Limited Activist Model

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Abstract

The study of political parties is essential to understanding the complex dynamics within a given country's political system. In this paper, we look at Rebecca A. Jeffs, John Hayward, Paul A. Roach, and John Wyburn's "Activist Model of Political Party Growth" which endeavors to model the growth of a political party through word-of-mouth activism. The authors draw on SIR models of epidemiology and previous studies of this technique applied to social diffusion scenarios. To understand this model, we parse through the various actors, variables, and assumptions that went into the creation of the model. We then analyze the authors' results and application of this model. Finally, we use a numerical solver to solve the equations and reproduce the author's graphs, adjusting optimized parameters to observe their effects on the dynamical system.

1 Introduction

Political parties are instrumental to the way a government functions within a country. The growth and decline of a political party is of interest as it can heavily influence the way in which policies are formed and implemented. There is a breadth of study that relates the size, growth, and effectiveness of political parties. Numerous studies have indicated a strong relationship between party member size and growth. Larger parties typically have more political influence in the governance of a country. However, the effectiveness of a party depends, not only on size, but also on the level of involvement of its members.

The goal of this paper is to model the growth of a political party through word-of-mouth activism. The focus of the paper is on how political parties grow in this manner, not why word-of-mouth activism is effective in contributing to party growth. There are many ongoing debates surrounding the effects of party size on participation. These indicate a potential limit to the size of a party with respect to its ability to keep members active. Aspects that affect this limit can be things such as increased complexity related to size or a potential limit on activism imposed by party leaders to maintain more oligarchical control, among other factors. Active membership is essential to maintain party growth, so these factors are in clear contention.

Party activists are the key players in a political party with respect to keeping members active and sustaining party growth. However, party growth is by no means the sole responsibility or the primary focus of activists. Moreover, party growth often occurs in brief spurts, often surrounding elections when activist attention is more directed toward engaging the electorate. In terms of party

size, smaller, growing parties often display more political activism, whereas larger parties are faced with more internal complexities and often lack appropriate funding to keep activists engaged and incentivized. In light of this information, the mathematical questions explored in this paper are: How does activism increase the growth of a political party? What are the constraints on a party's ability to grow? How do activists recruit new members of the party? In pursuit of these answers, the creators of this model draw from SIR models in epidemiology as previously applied to models of social diffusion, such as the spread of rumors, religious affiliation, and the like. They then modify and apply these techniques in order to understand the dynamics of political party membership.

2 Explanation of the Limited Activist Model

Table 1: *Actors in this model*

H	People who will not support the party
S	Number of people not opposed to joining the party (Susceptible)
I	Number of Recruiting Activists (Infectious)
A	Number of Non-Recruiting Activists
M	Number of Inactive Party Members

Table 2: *Other Variables Considered*

N	Adults eligible to vote
P	Total number of people in the party
C_p	Recruitment Potential
α	Leaving Rate
g	Fraction recruited
τ_i	Average time spent as a Recruiting Activist
τ_a	Average time spent as a Non-Recruiting Activist
R_p	Reproductive Potential
R_{epi}	Epidemic Threshold
R_{ext}	Extinction threshold
g	Fraction recruited

Equations of the Limited Activist Model:

$$\frac{dS}{dt} = -\frac{C_p}{\tau_i N} SI + \alpha M \tag{1}$$

$$\frac{dI}{dt} = g\frac{C_p}{\tau_i N}SI - \frac{I}{\tau_i} \tag{2}$$

$$\frac{dA}{dt} = (\frac{I+A}{P})(1-g)\frac{C_p}{\tau_i N}SI + \frac{I}{\tau_i} - \frac{A}{\tau_a} \tag{3}$$

$$\frac{dM}{dt} = (\frac{P-I-A}{P})(1-g)\frac{C_p}{\tau_i N}SI + \frac{A}{\tau_a} - \alpha M \tag{4}$$

Reproduction Thresholds:

