Definition. A **relation** R from A to B is a subset of $A \times B$.

$$R \subseteq A \times B$$

We say that x is *related* to y by R, and write xRy, if and only if

$$(x, y) \in R$$

We refer to set A as the **domain** and set B as the **co-domain** of the relation.

Definition. A **relation** R on a set A is a subset of $A \times A$.

$$R \subseteq A \times A$$

Definition. The **inverse** of a **relation** R, denoted as R^{-1}, is defined as

$$R^{-1} := \{(y, x) \in B \times A \mid (x, y) \in R\}$$

Definition. A relation R is **reflexive**, if and only if

$$\forall x \in A, \quad xRx$$

Definition. A relation R is **irreflexive**, if and only if

$$\forall x \in A, \quad x \not R x$$

Definition. A relation R is **symmetric**, if and only if

$$\forall x, y \in A, \quad xRy \to yRx$$

Definition. A relation R is **asymmetric**, if and only if

$$\forall x, y \in A, \quad xRy \to y \not R x$$

Definition. A relation R is **anti-symmetric**, if and only if

$$\forall x, y \in A, \quad xRy, yRx \rightarrow x = y$$

Definition. A relation R is **transitive**, if and only if

$$\forall x, y, z \in A, \quad xRy, yRz \rightarrow xRz$$

Definition. A relation R is **intransitive**, if and only if

$$\forall x, y, z \in A, \quad xRy, yRz \rightarrow x \not R z$$

Definition. A relation R is **strongly connected**, if and only if

$$\forall x, y \in A, \quad xRy \lor yRx$$

Definition. A relation R is **connected**, if and only if

$$\forall x, y \in A, \quad x \neq y \rightarrow xRy \lor yRx$$

Definition. An **equivalence relation** $\equiv$ is a relation that is reflexive, symmetric, and transitive.

Definition. Suppose A is a set and R is an equivalence relation on A. For each element a in A, the **equivalence class** of a, denoted $[a]$, is defined as

$$[a] := \{x \in A \mid xRa\}$$

Definition. A **total order** $\leq$ is a relation that is reflexive, anti-symmetric, transitive, strongly connected.

Definition. A **strictly total order** $<$ is a relation that is irreflexive, anti-symmetric, transitive, connected.