Welcome to Math 110

Professor K. A. Ribet

August 27, 2025

This is the room where it happens

We'll be a team each Monday, Wednesday and Friday

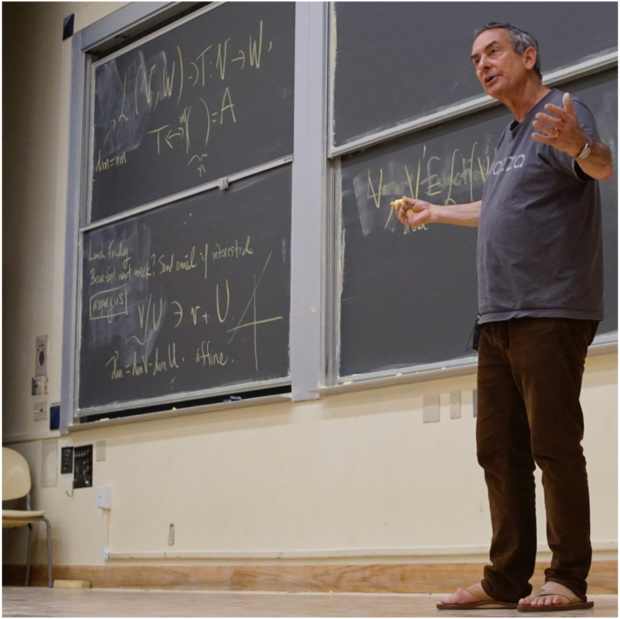
Photo shows Math 54 in fall, 2005

This is the room where it happens

We'll be a team each Monday, Wednesday and Friday

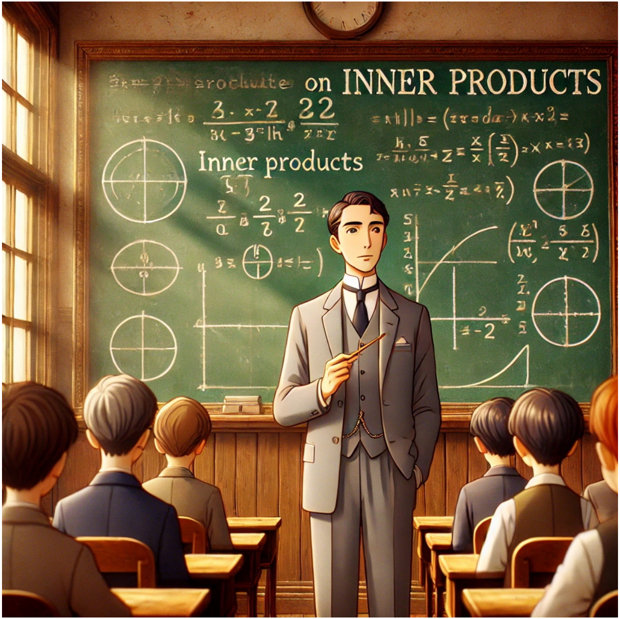
Photo shows Math 110 in spring, 2020

Another view: Math 110 — spring, 2020



Math 110 — spring, 2025

Practice session for this semester



No Course Capture

This Math 52 announcement is behind my decision not to publish course capture recordings this semester.

Come to class!

The slides that I project in class will be available on our `bCourses` site.

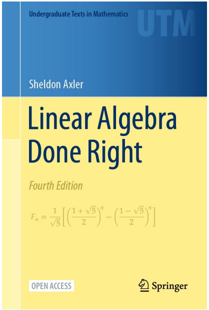
Put away your phones

I encourage you to read the New York Times article on this subject that was published on August 21.

Textbook

The fourth edition of Sheldon Axler's book Linear Algebra Done Right.

You can download it for free (Open Access) and can buy a print copy if you'd like one.



Please read the book's prefaces and acknowledgments ASAP.

Sheldon Axler

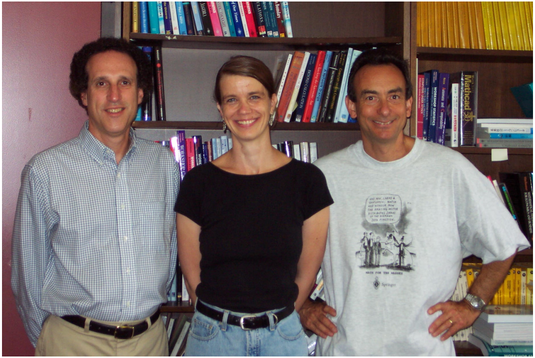


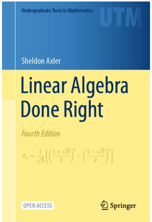
Photo from August, 2001 with me, Ina Mette and Axler

Sheldon Axler will visit us on November 21.