$$R_p = gC_p \tag{5}$$

$$R_{epi} = \frac{1}{\frac{S}{N}} = \frac{N}{S} \tag{6}$$

$$R_{ext} = \frac{1}{(1-\frac{H}{N})} \tag{7}$$

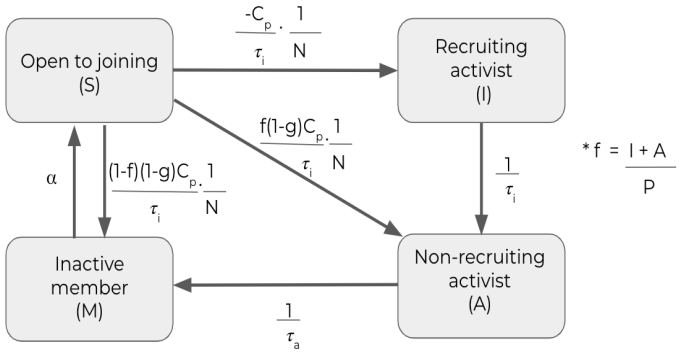


Figure 1: *Infographic representing rates of flow between groups.*

3 Summary of Author's Results

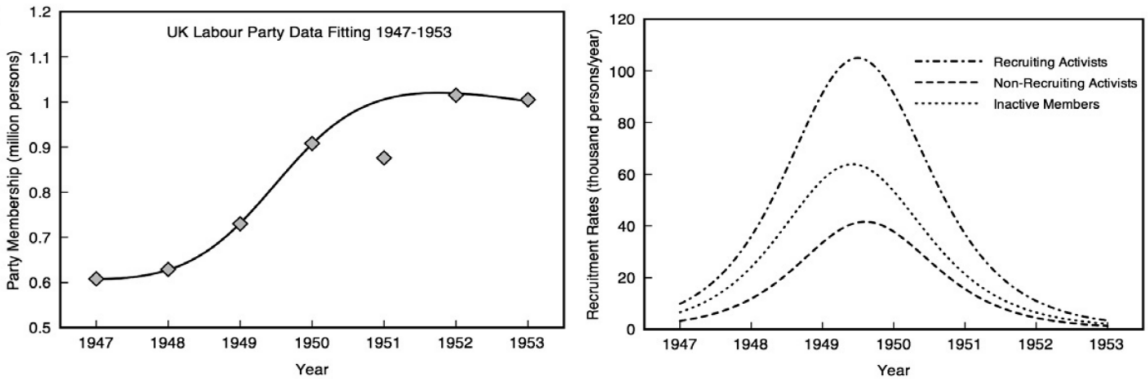
3.1 UK Labour Party 1944 - 1945

Table 3: *Labour Party Membership 1944-1954 [1]*

Year	1944	1945	1946	1947	1948	1949	1950	1951	1952	1953
Membership	266	487	645	608	629	730	908	876	1015	1005

Early data involving the UK Labour party can be divided into three significant intervals of time: 1944-1945, 1945-1946, and 1947-1953, the data for which can be viewed in (Table 3).

During 1944-1945, those who had been inactive during World War II renewed their membership, thereby increasing party membership from the war time low of 219,000. It is assumed that the number of activists is around 250,000 during this period.



(a) *Data fitting UK Labour Party Membership 1947–1953*

(b) *Recruitment rates to party membership sub-populations I, A, M*

Figure 2: *Raw Data for Party Members and Breaking down Member Types. Reprinted from [1].*

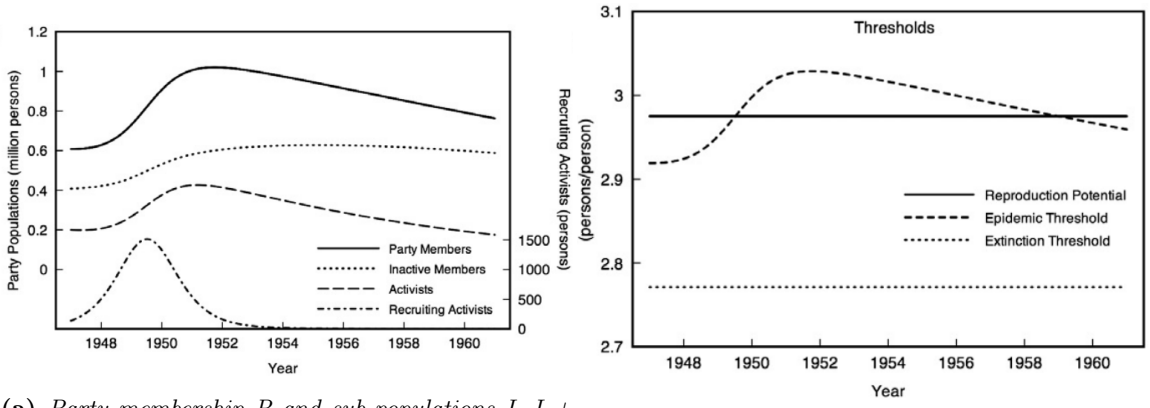
Applying the Limited Activist Model to this data, the reproduction potential remains under the epidemic threshold (Figure 3b). Growth in membership is a result of the high initial number of recruiting activists estimated at about I_0= 4,000. They estimate roughly 200,000 total activists in 1947 which is about a third of the party membership. These results are used for the third phase of growth from 1947-1953

For the campaign from 1947-1953, take N to be the UK electorate or the total population of legal adults. The total open population is composed of those who voted for the Labour party in 1945: O = S + I + A + M. Assume τ_a, the time as a non-recruiting activist, where $\tau_a = 10$ years since most activists are involved in at least one electoral campaign, typically 5 years, while some remain activists for life. The leaving rate, α, is based on the average post campaign membership decline of 3% per year leading up to 1960.

From 1954 to 1960, there was a high amount of volatility in membership so those values were not fit into this model as a differential equation would not have yielded accurate results.

The remaining parameters of this model were determined by least squares with the fraction of recruits who become recruiters represented by g, where g = 0.5. The parameter g is an example of a parameter that could not be estimated from the data or from known values. Instead, g was determined by testing values and seeing which value fit the data best. This same method was used to determine $A_{T0} = I_0 + A_0$ and τ_α.

This model was a good fit for party growth from 1947-1953 except for the data during 1951 in



(a) *Party membership P and sub-populations I, $I + A$, M*

(b) *Thresholds R_{epi}, R_{ext}, compared with reproduction potential R_p*

Figure 3: *Results of the Model on UK Labour Party 1947 - 1960. Reprinted from [1].*

which the party had poor election results. It showed the momentum recruitment activists had on the party's sustained growth up to the 1960's despite the fact that the number of recruiting activists peaked in 1949 (Figure 3a). The number of activists did not peak until 1951 in which they encompassed 42% of all party members. These numbers do not return to pre-campaign values of 1948 until 1959 which was long after the epidemic phase of the party's growth had occurred (Figure 2b). Graphing R_p (the reproductive potential) against the extinction and epidemic threshold demonstrates that R_p is well above the extinction threshold and is above the epidemic threshold during times of large growth and smaller pools of potential recruits (Figure 3b).

