

# OSM Bootcamp Econ - DSGE

Ari Boyarsky  
aboyarsky@uchicago.edu

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## 1 Part 1

### Exercise 1

Using Guess and Verify:  $C_t = \phi Y_t$  and  $K_{t+1} = (1 - \phi)Y_t$  such that  $C_t + K_{t+1} = Y_t = e^{z_t} K_t^\alpha$ . Then, we get the Euler:

$$\begin{aligned}\frac{1}{C_t} &= \beta \mathbb{E}_t \left\{ \frac{\alpha e^{z_{t+1}} K_t^{\alpha-1}}{C_{t+1}} \right\} = \beta \mathbb{E}_t \left\{ \frac{\alpha e^{z_{t+1}} K_t^{\alpha-1}}{\phi Y_{t+1}} \right\} = \beta \mathbb{E}_t \left\{ \frac{\alpha e^{z_{t+1}} K_t^{\alpha-1}}{\phi e^{z_{t+1}} K_{t+1}^\alpha} \right\} = \beta \mathbb{E}_t \left\{ \frac{\alpha}{\phi K_{t+1}} \right\} \\ &\implies \frac{1}{\phi e^{z_t} K_t^\alpha} = \beta \mathbb{E}_t \left\{ \frac{\alpha}{\phi K_{t+1}} \right\} \implies \frac{1}{e^{z_t} K_t^\alpha} = \beta \frac{\alpha}{K_{t+1}} \\ &\implies K_{t+1} = \beta \alpha e^{z_t} K_t^\alpha\end{aligned}$$

Which also implies that  $A = \beta \alpha$ .

### Exercise 2

$$\begin{aligned}\frac{-a}{1-l_t} + \frac{w_t(1-\tau)}{c_t} &= 0 \\ \frac{1}{c_t} &= \beta \mathbb{E} \left\{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_t^\gamma} \right\}\end{aligned}$$

### Exercise 3

The conditions are:

$$\begin{aligned}\frac{-a}{1-l_t} + \frac{w_t(1-\tau)}{c_t} &= 0 \\ \frac{1}{c_t} &= \beta \mathbb{E} \left\{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}} \right\}\end{aligned}$$

### Exercise 4

The conditions are:

$$\begin{aligned}\frac{-a}{(1-l_t)^\xi} + \frac{w_t(1-\tau)}{c_t} &= 0 \\ \frac{1}{c_t^\gamma} &= \beta \mathbb{E} \left\{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}^\gamma} \right\}\end{aligned}$$

## Exercise 5

The conditions are:

$$\frac{1}{c_t^\gamma} = \beta \mathbb{E} \left\{ \frac{(r_{t+1} - \delta)(1 - \tau) + 1}{c_{t+1}^\gamma} \right\}$$

In the steady state we get:

$$\frac{1}{c_{ss}} = \beta \frac{(r_{ss} - \delta)(1 - \tau) + 1}{c_{ss}^\gamma}$$

## Exercise 6

$Y_{ss} = 0.6855$  and  $I_{ss} = 0.2257$

## 2 Part 2

See included .ipynb files.