

The background of the slide is a composite image. It features a close-up of a human eye, looking directly forward. Overlaid on this image is a complex, glowing blue and white digital circuit pattern, resembling a printed circuit board (PCB) or a network diagram. The lines of the circuit are thin and intricate, weaving across the entire frame. The eye itself is detailed, with visible eyelashes and a clear iris. The overall color palette is dominated by the cool blues and whites of the circuit, with the natural tones of the eye providing a focal point.

# Introduction to Computer Vision

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**Kaveh Fathian**

Assistant Professor

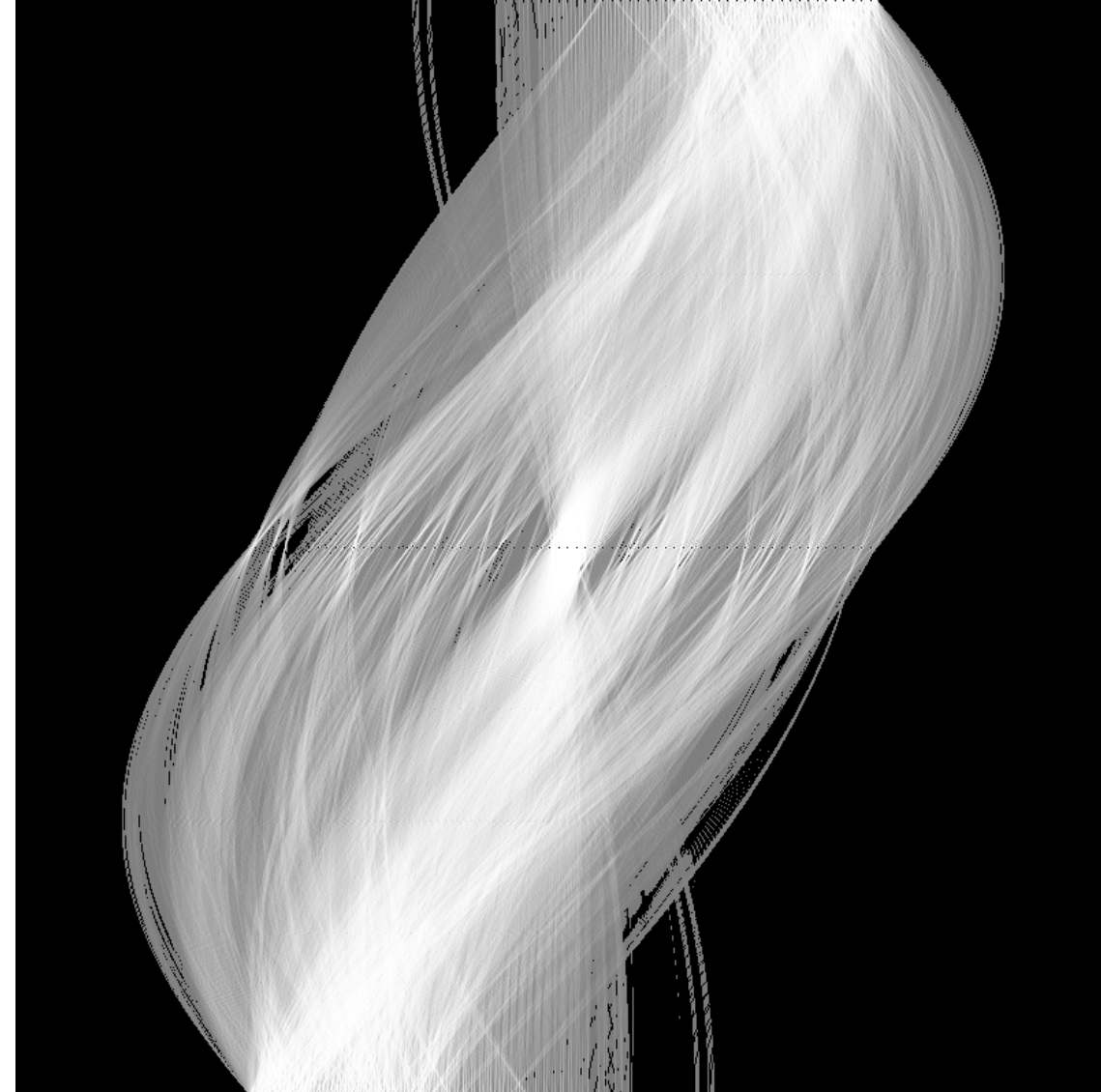
Computer Science Department

Colorado School of Mines

**Lecture 7**

# Learning Outcomes

- Hough Transform
- Line Detection



# Hough transform

Hough transform is a **generic framework** for detecting a parametric model

Example: Hough transform can detect lines

Why Hough transform? E.g., vs. Canny edge detection?

- Edges don't have to be connected
- Lines can be occluded
- Key idea: edges **vote** for the possible models

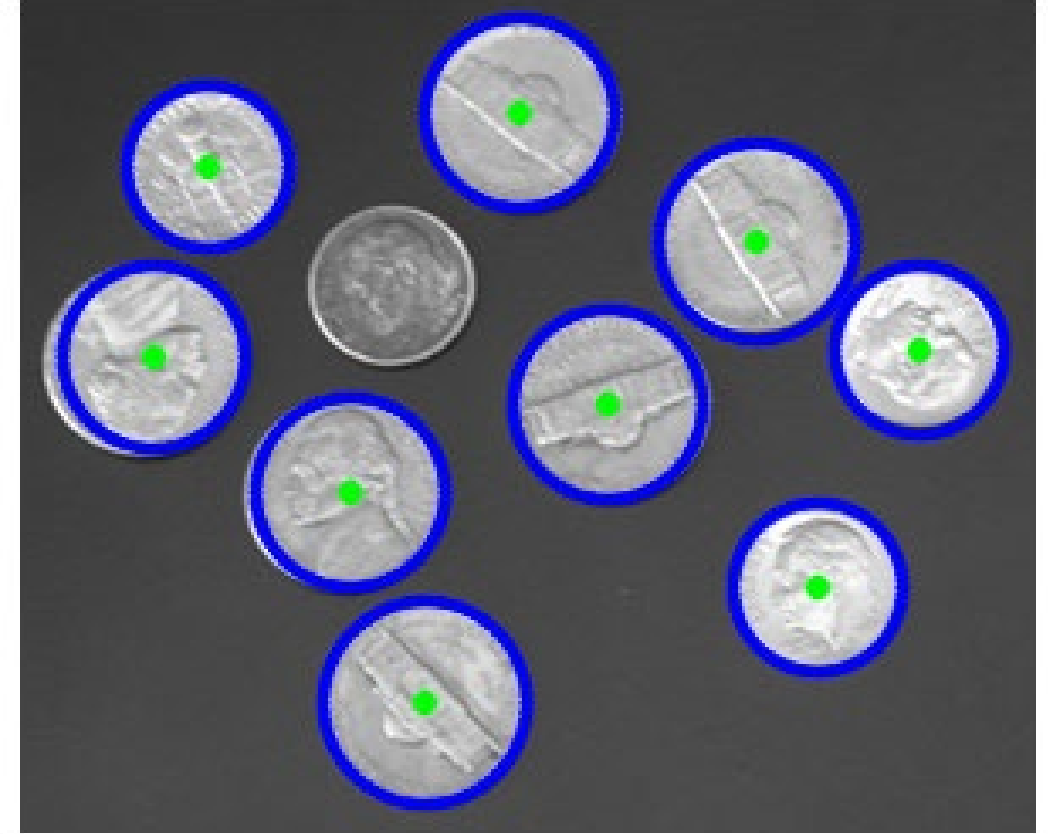


# Circles

source



detected circles



Example: Hough transform for circle detection  
What is its application?



