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OSM Bootcamp Math

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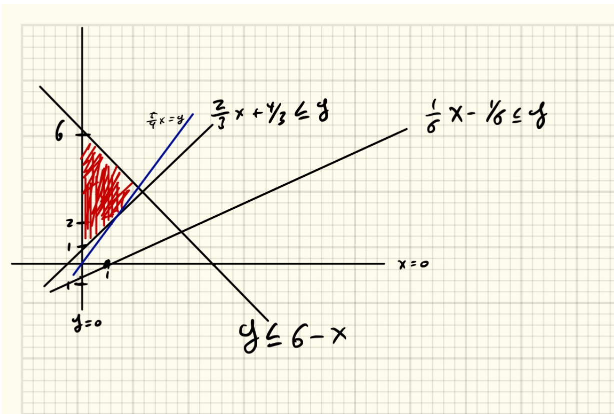
Exercise 8.1



Verticies: $(14/5, 3.2), (0, 6), (0, 4/3) \implies (14/5, 3.2)$ max and objective function is 1.2.

Exercise 8.2

1. $(0, 0) \implies 0, (4, 0) \implies 12, (0, 6) \implies 6, (6, 2) \implies 20$ Thus (6,2) is max.
2. $(0, 0) \implies 0, (27, 0) \implies 108, (0, 11) \implies 66, (5, 15) \implies 110$. So (5,15) is max.

Exercise 8.3

$$\max\ 12G + 10J - 8G - 7J$$
$$s.t.\ 15G + 10J \leq 1800$$
$$2G + 2J \leq 300$$
$$J \leq 200$$

Exercise 8.4

$$\begin{aligned}
\min 2x_1 + 5x_2 + 5x_3 + 2x_4 + 7x_5 + 9x_6 + 2x_7 + 4x_8 + 3x_9 \\
s.t.\ x_1 + x_3 \leq 10 \\
x_2 + x_6 + x_5 + x_4 - x_1 \leq 1 \\
x_2 - x_7 \leq -2 \\
x_8 - x_4 - x_3 \leq -3 \\
x_9 - x_5 - x_8 \leq 4 \\
-x_7 - x_6 - x - 9 \leq -10 \\
x_i \leq 6\ \forall\ i
\end{aligned}$$

Exercise 8.5

Dict	1					
x	y	s1	s2	s3	p	
1	3	1	0	0	0	15
2	3	0	1	0	0	18
1	-1	0	0	1	0	4
-3	-1	0	0	0	1	0

Dict	2					
x	y	s1	s2	s3	p	
0	4	1	0	-1	0	11
0	5	0	1	-2	0	10
1	-1	0	0	1	0	4
0	-4	0	0	3	1	12

Dict	1					
x	y	s1	s2	s3	p	
0	0	1	-0.8	0.6	0	3
0	1	0	0.2	-0.4	0	2
1	0	0	0.2	0.6	0	6
0	0	0	0.8	1.4	1	20

Exercise 8.7

Optimal value is 2 $x_1 = 0, x_2 = 2$

Exercise 8.8

$$\max -x_1 - x_2 - x_3 \ s.t. x_i \geq 0$$

Exercise 8.9

$$\max x_1 + x_2 + x_3 \ s.t. x_i \geq 0$$

Exercise 8.10

$$\max x_1 + x_2 + x_3 \ s.t. x_i \geq 0, 3 \geq x_3 \geq 2$$

Exercise 8.11

$$\max x_1 + x_2 + x_3 \ s.t. x_1 + x_2 + x_3 \geq 1, 0 \leq x_1, x_2, x_3 \leq 2$$

which we can write as the aux problem:

$$\max -x_0 s.t. - x_1 - x_2 - x_3 - x_0 \leq -1, x_i \geq 0$$

Exercise 8.12

Max value is 1 at x=(1,0,1,0)

Exercise 8.15

Proof.

$$c^T x = x^T x \leq x^T (Z^T y) = (Ax)^T y \leq b^T y$$

□

Exercise 8.17

$\max c^T x \ s.t. \ Ax \leq b, x \geq 0 \implies \min b^T y \ s.t. A^T y \geq c, y \geq 0 \implies \max -b^T y \ s.t. \ - A^T y \leq -cy \geq 0 \implies \max c^T z \ s.t. \ Az \leq b, z \geq 0$

Exercise 8.18

The primal problem is solved with a max value of 1.75. Then the dual problem is:

$$\min 3y_1 + 5y_2 + 4y_3 \ s.t. \ 2y_1 + y_2 + 2y_3 \geq 1, y_1 + 3y_2 + 3y_3 \geq 1, y_i \geq 0$$

Then the min is attainted at y=(0.25,0,0.25,0) with value 1.75.