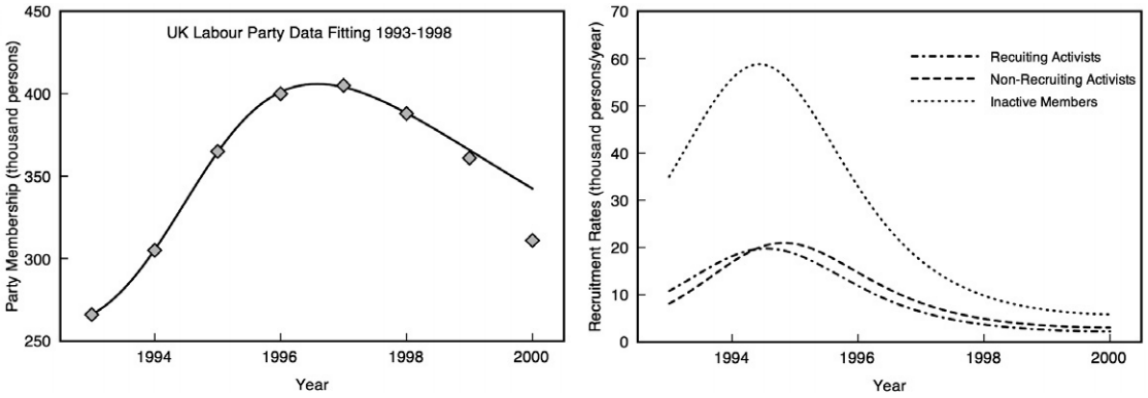
3.2 UK Labour Party 1993 - 1997

John Smith was handed leadership of the UK Labour Party in 1992, and started a recruitment campaign to revitalize the party with the goal of winning the 1997 election. Efforts were a success, culminating in a large enough party membership to win in 1997. In (Figure 4a), the model depicts the significant growth of the party from 1993-1997 leading up to the election.

Due to conflict and poor engagement, the fraction of activists is significantly lower and the leaving rate ($\alpha = 15\%$) is higher compared to the 1940s. The fraction of recruits that become activists (g = 0.2) is significantly smaller due the ultimate goal of increasing size enough to win the election, not necessarily maximizing activism.

Notably, the largest recruitment of inactive members peaks in late 1994 (Figure 4b), around the same time of the peak in recruiting activists. Despite the number of recruiting activists less than doubling in size as seen in (Figure 5a), the spike in total members is large enough to win the election, reaching its maximum in the 1997 election year.

Looking at (Figure 5b), the reproduction potential R_p starts out above the epidemic threshold indicating an epidemic of recruiting members. Beginning in 1994, R_p remains below R_{epi}, linearly

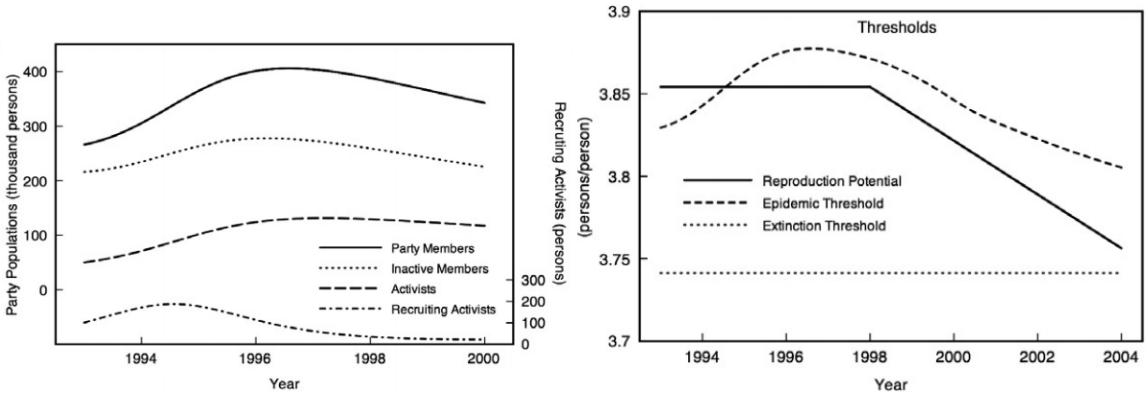


(a) *Data fitting 1993–1998, showing deviation of model from post 1998 decline*

(b) *Recruitment rates to party membership sub-populations I, A, M*

Figure 4: *Data Fitting and Analyzing Movement of Activists. Reprinted from [1].*

decreasing post-election from 1998 onward. In order to model the decline of the party post-election, parameters are modified including a decrease in reproduction potential and an increase in leaving rate. This decline is likely due to a shift in focus from party membership to government. Reproduction potential remains below the epidemic threshold, and approaches the extinction threshold over time.



(a) *Party membership P and sub-populations I, I + A, M*

(b) *Thresholds R_{epi}, R_{ext}, compared with reproduction potential R_p, assuming falling R_p, and rising leaving rate α, to account for post 1998 drop*