# Image and parameter space

variables

$$y = mx + b$$

parameters

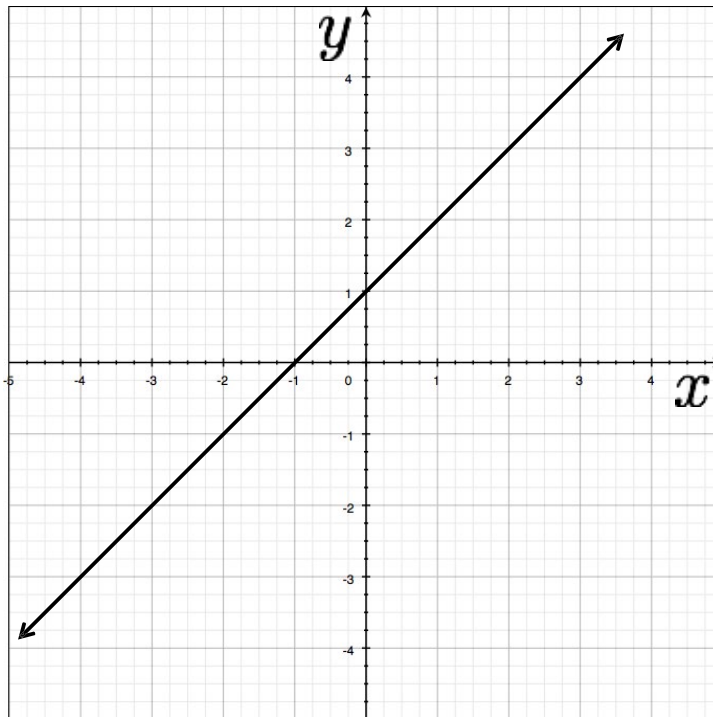


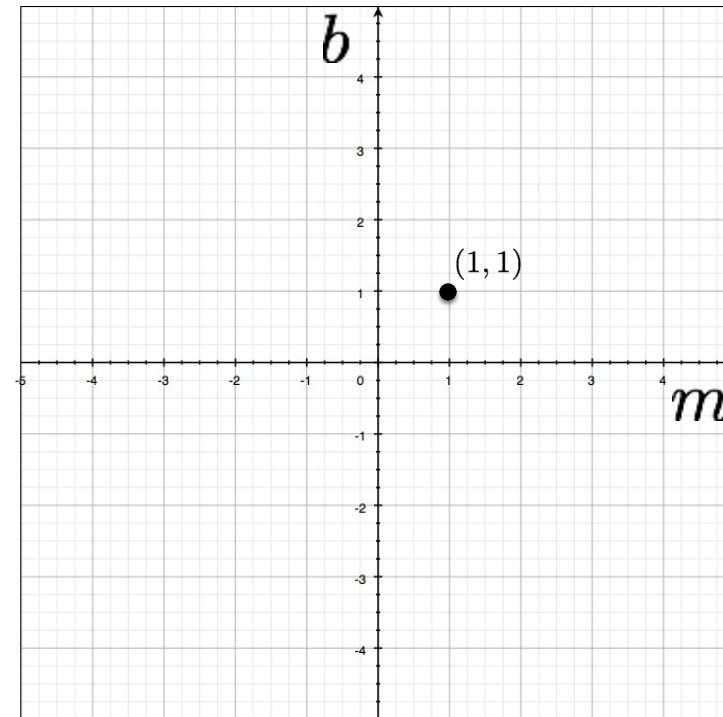
Image space

a line  
becomes a  
point

variables

$$y - mx = b$$

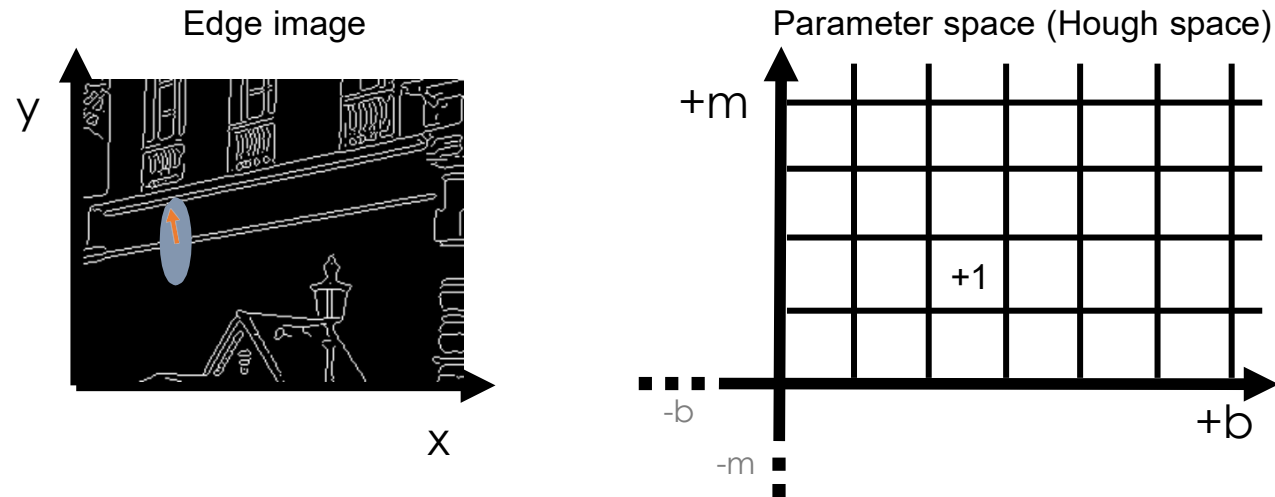
parameters



Parameter space

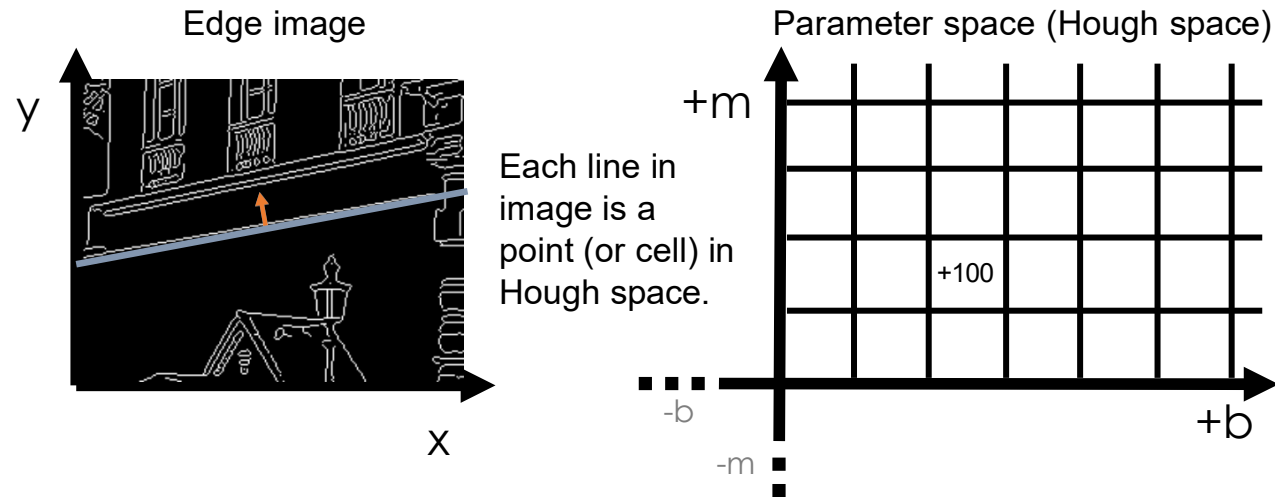
# Hough Transform: Outline

- Create a *grid* of candidate  $m, b$  parameter values: **accumulator**
  - Why a grid?
  - $m, b$  are continuous; grid discretizes into hypotheses.
- Each edge pixel votes for a set of parameters, which increments those values in grid



