Proof of Inter-Window Semantics

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For our mapping approach proposed in *Bridging the Gap: Complex Event Processing on Stream Processing Engines*, we first derive formal definitions for Simple Event Algebra [7] operators by modifying well-defined operators of the active database representative Snoop [3]. Then, we map our derived operator definitions to one (1:1 Mapping) or a combination of relational algebra operators (1:n Mapping), defined by Codd et al. [4, 5]. In the remainder, we elaborate on the correctness of our mapping approach, i.e., the detection of all pattern matches using our mapping. As our derived SEA operator definitions are equivalent to the definitions of its relation counterpart, the question of correctness boils down to our modifications applied to the operators of Snoop [3]:

Snoop defines its operators as Boolean functions to detect patterns in a point-in-time manner. The Boolean function P(ts) returns $true$ if an (complex) event $e \in T$ occurs at the point in time ts, else $false$ [3].

$$P(ts) = \begin{cases} True, \text{ iff } e \in T \wedge \text{e.ts = ts} \\ False, \text{else} \end{cases} \quad (1)$$

In order to match the specification of SEA and the stream processing paradigm CEP, we adjust Snoop's general operator semantics with two *operator modifications*:

Modification 1: In order to handle high-frequent and unbounded streams, stream processing uses windows to create finite substreams of length W [2, 6]. We use ts_b and ts_e to define an arbitrary time interval $[ts_b, ts_e)$ so that $W = ts_e - ts_b$. Thus, we adjust the input of P to a set of events E = $\{e_1,...,e_n\}$, where for each event e_i it is true that $e_i.ts \in [ts_b, ts_e)$. We modify Eq. 1 as follows:

$$[P]_{ts_b}^{ts_e} = \begin{cases} True, \text{iff } \exists e_i \in T \wedge e_i.ts \in [ts_b, ts_e) \\ False, \text{else} \end{cases} \quad (2)$$

Modification 2: To fulfill the closure properties of SEA, we further need to modify the output of the function $[P]_{ts_b}^{ts_e}$ (Eq. 2). In particular, the function either returns the set of events e_i satisfying all constraints or an empty set and no Boolean value.

$$[P^*]_{ts_b}^{ts_e} = \begin{cases} \{e | e \in T \wedge e.ts \in [ts_b, ts_e)\} \\ \emptyset, \text{else} \end{cases} \quad (3)$$

Applying both modifications to the operator definitions of Snoop, yield the final definitions of our operators. While, on the one hand, we modify in- and output semantics, Modification 1 incorporates the window operators that discretize the stream(s) into a finite substream, i.e., a set of tuples [1]. As a relation is also defined as a set of tuples, the intra-window semantics of our operators, i.e., the operation applied to the set of tuples assigned to single window, are semantically equivalence to relational algebra operators [1, 8]. In particular, Eq. 3 contains the intra-window semantics, i.e., the computations applied on the finite substream S_k. To guarantee the correctness of our modifications, i.e., the detection of all complex events, we also need to analyze the inter-window semantics that define how subsequent windows are created and, thus, how the stream S is discretized. To this end, let us consider a pattern p, which is composed of operators OP and a time window of length W. The semantics of different operators OP, i.e., sequence, conjunction, disjunction, iteration, and negation, are formally defined by our derived definitions and applied to each finite substream $S_k \in S$. In particular, $S_k = [S]_{ts_{e_k}}^{ts_{b_k}}$, where $[ts_{b_k}, ts_{b_e})$ describes an arbitrary time interval for which is true that $ts_{e_k} - ts_{b_k} = W$. Furthermore, the window operator contains the semantics of how subsequent substreams S_{k+m} are created. For our mapping, we use time-based sliding windows, which, in addition to W, define a slide size s that declares how often new windows start. Thus, sliding windows create the following sequence of potentially overlapping substreams: $S_{k+m} = [S]_{ts_{e_{k+m}}}^{ts_{b_{k+m}}}$, where $ts_{k+m} = ts_k + s * m$, respectively $(m \in \mathbb{N})$. Furthermore, following our mapping directives, our mapping requires sliding per tuple, thus, $s = 1$ for slide-by-tuple windows or is smaller or equal to the frequency of the stream with the highest arrival rate in p. Finally, a complex event $ce(e_1,...,e_n)$ is a pattern match, i.e., the composition of all participating events e_i. For simplification, we consider that the events $e_i \in ce$ are in temporal order and $e_1.ts$ is the smallest and $e_n.ts$ the largest timestamp in ce $(ts \in \mathbb{N})$. Thus, ce is a valid match of p if $e_n.ts - e_1.ts < W$. Furthermore, ce is detected, if $e_i, e_n \in S_k$ and thus, $e_1.ts, e_n.ts \in [ts_{b_k}, ts_{e_k})$.

Theorem 0.1. *Let us consider the pattern p is applied to the stream S and yields a complex event ce. Then, there exists at least one substream $S_k = [S]_{ts_{e_k}}^{ts_{b_k}}$ such that $e_i, e_n \in S_k$.*

Proof: As e_1 and e_n are two consecutive events for which it is true that $e_n.ts - e_1.ts < W$, we define $e_n.ts = e_1.ts + q$, where $0 \leq q < W$ and show that for both corner cases, i.e., $q = 0$ and $q = W - 1$ a substream S_k is created by the sliding window operator.

Case 1 $q = 0$: If $e_1 \in S_k \Rightarrow e_1.ts \in [ts_{b_k}, ts_{e_k})$
$\Rightarrow e_n.ts = e_1.ts + 0 = e_1.ts \Rightarrow e_n.ts \in [ts_{b_k}, ts_{e_k})$
$\Rightarrow e_1, e_n \in S_k$ □

Case 2 $q = W - 1$: For case 2, the complex event ce is only detected in S_k, if $e_1.ts = ts_{b_k}$ as otherwise $e_n.ts = e_1.ts + x + W - 1 > ts_{e_k}$. To this end, first, let us consider $e_1 \in S_k \wedge e_1.ts = ts_{b_k}$.

Case 2.1. $q = W - 1 \wedge e_1.ts = ts_{b_k}$: If $e_1.ts = ts_{b_k} \Rightarrow e_1 \in S_k$
$\Rightarrow e_n.ts = e_1.ts + W - 1 = ts_{b_k} + W - 1 = ts_{e_k} - 1 \Rightarrow e_n.ts \in [ts_{b_k}, ts_{e_k})$
$\Rightarrow e_n \in S_k$ □

Second, let us consider that e_1 occurs at $ts_{e_k} - 1$, i.e., e_1 is the last event considered in S_k. It follows, that $e_n.ts \notin S_k$, because $ts_{e_k} + W - 1 > ts_{e_k} - 1$. Thus, we need to ensure that there exists a S_{k+m} in which e_1 and e_n occur, i.e., $\exists S_{k+m}$ so that $e_1.ts = ts_{e_k} - 1 \wedge e_1.ts = ts_{b_{k+m}}$.

Case 2.2. $q = W - 1 \wedge e_1.ts = ts_{e_k} - 1 \wedge e_1.ts = ts_{b_{k+m}}$:
$ts_{e_k} - 1 = ts_{b_{k+m}}$
$\Rightarrow ts_{e_k} - 1 = ts_{b_k} + s * m$
$\Rightarrow ts_{e_k} - ts_{b_k} - 1 = m$
$\Rightarrow W - 1 = m$

Thus, the match ce is detected $W - 1$ windows after e_1 occurred and e_n occurs the first time. □

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