Chaotic Attractors with Circuits

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PHY 320

Background

This research was done for the Chua (Double Scroll) Attractor, Lorenz attractor, and the Rössler attractor. These attractors are all represented by three nonlinear ordinary differential equations, each with specific coefficients and initial conditions. In this case, these are strange attractors, where chaotic behavior is found in a subset of phase space. This means that a large set of initial conditions leads to orbits that converge to this chaotic region. Chua's circuit is a simple electronic circuit that exhibits classic chaotic behavior, actually the simplest circuit to do so. Similarly, the Lorenz and Rössler attractor can also be recreated as circuits.

By plotting the system of equations in 3D the attractor appears. Varying the coefficients of the systems of equations changes the phase it is in. For example, the Chua circuit exhibits a period doubling route to chaos. By changing the coefficients, one can find the period two, four, and chaotic attractor. Then, using LTSpice, plots of the physical attractor were created by plotting it in phase space. This was done by plotting either the voltage at two different points in the circuit against each other, or the voltage and a current. This created similar plots to the ones made using the system of equations. By varying the physical components of the circuit, such as inductance and capacitance, the phase also changed.

Objective

The goal was to find what coefficients of the system of equations matched the physical traits of the circuits. This was done by comparing the plots of the system of equations on google CoLab to the ones generated on LTSpice and looking for similarities. This can be useful as it allows us to know what values (resistance, capacitance, inductance) of the circuits would be needed to create the different phases, for example period doubling for the Chua circuit.

Physical Systems

As stated, the attractors can be modeled by a system of three differential equations. When plotted in phase space is when the shape of the attractor is realized. The equations are as follows.

$$dx/dt = a*(y-x-H)$$
$$dy/dt = x - y + z$$
$$dz/dt = - * y$$

$$H = C\_2*x + 0.5*(R-C\_2)*(abs(x+1)-abs(x-1))$$

$a = 15.6$ $b = 25.8$; $R = -1.143$ $C\_2 = -0.714$

H is needed for the nonlinear resistor

Figure One: Chua Circuit Equations

$$dx/dt = s(y - x)$$

$$dy/dt = rx - y - xz$$

$$dz/dt = xy - bz$$

$s = 10$ $r = 28$ and $b = 8/3$

Figure Two: Lorenz Attractor

$$dx/dt = -(y(t) + z(t))$$

$$dy/dt = x(t) + ay(t)$$

$$dz/dt = b + x(t)z(t) - cz(t)$$

$a = 0.2, b = 0.2, c = 5.7$

Figure Three: Rössler Attractor

When plotted with the listed coefficients, these create the following attractors.This was done using Google Colab.