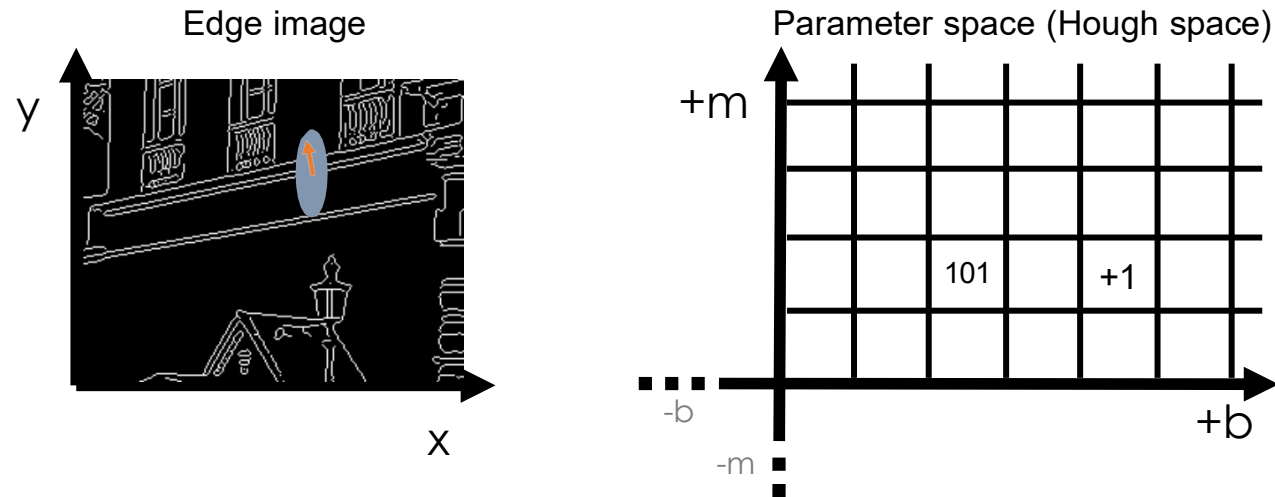
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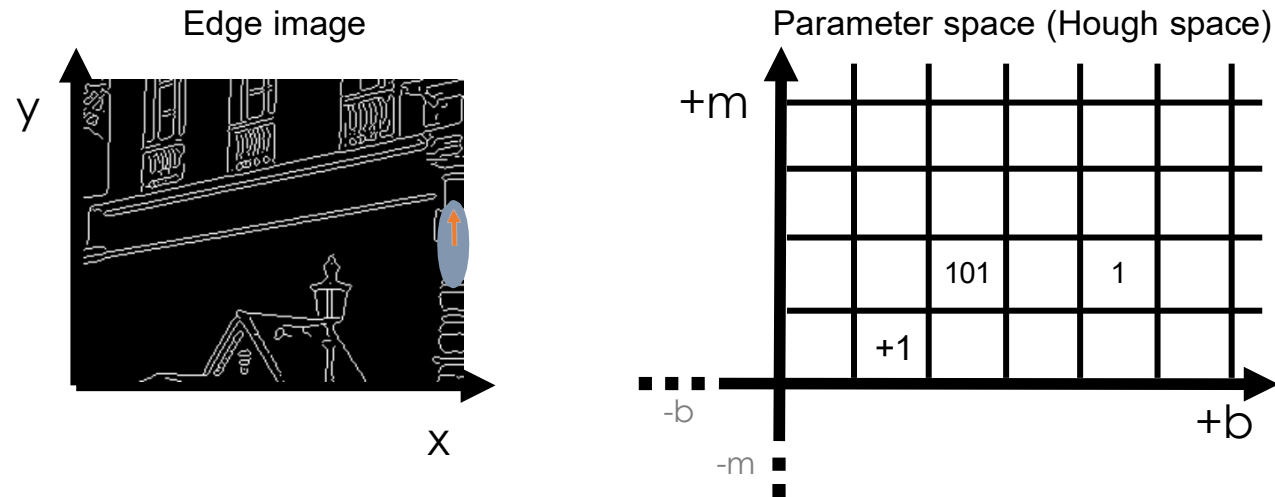
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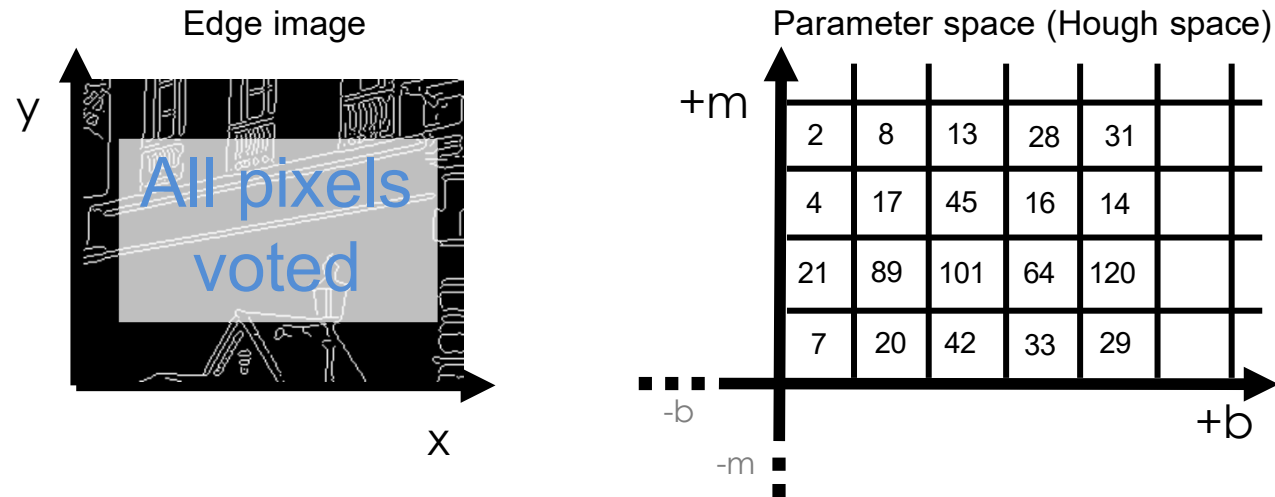
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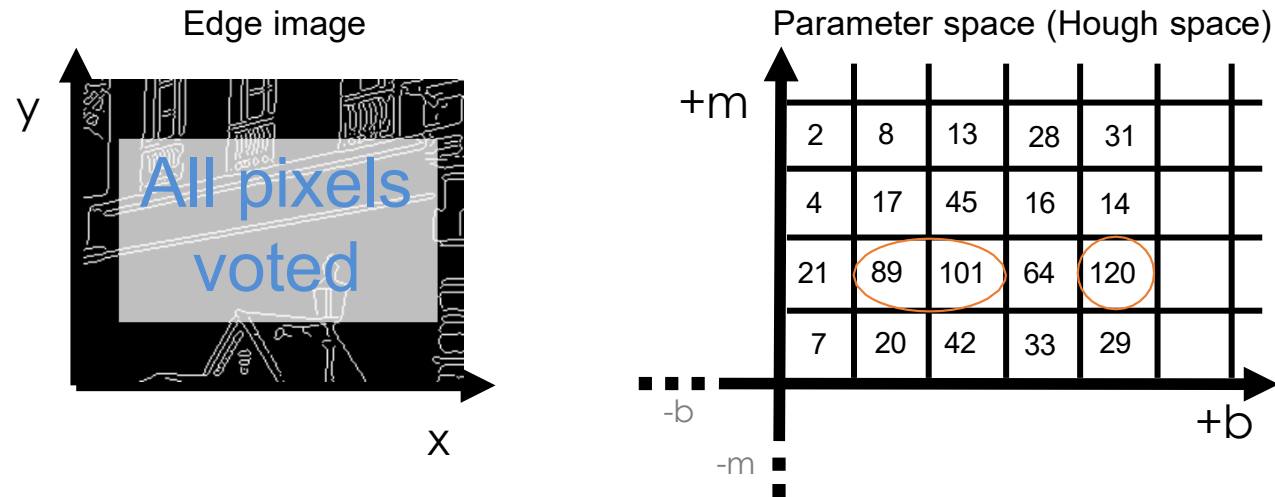
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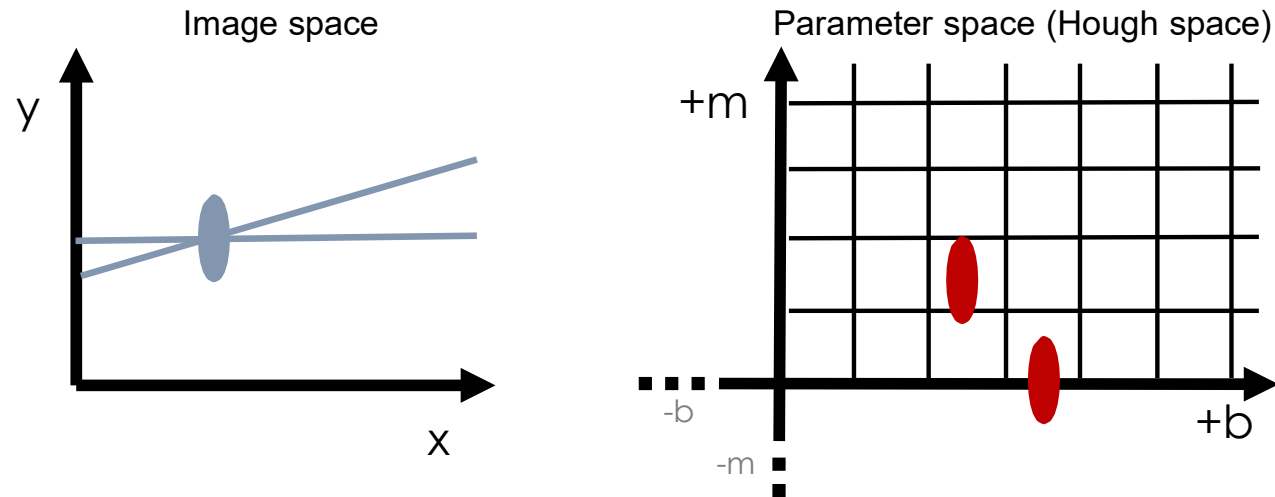
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- Find maxima – our line candidates.



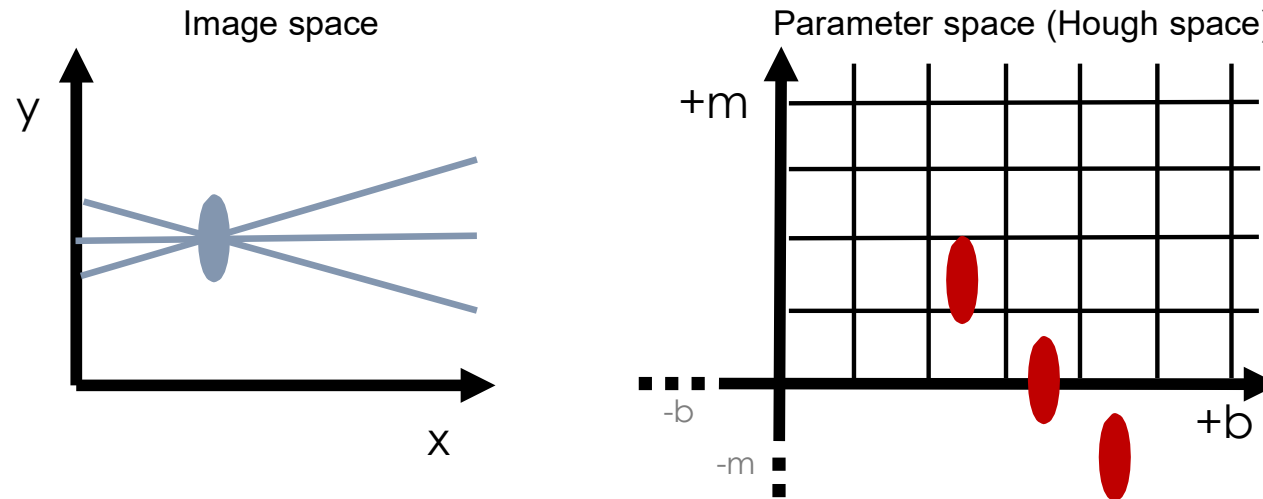
# Hough Transform: Step back

- Hough space represents all possible lines.
- With gradient information constraint:
  - Edge is single point in Hough space.
- Without gradient orientation information?



