Homework 06: Feature Matching; Hough Transform; Optical Flow

Kaveh Fathian, Email: fathian@ariarobotics.com

Handout: 2025-10-06

Due: 2025-10-13, 11:59pm, on Canvas

General Instructions:

- You should solve the homework and submit your report **individually**. Identical submissions will receive a grade of zero.
- Getting help from others or checking your answers with other students (not the TAs) is okay and encouraged.
- Ask any questions on **Ed Discussion** (instead of emailing).
- **Before** the homework due date, TAs are strictly prohibited from **pre-grading** your homework. Do not expect the TAs to help you verify if your answers are correct or give you the problem solution.
- **After** the homework due date, if you do not know how to solve a problem, reach out to the TAs. They will walk you through the solution and help you understand it. Note that homework solutions will **not** be posted because some problems will be used in next year's class.
- **Exams** may contain questions related to homework, so make sure you learn how to solve the homework problems correctly.
- The deliverables are outlined for each problem, and you should carefully **follow the instructions**. Failing to follow instructions will result in **points being subtracted**.
- You will submit a **single PDF** file to Canvas as your homework report. The PDF must contain your **answers** and any requested **outputs** (e.g., printouts, snapshots of code, or GUIs). If requested, follow the instructions specified by the problem to provide your **code** (e.g., in a compressed .zip or .tar file) in addition to the PDF file.
- **Grading:** Each homework in this class will contribute **5pts** to your final grade (there will be 12 homework assignments, each 5pts, leading to 60pts for all assignments). A detailed grading **rubric** will be posted on **Canvas** after the homework due date. Any bonus points will be added to your overall course bonus points, which will be added to your final grade.
- **Late submission:** Late or missed submission will not be accepted and will receive a grade a zero. Any excused absence must be documented and disclosed to the instructor (extensions will be granted on a case-by-case basis). Three or more missed homework lead to an INC grade.

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EXERCISE 1 (2pts) – The objective of this problem is to develop a high-level understanding of the SIFT algorithm. Given a keypoint at the center of a 3x3 pixel window on the image, you should compute a descriptor for the keypoint by following the instructions below. The magnitude and direction (in degrees) of the gradient for each pixel in the window is given below.

Grad_magnitude =

1	**0**	**1**
1	**0**	**1**
1	**1**	**0**

Grad_angle =

100	**20**	**300**
300	**100**	**300**
300	**20**	**200**

Step 1: Similar to SIFT algorithm, estimate the orientation of the keypoint via creating a histogram from gradient directions in the 3x3 window. Use bins $[0,90]$, $[90,180]$, $[180, 270]$, $[270,360]$ for the histogram. Report the histogram values in the table below:

Histogram bin	$[0,90]$	$[90,180]$	$[180,270]$	$[270,360]$
Histogram value				

Step 2: Normalize the orientation by rotating the 3x3 window using either a rotation of $\mathbf{0}$, $\mathbf{90}$, $\mathbf{180}$, or $\mathbf{270}$ degrees. That is, the orientation histogram of the rotated window must have the largest value in the bin $[0,90]$. Report the ***rotation value***, and the orientation histogram ***after the rotation*** in the following table:

Histogram bin	$[0,90]$	$[90,180]$	$[180,270]$	$[270,360]$
Histogram value				

In addition, report the gradient ***magnitude*** and ***angle*** of the ***rotated window*** in the following 3x3 table:

Grad_magnitude =

?	**?**	**?**
?	**?**	**?**
?	**?**	**?**

Grad_angle =

?	**?**	**?**
?	**?**	**?**
?	**?**	**?**

Note that all angles must be in the $[0, 360]$ range.

Step 3: Similar to SIFT algorithm, use the rotated window gradient to construct a descriptor vector from the sum of gradient magnitudes. Use bins $[0,90]$, $[90,180]$, $[180, 270]$, $[270,360]$ for the angle histogram, and report the descriptor vector in the following table:

Histogram bin	$[0,90]$	$[90,180]$	$[180,270]$	$[270,360]$
Descriptor value				

Deliverables: All tables and values requested in Steps 1-3.

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EXERCISE 2 (3pts) – The objective of this problem is to understand how feature points are matched using their descriptors. Consider keypoints k_1 and k_2 in image I with descriptor vectors respectively as

$$v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, v_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}.$$

On image I', we have keypoints k'_1, k'_2 and k'_3, with descriptor vectors respectively as

$$v'_1 = \begin{bmatrix}0\\0\\1\end{bmatrix},\ v'_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}, v'_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}.$$

Step 1: Using ***squared*** Euclidean norm as distance, i.e., $d(v, v') = \|v - v'\|^2$, compute the pairwise distance between all descriptors and report the results in the table below:

Pairwise descriptor distance	v'_1	v'_2	v'_3
v_1			
v_2			

Step 2: Match each keypoint in image I to a keypoint in image I' based on closest descriptor. If there is a tie, either match is acceptable. Prune matches by computing the Low's ratio and removing bad matches using a threshold of $\mathbf{0.7}$. Report the results in the following table (use "none" for pruned matches):

keypoint in image I	Matched keypoint in I' before ratio test	Low's ratio	Matched keypoint in I' after ratio test
k_1			
k_2			

Step 3: Similar to Step 2, match each keypoint in image I' to a keypoint in image I based on closest descriptor. Report the results in the following table:

keypoint in image I'	Matched keypoint in I before ratio test	Low's ratio	Matched keypoint in I after ratio test
k'_1			
k'_2			
k'_3			

Step 4: Are the matches bidirectional? That is, if keypoint k is matched to keypoint k', does k' also matches to k? If yes, is that always the case? Given a set of keypoint matches from image I to I' and vice versa, how can we select matches that are bidirectional?

Deliverables: Completed tables for steps 1-3 and answer to questions for step 4.