LESSON 3

INTRO TO PATH-DEPENDENT OPTIONS

Derivative Pricing - Module 2

WORLDQUANT UNIVERSITY

Outline

- Intro to Path-dependent options.
- Problems in Binomial model for path-dependent options.
- Potential solutions & computational costs' trade-off.

Path-dependent options

So far, we have used the binomial tree to price options with conventional payoffs:

$$\Rightarrow (S_t - K)^+ \text{ for call options and } (K - S_t)^+ \text{ for put options.}$$

However, one of the main advantages of derivative instruments is they're highly customizable for *ad hoc* needs:
$\Rightarrow$ Most clients will demand OTC instruments with **non-conventional payoffs**!

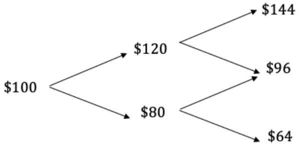
One example of such payoff is the **Asian option**:

- Payoff of Asian options is dependent on the arithmetic average of stock price:
  $\rightarrow (S_{Avg.} - K)^+$ for Asian Call, $(K - S_{Avg.})^+$ for Asian Put.
- This implies that we need to compute the average price, $S_{Avg.}$, for ALL paths
- While simple for the 'toy examples' that we have been using to illustrate concepts, there are a bunch of computational problems related to it
- We will use the Asian option to illustrate these, but note that problems can be extrapolated to any context with path-dependent options

Asian option on Binomial Model

Picture an Asian Call option under a binomial model with the following data:

$S_0 = 100,\ K = 90,\ T = 1,\ r = 0\%,\ u = 1.2,\ d = 0.8,\ N = 2\ \ \rightarrow p = 0.5$



Path	Avg. Price	Asian Call Payoff
{u, u}	121.33	31.33
{u, d}	105.33	15.33
{d, u}	92	2
{d, d}	81.33	0

Values for the Asian Call option at step 1 in the tree will be:

$$C_1^u = 0.5 \times 31.33 + (1 - 0.5) \times 15.33 = \$23.33$$
$$C_1^d = 0.5 \times 2 + (1 - 0.5) \times 0 = \$1$$

So, value of the Asian option today will be:

$$C_0 = 0.5 \times 23.33 + (1 - 0.5) \times 1 = \$12.165$$

Asian option main takeaways

Takeaways from last example on Asian option:

- Note that here number of terminal prices $\neq$ number of option payoffs $\rightarrow$ Path matters
- To get option payoff we have to compute all paths $\rightarrow \#Paths = 2^N$
- Ideally, the more paths we compute the merrier
  - Improve accuracy?
  - Computational cost?

    $\Rightarrow$ If you haven't yet, try running one of the previous codes for large N!

    $\Rightarrow$ Picture you want $N = 100$ steps. That means 1.27×10^{30} paths!
- While conceptually it is easy to price Asian options using the binomial tree, implementing it in practice is way more difficult
- These problems are common to all types of path-dependent options/derivatives:
  - Barrier options; Lookback options; Binary options; ...

Summary of Lesson 3

In Lesson 3 we have seen some important concepts:

- Path-dependent options and unconventional payoffs
- Use of the binomial model to price options with path-dependent payoff
- Computational problems arising from using binomial model with path-dependent derivatives

$\Rightarrow$ **TO-DO NEXT**: This Lesson does not have an associated Jupyter Notebook. Move to Lesson 4, and its corresponding Notebook!

$\Rightarrow$ In the next lesson we introduce an alternative methodology to alleviate the computational problems embedded here: Monte-Carlo methods.