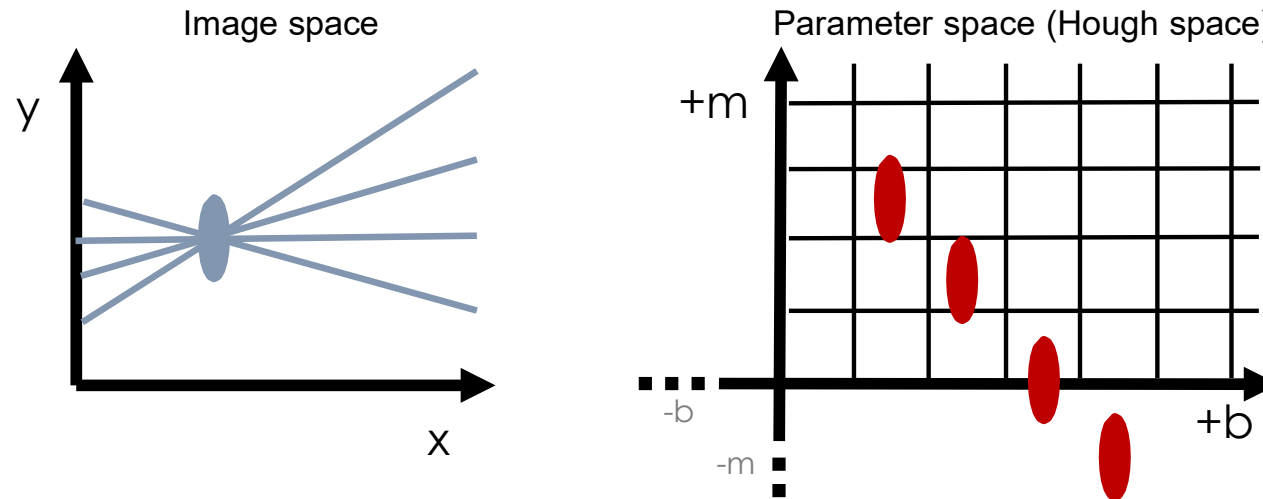
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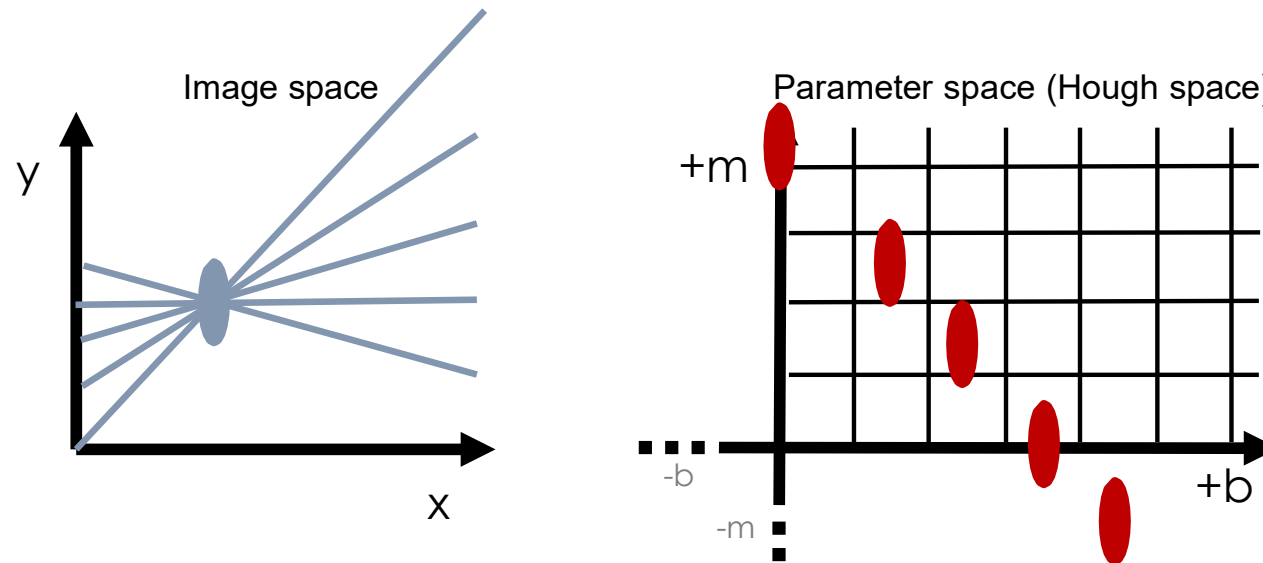
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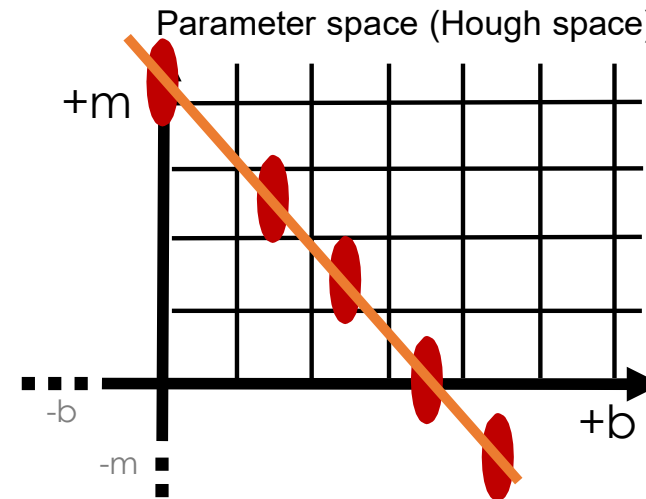
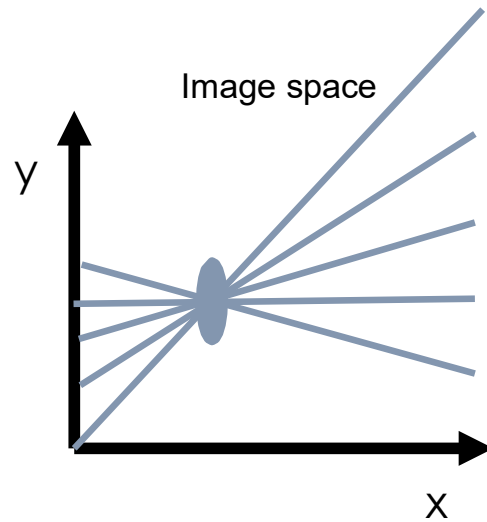
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# Hough Transform: Step back

- Hough space represents all possible lines.
- With gradient information constraint:
  - Edge is single point in Hough space.
- Without gradient orientation information?
  - Point is line in Hough space



# Image and parameter space

variables

$$y = mx + b$$

parameters

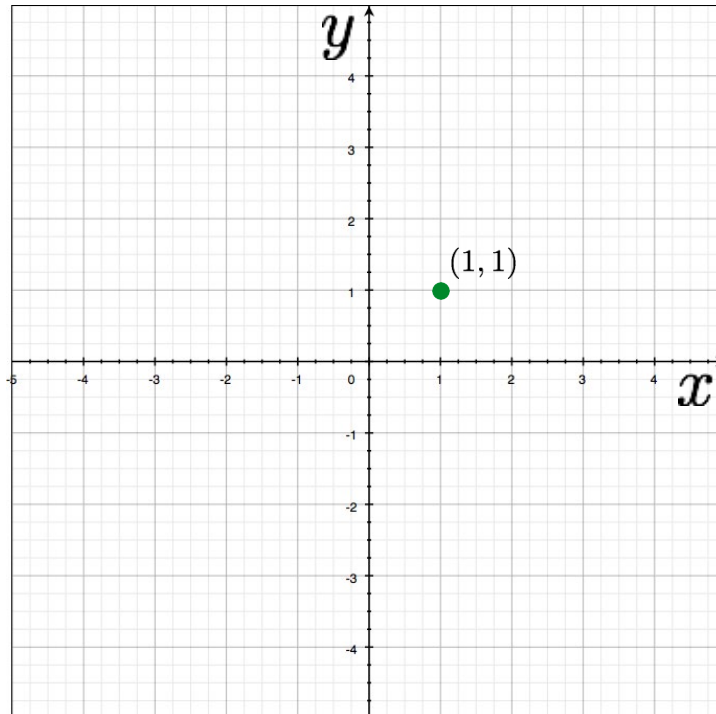
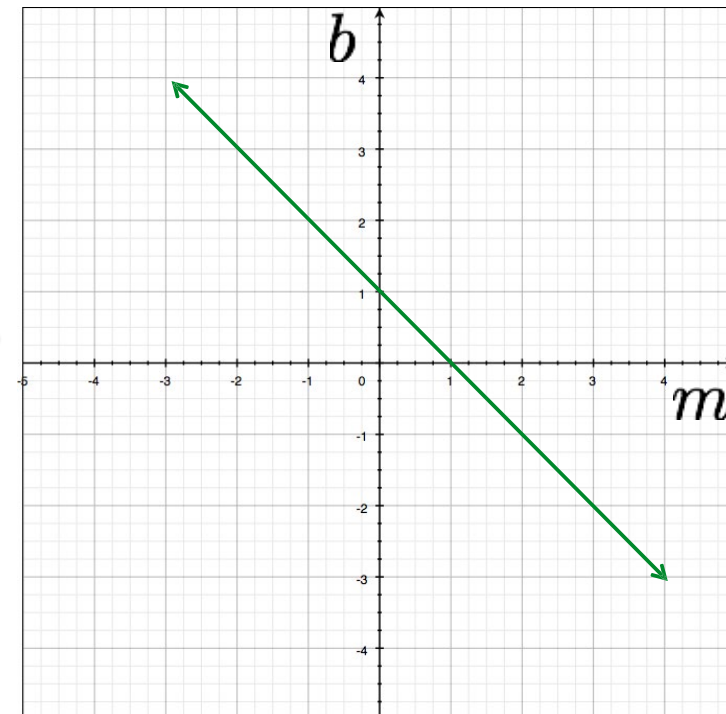


Image space

variables

$$y - mx = b$$

parameters



Parameter space

a point  
becomes a  
line

# Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

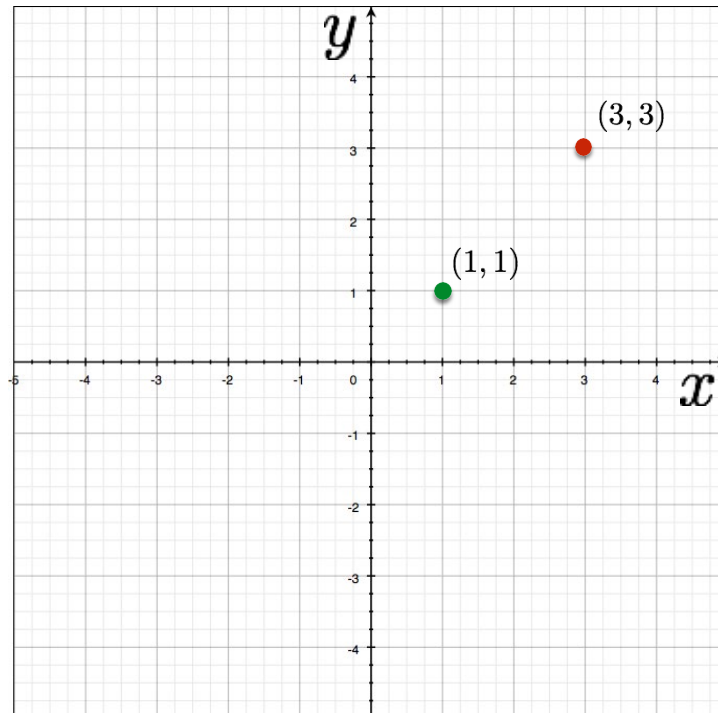
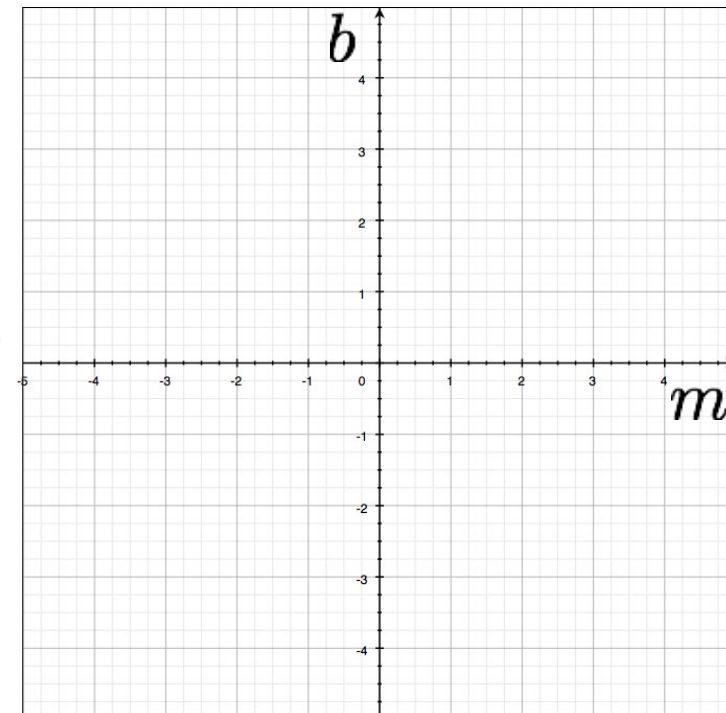
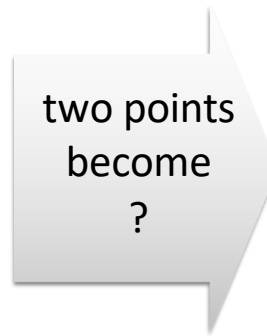