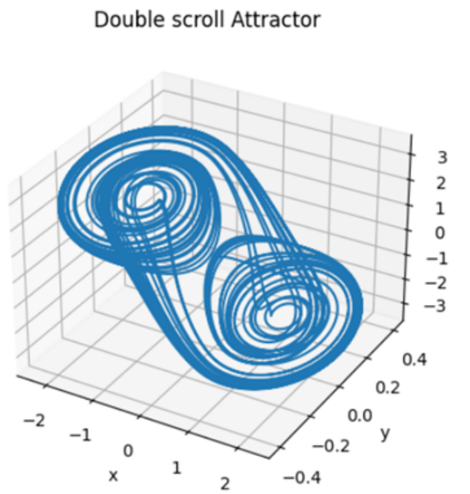


Figure Four: Double scroll attractor using Chua’s equations

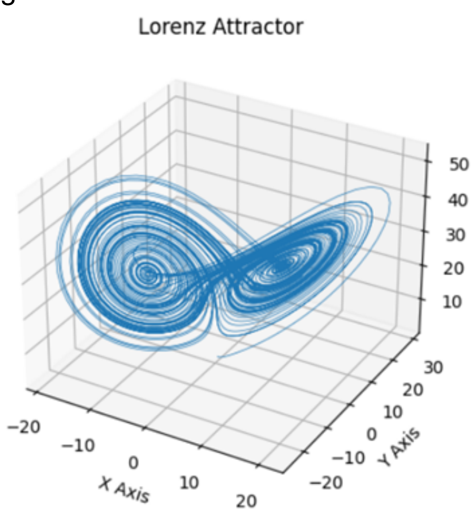


Figure Five: Lorenz attractor

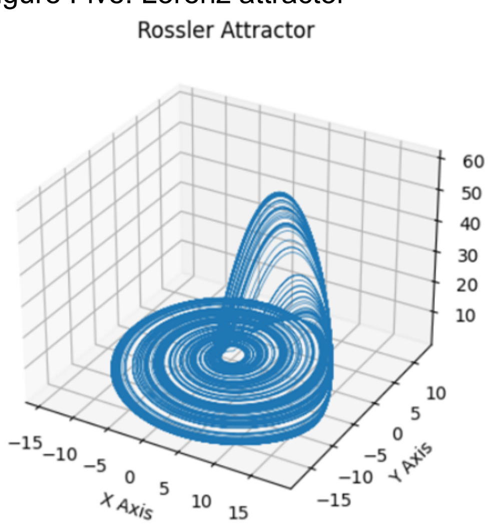


Figure Six: Rössler attractor

Circuit Diagrams

Each attractor can be simulated physically using circuits. Due to limited time and capabilities, these were recreated digitally using LTSpice, a circuit simulation software.

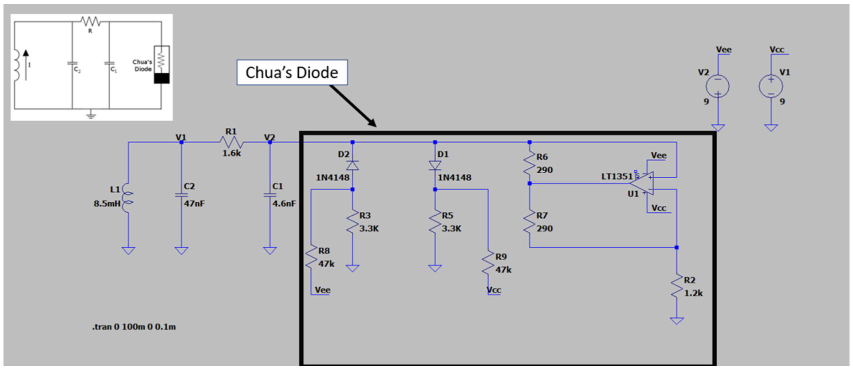


Figure Seven: Chua's Circuit

The Chua's circuit consists of resistors, capacitors, and a nonlinear resistor known as a Chua;s Diode. This means that it does not follow ohm's law, and its IV plot will not be linear.

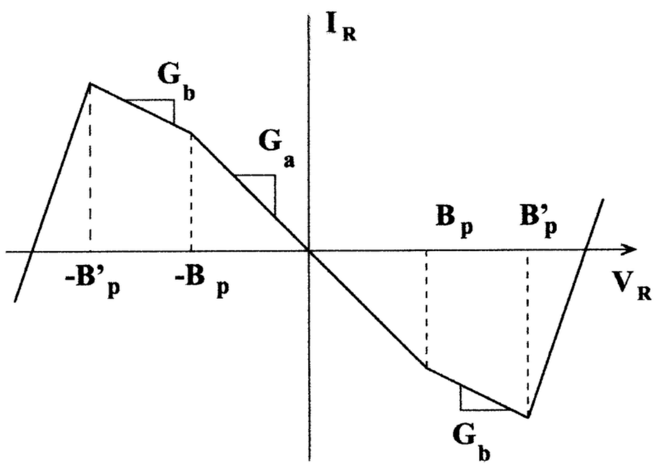


Figure Eight: Chua's Diode IV plot

As you can see, this is not a linear plot, and is a cause of the chaotic motion of the circuit.

