Introduction to Computer Vision

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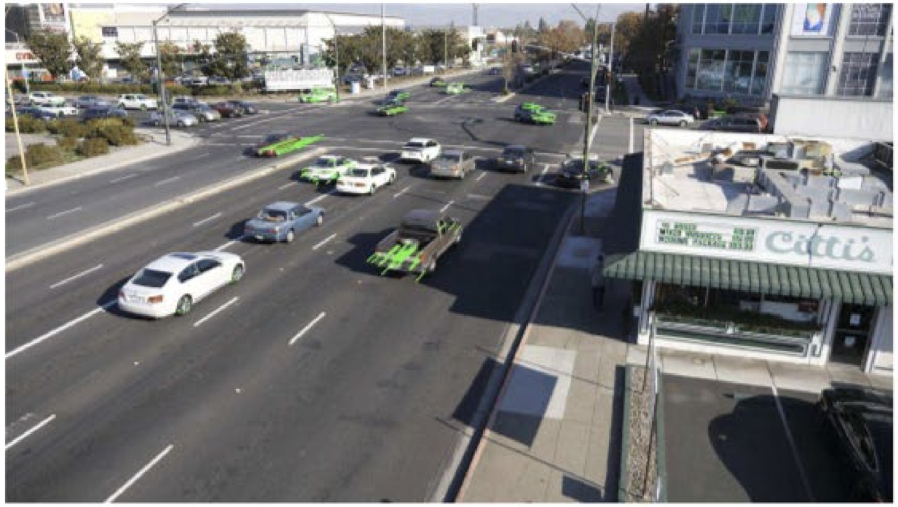
Lecture 8

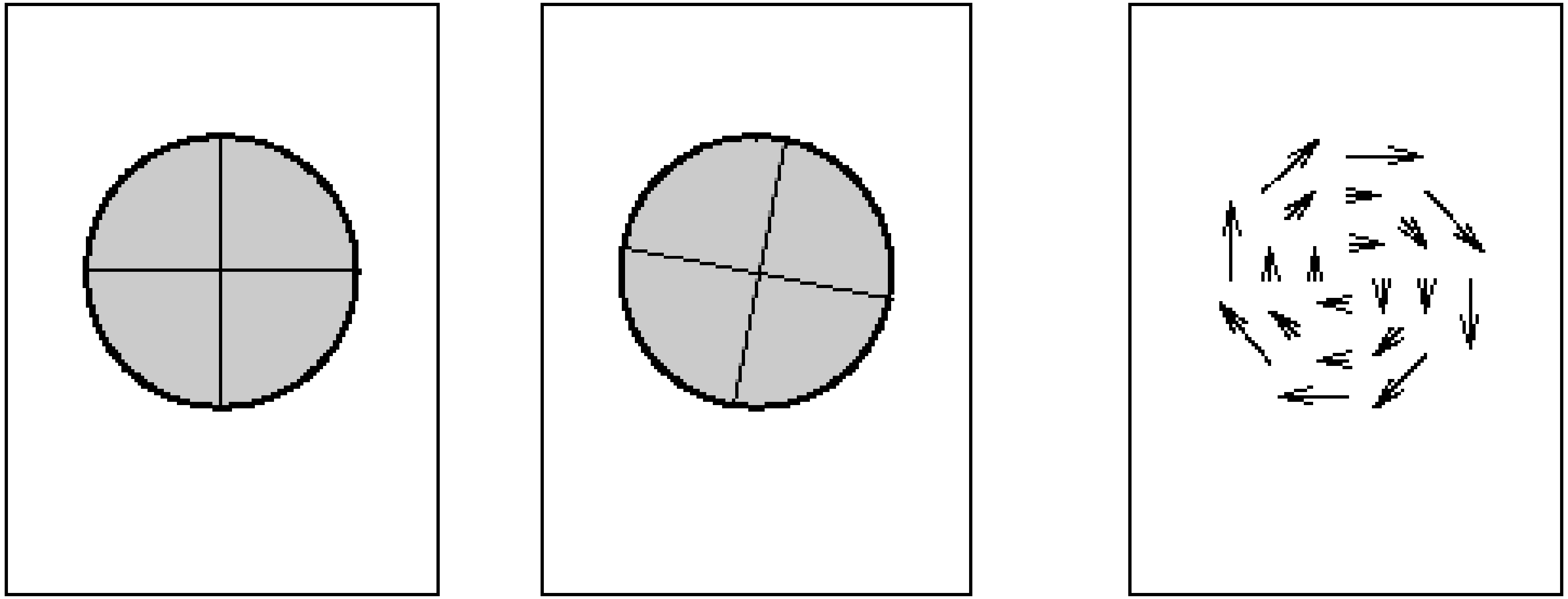
Learning Outcomes

- Optical Flow



Optical Flow



Fundamental Equations of Computer Vision

1. Image Filtering $h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$

2. Optical Flow $I(x,y,t)=I(x+\Delta x, y+\Delta y, t+\Delta t)$

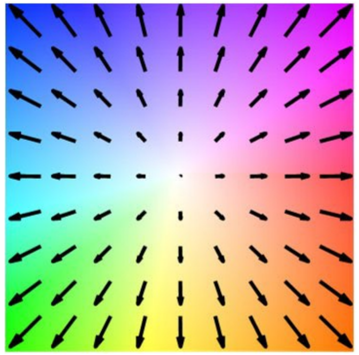
3. Camera Geometry $\mathbf{x}=\mathbf{K}\left[\mathbf{R}\ \mathbf{t}\right]\mathbf{X}$ $\quad x^T F x' = 0$

4. Machine Learning $\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x}\in S_i} \|\mathbf{x}-\boldsymbol{\mu}_i\|^2.$ $\quad y=\varphi(\sum_{i=1}^{n} w_i x_i + b) = \varphi(\mathbf{w}^T\mathbf{x}+b)$