His X feed announces sales and news relevant to LADR.

What is this course about?

- Vector spaces (mostly finite-dimensional)
- Maps between them (linear maps)
- Vector spaces made from other vector spaces (e.g., subspaces, quotients, dual spaces,...)
- Eigenvalues, eigenvectors, diagonalizability
- Inner products ($\Longrightarrow$ distances, geometry)
- Spectral theorems (real and complex)
- Jordan canonical form?



What Google Gemini thinks I should say

I want to be upfront: linear algebra can be one of the most challenging courses you'll take in your undergraduate career. The concepts can be abstract, and the way we prove things might be new to you. But if you put in the work, this will also be one of the most rewarding courses you take. It will fundamentally change the way you think about mathematics, and it will give you a powerful set of tools that you'll use for the rest of your academic and professional lives. We'll learn to see the world in terms of vectors, spaces, and transformations.

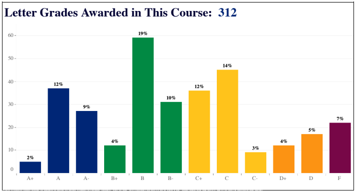
“This approach is honest and motivational. It acknowledges the difficulty without being intimidating. By promising a high reward for the challenge, it frames the course as a valuable, transformative experience rather than just a hurdle to clear. It builds a sense of partnership and mutual respect."

Grades

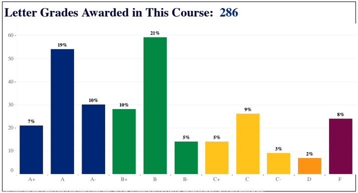
1. Your letter grade will be a monotone function of your composite grade (higher numbers $\Longrightarrow$ better or equal grades).
2. As long as you remain engaged with the course, you are most likely to get some flavor of an A or a B.

Lower grades come from disengaging (e.g., not participating or no longer thinking about the material).

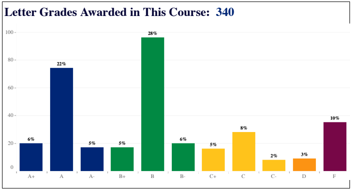
Recent Math 110 Letter Grades



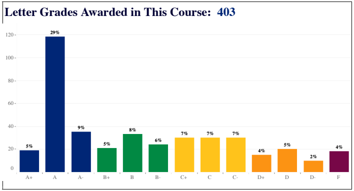
Fall, 2023



Spring, 2024



Fall, 2024



Spring, 2025

Charts from a public campus website

What is the composite grade?

- Quizzes 20%
- Homework 10%

Exams 70%:

- First midterm 20%
- Second midterm 20%
- Final exam 30%

☛ Your exam score is the *max* of:

- MT1 + MT2 + Final
- (MT2 + final) $\times$ 70/50
- (MT1 + final) $\times$ 70/50

A student who misses one midterm and gets full credit on the other two exams will earn an exam score of 70/70.

Midterms and final

The schedule of exams is as follows:

- First midterm, Monday, September 29 in class
- Second midterm, Friday, October 31 in class
- Final exam, Monday, December 15, 7–10 PM

There are no makeup exams.

If you miss one midterm because you're sick or for other reasons, your exam score will be computed using the remaining two exams (see below).

If you can't attend the final for a documentable compelling reason, you can request an incomplete grade.

Exams

The midterms are 50-minute 8:10 AM sprints that will test basic understanding early in the morning.

The final exam allows time for probing and thoughtful questions.

Past exams are on `bCourses`.

Exams

No devices but you can bring in a page of notes. One page for each exam. First MT tests the first third of the course. Second MT tests mostly the second third of the course. Final tests everything.

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly, **using complete English sentences** (and not just symbols). Remember that the paper you hand in will be your only representative when your work is graded. Please write your name clearly on each page of your exam. Please hand in all pages that you have received (including this cover sheet), preserving the order of the pages. You do *not* need to hand in the sheet of notes that you brought to the exam.

Write proofs in full sentences.

Homework

Weekly homework will be due primarily on Friday evening. The first assignment will be due on September 5. Some problems will be taken from Axler's book, while others will be invented by me (or cribbed from alternative sources).