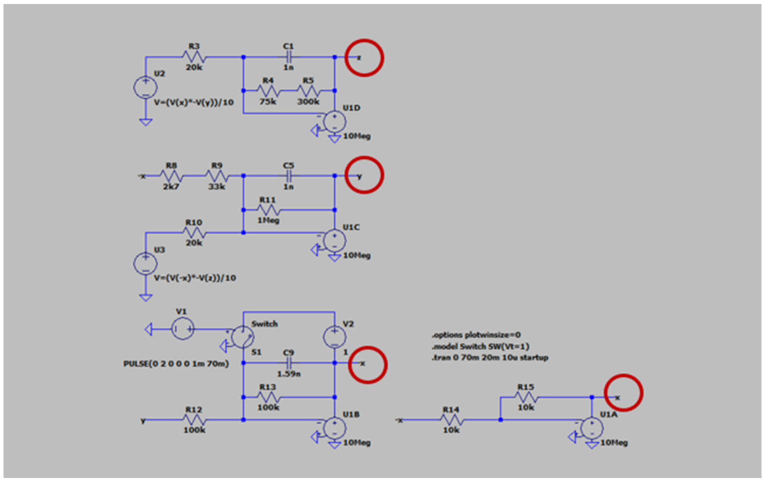


Figure Nine: Lorenz Circuit

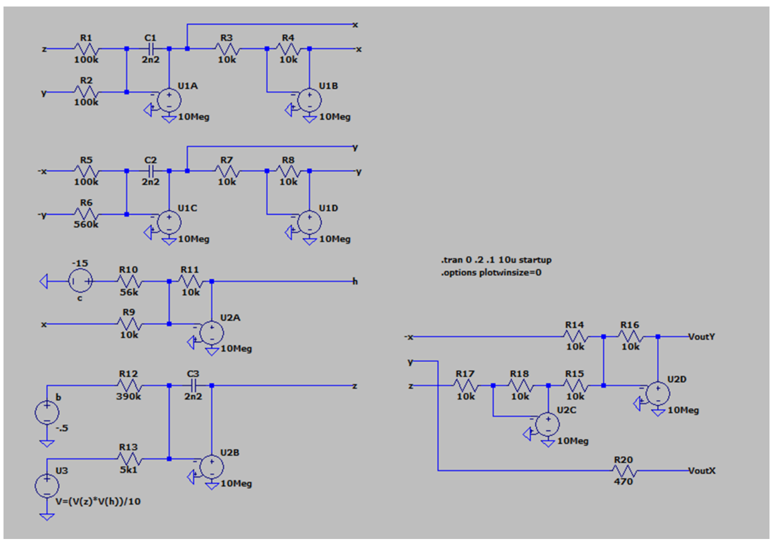
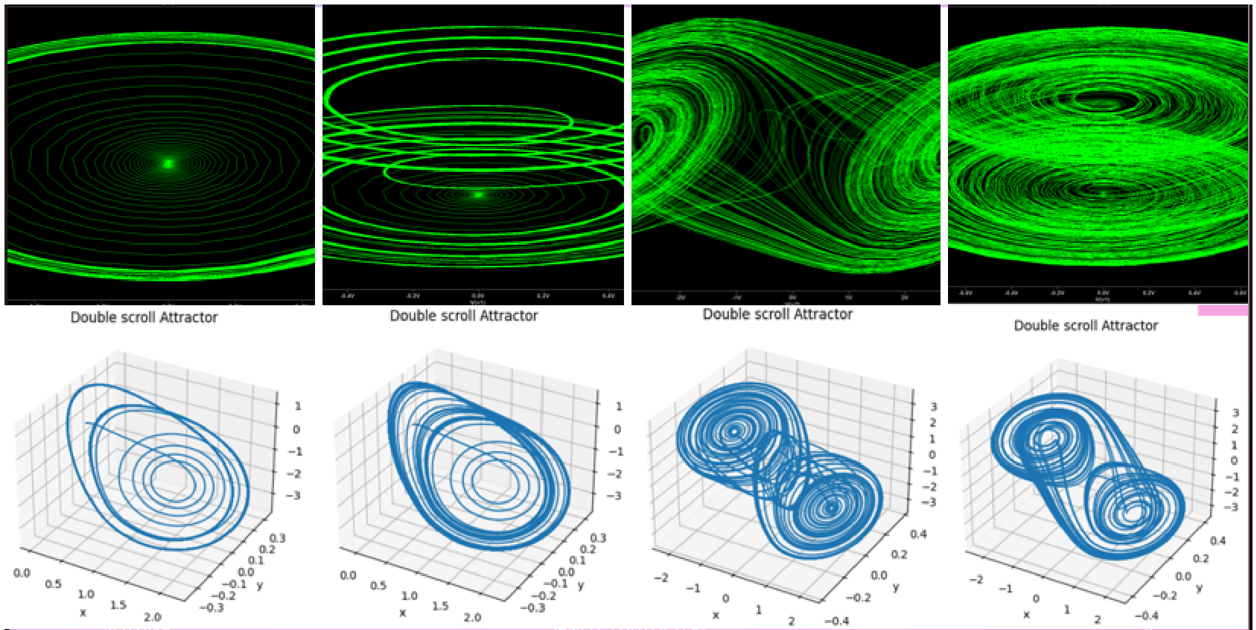