Optical Flow: Dense Correspondence Over Time

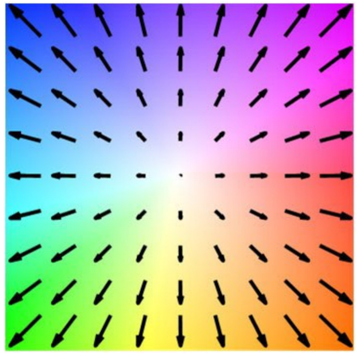
Input [Liu *et al.* CVPR'08]

Optical flow (2D motion vector)



Color key
[Baker *et al.* IJCV'11]

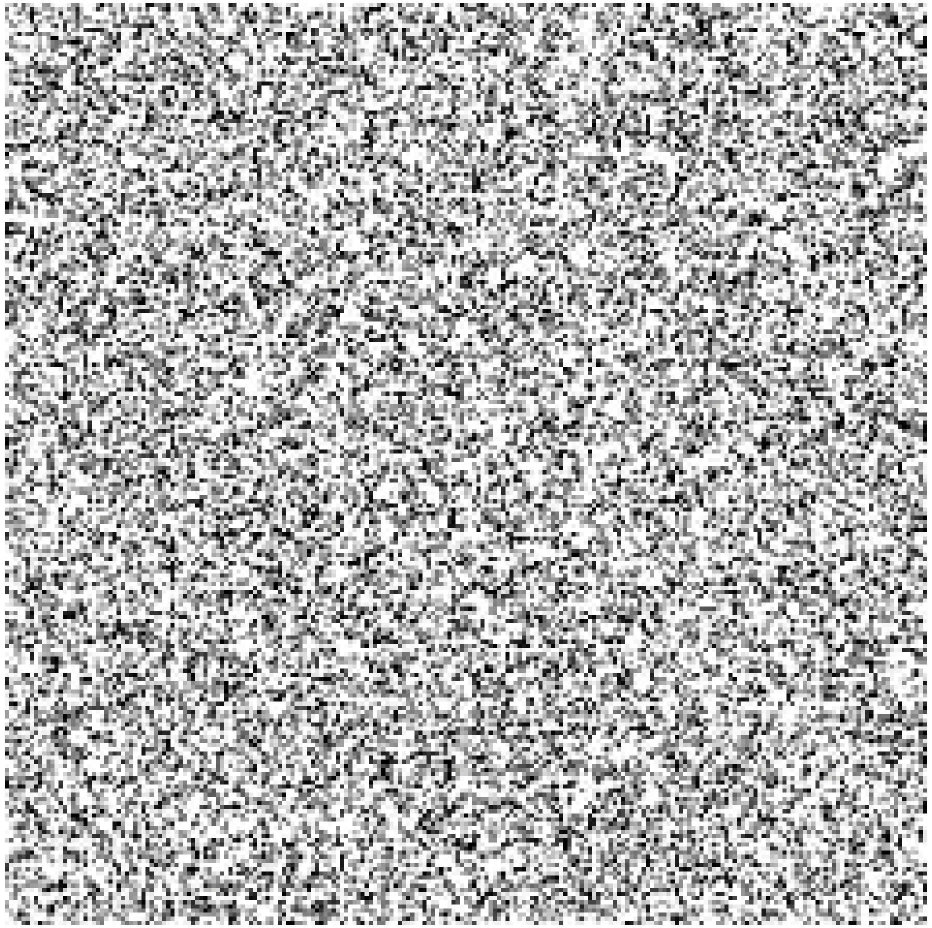
https://www.youtube.com/watch?v=e_TeY6QRp4c



Color key

Correspondence and perceptual organization

Sometimes correspondence/motion is the only cue



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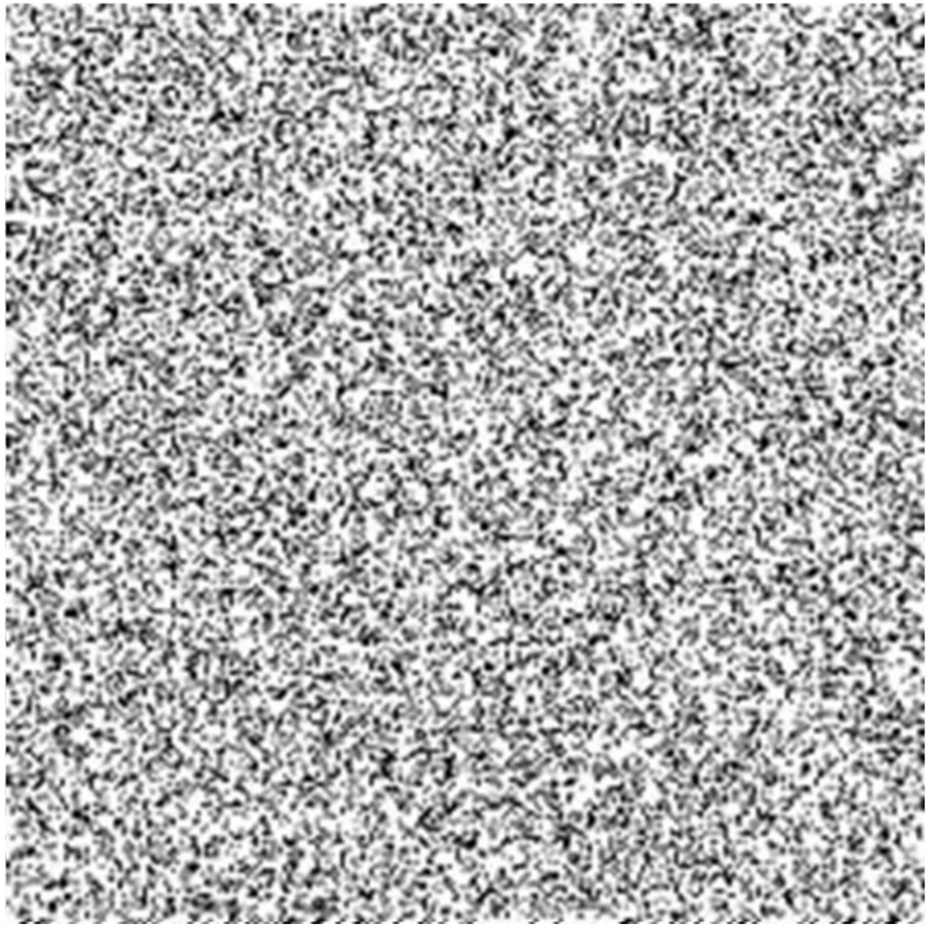


https://www.youtube.com/watch?v=mNrqvcS0oI0

Applications of optical flow

Video Superresolution
[Liu & Sun CVPR 2011, TPAMI 2014]