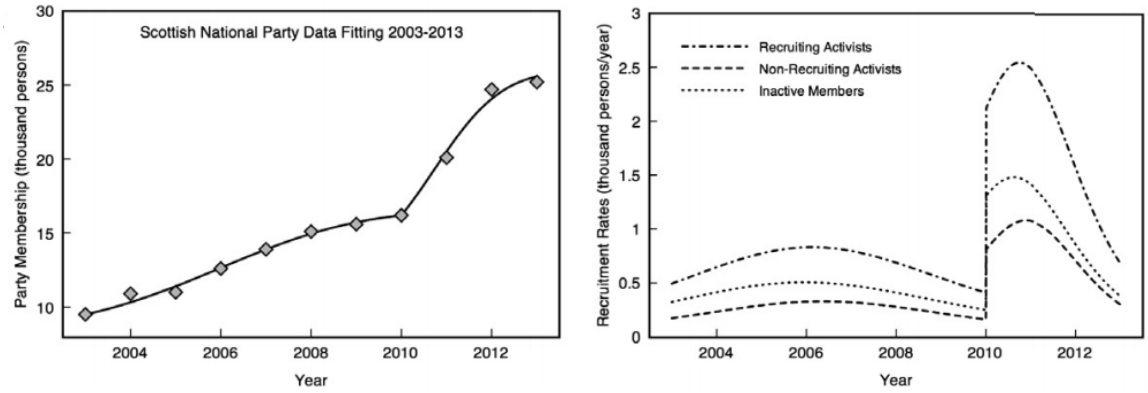
Figure 5: *Results of the Model on UK Labour Party 1993 - 1998. Reprinted from [1].*

3.3 Scottish Nationalist Party (SNP) 2003 - 2013

In 2003, the Scottish Nationalist Party failed to gain power in Scotland for a second time in succession and lost a large number of members. After changing leadership to Alex Salmond in 2004, the party experienced two surges in party membership, once between 2003 and 2010, which allowed them to achieve power as a minority government, and another between 2010 and 2013, which led them to win the 2011 Scottish Parliament elections.

The Limited Activist model was applied to the party data from these two phases (2003-2010, and 2010-2013) separately. The overall growth was separated into two time intervals because there was a great jump in the number of initial recruiting activists, represented by I_o, in 2010 (Figure 7a) and a new initial value was needed. Also, party growth had almost finished by 2010 as recruitment rates for all types of members had dropped to their initial values (Figure 6b).

For both periods, the initial total population was taken from the typical Scottish electorate size from 2003-2014 as it remained relatively stable throughout. As the Scottish Nationalist Party pursued a similarly positive recruitment drive as the UK Labour Party in the 1940s, initial values for variables were assumed to be the same as 1940s Labour: the time as a non-recruiting activist $\tau_a =$ 10 years, the initial fraction of activists in the party $\frac{A_{\tau 0}}{P_0} = 0.33$, fraction of recruits who become recruiters g = 0.5, and leaving rate $\alpha = 0.05$.

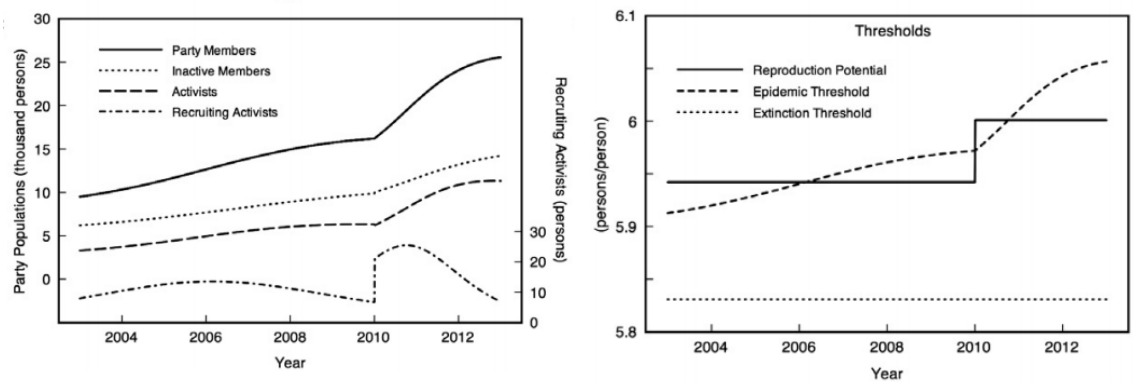


(a) *Data fitting in two phases, 2003–2010 and 2010–2013*

(b) *Recruitment rates to party membership subpopulations I, A, M*

Figure 6: *Data Fitting and Analyzing Movement of Activists. Reprinted from [1].*

The model was a good fit for SNP's growth throughout the two phases. As seen in (Figure 6a), between 2003 and 2007, party membership increased by 70% mainly due to the faith in the changed leadership, with membership growth slowing down and near plateauing before 2010. In (Figure 7b), R_p is above R_{epi} up to a bit past 2006, indicating an epidemic phase. After 2006, R_p levels are below R_{epi}, indicating the end of the epidemic phase, which aligns with the party's fast and later-slowing-down growth as shown in (Figure 6a) and (Figure 7b).



(a) *Party membership P and sub-populations I, I + A, M*

(b) *Thresholds R_{epi}, R_{ext}, compared with reproduction potential R_p*

Figure 7: *Results of the Model on Scottish Nationalist Party 2003 - 2013. Reprinted from [1].*

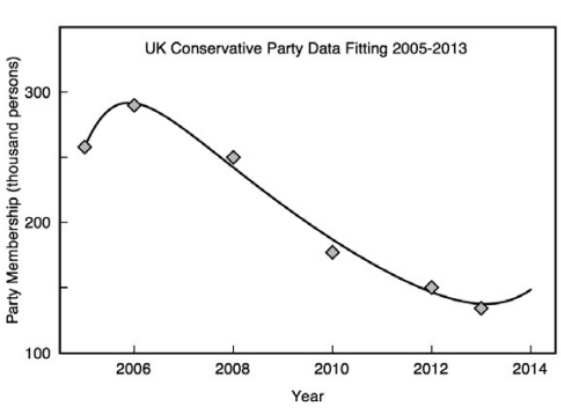
In the second phase, party membership increased by 50% in three years, mainly due to the jump in numbers and effectiveness of recruiting activists in 2010, in preparation for the 2011 Scottish parliament elections . The number of recruiting activists increased from less than 1000 to around 3000 (Figure 6a). Recruitment rates for recruiting activists rose from around 0.1 to values past 2 (Figure 6b). Between 2010 and 2011, R_p is above R_{epi} levels, indicating an epidemic phase of recruitment and becomes lower than R_{epi} levels afterwards, indicating a period of decreasing recruitment between 2011 and 2013 (Figure 7b). This also matches the data trends in (Figure 6a), (Figure 6b), and (Figure 7b).

