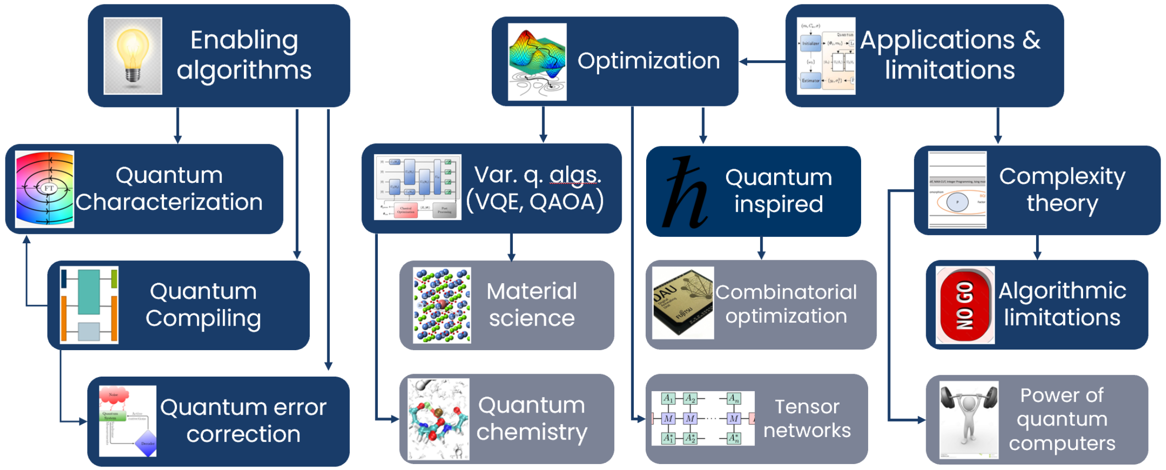
On the limitations and possibilities of practically relevant quantum advantages



TUHH Hamburg University of Technology

Martin Kliesch

ARIC Brown Bag Session, June 24, 2025

What does it take to achieve a practically relevant quantum advantage?

TUHH

1) Q. hardware

2) An application with a separation

3) Implementation (whole stack)

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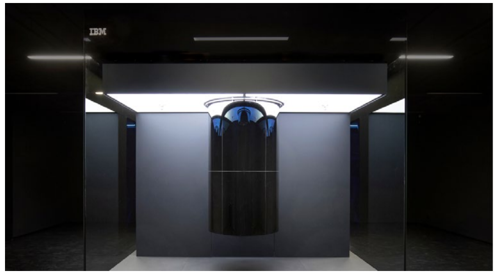
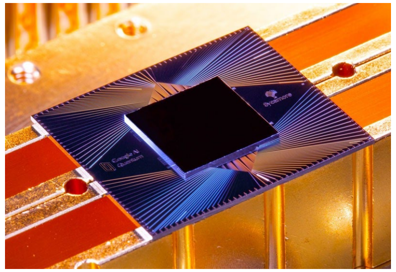
- Qubit number

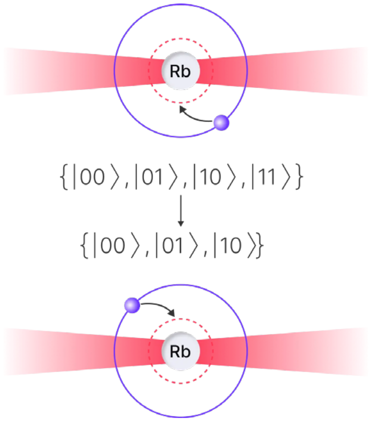


Superconducting qubits [IBM]

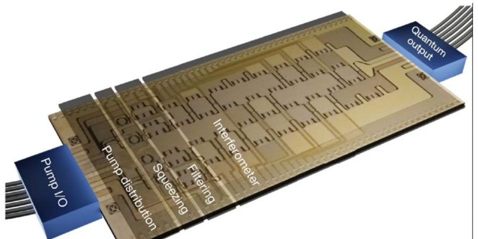


[Google]

2) An application with a separation

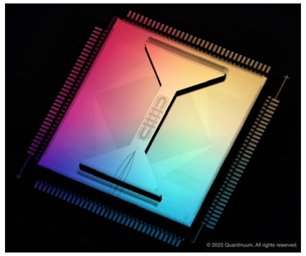


Neutral atoms [QuEra]



Photonics
[Arrazola et al, ature'21]

3) Implementation (whole stack)



Trapped ions
[Quantinuum]

What does it take to achieve a practically relevant quantum advantage?

