LESSON 3

DELTA AND DELTA HEDGING

Derivative Pricing - Module 1

WORLDQUANT UNIVERSITY

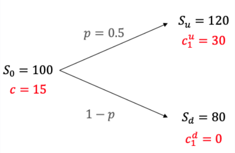
Outline

- Revisiting the delta concept formally
- Delta Hedging
- Putting it all together under the binomial framework

Delta: What We Already Know

In previous lessons, we showed how to obtain risk-neutral probabilities by constructing a replicating portfolio. We did this by calculating the ratio of the change in option price vs. the change in the price of the underlying asset. That is the concept behind **delta (Δ)**.

Picture the following one-step binomial tree with $r = 0$ for a call option with $K = 90$:



Delta (Δ) measures the change in the price of the option with respect to the change in the price of the underlying asset.

$$\Delta_0 = \frac{c_1^u - c_1^d}{S_u - S_d} = \frac{30 - 0}{120 - 80} = 0.75 \tag{1}$$

Delta: Extending the Tree

Things get more complicated as you extend the tree to contain more nodes, but the intuition on Δ remains the same.

Think of a tree similar to the one before but with N=2:

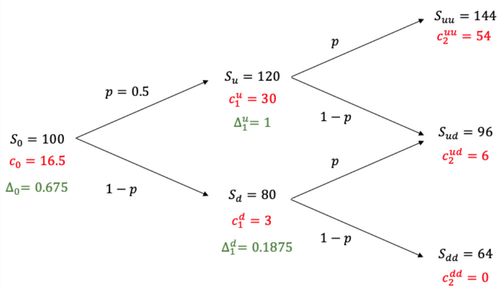


Figure: Binomial tree with stock prices (black), call option prices (red), and delta (green).

Delta and Delta Hedging

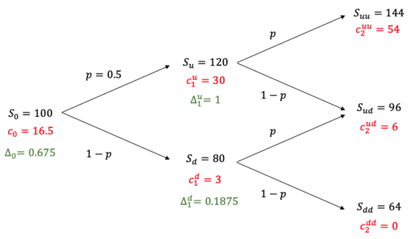
Delta is a classic example of a so-called *Greek* in derivatives. These Greeks are essentially variables that measure different sensitivities of option prices to several dimensions. We will dig deeper into these in upcoming modules.

For now, let's review a few features from delta to highlight:

- Δ measures option price sensitivity to changes in the price of the underlying.
- One important application is to construct *replicating portfolios* that are risk-free (remember previous lessons).
- The latter implicitly means that we can use Δ as a measure to **hedge** the exposure of our portfolio to underlying price changes $\rightarrow$ **Delta Hedging**

Let's revisit the example in the previous slide to understand the role of Δ in constructing a hedged position...

Delta Hedging: An Example



$\Rightarrow$ What would the hedge look like? (*Assume that the underlying asset is a stock and that we are the bank selling this call*)

- At $t = 0$, we need to hold 0.675 shares of the stock.
- At $t = 1$, we will need to hold 0.1875 shares if the underlying price goes down (S_d) or 1 share if it moves up (S_u).

Delta Hedging: An Example

- How would this **Delta-hedge** work? $\rightarrow$ Example for seller & path 'du':

	$t=0$	$t=1$	$t=2$	Total
Underlying (stock) price	\$100	\$80	\$96	
Call option	\$16.5	\$3	\$6	
Δ hedge	0.675	0.1875		
Stock portfolio value	\$67.5	\$28.5	\$18	
Cash account	-\$67.5	+\$39	+\$18 - \$6	**-\$16.5**

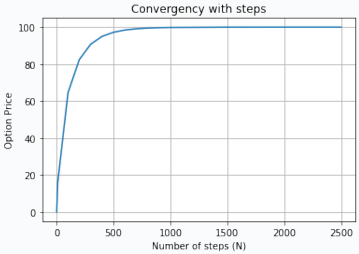
At $t = 0$, we buy 0.657 shares of the underlying ($0.675 \times 100 = \$67.5$).
At $t = 1$, we sell 0.4875 shares ($= 0.675 - 0.1875$) to achieve the $\Delta = 0.1875$. For that, we obtain \$39 ($= 0.4875 \times 80$). The value of our stock portfolio at $t = 1$ is therefore \$28.5 ($= 67.5 - 39$) and we own 0.1875 shares.
At $t = 2$, our 0.1875 shares of the underlying will be worth \$18 ($= 0.1875 \times \$96$). Since $t = 2$ is the maturity of the option contract, we get that \$18 from selling the stock. But the option buyer will come to collect its payoff, which we, as the seller, will have to pay (\$6).
You can easily see that it is no coincidence that the total cost of the hedge is exactly the price of the call option at $t = 0$!.

- But where is the profit for the Bank? $\rightarrow$ Fees!

Generalization to Any N

Now that we have understood the basics of pricing options through a binomial model and delta-hedging our position, let's check its power by increasing N.

The accompanying Jupyter Notebook contains a testing example of this based on the previous framework:



- Seems absurd that the call option price actually converges to S_0...Why?
- **Answer:** Careful with volatility! We need to adjust variability (i.e. u and d) to N.

Summary of Lesson 3

In this lesson, we have learned how to:

- Define and develop the concept of delta and delta hedging
- Calculate delta in the binomial model framework
- Use the binomial model for constructing delta-hedged portfolios
- Dynamically adjust the hedging in the binomial tree

⇒ **TO-DO NEXT**: In the Jupyter Notebook accompanying this lesson, you will find an example of how to construct delta hedging in the binomial tree in Python.

⇒ In the next lesson, we will go deeper into how to calibrate the binomial model to recognize volatility of the asset and revisit all the features covered in this module under a complete framework.