Figure Ten: Rössler Circuit

These two circuits are made of summing amplifiers, integrators, and analog multipliers (as well as resistors and capacitors). A summing amplifier is pretty straightforward, it just takes multiple inputs and gives their sum as the output. An integrator takes the time integral of the input, and an analog multiplier gives the negative product of its inputs as the output.

Finding the Coefficients and Circuit Values

Chua Circuit



Beta = 35   L = 6.5mH ; Beta = 33.8 L = 7mH ; Beta = 31   L = 8mH   ; Beta = 25 L = 8.5mH

Figure Eleven: By changing the Beta coefficient of the system of equations and the inductance of the circuit, one can see the period doubling route to chaos of the chua circuit occurs.

The plots shown in figure eleven were made through the circuit simulations on LTSpice on the top, which is technically a virtual oscilloscope. On the bottom, they were created by running the systems of equations on google CoLab.

By looking at figure eleven, one can see that period two can be found in the system of equations when beta is set at 35 and in the circuit when the inductance is 6.5 mH (millihenries). Period four seems to be at 7 mH for the circuit, and with beta set to 33.8 in the equations. At Beta set to 31 the system enters chaos, and the same for when the inductance is at 8mH. Finally, the system enters the attractor when Beta is 25 and the inductance is set to 8.5 mH.