1) Q. hardware

- Qubit number
- Connectivity & gate set
- Noise
  → Circuit depth
- QPU clock rate (incl. QEC ≥ 10.000 x slower than CPUs)

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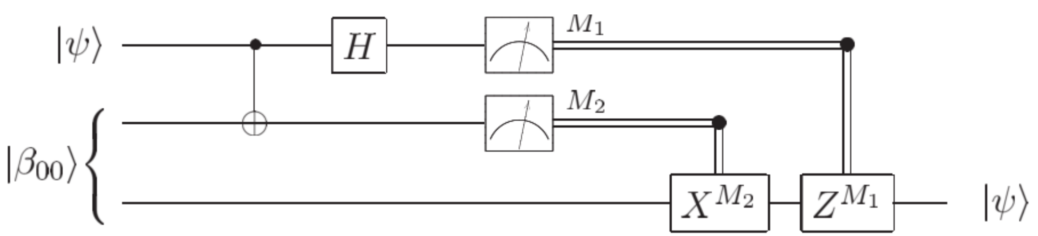
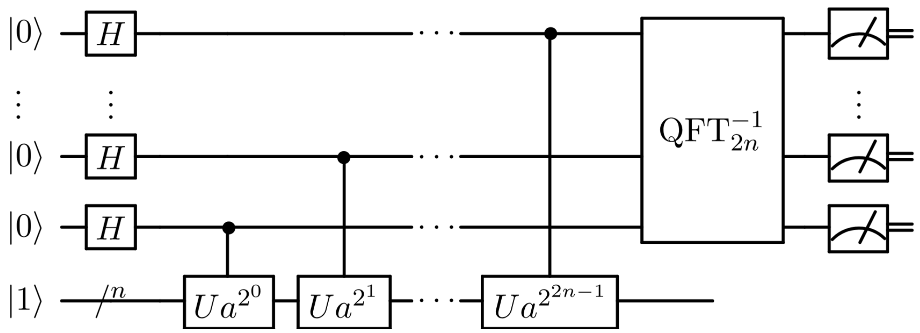
3) Implementation (whole stack)

Quantum circuits





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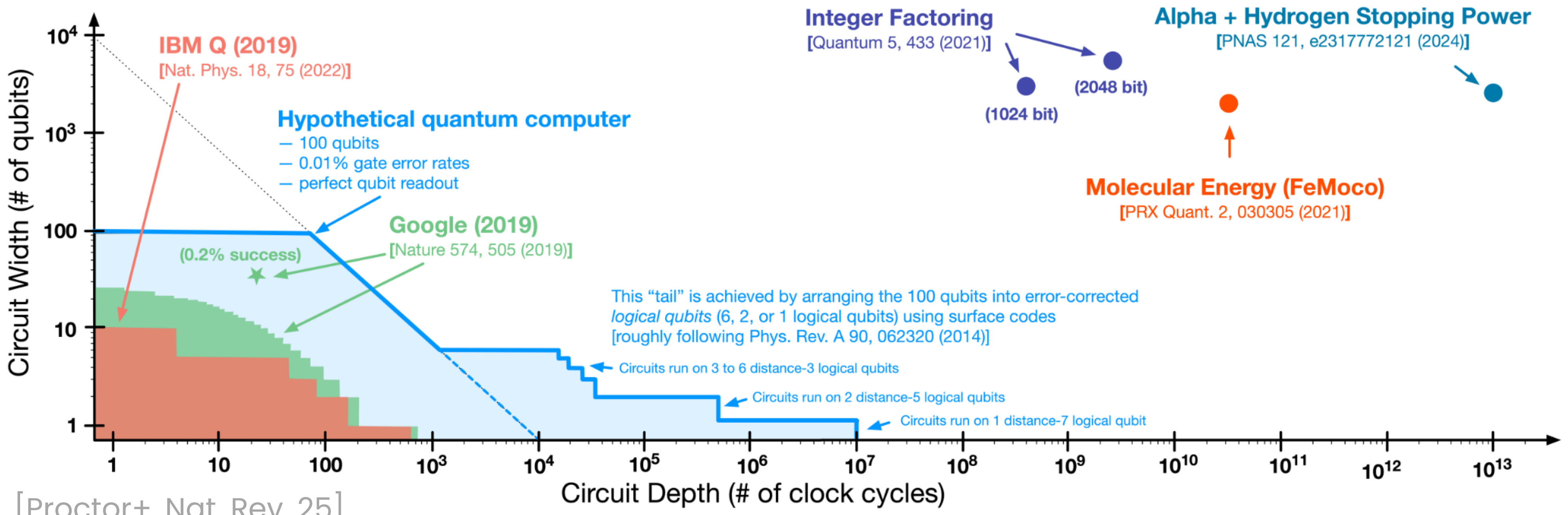