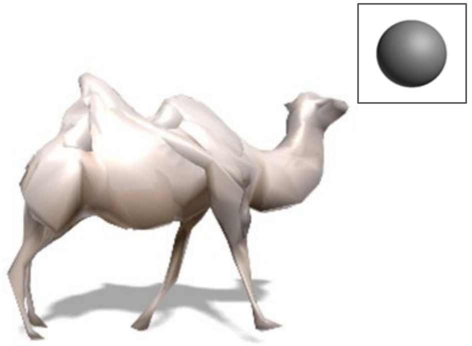
Applications of optical flow

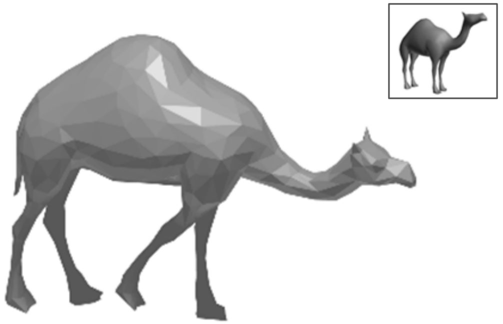
https://youtu.be/MjViy6kyiqs?si=zXenEX6O6VJkEaQF

Super SloMo [Jiang, Sun, *et al.* CVPR 2018 Spotlight] Incorporated into **NVIDIA NGX** SDK for the Turing GPU.

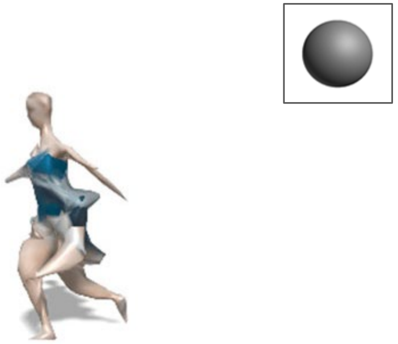
Face Unblur for Pixel 6

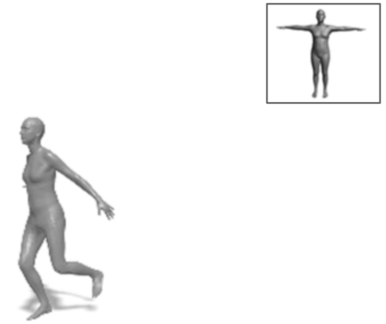
Articulated shape reconstruction from a monocular video





<u>A-CSM</u>: Kulkarni *et al.* CVPR 2020

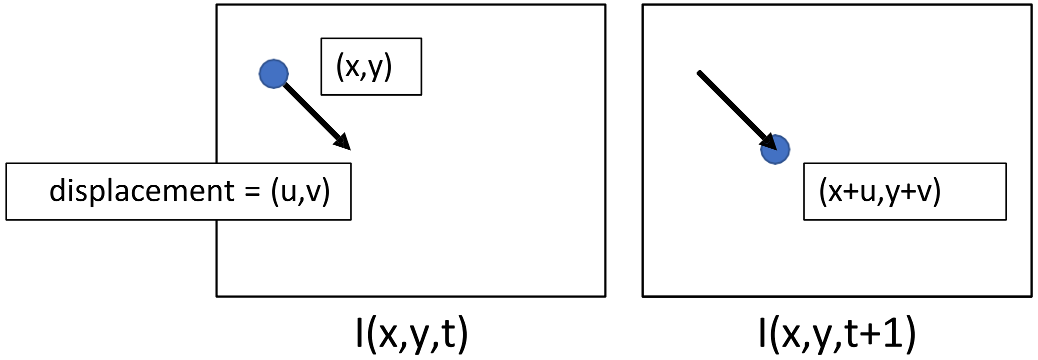




<u>VIBE</u>: Kocabas *et al.* CVPR 2020

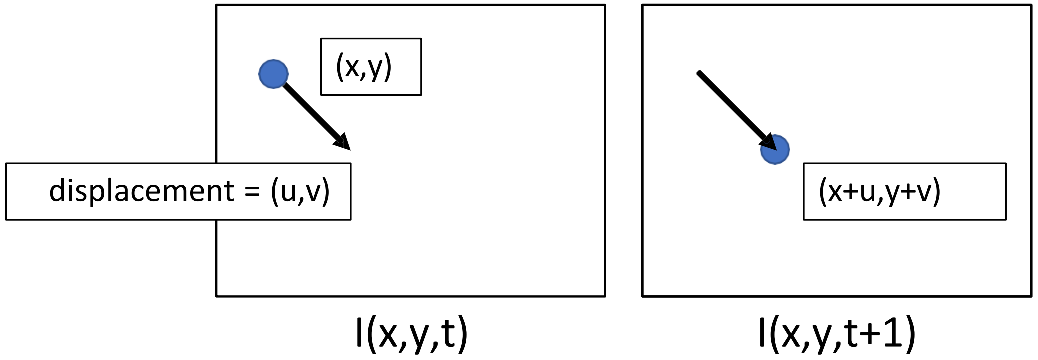
Challenge: Solving non-rigid 3D shape from 2D measurements without template or category prior is highly *under-constrained*

Optical Flow



Brightness constancy: $I(x, y, t) = I(x + u, y + v, t + 1)$

Optical Flow



Brightness constancy: $I(x, y, t) = I(x + u, y + v, t + 1)$

Recall Taylor Expansion: $I(x + u, y + v, t + 1) = I(x, y, t) + I_x u + I_y v + I_t \ldots$

Optical Flow Equation

$$
\begin{aligned}
I(x+u, y+v, t+1) &= I(x, y, t) \\
0 &= I(x+u, y+v, t+1) - I(x, y, t) \\
&\approx I(x, y, t) + I_t + I_x u + I_y v - I(x, y, t) \\
&= I_t + I_x u + I_y v \\
&= I_t + \nabla I \cdot [u, v]
\end{aligned}
$$

Taylor Expansion

When is this approximation **bad**?