Image space



Parameter space

# Image and parameter space

variables

$$y = mx + b$$

parameters

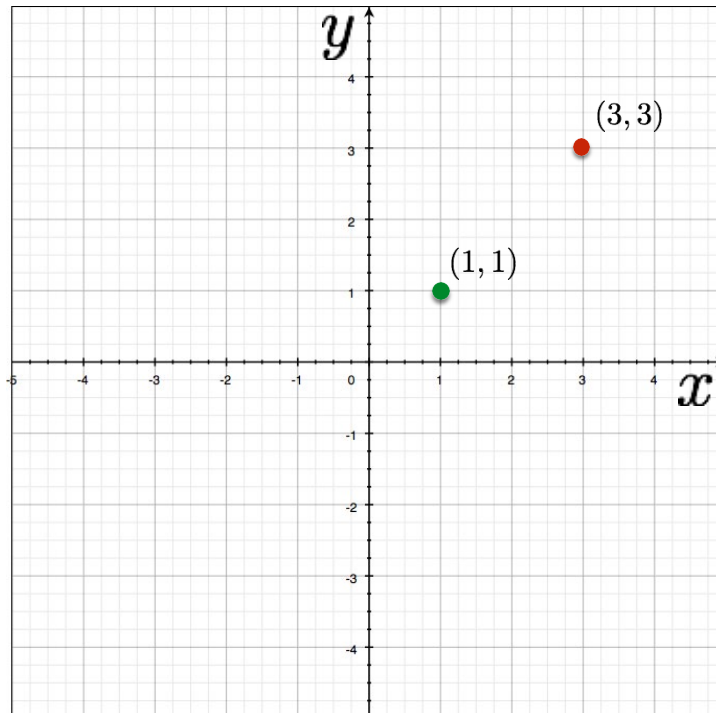


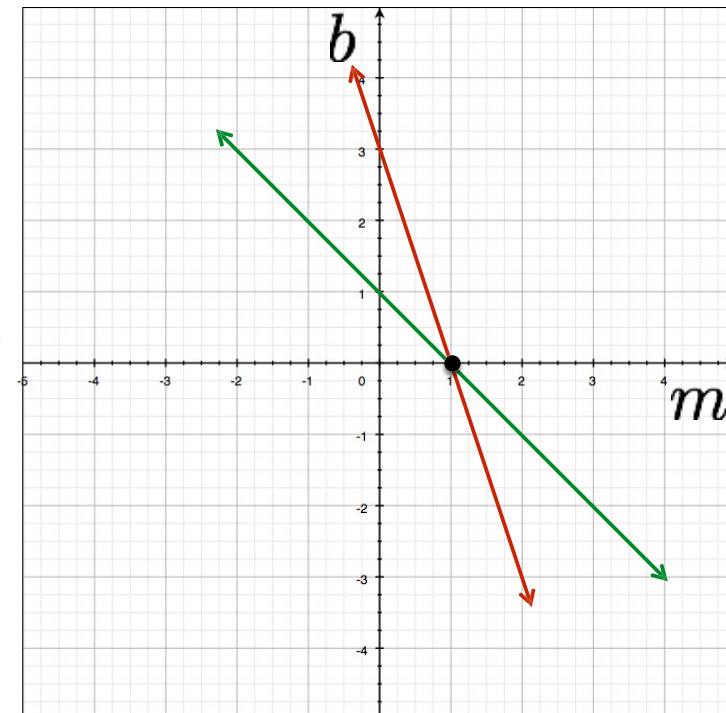
Image space

two points  
become  
?

variables

$$y - mx = b$$

parameters



Parameter space

# Image and parameter space

variables

$$y = mx + b$$

parameters

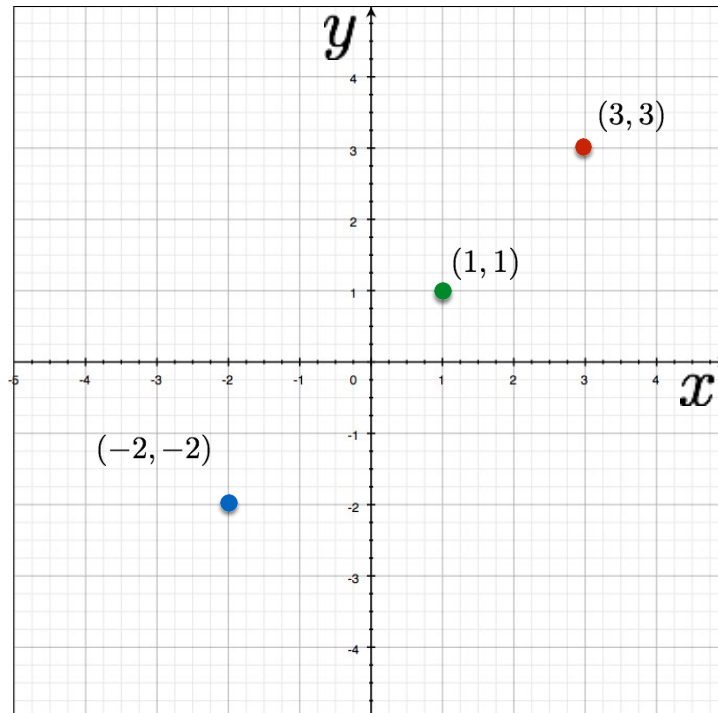


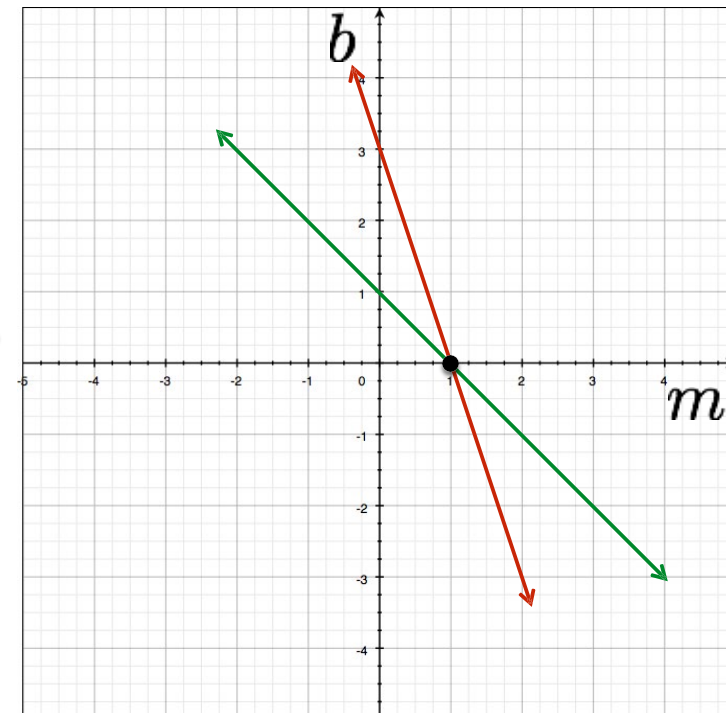
Image space

three points  
become  
?

