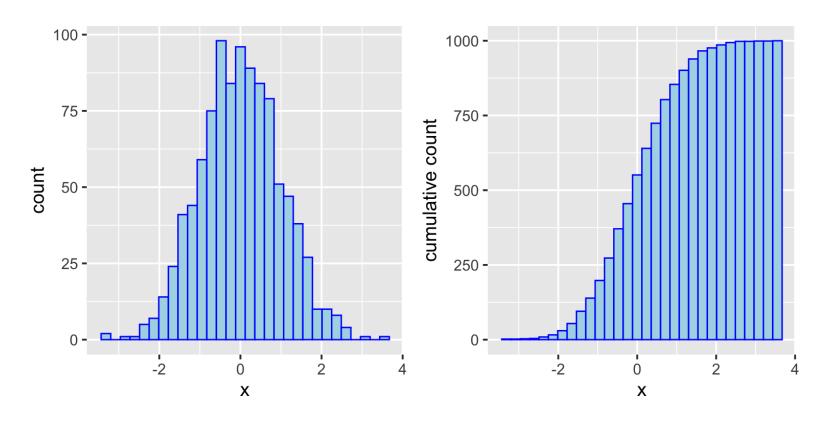


## Frequency histogram





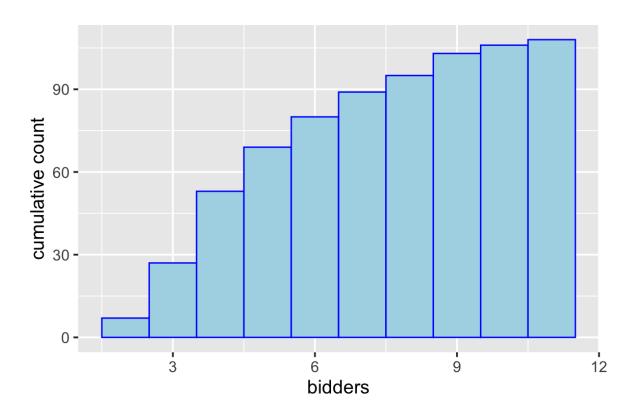
Class		Freq	CumulativeFreq
30	0-400	1	1
40	0-500	4	5
50	0-600	5	10
60	0-700	4	14
70	0-800	4	18



### **EXERCISE**

(based on #17, p. 26) Construction industry data:

bidders contracts			
2	7	a) What proportion of the contracts involved at most five bidders?	
3	20		
4	26	b) What proportion of the contracts involved between five and ten bidders, inclusive?	
5	16		
6	11	c) Draw a cumulative frequency histogram.	
7	9		
8	6		
9	8		
10	3		
11	2		



### Sample and population means

population mean:  $\mu$  = sum of N population values / N

sample mean: 
$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

population median:  $\widetilde{\mu}$ 

sample median:  $\widetilde{x}$ 

## Measures of variability

#### deviations from the mean

$$x_1 - \overline{x}$$
,  $x_2 - \overline{x}$ , etc.

Data: 3, 8, 11, 14

Mean: 9

value deviation deviation<sup>2</sup>

3

-6

36

8

-1

1

11

2

4

25

14

5

Sum of squared deviations

 $S_{xx}$ : 36 + 1 + 4 + 25 = 66

**Population variance** 

$$\sigma^2 = 66/4 = 16.5$$

$$\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2 / N$$

## Sample variance

#### **Sum of squared deviations:**

$$S_{xx}$$
: 36 + 1 + 4 + 25 = 66

#### Sample variance:

$$s^2 = 66 / 3 = 22$$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

#### Why n-1?

Short answer: using **n** would result in an underestimation, since the values in the sample are closer to the sample mean than to the true population mean (which we don't know)

### Standard deviation

### **Square root of variance**

- Population s.d. =  $\sqrt{\sigma^2}$
- Sample s.d. =  $\sqrt{s^2}$
- same units as original values

### EXERCISE (p. 47, #62)

Consider the following information on ultimate tensile strength  $(lb/in^2)$  for a sample of n = 4 hard zirconium copper wire specimens:

```
\overline{x} = 76,831

s = 180

smallest x_i = 76,683

largest x_i = 77,048
```

Set up equations to determine the values of the two middle sample observations. *Do not solve.*