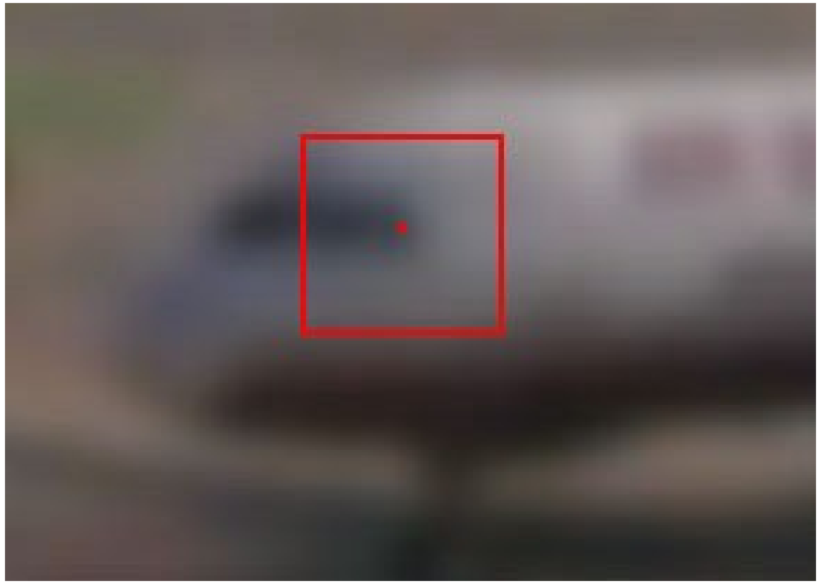
u or v big

Optical Flow Equation

Brightness constancy equation

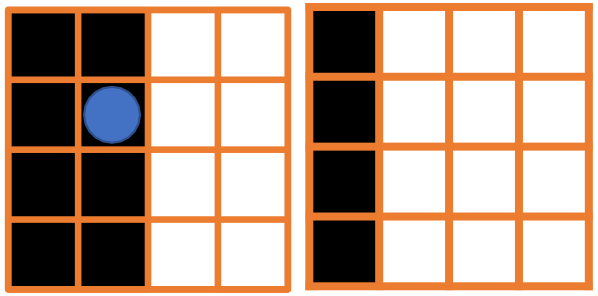
$$I_x u + I_y v + I_t = 0$$

What do static image gradients have
to do with motion estimation?



Brightness Constancy Example

$$I_x u + I_y v + I_t = 0$$



t   t+1

It = 1-0 = 1

@  Iy = 0

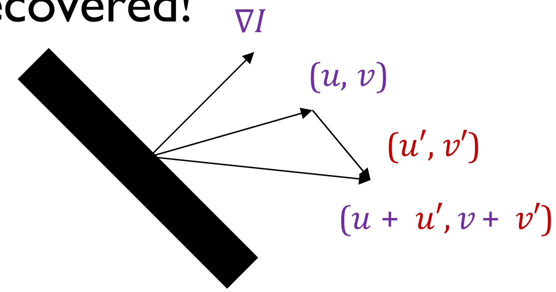
Ix = 1-0 = 1

What's u? **What's v?**

The Brightness Constancy Constraint

$$I_x u + I_y v + I_t = 0$$

- Given the gradients I_x, I_y and I_t, can we uniquely recover the motion (u, v)?
  - One equation, two unknowns
  - Suppose (u, v) satisfies the constraint: $\nabla I \cdot (u, v) + I_t = 0$
  - Then $\nabla I \cdot (u + u', v + v') + I_t = 0$ for any $(u\ , v\ )$ s. t. $\nabla I \cdot (u\ , v\ ) = 0$
  - Interpretation: the component of the flow perpendicular to the gradient (i.e., parallel to the edge) cannot be recovered!



Solving Ambiguity – Lucas Kanade

- 2 unknowns [u,v], 1 eqn per pixel How do we get more equations?
- Assume **spatial coherence**: pixel’s neighbors have move together / have same [u,v]
- 5x5 window gives 25 new equations

$$I_{\mathrm{t}} + I_{\mathrm{x}} u + I_{\mathrm{y}} v = 0$$

$$\begin{bmatrix} I_{\mathrm{x}}(p_1) & I_{\mathrm{y}}(p_1) \\ \vdots & \vdots \\ I_{\mathrm{x}}(p_{25}) & I_{\mathrm{y}}(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_{\mathrm{t}}(p_1) \\ \vdots \\ I_{\mathrm{t}}(p_{25}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

Solving for u,v

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{25}) \end{bmatrix} \qquad \underset{25x2}{\boldsymbol{A}} \; \underset{2x1}{\boldsymbol{d}} = \underset{25x1}{\boldsymbol{b}}$$

What's the solution?

$$(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})\boldsymbol{d} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} \quad \rightarrow \quad \boldsymbol{d} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b}$$

Intuitively, need to solve (sum over pixels in window)

$$\underbrace{\begin{bmatrix} \sum \boldsymbol{I_x I_x} & \sum \boldsymbol{I_x I_y} \\ \sum \boldsymbol{I_x I_y} & \sum \boldsymbol{I_y I_y} \end{bmatrix}}_{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum \boldsymbol{I_x I_t} \\ \sum \boldsymbol{I_y I_t} \end{bmatrix}}_{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b}}$$