variables

$$y - mx = b$$

parameters



Parameter space



# Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

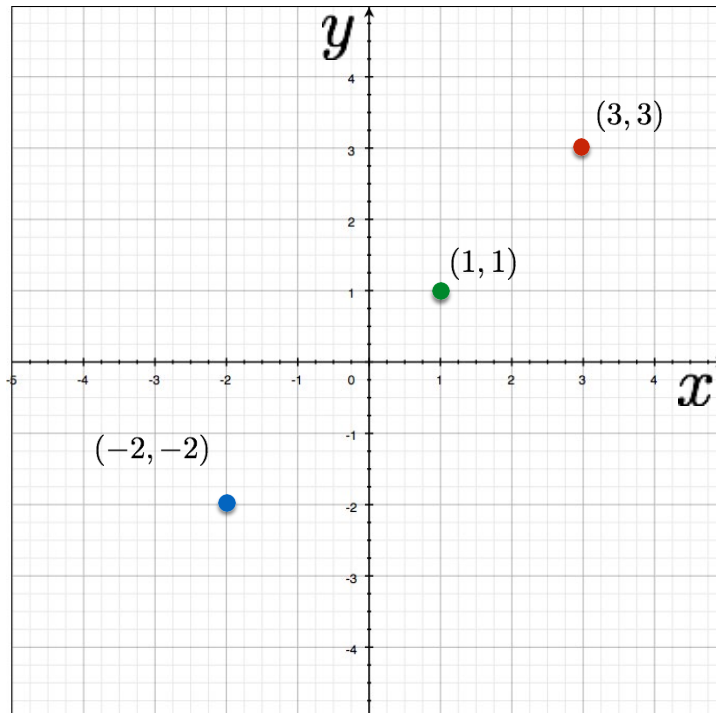
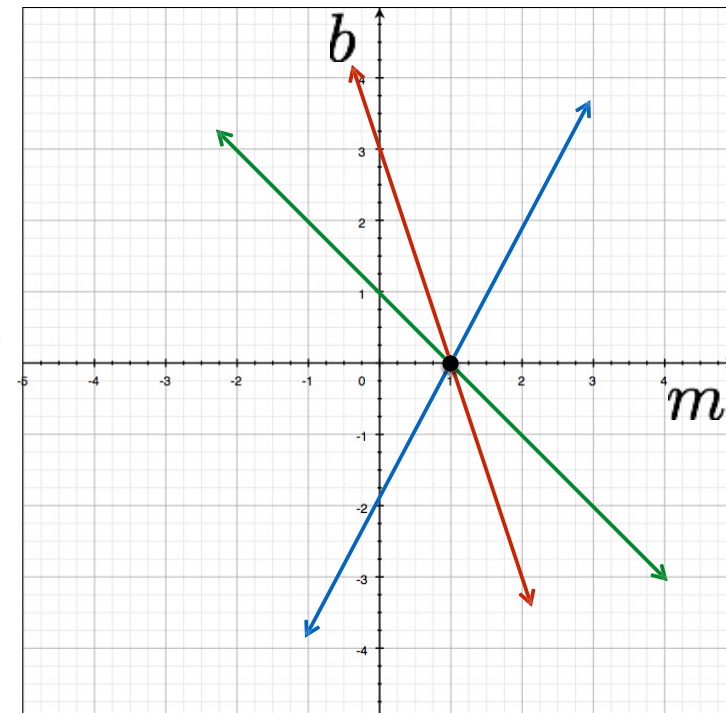


Image space

three points  
become  
?



Parameter space

# Image and parameter space

variables

$$y = mx + b$$

parameters

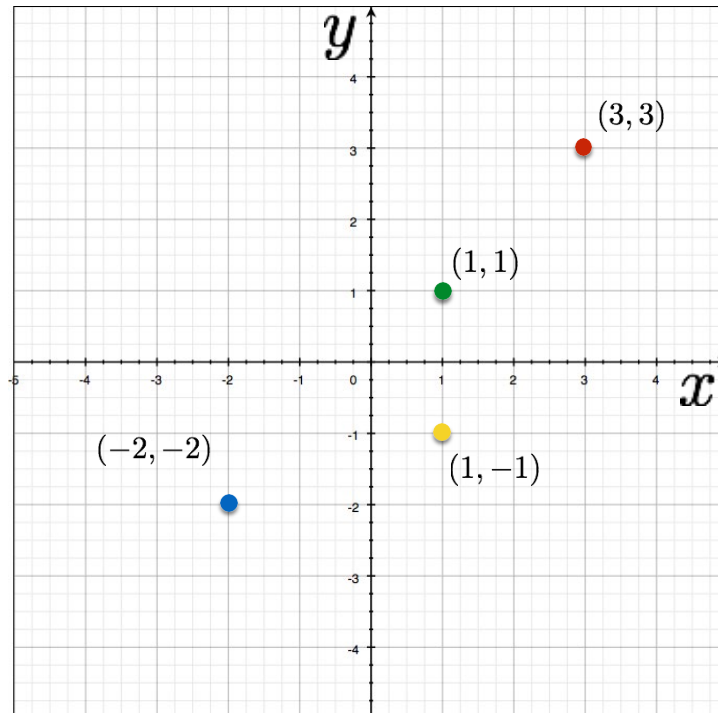
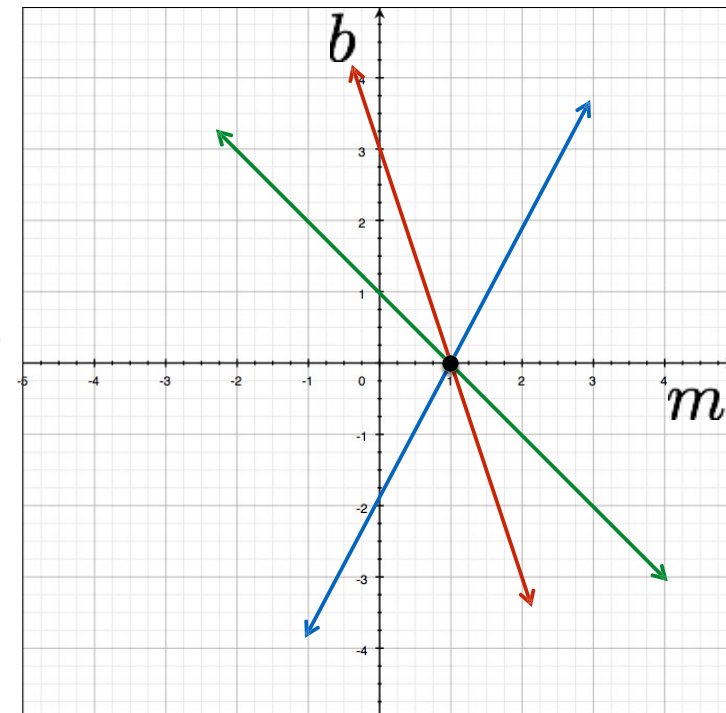


Image space

variables

$$y - mx = b$$

parameters



Parameter space

four points  
become  
?

# Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

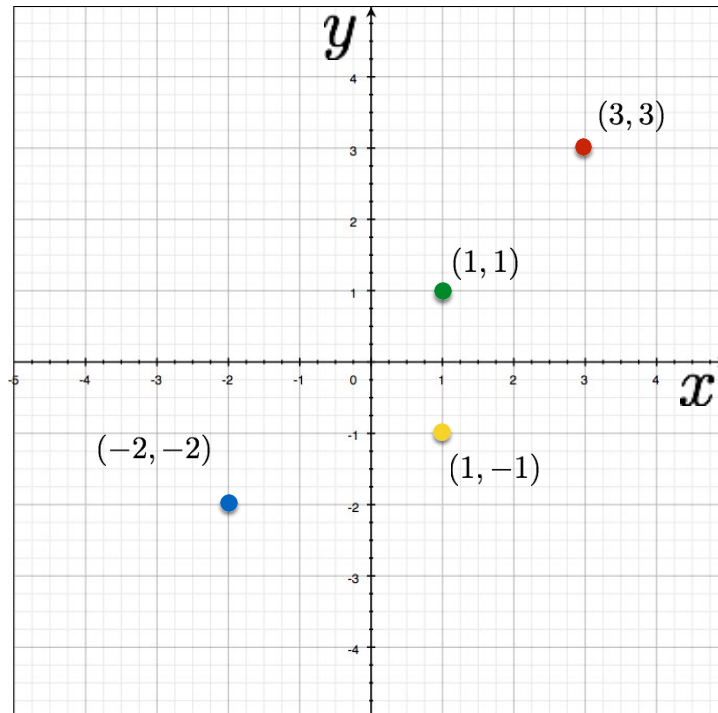
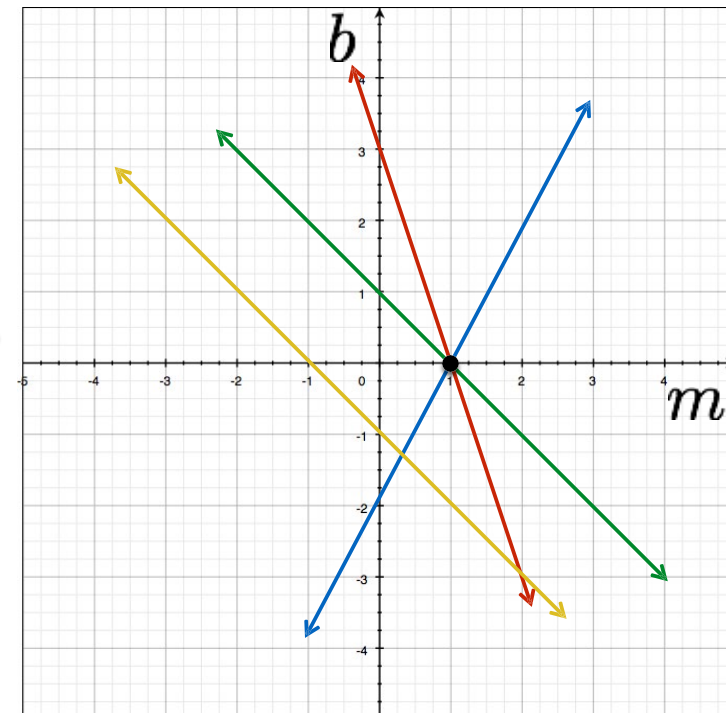


Image space

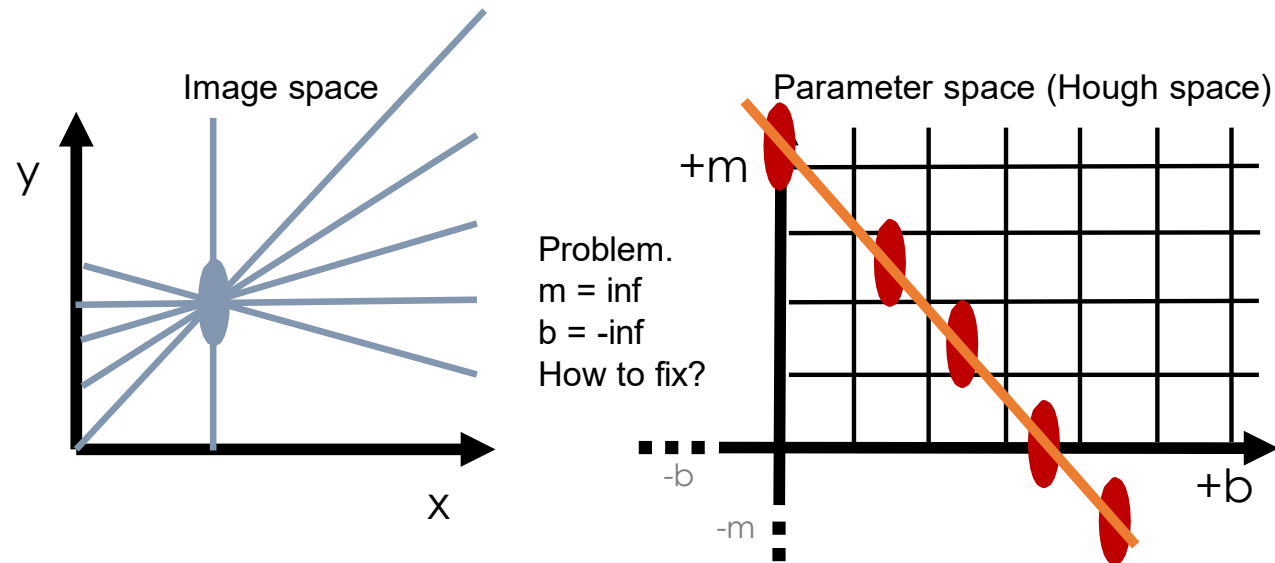
four points  
become  
?



