LESSON 2

OPTION PRICING AND PUT-CALL PARITY

Derivative Pricing - Module 1

WORLDQUANT UNIVERSITY

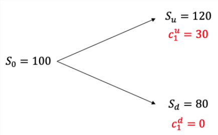
Outline

- Replicating Portfolio for Call and Put Options
- Pricing Put Options
- Put-Call Parity

Generalizing: Replicating portfolios

⇒ An option (call or put) can be replicated with a self-financed portfolio combining:

1. A position (long or short) in shares
2. A position in a risk-free bond (lend or borrow)



With X number of shares and B amount in the risk-free bond, such that (assume r=0 for simplicity):

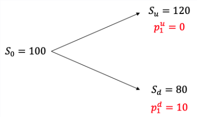
$$XS_u + B = c_1^u \quad \rightarrow X120 + B = 30$$
$$XS_d + B = c_1^d \quad \rightarrow X80 + B = 0 \Rightarrow \text{Solving these: } X = 0.75, \quad B = -60$$

These numbers are no coincidence to what we previously used.

- At t=0 we would borrow \$60 ($Be^{-rT}, r = 0$), and invest \$75 dollars in the stock ($= 0.75 \times S_0$).
- This would yield exactly the same payoff as the call option.
- Part of the portfolio is self-financed (\$'s borrowed at r). The \$15 not financed is, precisely, the price of the call option today.

Replicating Portfolios for Put Option

We can do the same thing for a European put option with the same strike, $K = 90$:



- Check that you know how to obtain the replicating portfolio.

It is easy to check that, for the put case, $X = -0.25$ and $B = 30$.

$\rightarrow$ Likewise, the difference in the self-financed portfolio is the value of the option at t=0:

$$-0.25S_0 + 30e^{-rT} = p_0 \ \rightarrow p_0 = -0.25 \times 100 + 30 = \$5$$

$\rightarrow$ The same value as via backward induction with risk-neutral probabilities:

$$p_0 = e^{-rT}[\mathbf{p}p_1^u + (1 - \mathbf{p})p_1^d]$$

Please, do not mix up $\mathbf{p}$ (risk-neutral prob) with p_t (value of the put option)!

Put-Call Parity

Let's now explore the relationship between the value of the call and put options.

- To do so, we are going to build on the previous replicating portfolios:

$$0.75S_0 - 60e^{-rT} = c_0$$
$$-0.25S_0 + 30e^{-rT} = p_0$$

which we can put as:

$$0.75S_0 - 60e^{-rT} - c_0 = -0.25S_0 + 30e^{-rT} - p_0$$

and rearranging:

$$c_0 + 90e^{-rT} = S_0 + p_0$$

but the 90 is no coincidence; it is the strike of both options, K:

$$c_0 + Ke^{-rT} = S_0 + p_0$$

The latter expression is known as put-call parity.

Summary of Lesson 2

In this lesson, we have learned:

- Constructed replicating portfolios for call & put.
- Use of a binomial model to simulate the behavior of the underlying asset.
- Option payoffs and riskless portfolios in a binomial tree.
- Use of the no-arbitrage argument to find option price.
- Risk-neutral valuation.

⇒ **TO-DO NEXT**: In the Jupyter Notebook accompanying this lesson, you will find an example extending the binomial tree from Lesson 1. We will use the extended tree to do a more complex pricing of options and verify whether put-call parity holds.

⇒ In the next lesson, we will continue building on the concepts of replication and risk exposure of an investor with a position in an option. Moreover, can investors hedge their risk exposure?