Challenges for traditional methods

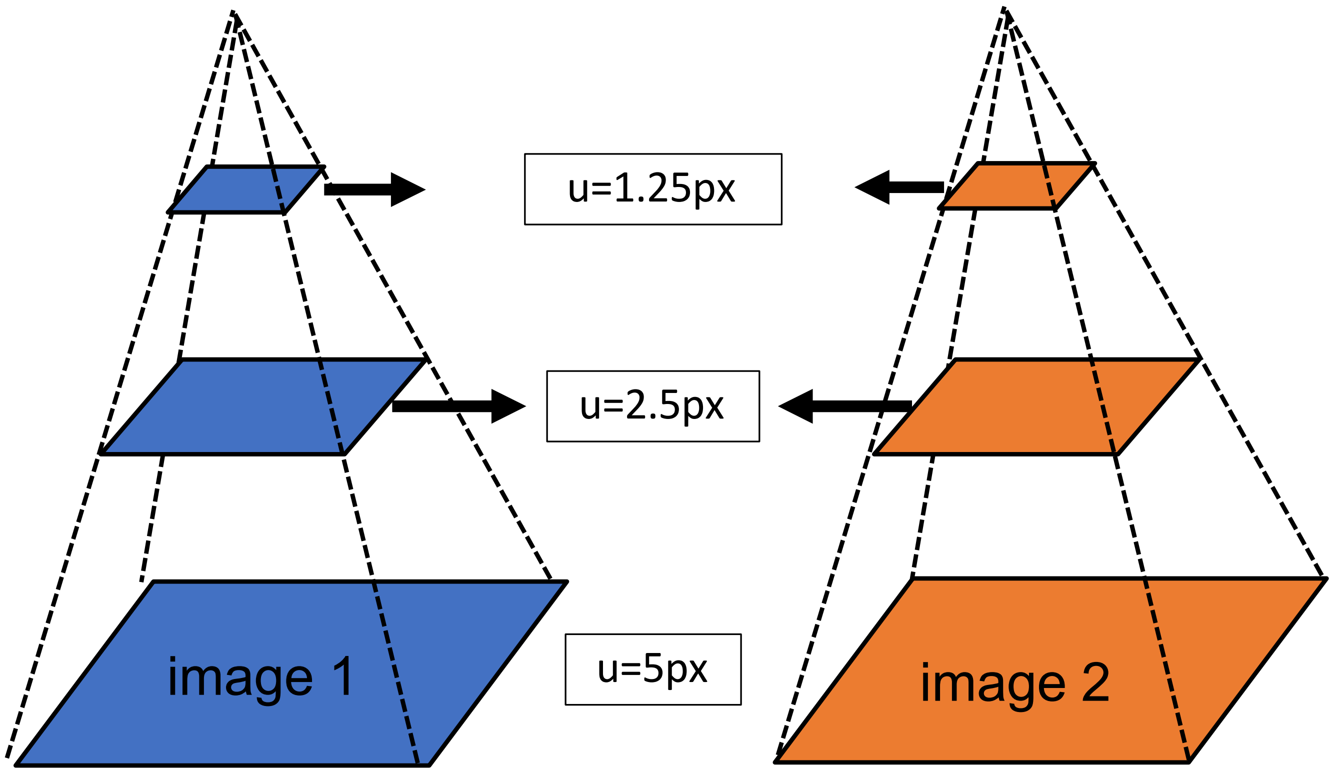
Large motion, motion blur

Textureless regions

Occlusions

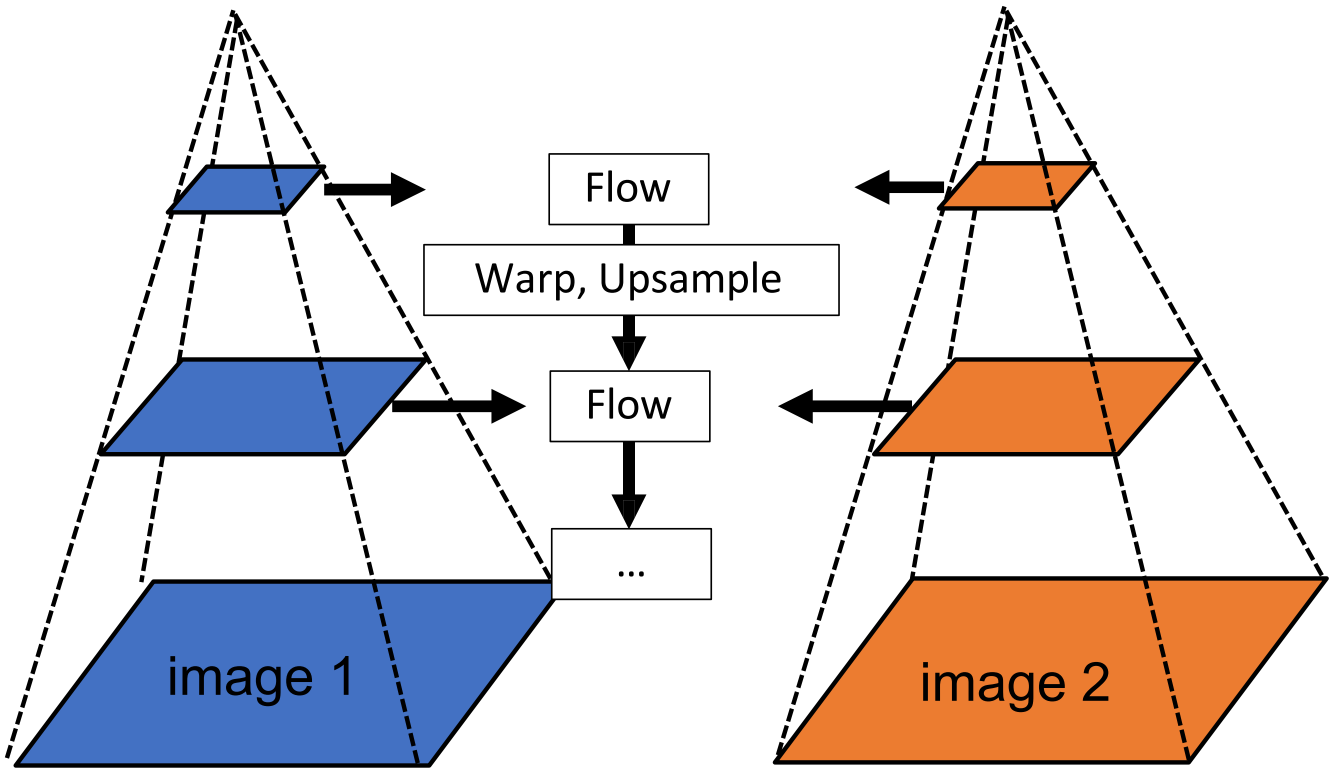