However, the model predicts that R_p levels remain below R_{epi} levels after 2011 and become farther and farther from the R_{epi} values, indicating a decreasing and ending party growth by 2014 (Figure 7b). Yet, the party doubled its membership numbers before the end of 2014 (Figure 6a), which the model failed to predict.

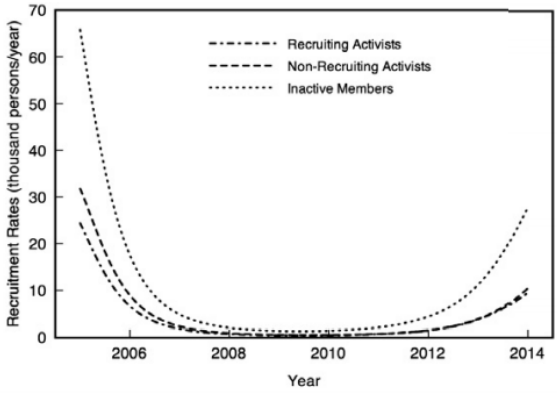
3.4 UK Conservative Party 2005 - 2012

The final application of the limited activist model is the UK Conservative Party's brief rise and sustained decline between 2005 and 2012. The party became weak after losing two successive general elections, therefore the duration of a non-recruiting activist is assumed to be shorter than the previous three applications, with $\tau_a = 5$ years, and the fraction of recruits that become recruiters is assumed to be low, with g = 0.2. Initial fraction of activist is also lower than the one-third previously used and the leaving rate is high, with $\alpha = 0.2$.

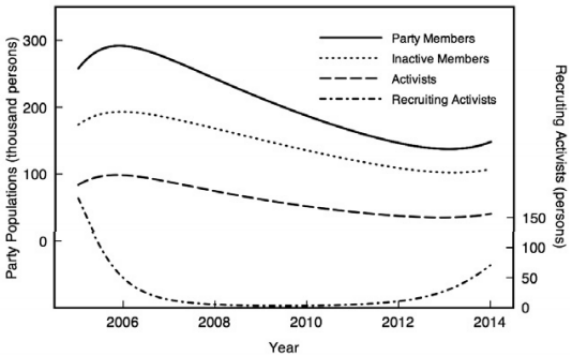
(Figure 8c) shows that there was a short period of party growth between 2005 and 2006. During that time period, reproduction potential R_p is lower than the epidemic threshold R_{epi} (Figure 8d), indicating that the small growth was due to a large number of activists and not their effectiveness in recruiting.



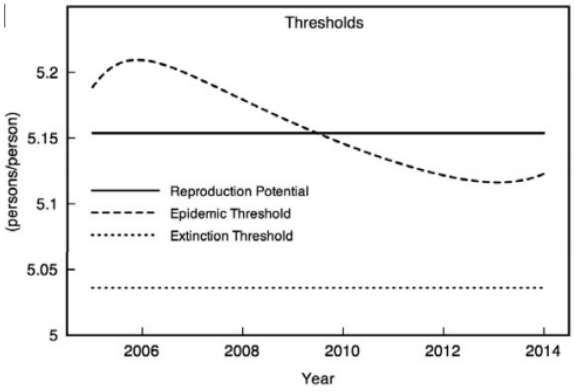
(a) *Data fitting 2005–2013, extrapolated to 2014*



(b) *Recruitment rates to party membership sub-populations I, A, M*



(c) *Party membership P and sub-populations I, I + A, M*



(d) *Thresholds R_{epi}, R_{ext}, compared with reproduction potential R_p*

Figure 8: *Data Fitting and Analyzing Movement of Activists. Reprinted from [1].*

Between 2006 and 2012, the party experienced a sustained decrease in party membership. This can be explained by not only a high leaving rate but also extremely low recruitment rates (Figure 8b) and the sharp drop in the number of recruiting activists (Figure 8c), which lead to a proportionally more active party with the loss of inactive members and no new recruitment. The dynamic effects from these changes most probably lead to the short-term rapid decline experienced by the party.

After 2009, R_p is higher than R_{ext} and R_{epi}, predicting not extinction but a return to growth for the Conservative Party from 2014 (Figure 8d). This may be due to the proportionally more active structure of the party. A tentative figure of 149,800 for 2014 is in line with the model's prediction. Therefore, the model is a decent fit for the overall trajectory of the UK Conservative Party's growth and decline over this time period.