Parameter space

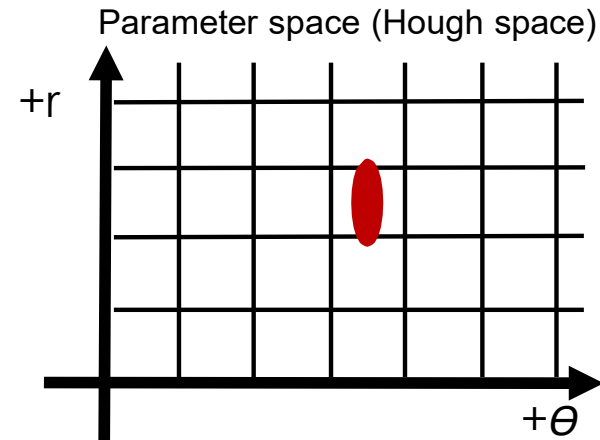
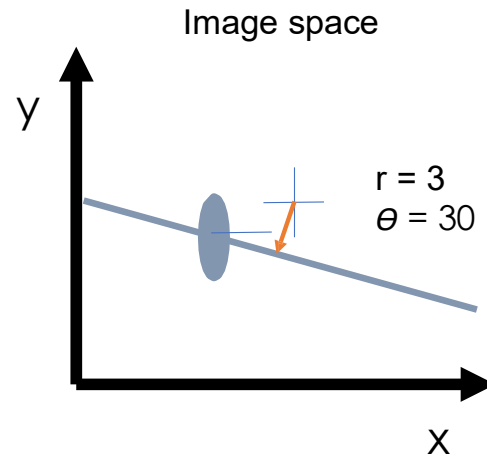
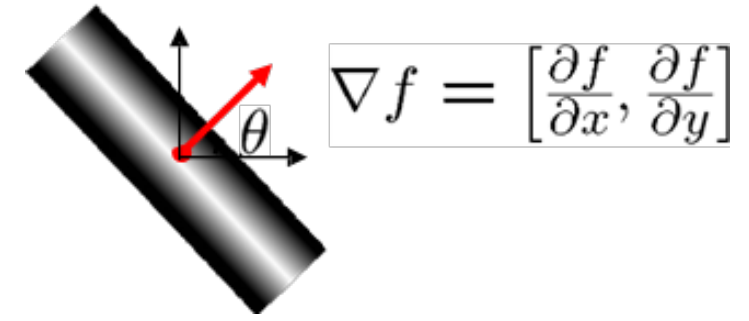
# Hough Transform: Step back

- Hough space represents all possible lines
- With gradient information constraint:
  - Edge is single point in Hough space
- Without gradient orientation information?
  - How big is Hough space?



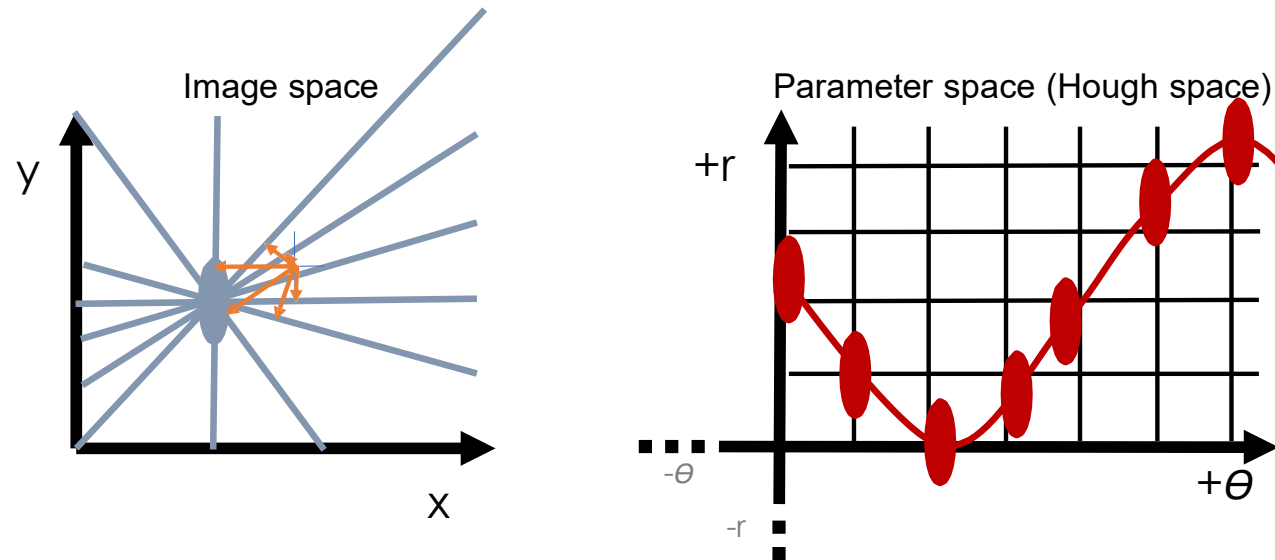
# Hough Transform: Line Normal Form

- Use  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$ 
  - Space is 0 to 360
- Use  $r$  = distance to line from some origin
  - $r_i = x_i \cos \theta_i + y_i \sin \theta_i$
  - Space is  $\pm \sqrt{\max_x^2 + \max_y^2}$



# Hough Transform: Line Normal Form

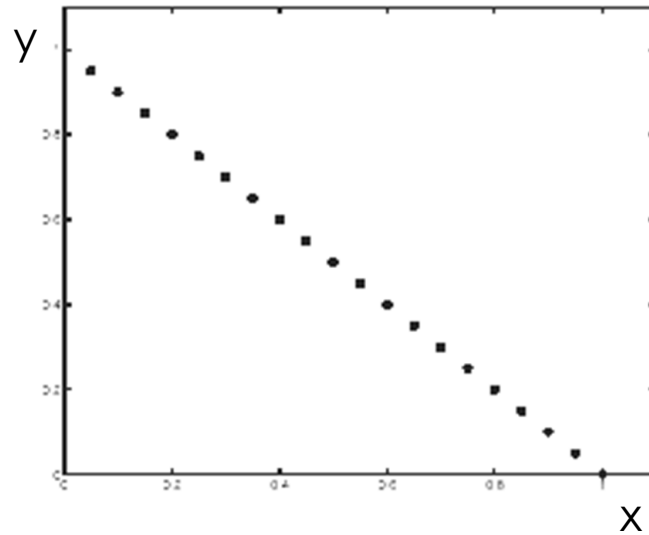
- In this line form, unoriented edge draws a sinusoid in Hough space:



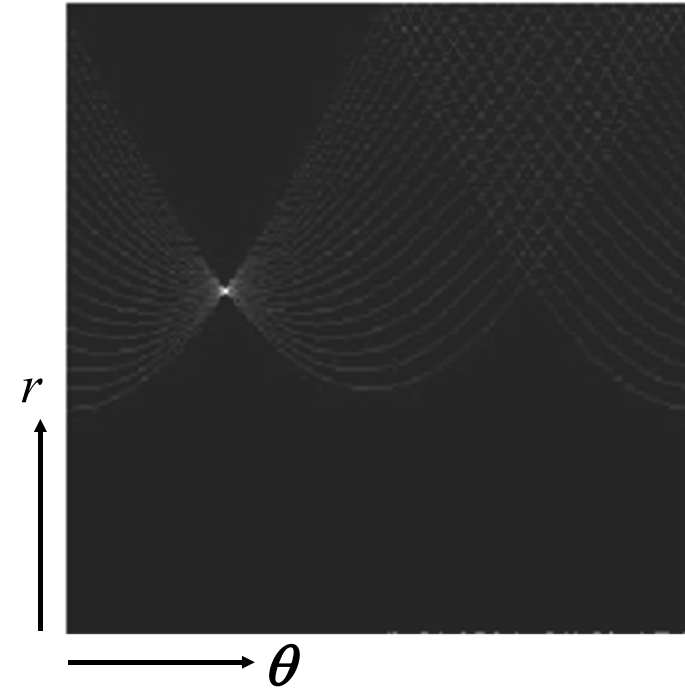


# Hough transform - experiment

Next few images *ignore* edge orientation.  
Each point is one sinusoid.