Homework to be uploaded to `Gradescope` and will be due at 11:59 PM on the due date. It's OK to typeset your paper with LaTeX and also OK to write by hand.

Collaboration is encouraged — but please acknowledge your collaborators. It's also fine to discuss homework problems with me or our GSI or to post about problems. *No AI, though!*

The homework grade will be based primarily on completion. In addition, you will get detailed feedback from the grader on selected problems.

Your prof

As possibly explained in an Ed Discussion post:

- I'm the Math 110 guy this semester.
- I'm the president of the Faculty Club (where you can come to lunch on your own any weekday).
- I'm a "Senior Fellow" with the Miller Institute for Basic Research in Science.
- I'm a Campus Faculty Partner with UC Berkeley Residential Life.

There will be occasional minor conflicts; for example, I like to have office hours on Thursday mornings, but there's a Faculty Research Discussion Panel on Thursday, September 4 from 11 AM to 1 PM in the Unit 3 All Purpose Room. (You can come!)

Your prof at the dining halls

- Today (Wednesday) at Cafe 3 at 11:40 AM
- Tomorrow (Thursday) at Crossroads at 12:15 PM

Office Hours

885 Evans Hall

Mondays, 1:30–3 PM
Thursdays, 10:30–noon.

Office full $\Longrightarrow$ possible move to a nearby classroom

First office hour *tomorrow* at 10:30 AM.

No office hour next Monday (Labor Day) or Thursday, September 4 (Faculty Research Panel). Sad! What can we do about that?

Special office hour, Friday, September 5, 10:30–noon in 885 Evans

Take it from the Daily Cal

CAL DAY 2015 THURSDAY, APRIL 16, 2015

Learning from professors

BY STEPHANIE WANG | STAFF LAST UPDATED APRIL 18, 2015

I found myself fidgeting in front of a professor's office, ready to burst into tears at any second. The door was closed, and the entire hallway was silent. I had missed my morning final exam when my alarm failed to go off. I couldn't believe it. Everyone knows you automatically get an F if you don't take the the final exam. Gathering all the remaining courage left in me and clinging on to the last bit of hope, I raised my hand and knocked. The professor swiftly opened the door and invited me to come in. I had sent an email earlier, and of course, he recognized me. Who else would be knocking at the start of summer break? In the following moments, I was the most nervous and scared I have ever been during school.

Instead of consisting of him demanding answers from me immediately, our conversation started off with him asking me how I was feeling and telling me that I must have felt awful. I was surprised by how nice and reassuring he was. After I explained what happened, we moved on to my future plans. He told me that all of us college students are still young and that no one is better than others.

78 1 0

Get to know your professors. Introduce yourself at office hours. Don't be shy.

In class and office hours, ask lots of questions. Your fellow students will be grateful.

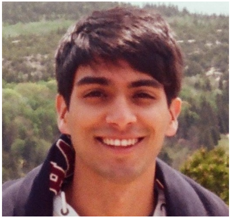
Office hours are inviting and accessible

I endorse the principles laid out by the Center for Teaching & Learning. You can stop by to introduce yourself or to chat about whatever is on your mind. Many students come to meet other students and to find out what's on *their* minds.

Introducing our GSIs

Will Fisher*

Chan Bae



Philip LaPorte

Jacob Parish

Peter Rowley

* Will Fisher is *Head GSI*

Introducing your classmates

- an Applied Math and Data Science transfer student
- a transfer student... and really excited to begin my first semester here at UC Berkeley
- an Applied Math major... excited to take Math 110 this semester. I had one of my friends take it and he said he enjoyed it, even though he did not enjoy his professor
- an incoming Sophomore here at Cal pursuing an applied math major + design innovation minor
- am going into my fourth year of undergrad and second year at Berkeley
- a transfer student incoming to UC Berkeley for the Fall, with the intent of dual majoring in the Physics and Neuroscience programs

How to study and succeed in this course

Invest yourself in our Ed Discussion page.

Are we going to do any math today?

We can try. . . .

Fields

A field (as studied in Math 113) is a commutative system where you can add, subtract, multiply and divide (except by 0).

Math 55: you get a field by considering the set of integers mod a prime number. For example, to find a multiplicative inverse of 23 (mod 101), use the Euclidean algorithm to find x such that $23x \equiv 1 \pmod{101}$. (Spoiler: $x = 22$ because $22 \times 23 = 506$.)

In our course, a field is either **R** or **C**. It's called **F**.

Real numbers

The familiar decimal numbers, positive, negative or 0.