Lighting changes, noise …

Coarse-to-fine optical flow estimation

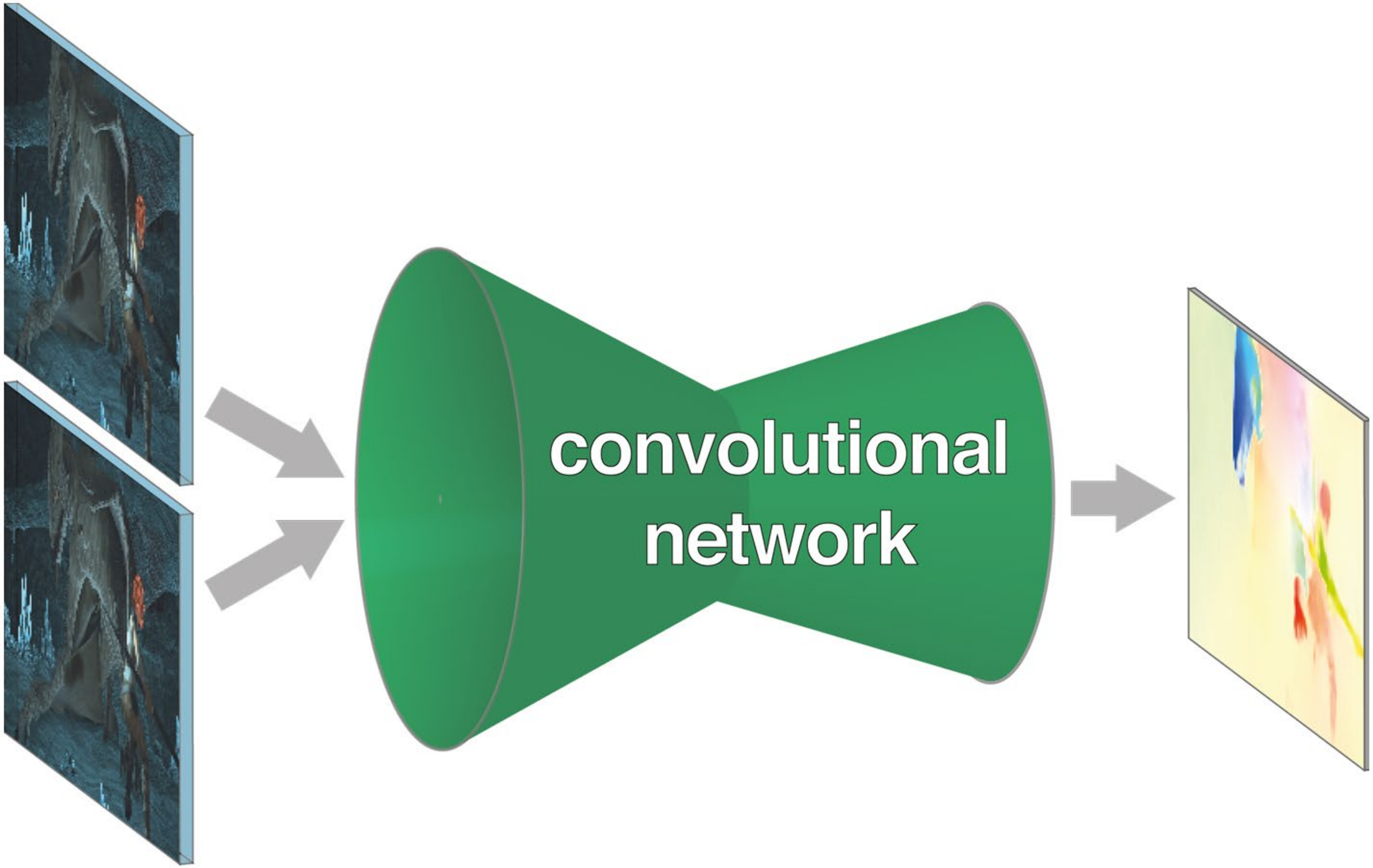


Typically called Gaussian Pyramid

Coarse-to-fine optical flow estimation