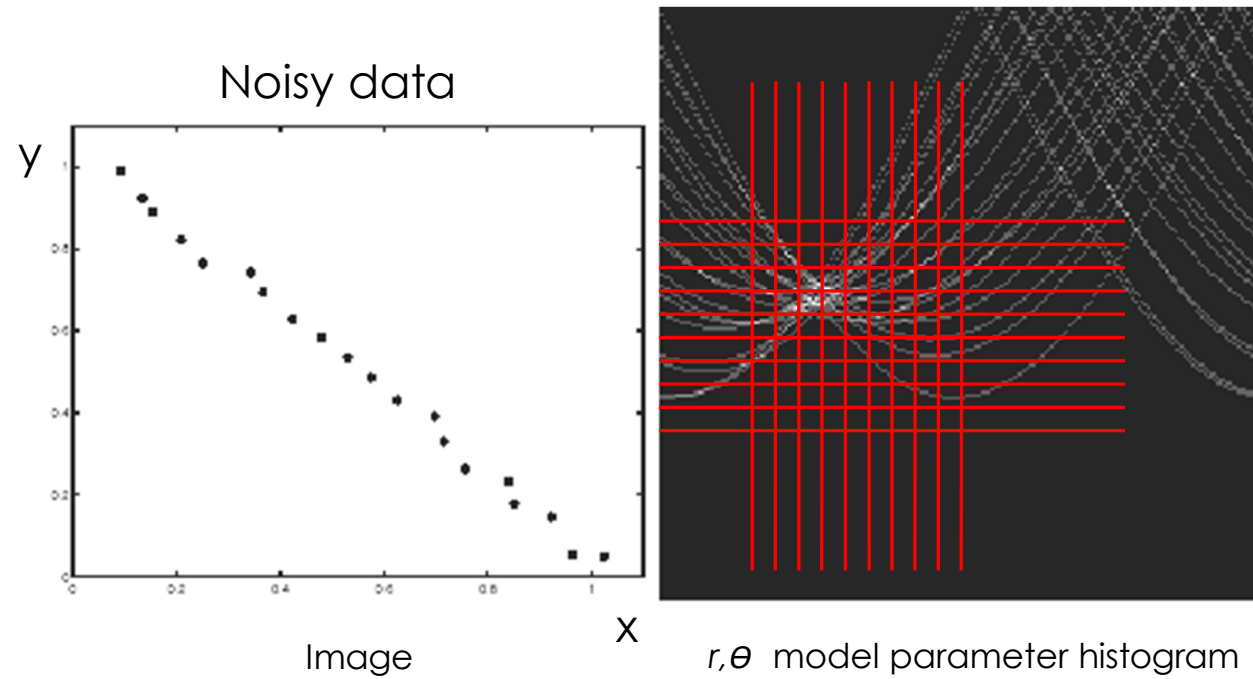


Image



$r, \theta$  model parameter histogram

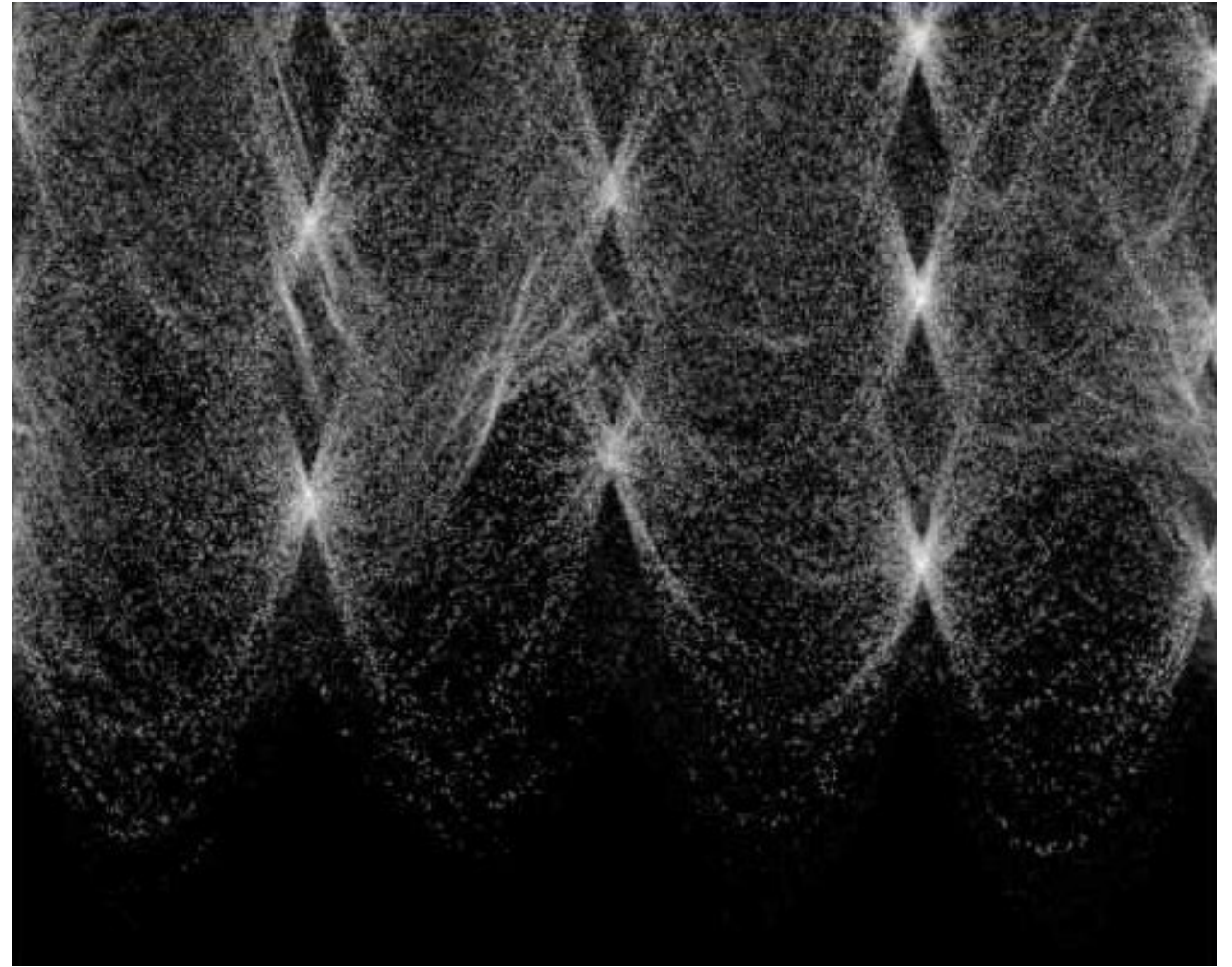
# Hough transform - experiment



Need to adjust grid size or smooth

- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

# Hough transform example

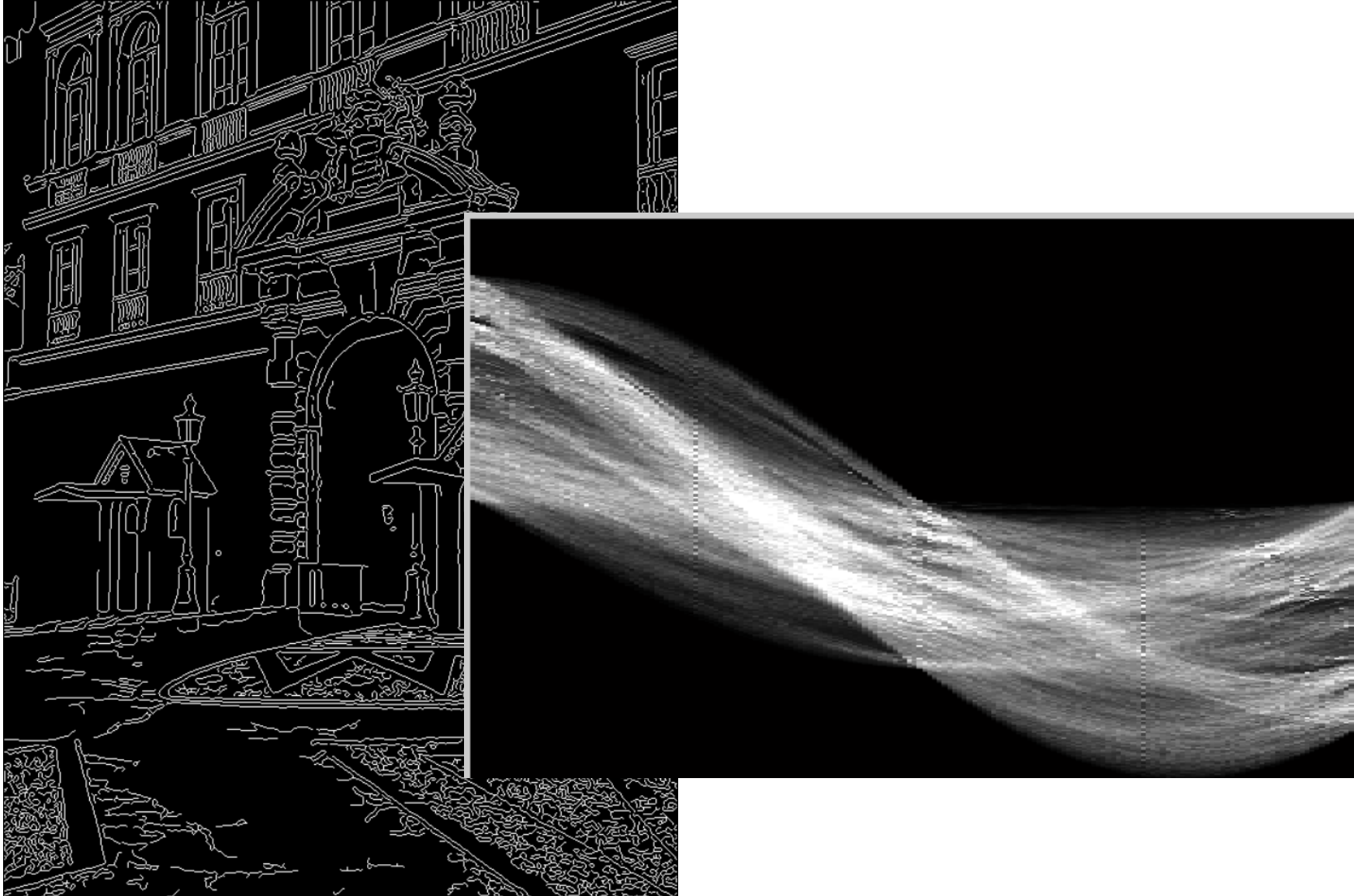




# Image → Canny



# Canny → Hough votes



# Hough votes $\rightarrow$ Edges

Find peaks and post-process.