4 Analysis & Extension

To start, we inputted the initial values for the constants (Table 4) based on the data used in the UK Labour Party 1940-1960 research study. Substituting these values into the system of ODE's

Table 4: *Parameter Values*

N	33.24 million	P_0	608,000
$I_0 + A_0$	200,000	$N - H$	11.995 million
τ_a	10 years	τ_i	0.0144
α	0.05	g	0.5
C_p	5.9501	I_0	139

(Equations 1,2,3,4) we get a system of equations:

$$\frac{dS}{dt} = -\frac{C_p}{\tau_i N}SI + \alpha M \Rightarrow \frac{dS}{dt} = -0.00001243(SI) + 0.05M$$

$$\frac{dI}{dt} = g\frac{C_p}{\tau_i N}SI - \frac{I}{\tau_i} \Rightarrow \frac{dI}{dt} = 0.000006215(SI) + 69.44I$$

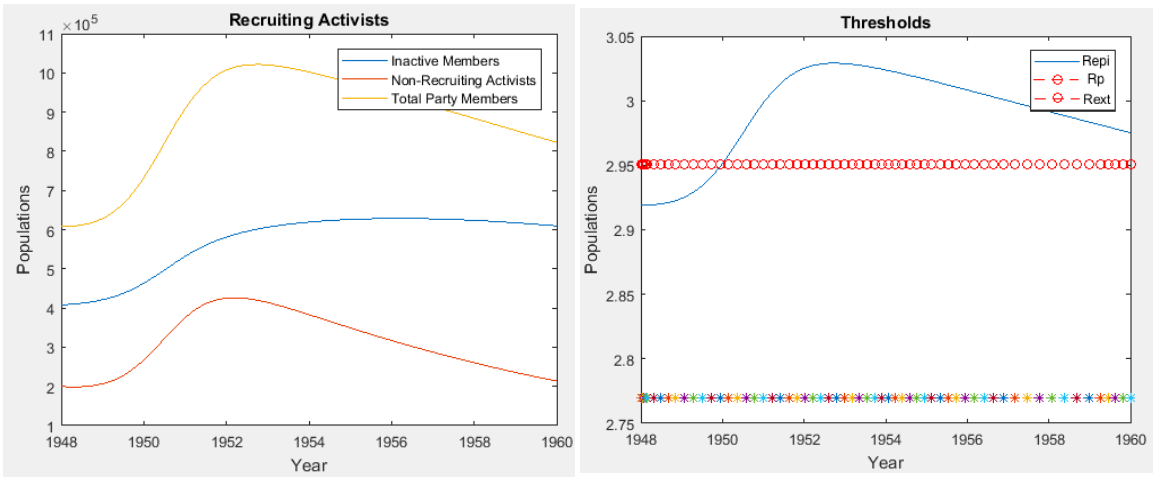
$$\frac{dA}{dt} = (\frac{I+A}{P})(1-g)\frac{C_p}{\tau_i N}SI + \frac{I}{\tau_i} - \frac{A}{\tau_a} \Rightarrow \frac{dA}{dt} = 0.000006215(SI)(\frac{I+A}{P}) + 69.44I - 0.1A$$

$$\frac{dM}{dt} = (\frac{P-I-A}{P})(1-g)\frac{C_p}{\tau_i N}SI + \frac{A}{\tau_a} - \alpha M \Rightarrow \frac{dM}{dt} = 0.000006215(SI)(\frac{P-I-A}{P}) + 0.1A - 0.05M$$

Taking this system of equations, we inputted them into Matlab utilizing the ode45 differential equation solver. After solving the four equations, we made two graphs modelling the number of party members, activists, and inactive members as well as a graph of the growth potential versus the epidemic and extinction threshold (see Figure 9).

To examine how the Recruitment Potential (C_p), Duration Recruiting Activist (τ_i), and Initial Recruiting Activist (I_0) was optimized in the original model, we changed the optimized values and analyzed the effects it had on the model (Figure 10).

To start, we examined how C_p, the number of susceptibles recruited by a single recruiting activist, affected the model using C_p=2 which is lower than the optimized value and C_p=7 which is higher than the optimized value. When C_p is decreased (Figure 10*a*, 10*d*), party population declines. This is because the growth in activists and inactive members declines while the growth of the number of susceptibles increases. So more people are leaving the party then joining the party leading to an overall decline in party members over time. However, the increase in the number of susceptibles leads to greater reproductive potential putting it above the epidemic threshold.



(a) *Party Membership Regraph*

(b) *Potential Thresholds Regraph*

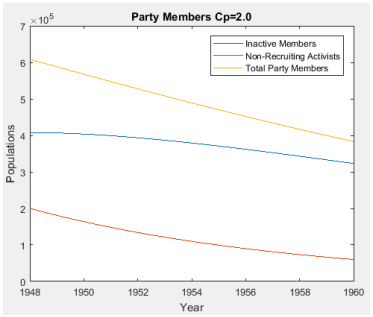
Figure 9: *Reproduction of the Results of the Model on UK Conservative Party 1944 - 1945*

When C_p is increased (Figure 10*c*, 10*f*), growth in activists and party members increases leading to long term sustained growth with reproductive potential well below the epidemic threshold. Despite the high growth rate, the party population becomes more entrenched in inactive members as opposed to activists (Figure 10*c*) which leads to a more inactive party which puts growth well below the epidemic threshold. The epidemic threshold is so high in (Figure 10*f*) since R_{epi} is a ratio of $\frac{N}{S}$ and an increase in C_p decreases S leading to an increase in R_{epi}. This is intuitive since an increase in the number of susceptibles recruited by a recruiting activist decreases the pool of potential recruits since many have already joined to party so it is much harder to have an epidemic growth rate. When the party has become saturated, party growth declines.

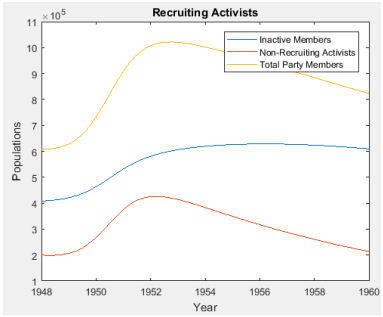
Next, we varied τ_i, the time spent as a recruiting activist (Figure 11). When τ_i is decreased, the number of recruiting activists and susceptibles decline, signifying an increase in inactive party members as opposed to activists, which is demonstrated in (Figure 11*d*). The increase in the proportion of inactive to active members puts the reproductive potential below the epidemic level (Figure 11*a*) until the proportion of activists reaches a low in which case, the party is ready for growth (with epidemic potential).