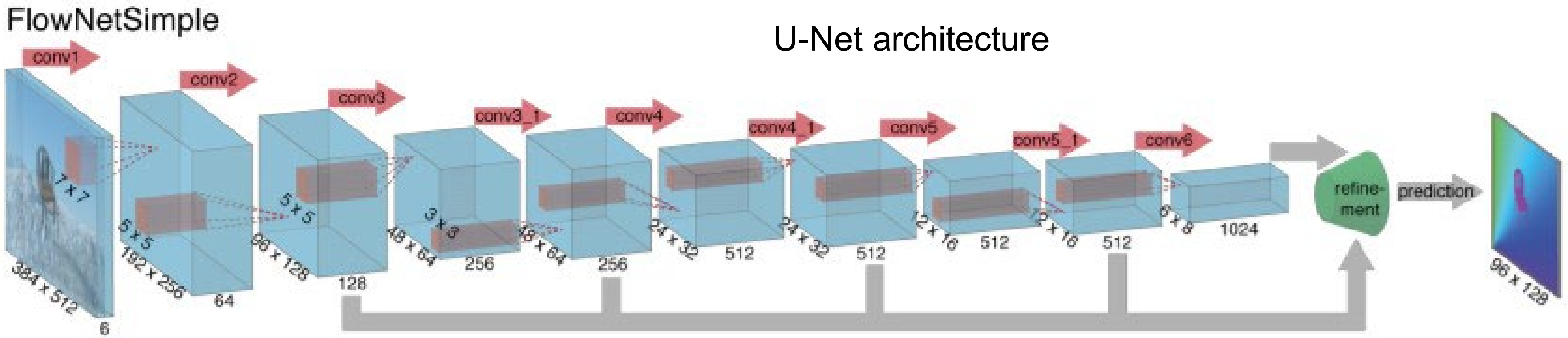


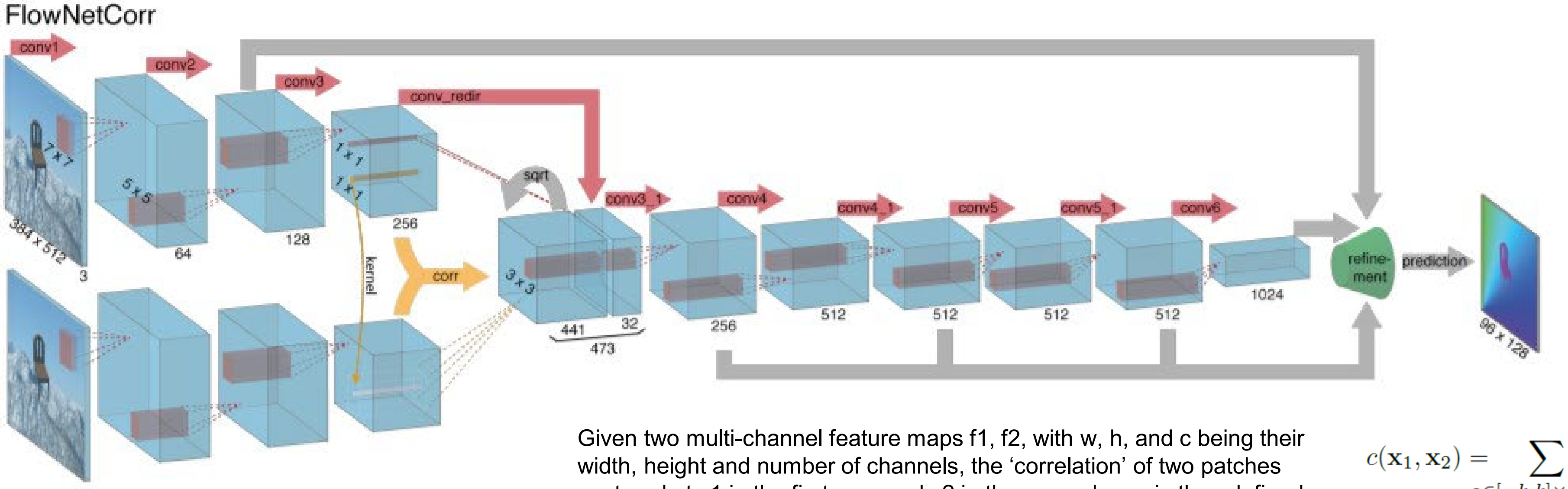
Typical DL Pipeline for Optical Flow



FlowNetS: mapping from images to flow

[Dosovitskiy *et al.* ICCV'15]

