LESSON 3

BLACK-SCHOLES AND MONTE CARLO METHODS

Derivative Pricing - Module 4

WORLDQUANT UNIVERSITY

Outline

- Risk-neutral valuation in the Black-Scholes world: Price dynamics?
- Monte Carlo methods for option pricing
- Convergence between Black-Scholes and Monte Carlo

Risk-neutral valuation in the Black-Scholes world

In the last lesson, we derived the option pricing formulas from the Black-Scholes model under certain price dynamics for stock prices:

$$dS = S\left(\mu dt + \sigma dW_t\right)$$

However, as we mentioned a few times already, we are working in a risk-neutral world:

- Where we work with risk-neutral problems
- Where the return earned by an asset is r, the risk-free rate

$\Rightarrow$ Remember that these two changes offset each other.

Hence, to work in a risk-neutral setting, our Brownian Motion SDE will be:

$$dS = S\left(rdt + \sigma dW_t\right),\ discretized\ by\ \rightarrow S_T = S_t\ e^{\left(\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}Z\right)}$$

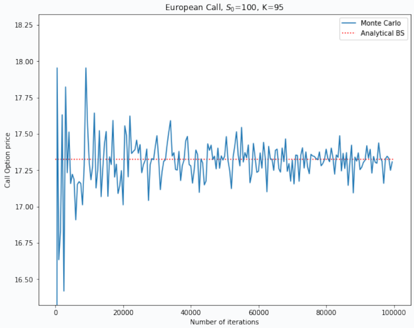
with $Z \sim \mathcal{N}(0,1)$

Monte Carlo methods and convergence with Black-Scholes

You are already familiar with Monte Carlo methods and their powerful applications for option pricing.

In this lesson, we are going to revisit this technique to verify that, under the risk-neutral world price dynamics in the Black-Scholes world, we are able to replicate the results from the closed-form analytical solution.

- Now, go to the Jupyter Notebook associated with this lesson to check this out.



Summary of Lesson 3

In Lesson 3, we've looked at:

- Monte Carlo methods in the Black-Scholes world
- Convergence between Monte Carlo method and Black-Scholes analytical price

$\Rightarrow$ **TO-DO NEXT**: Now, please go to the associated Jupyter Notebook for this lesson if you haven't done so yet.

$\Rightarrow$ In the next lesson, we will see how to apply all these techniques to a different underlying asset price process.