We grew up with them, but see Math 104 for more info.

The field of complex numbers

A complex number is a sum $a+bi$, where a and b are real numbers and i satisfies $i^2=-1$. Thanks to 0, every real number is also a complex number; for instance $110=110+0\cdot i$.

The complex numbers form a *field* because we can use the "rationalize the denominator" trick to invert nonzero complex numbers:

$$\frac{1}{a+bi}=\frac{a-bi}{(a+bi)(a-bi)}=\frac{a}{a^2+b^2}+\frac{-b}{a^2+b^2}\cdot i.$$

For example, to find the inverse of $1-i$, we note that $(1-i)(1+i)=1^2-i^2=2$, so that $\frac{1}{1-i}=\frac{1}{2}(1+i)=\frac{1}{2}+\frac{1}{2}i$.

NAME: **STUDENT ID:**

g. (3 pts) The complex number $\frac{7+i}{1-i}$ can be written as $a+bi$ for some real numbers a and b.

What is a?

What is b?

What is $|a+bi|$?

We all can do this problem from a 2022 Math 1B final. Multiply $7+i$ by the inverse that we just found.

Vectors

Vector spaces are abstractions of the spaces $\mathbf{R}^n$ (and $\mathbf{C}^n$) that we saw in Math 54.

When we start the subject, vectors are arrows in n-space (having magnitude and direction, as one says), but we slide them so they come out of the origin $(0, \ldots, 0)$. Then what matters is where they end up, which is some point $(a_1, \ldots, a_n)$. The vectors in $\mathbf{F}^n$ are n-tuples of elements of $\mathbf{F}$. Said otherwise, the vector space $\mathbf{F}^n$ is the set of n-tuples of elements of $\mathbf{F}$.

If $n = 0$, there's only the empty n-tuple, (   ).

For $n \geq 0$, the n-tuple whose only entries are 0 is called 0.

Thus $\mathbf{F}^0$ consists of a single element: 0.

Operations on vectors

As you may remember from Math 54, $\mathbf{F}^n$ has two salient structures: vector addition and scalar multiplication.

Addition:

$$(a_1, \ldots, a_n) + (b_1, \ldots, b_n) = (c_1, \ldots, c_n),$$

where $c_j = a_j + b_j$ for each j.

Scalar multiplication:

$$\lambda \cdot (a_1, \ldots, a_n) = (\lambda a_1, \ldots, \lambda a_n).$$

Here, λ is an element of $\mathbf{F}$ (a "scalar").

> *A vector space over* $\mathbf{F}$ *is a set with an addition and a scalar multiplication. These operations are constrained to satisfy a pile of axioms that are familiar for* $\mathbf{F}^n$*.*

Axioms for addition

A vector space is a set V with an addition $v, w \longmapsto v + w$ that's commutative

$$v + w = w + v$$

and associative

$$v + (w + z) = (v + w) + z.$$

There's a vector $0 \in V$ such that $v + 0 = v$ for all $v \in V$. For each $v \in V$, there's a vector $-v$ such that $v + (-v) = 0$.

For fans of Math 113, these axioms state that V with its addition is an abelian group.

A first exercise is to prove that 0 is unique and that $-v$ is unique for each v. (I used the **p** word.)

Axioms for multiplication

The set V also has a scalar multiplication

$$\lambda \in F, v \in V \longmapsto \lambda \cdot v \in V.$$

The remaining axioms:

- $1 \cdot v = v$;
- $(\mu\lambda) \cdot v = \mu(\lambda \cdot v)$;
- $(\mu + \lambda)v = \mu \cdot v + \lambda \cdot v$;
- $\lambda(v + w) = \lambda v + \lambda w$.

Comments:

The “dot” in scalar multiplication is optional and tends to disappear.

My bad: I didn’t mention the “for all. . . ” quantification in each axioms.

Examples of vector spaces

First off, $\mathbf{F}^n$ for all $n \geq 0$.

Secondly, the set of all "infinite vectors" $(a_1, a_2, a_3, \ldots)$ with entries in $\mathbf{F}$. Axler calls this $\mathbf{F}^\infty$. Note that $\mathbf{F}^\infty$ is the set of all functions

$$\{1, 2, 3, \ldots\} \longrightarrow F.$$

More generally, if S is a set, the set of functions

$$S \longrightarrow \mathbf{F}$$

is a vector space $\mathbf{F}^S$. We add functions in the natural way ("pointwise") and we multiply functions by scalars in the way that you'd guess.

