LESSON 1

THE TRINOMIAL MODEL

Derivative Pricing - Module 3

WORLDQUANT UNIVERSITY

Outline

- Extending the Binomial Model
- Revisiting No-Arbitrage Argument in Trinomial Trees
- Pricing Problems Associated with Trinomial Model

Extending the Binomial Model

In the previous modules, we dealt extensively with the different features associated with pricing various types of options in the binomial model.

In this module, we extend the binomial model to consider different types of stock price movements:

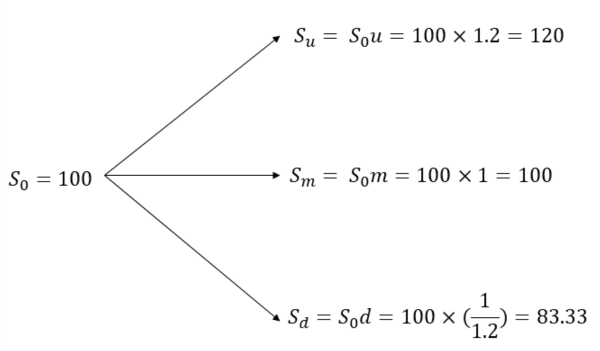
- Ideally, you would like a continuous-time framework in which underlying stock prices can move any direction in any magnitude to capture the underlying behavior.
- While we will get there through the course, there are many problems that arise once we depart from the binomial model setting.
- In order to **better understand what these are and why they arise**, we will naturally extend the binomial model to a **trinomial model**:

  $\Rightarrow$ That is, prices could move in 3 different directions:

  - Upward movement, u.
  - Mid (or no-) movement, m. (typically $m = 1$)
  - Downward movement, d (typically $d = \frac{1}{u}$).

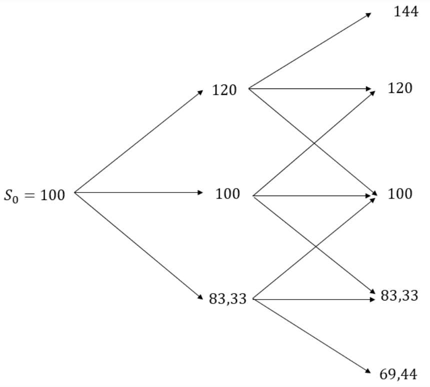
The Trinomial Model

Here, you have an example of how a trinomial model with 1 step ($N = 1$) will look:
$u = 1.2$; $d = \frac{1}{u}$; $m = 1$; $N = 1$; and $S_0 = 100$



The Trinomial Model

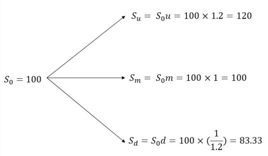
Here, you have an example of the same trinomial tree with N=2:



No-Arbitrage in Trinomial Model

Let's now dig deeper on how to apply risk-neutral valuation in the Trinomial tree.

We will deal with 3 probabilities here: p_u, p_m, and p_d, associated with corresponding movements:



From absence of arbitrage and risk-neutral probabilities, these are the conditions that have to be met:

(1) $p_u + p_m + p_d = 1$

(2) $S_0 = e^{-rdt}[p_u \times S_u + p_m \times S_m + p_d \times S_d]$, $\rightarrow$ *Martingale condition* (more on this later)

$\Rightarrow$ Does this system of equations have a solution?

No-Arbitrage in Trinomial Model: FTAP I

The previous system of equations has, indeed, multiple solutions.

$\rightarrow$ Because # unknowns > # equations.

$\rightarrow$ There exist multiple sets $\{p_u, p_m, p_d\}$ that constitute a solution.

$\rightarrow$ For any of these sets of $\{p_u, p_m, p_d\}$, we can say such a market is arbitrage-free.

What we have implicitly derived here is the first Fundamental Theorem of Asset Pricing (**FTAP I**)

Fundamental Theorem of Asset Pricing (FTAP) I

If there exists a set of measures $\{p_u, p_m, p_d\}$, the market is arbitrage-free.

But for our pricing purpose, another important problem arises:

$\Rightarrow$ How can we select the set of risk-neutral probabilities to use among infinite measures?

Summary of Lesson 1

In lesson 1, we have:

- Extended the binomial tree to augment diversity of stock price movements.
- Introduced the trinomial tree as a natural next step.
- Gotten a first glance at the potential problems with replication in a trinomial setting.

⇒ **TO-DO NEXT**: Now, please go to the associated **Jupyter Notebook** for this lesson to check that you know how to build a trinomial tree using Python. This is not as easy as it is for the binomial model. You will also see there an example on how to match volatility to stock price movements in a trinomial tree.

⇒ In the next lesson, we will continue to deal with the problems associated with replication in trinomial trees.