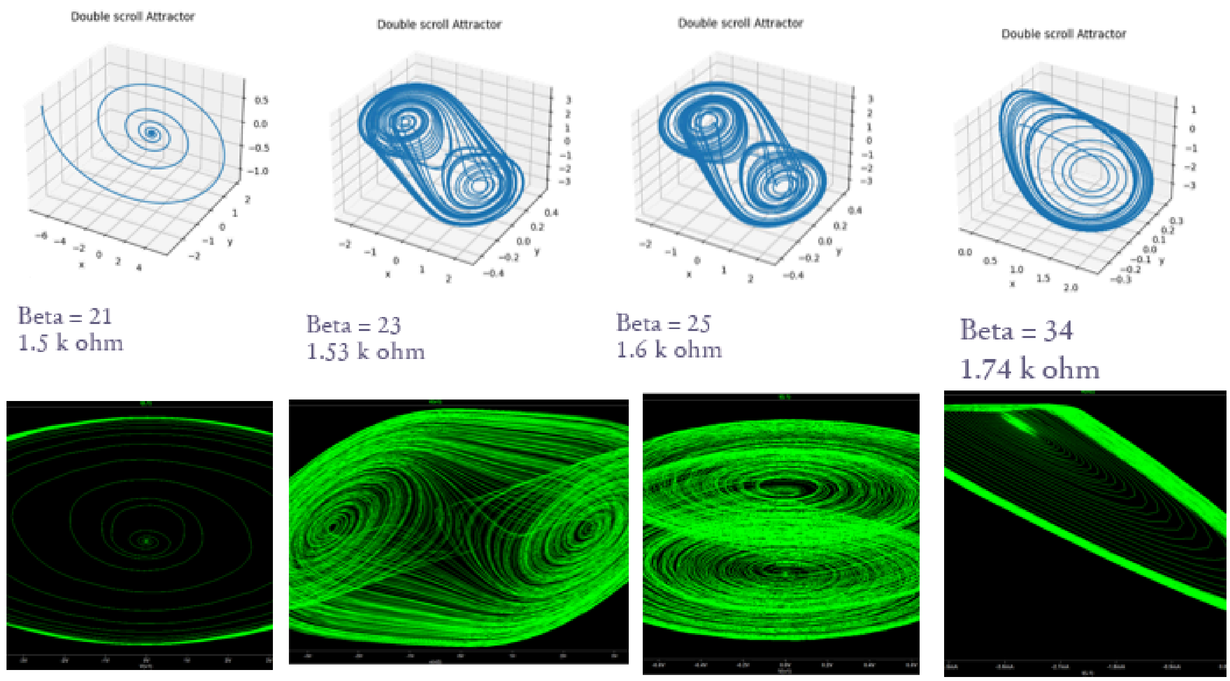
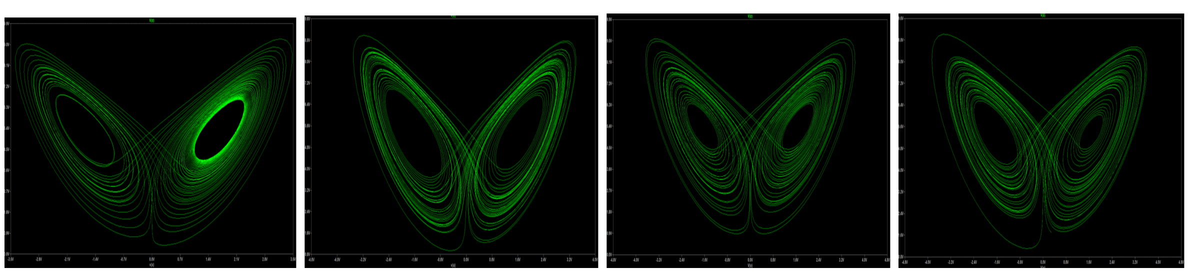
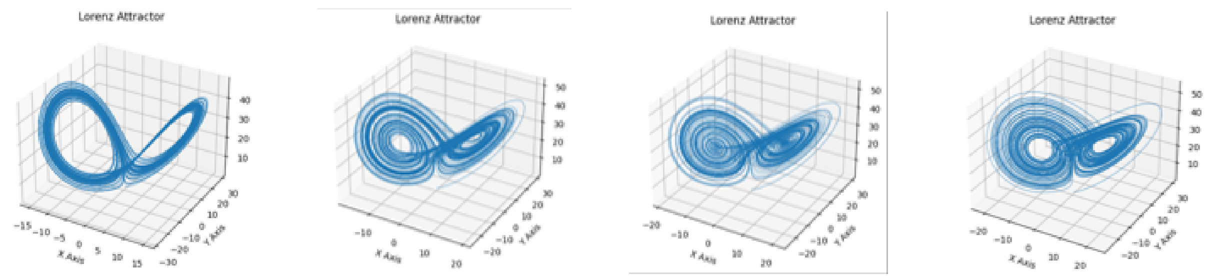


Figure twelve: Varying resistance of one resistor and beta

It was harder to find period doubling by varying resistance. Figure twelve shows the closest approach to the attractor before actually entering it. When beta is 21, the circuit seems to be the same as when resistance is set to 1.5 kOhms(Kilohms). It is very evident the circuit is at the same spot when beta is 23, and resistance is 1.53 kOhms, but at this point the circuit is already chaotic. It is again in the double scroll attractor when resistance is 1.6 k Ohms and beta is 25. From figure eleven, we know that when beta is 35 the system is in period 4. The right of figure twelve has beta at 34, and this appears to be right after period 4, right before it enters the attractor. This occurs physically when the circuit is at 1.74 kOhms.