When τ_i is increased, the growth of the recruiting activists declines while the growth of the number of susceptibles increases. The decline in activists as seen in (Figure 11*c*) as well as the increase in inactive members later on results in similar behavior as when τ_i is decreased. The transition to a more proportionally inactive party population results in reproductive potential below the epidemic level (Figure 11*f*). However, as time goes on, the epidemic threshold decreases as the proportion of activists decreases indicating future growth.

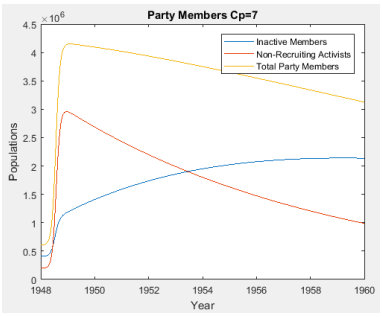
Lastly, we modified I_0, the initial number of recruiting activists. Varying I_0 has the least effect on party growth and potential. Starting with a low number of initial recruiting activists results



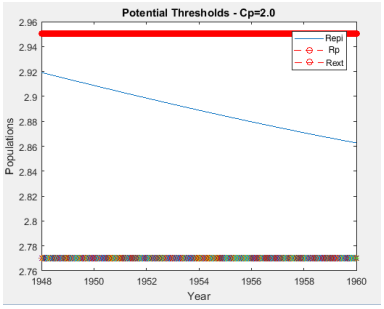
(a) *Party membership when C_p is decreased*



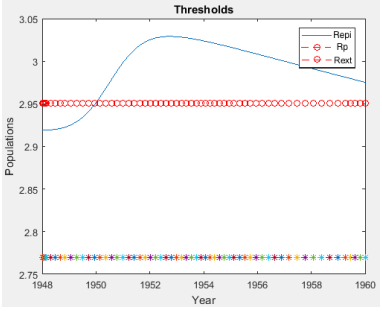
(b) *Party membership for optimized C_p*



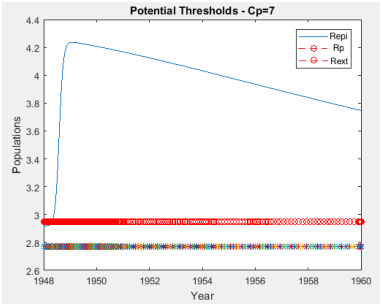
(c) *Party membership when C_p is increased*



(d) *Potentials when C_p is decreased*



(e) *Potential for optimized C_p*

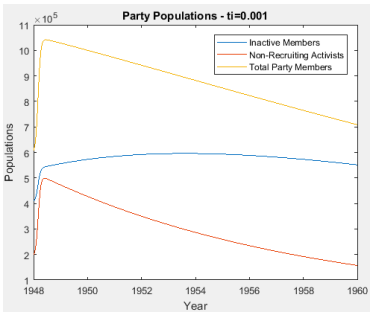


(f) *Potentials when C_p is increased*

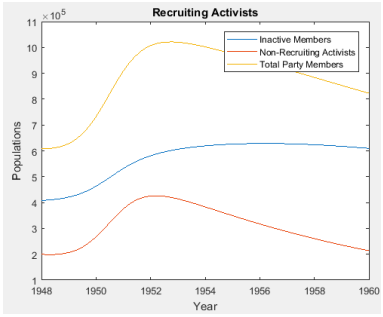
Figure 10: *Results when modifying C_p in the model*

in slower growth with the curves in (Figure 12*a*, 12*d*) shifted slightly to the right. When I_0 is increased, (Figure 12*d*, 12*f*) growth peaks faster but is sustained for a similar amount of time.

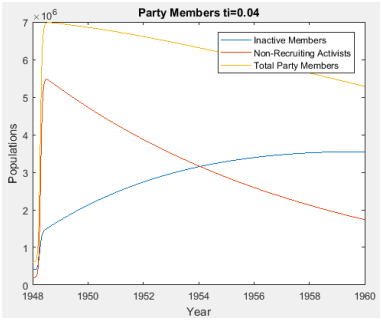
By experimenting with the values of these optimized variables, we were able to see the various effects on the model and how the model was fit to the data. We found that manipulating C_p and τ_i had the greatest effect on the model, while manipulating I_0 only superficially affected party growth. The effects of increasing and decreasing C_p and τ_i results in similar behavior in the model. Additionally, reproducing the graphs used in the study gave us a better understanding of the dynamics between the different groups and how the different parameters affect party growth.



