LESSON 1

BASICS OF FINANCIAL DERIVATIVES

Derivative Pricing - Module 1

Outline

- What is a financial derivative?
- Revisiting the main features of option contracts
- Intro to binomial model for equity option pricing

Financial Derivatives: What are they?

During the Financial Markets course, you got familiar with financial derivatives and learned about their powerful applications in the real world for players on the **buy-side** (e.g., hedge funds) and the **sell-side** (e.g., investment banks) of the industry.

- Investors find these instruments convenient for a variety of reasons (e.g., hedging, leverage) → Volume traded on **equity option market:**

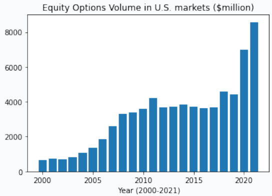


Figure: Constructed with data from Options Clearing Corporation

In this course, you'll learn the basics of pricing some of these instruments.

Financial Derivatives: What are they?

Given their importance, we will focus on **equity options** during most of the course. Some important features of these contracts that we need to know:

- Call vs. Put
- Moneyness & Liquidity → OTM/ATM/ITM?
- European vs. American → early exercise?
- Non-linear payoffs
- Leverage → e.g., 100 shares of underlying stock
- Over-the-Counter (OTC) vs. Exchange-Traded → credit risk?
- Exposure to volatility → e.g., Butterfly / Straddles

An **option contract** is a contingent claim on an underlying asset with a pre-defined payoff:

- Call Option: $(S_t - K)^+ = max(S_t - K; 0)$,
  where S_t is underlying stock price at time t and K is the option's strike price.
- Put Option: $(K - S_t)^+ = max(K - S_t; 0)$

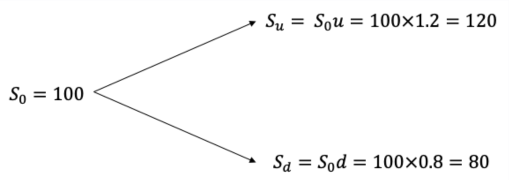
Modeling Underlying Stock

The essential characteristic of a derivative is that its payoff depends on the evolution of an **underlying asset** (e.g., a stock for equity options).

- Thus, a key aspect for pricing (and our first step) is to model the future evolution of the underlying asset.

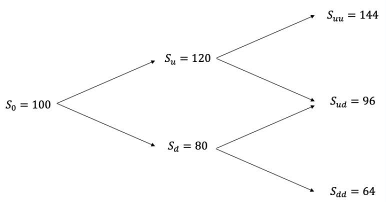
Probably one of the simplest and more popular techniques for modeling underlying stock price evolution is the **binomial tree** (Cox, Ross, and Rubinstein, 1979):

- A binomial model assumes that stock prices can make, at any point in time, upward (u) or downward (d) movements. For example, for $u = 1.2$ and $d = 0.8$:



Binomial Tree

We can expand this tree to recognize any amount of steps (e.g., $N = 2$)...

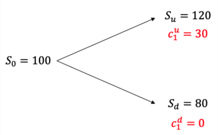


What is the information known at each node? $\rightarrow$ concept of **filtration!**

- For example, if we are on $S_{ud} = 96$, we know that the stock price climbed on the first step and descended on the second. For $S_t = 96$, then, filtration is $\{u, d\}$.
- What is the filtration of $S_t = 64$?
- This will be an important concept moving forward especially when we get to probability theory.

Binomial Tree & Call Option

Once we have an underlying stock price evolution, getting the payoff of any simple option is very easy. Picture a European Call Option with strike price $K = 90$, maturity $T = 1$ *year*, and a 1-step ($N = 1$) binomial tree:



Given payoffs, basic financial intuition would tells us that the **fair value** of such an option is just the present value of expected payoff, which raises two important questions:

- For *expected payoff*, what are the real-world probabilities of u and d happening?
- For *present value*, what is the discount rate that we should use?

The answers to both of these questions rest on the concept of **risk-neutral valuation**.

No-Arbitrage Argument

Let's ask ourselves the following: Can we construct a portfolio that yields a riskless payoff?

- Consider a portfolio with X amount of shares of the underlying and a short position in the call option (i.e., seller of the option).
- Call payoff can move by \$30 ($c_1^u - c_1^d$) while underlying price moves by \$40 ($S_u - S_d$).
- Thus, we need to hold 0.75 underlying shares ($= 30/40$) to be exposed to future movements like the call option.

What will be the value of such a portfolio in the future ($T = 1$)?

- If the stock climbs to \$120, we have a 'u' scenario:

$$\$120 \times 0.75 - \$30 = \$60$$

- If the stock drops to \$80, we have a 'd' scenario:

$$\$80 \times 0.75 - \$0 = \$60$$

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No-Arbitrage & Option Pricing

Investors can build a portfolio that yields a value with certainty making it riskless.

- That \$60 is the value of the portfolio one year from now.
- Since it is a riskless portfolio, we can discount at risk-free rate $\rightarrow$ Suppose $r_f = 10\%$ a year:

$$\text{Portfolio Value Today} = 60 \times e^{-0.1 \times 1} = \$54.29$$

- **Under no-arbitrage**, if underlying today is \$100, Call today (C_0) will be worth...

$$\text{Portfolio Value Today} = \$100 \times 0.75 - C_0 = \$54.29 \rightarrow C_0 = \$20.71$$

This exercise provides **two important results** for option pricing:

- We can price an option as part of a riskless portfolio $\rightarrow$ Risk-free discount rate
- Under these conditions, we can obtain implied probabilities (risk-neutral).

The latter two concepts are the foundation of risk-neutral valuations.

$\rightarrow$ We can think of investors as risk-neutral!

Intro to Risk-Neutral Probabilities

Absent arbitrage, we get the value of riskless portfolio & call option $\rightarrow$ Probabilities?

- Present value of our riskless portfolio is the portfolio cost today. For u:

$$S_0X - C_0 = (S_0uX - c_1^u)e^{-rT} \rightarrow C_0 = S_0X(1 - ue^{-rT}) + c_1^u e^{-rT}$$

- We know that $X = \frac{c_1^u - c_1^d}{S_u - S_d}$, so substituting and rearranging:

$$C_0 = S_0 \frac{c_1^u - c_1^d}{S_0u - S_0d}(1 - ue^{-rT}) + c_1^u e^{-rT} \rightarrow C_0 = \frac{c_1^u(1 - de^{-rT}) + c_1^d(ue^{-rT} - 1)}{u - d} \rightarrow$$

which can be written as:

$$C_0 = e^{-rT}[pc_1^u + (1-p)c_1^d], \text{ where } p = \frac{e^{rT} - d}{u - d}$$

- That, in our previous example with $u = 1.2$, $d = 0.8$, and $r_f = 10\%$ would be:

$$p = \frac{e^{0.1 \times 1} - 0.8}{1.2 - 0.8} = 0.7629$$

This (p) is our risk-neutral probability measure.

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Summary of Lesson 1

In this intense Lesson 1, we have covered several important concepts:

- Basic features of financial derivatives (options in particular)
- Binomial model to simulate the behavior of the underlying asset
- Option payoffs and riskless portfolio in binomial tree
- No-arbitrage argument to find option price
- Intro to risk-neutral valuation

⇒ **TO-DO NEXT**: In the Jupyter Notebook accompanying this lesson, you will find an example of how to build a function to construct a binomial tree in Python. Here, you will also see how, using backward induction, we can get the price of the call option.

⇒ In the next lesson, we will go deeper into the replication and risk-neutral valuation ideas by considering a put option and its relationship to the call option case.