Lorenz Attractor





S = 3.5 | S = 7.5 | S = 10 Attractor | S = 15

C = 0.79 nF | C=0.9 nF | C=1nF | C=1.2nF

Figure Thirteen: These were generated varying the S coefficient of the system of equations, and the capacitance of one of the capacitors.

The Lorenz system doesn't exhibit period doubling the way the Chua circuit does, so I just matched the shape. This attractor is called the Lorenz butterfly, since it is shaped like one. This is a useful attractor, since a small change in initial conditions will result in a large change in the output. When S is set to 3.5, and the capacitance at 0.79 nF(nanofarads) the plots look pretty similar. I believe they may be inverted. You can see a really bright green region on the right of the LTSpice plot, which means it is more dense with points. The same can be seen on the bottom plot, except on the left. As you go to S is 7.5 and 10, and capacitance from 1 nF to 1.2 nF, you can see the attractor become more filled in. At 1nF and S at 10, the full Lorenz butterfly attractor is exhibited. When S is 15 and capacitance is 1.2nF, the system is leaving the attractor but still chaotic.

These particular plots seemed to match the best.

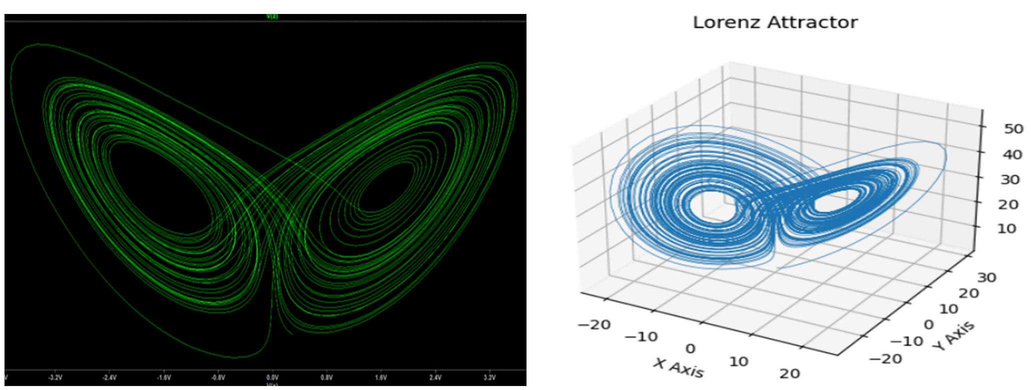


Figure Fourteen: Plots of the lorenz attractor with S = 15 and C = 1.2 nF

These were in the previous figure, but pointing out their similarities is easier when blown up. Both of them have the sort of stray strand, just on the opposite side, since they appear to be mirrored images. They both also become less filled in than the attractor is.