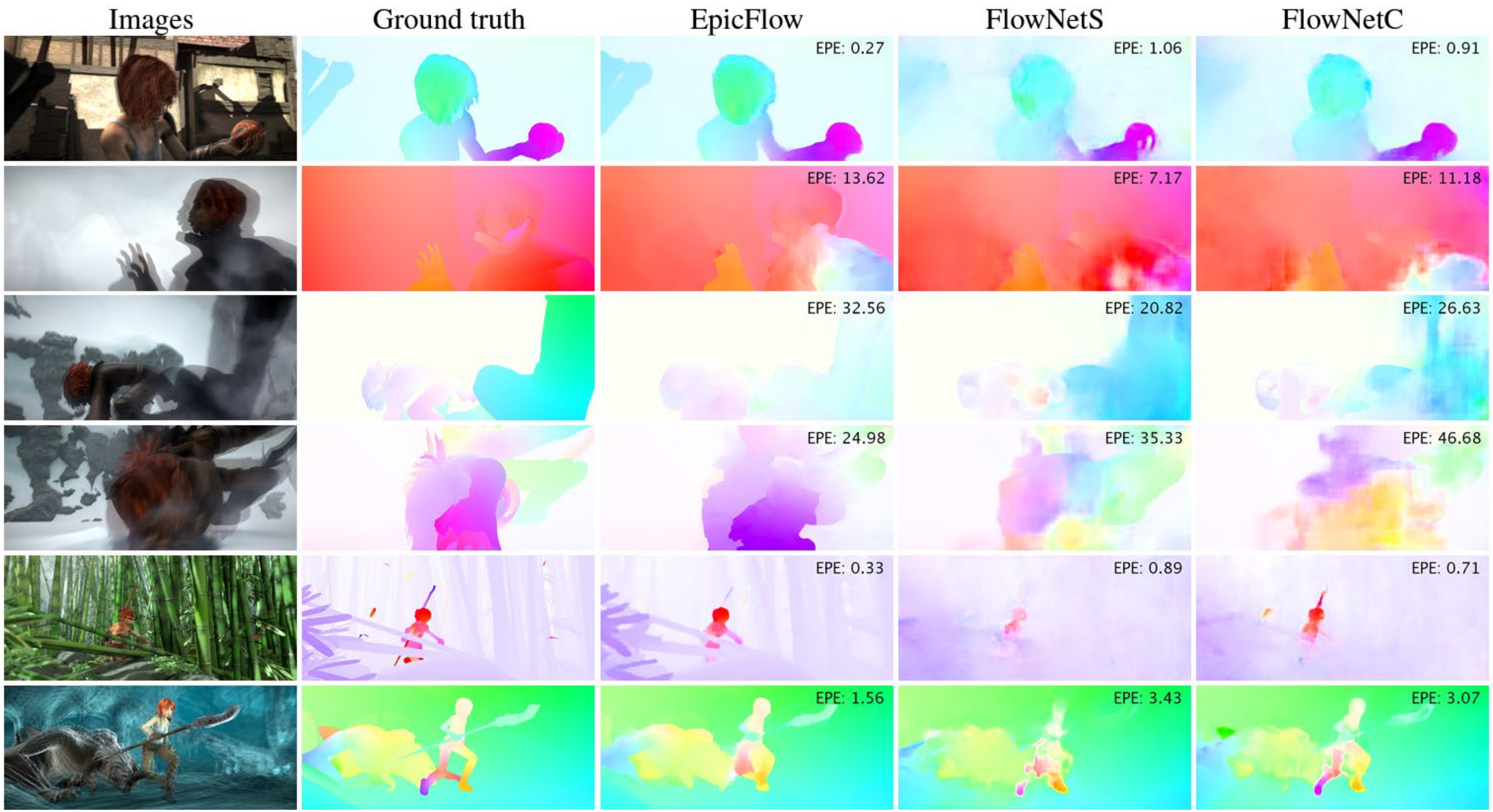




Given two multi-channel feature maps f1, f2, with w, h, and c being their width, height and number of channels, the 'correlation' of two patches centered at x1 in the first map and x2 in the second map is then defined as:

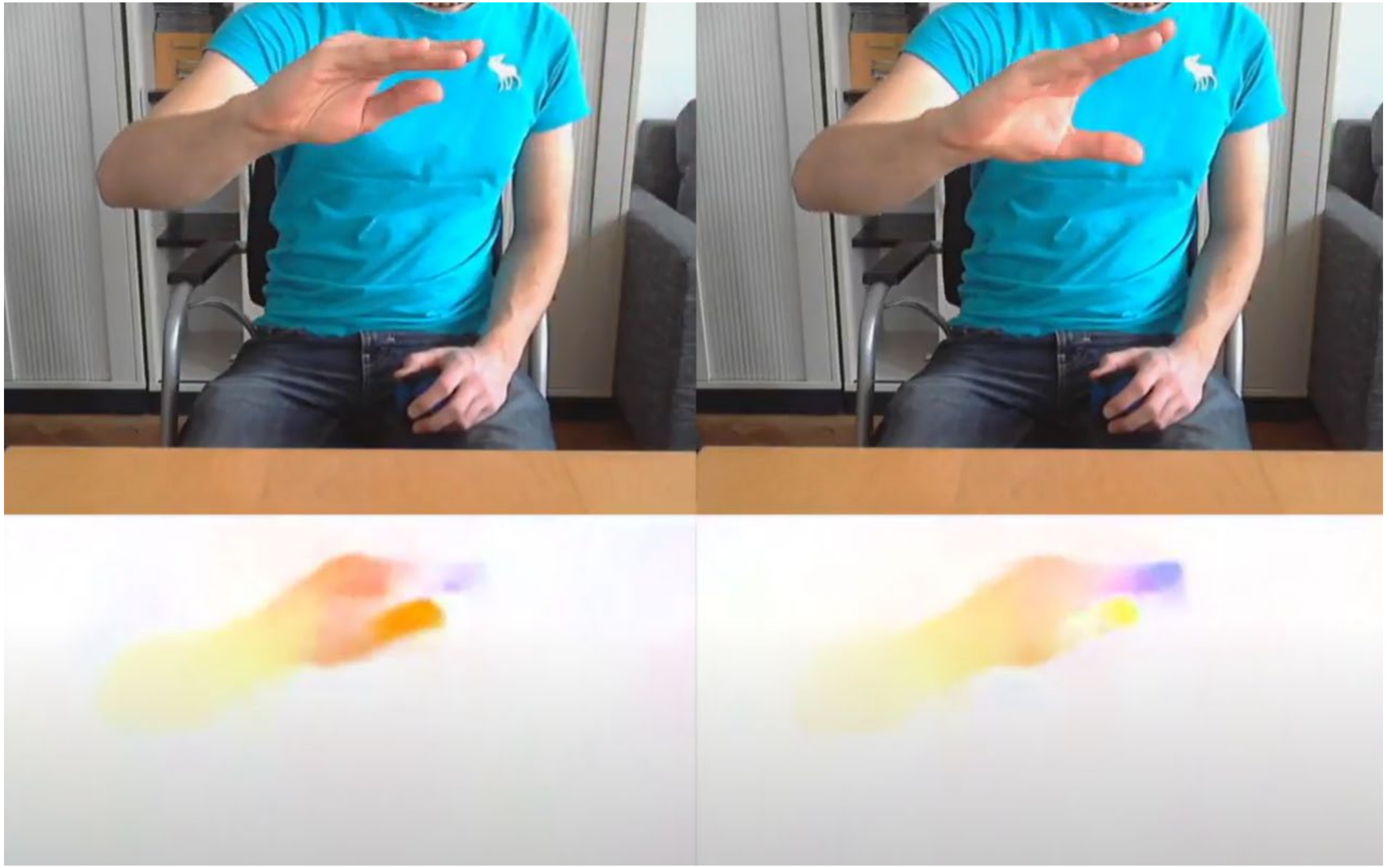
$$c(\mathbf{x}_1, \mathbf{x}_2) = \sum_{\mathbf{o} \in [-k,k] \times [-k,k]} \langle \mathbf{f}_1(\mathbf{x}_1 + \mathbf{o}), \mathbf{f}_2(\mathbf{x}_2 + \mathbf{o}) \rangle$$

FlowNetS: mapping from images to flow



EPE: endpoint error

FlowNetS: mapping from images to flow



https://youtu.be/k_wkDLJ8lJE?si=wJj1RePbw-NA75lU