Function space examples

The set of all functions $\mathbf{R} \longrightarrow \mathbf{R}$ is a vector space over $\mathbf{R}$. (We know how to add two functions or to multiply a function by 42.)

So is the set of all continuous functions $\mathbf{R} \to \mathbf{R}$. And so is the set of all differentiable functions $\mathbf{R} \to \mathbf{R}$.

Finally, so is the set of all twice differentiable functions $f : \mathbf{R} \to \mathbf{R}$ satisfying the differential equation

$$y'' - 3y' + 2y = 0.$$

Two such functions are e^x and e^{2x} (if I'm not mistaken).

Polynomials

For $m \geq 0$, a polynomial of *degree m* is an expression

$$a_0 + a_1 z + \cdots + a_m z^m$$

with the a_j in $\mathbf{F}$ and a_m nonzero. A polynomial of degree $\leq m$ is an expression $a_0 + a_1 z + \cdots + a_m z^m$ with the a_j in $\mathbf{F}$ and a_m possibly 0.

The set of polynomials of degree $\leq m$ is denoted $\mathcal{P}_m(\mathbf{F})$. This set is a vector space under the natural addition and scalar multiplication that I'll describe.

Note that

$$\mathcal{P}_m(\mathbf{F}) \longleftrightarrow \mathbf{F}^{m+1}, \qquad a_0 + a_1 z + \cdots + a_m z^m \longleftrightarrow (a_0, a_1, \ldots, a_m).$$

The union $\mathcal{P}(\mathbf{F}) := \bigcup_{m \geq 0} P_m(\mathbf{F})$ is the set of polynomials over $\mathbf{F}$ of all degrees. It's again a vector space over $\mathbf{F}$.

We can view $\mathcal{P}(\mathbf{F})$ as the set of sequences

$$(a_0, a_1, a_2, \ldots,)$$

of elements of $\mathbf{F}$ with the property that there's an $m \geq 0$ such that $a_j = 0$ for $j > m$. These are the sequences with only a finite number of nonzero entries. These are the sequences that are "eventually 0."

Comparing $\mathcal{P}(\mathbf{F})$ with $\mathbf{F}^\infty$

The space $\mathcal{P}(\mathbf{F})$ is the set of sequences that are eventually 0. It doesn't matter whether we call the first entry a_0 or a_1. Every element of $\mathcal{P}(\mathbf{F})$ is also an element of $\mathbf{F}^\infty$:

$$\mathcal{P}(\mathbf{F}) \hookrightarrow \mathbf{F}^\infty.$$

For example, the polynomial $1 - x + x^3$ can be regarded as the sequence $(1, -1, 0, 1, 0, 0, \ldots, 0, \ldots)$, which is an element of $\mathbf{F}^\infty$.

After we define the notion of a *subspace*, you will agree that $\mathcal{P}(\mathbf{F})$ is a subspace of $\mathbf{F}^\infty$. (I hope you will, anyway.)

Another example: null spaces of matrices

Suppose that A is an $m \times n$ matrix of elements of $\mathbf{F}$. Let

$$V = \{\, x \in \mathbf{F}^n \mid Ax = 0 \,\}.$$

Thus V is the set of solutions of m homogeneous equations in n unknowns, and we have

$$V \hookrightarrow \mathbf{F}^n.$$

After we define the notion of a *subspace*, you will agree that V is a subspace of $\mathbf{F}^n$. (I hope you will, anyway.)

For the moment, let's think about the fact that V is a set with an addition and scalar multiplication and is definitely a vector space over $\mathbf{F}$ on its own steam (i.e., without thinking too much about $\mathbf{F}^n$).

Consequences of the axioms

You can find (in the text, including the exercises) lots of consequences of the axioms. A sample:

- For $\lambda \in F$ and $v \in V$: if $\lambda v = 0$, then either $v = 0$ or $\lambda = 0$ (or both).
- For each $v \in V$, $-(-v) = v$.
- For $v \in V$, $(-1) \cdot v = -v$.

The first statement amounts to the implication

If $\lambda v = 0$ and λ is nonzero, then $v = 0$.

To prove it, assume the hypothesis of the implication, namely that $\lambda v = 0$ and λ is nonzero. Then λ is invertible in **F**. Since $\lambda v = 0$, $0 = \frac{1}{\lambda}(\lambda v)$. By the associativity of multiplication,

$$0 = (\frac{1}{\lambda} \cdot \lambda)v = 1 \cdot v = v.$$