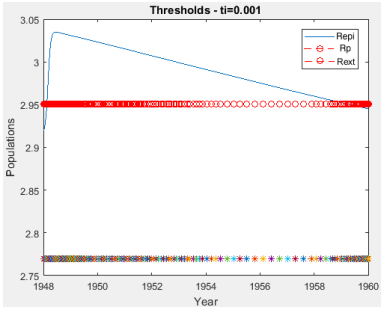
(a) *Party membership when τ_i is decreased*



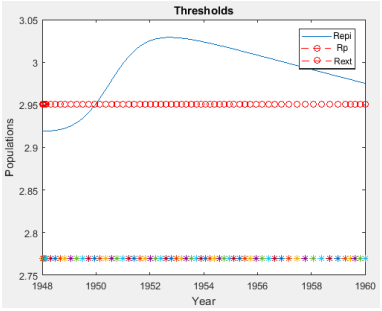
(b) *Party membership for optimized τ_i*



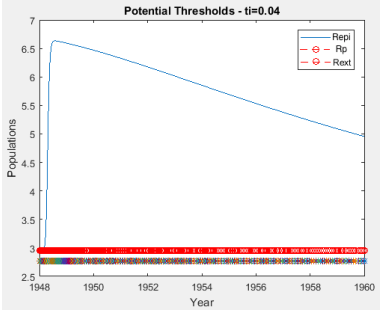
(c) *Party membership when τ_i is increased*



(d) *Potentials when τ_i is decreased*

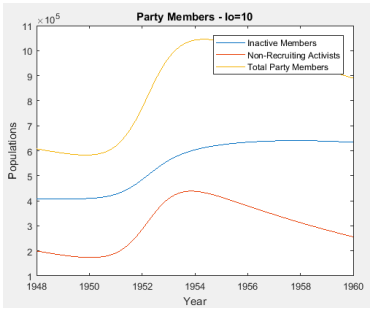


(e) *Potential for optimized τ_i*

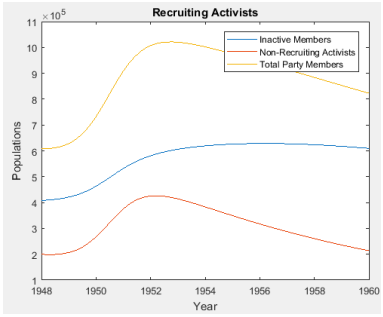


(f) *Potentials when τ_i is increased*

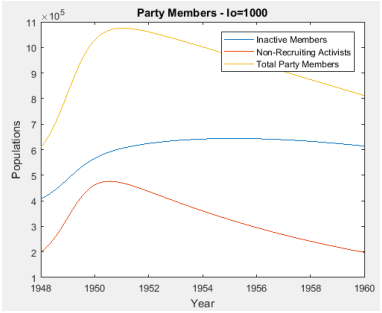
Figure 11: *Results when modifying τ_i in the model*



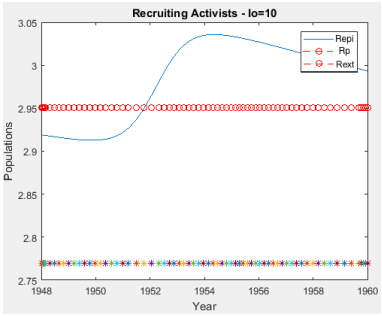
(a) *Party membership when I_0 is decreased*



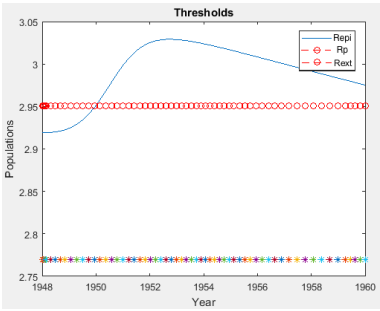
(b) *Party membership for optimized I_0*



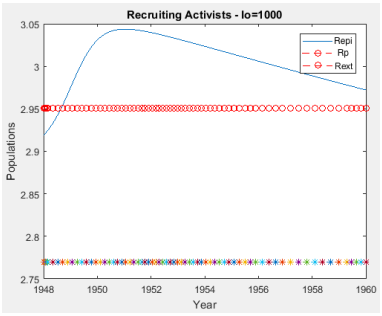
(c) *Party membership when I_0 is increased*



(d) *Potentials when I_0 is decreased*



(e) *Potential for optimized I_0*



(f) *Potentials when I_0 is increased*

Figure 12: *Results when modifying I_0 in the model*

5 Conclusion

The paper [1] set out to model the growth of political parties by word of mouth activism. The standard SIR model was not able to model the political party dynamics due to the short duration of the recruiting period. Thus, the updated Limited Activist Model solved this problem by dividing party members into recruiting and non-recruiting members, and dividing non-members into those open to persuasion and those hardened against the party's beliefs. This model was able to fit four different datasets of UK political parties convincingly well.

In all four application areas, R_p (reproduction potential) of the political parties were above R_{ext} (extinction threshold), thus despite party decline in each case, none of the parties studied showed an indication of extinction. After a period of decline, the parties experienced a return to growth, with two out of the four applications showing a return to political power for the party. The model suggests this may be associated with the natural recycling of party members and its effect on activism within the party. Understandably, the model could not account for abrupt changes caused by external political events such as the Scottish independence referendum.

Moreover, the fraction of total activists who contribute to growth is represented by $\frac{I+A}{P}$ is crucial for representing how active the party is as a whole. The fraction was considerably different in all four of the models. The more proportionally active the party, the more recruits who become active participants themselves. With the results in (Section 3) and (Section 4), the hypothesis that political parties grow through the action of activists recruiting via word of mouth was supported.

In our extension of the author's analysis, we varied the optimized variables derived in the study, C_p, τ_i, and I_0. We found that manipulating these variables resulted in changes in the growth and the decline of the party, which we were able to observe by reproducing party membership and potential thresholds graphs. We were successfully able to distinguish the qualitatively different results in party growth obtained from simulating each of these models.

References

[1] R. A. Jeffs, "Activist model of political party growth," *Physica A: Statistical Mechanics and Its Applications*, vol. 394, pp. 359–372, 2016.

[2] M. W. Reichelt and L. F. Shampine, "ode45," *The MATLAB ODE Suite*, 1984-2017.

[2]