Problem Set #1

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OSM Bootcamp Econ: Dynamic Programming

June 24, 2018

Exercise 1

1. p_t, B, prices at each period, and endowment of oil B
2. B' barrels left in next period
3. $B' = B - q$ where q is quantity consumed during the first period.
4. Sequence problem: $V = \max_{B'} \sum_{t=1}^{\infty} (\frac{1}{1+r})^{t-1} p_t (B - B')$
   Belman Equation: $V(B) = \max_{B'} p(B - B') + (\frac{1}{1+r})V(B')$
5. FOC: $-p + (\frac{1}{1+r})V'(B') = 0$
   $\frac{\partial V}{\partial B} = p - p\frac{\partial B'}{\partial B} + (\frac{1}{1+r})\frac{\partial V}{\partial B'}\frac{\partial B'}{\partial B} = p + [(\frac{1}{1+r})\frac{\partial V}{\partial B'} - p]\frac{\partial B'}{\partial B} = p \implies p = (\frac{1}{1+r})p'$
6. If $p_{t+1} = p_t$ we would always sell in the first period. To see this simply plugging into the euler equation yields $r = 0$. Thus, for the individual to be indifferent between the two periods requires 0 intrest. If $r > 0$ then it implies that we would sell in the first period to maximize the value. If $p_{t+1} > (r+1)p_t \implies p_t < (\frac{1}{1+r})(r+1)p_{t+1}$ which implies sell in the next period since the price in the next period will overcome the discount rate. For an interior solution we would want $p_t = (1+r)p_{t+1}$ which would imply indifference between the first and second period, then the owner might choose to sell some in the current period and some more in the second period given indifference in value from the prices.

Exercise 2-4

See Jupyter Notebook in Folder.