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OSM Bootcamp Math

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9.1

Suppose by way of contradiction that $Ly \neq Lx$. Then x is not a minimizer if it is less than. If it is greater than then $L(2x - z) = Lx + L(x - z) < Lx$ and so x is not a minimizer.

9.2

$(Ax-b)^T(Ax-b) = x^TA^TAx - 2b^TAx + 2b^Tb$. Then noticing that A^TA is pos semi def and taking the FOC yields $A^TAx = A^Tb$.

9.3

Steepest descent computes the gradient then goes a certain distance in that direction to minimize the objective. Newtons method starts with a guess and uses newtons method to find the roots of the derivative i.e. crit points. Conjugate gradient uses steepest descent but looks at the Q-conjugate direction.

9.10

Suppose x is a minimzer then $f(x) = 0 \implies Qx - b = 0 \implies x = Q^{-1}b$ Then applying newtons method yields the result.

9.12

$Bv = (A + uI)v = Av + uIv = \lambda v + uv = (\lambda + u)$ where lambda is arbitrary.

9.16

Letting $A = A_k^{-1}, B = y - A_k s_k, C = \|s_k\|^2, D = s_k^T$. Then $A_{k+1} = A_k^{=1} + \frac{(s_k - A_k^{-1}y_k)s_k^TA_k^{-1}}{s_k^TA_k^{-1}y_k}$

9.18

$\phi'(a) = -Df(x + a_kd_k)d_k = [x_k^TQ - b^T]d_k - (a_kd_k)^TAd_k = r_k^Td_k - a_k(d_k^TQd_k) \implies a_k = \frac{r_k^Td_k}{d_k^TQd_k}$