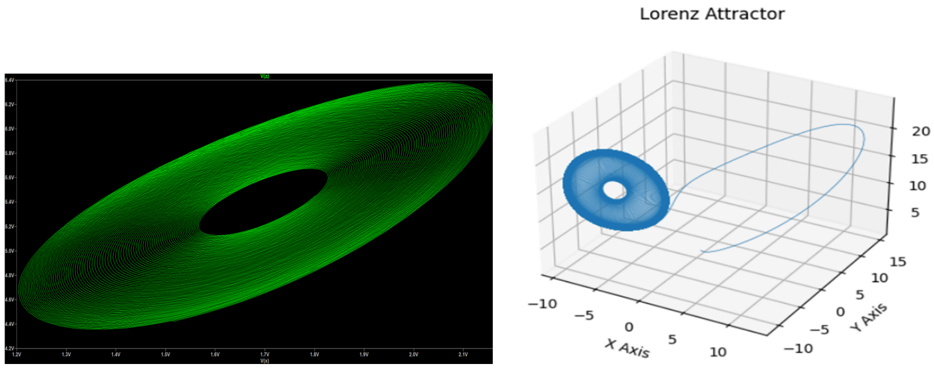
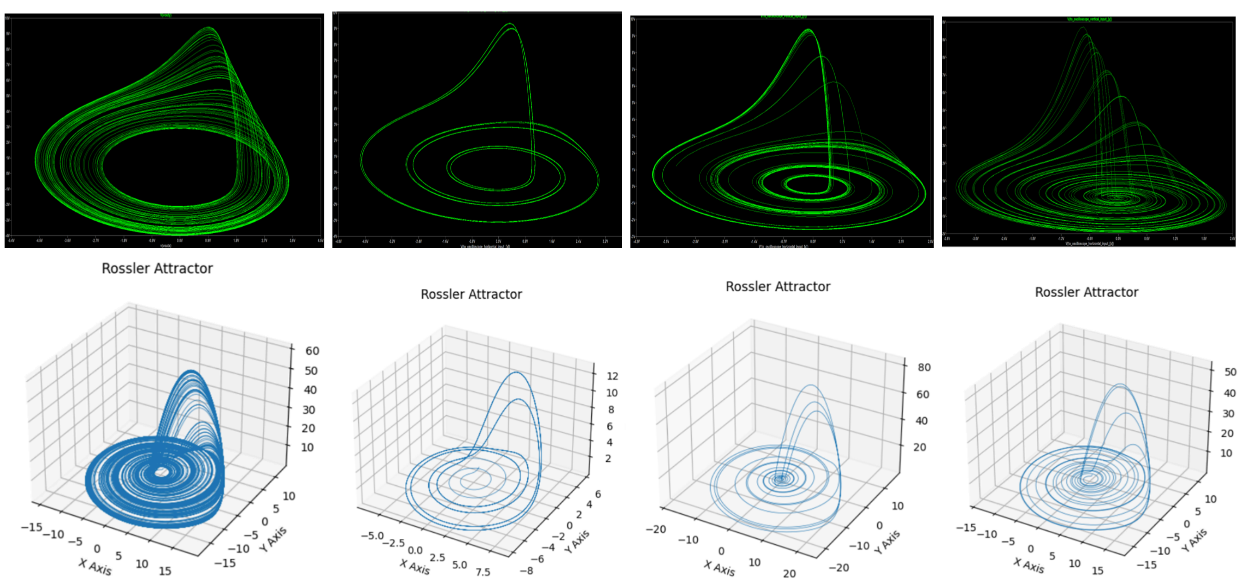


Figure Fifteen: R = 14.9563 and C = 0.78nF

Figure fifteen interested me as this was as close as I could get without entering the butterfly shaped attractor. These also appear to be the exact same shape. This could be useful, since without the butterfly shape the system doesn’t have the same drastic dependence on initial conditions.

Rössler Attractor

Similar to the Chua circuit, this attractor also has a period doubling route to chaos. However, it has period 1, period 2, period 4, chaotic attractor, period 3, period 6, and chaotic attractor again.



Attractor C = 9 R = 10 kΩ | Period 2 C = 4 R = 12.5 kΩ | Period 3 C = 12 R = 15 kΩ | Period 4 C = 8.5 R = 18 kΩ

Figure Sixteen: Varying resistance and C coefficient for the Rössler attractor.

While there are a few more periods, I was only able to successfully simulate period 2, 3, and 4. There are also two different phases of the attractor, but I could only get one. By looking at figure sixteen, you can see setting C to 4 and resistance to 12.5 kOhms (Kilohms) will give period 2, setting C to 12 and Resistance to 12.5 gives period 3, and setting C to 8.5 and resistance to 18 kOhms is period 4. I assume if you keep going, you could find the other attractor or period 6, but that was not what happened. Since I was only carrying one resistor in the whole circuit, it could be prohibiting me from finding the other periods. Changing multiple would probably be necessary to find them.

Future Research

While I was successfully able to find some physical values of the circuits that match that of the system of equations, there are definitely more to be found. For example, I could try to vary every coefficient of the system of equations and see if they have periods, or just try to match their different shapes with that of the circuit's output. I also only varied a few of the components from the circuits, but each had so many that could have been varied to create different results. In addition, the points from the plots generated in LTSpice could have been used to calculate the Lyapunov exponents to see how close they are to the actual known values.