[Proctor+, Nat. Rev. 25]

NISQ vs. FTQC

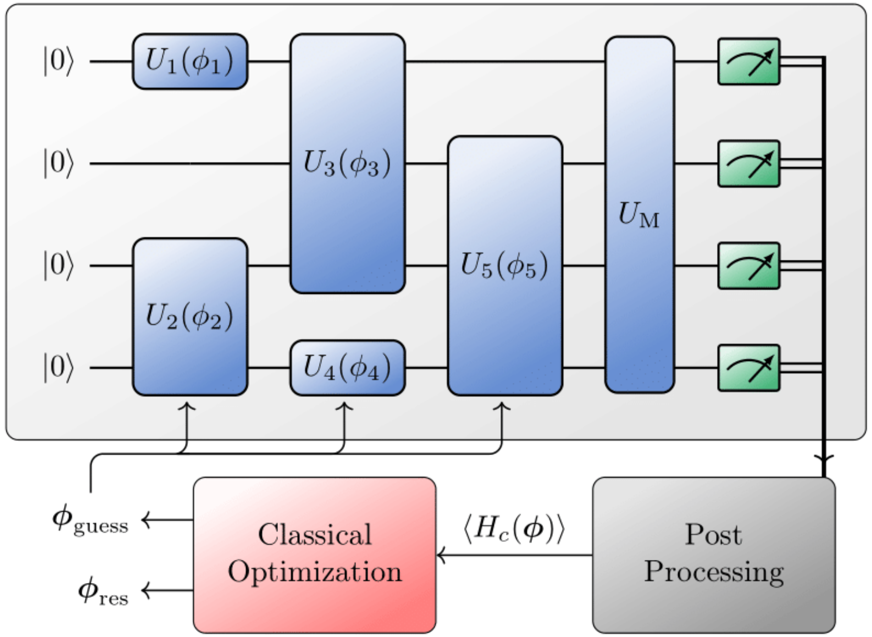
TUHH

Noisy intermediate scale quantum (NISQ) era [Preskill'18]

- ~ 50 to few 100 qubits
- Noisy devices
  without quantum error correction (QEC)

Example algorithms

- Overview [Bharti+, Rev. Mod. Phys.'22]
  + Myths [Zimboras+, arXiv:2501.05694]
- Variational quantum algorithms (VQAs)
  [Cerezo+, Nat. Rev. Phys.'21]
- ...



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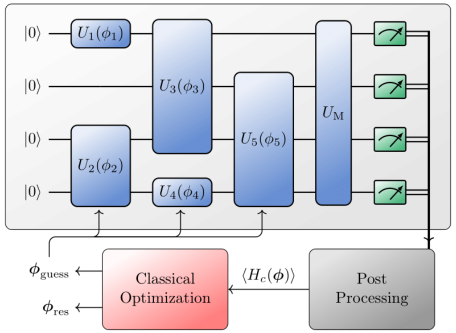
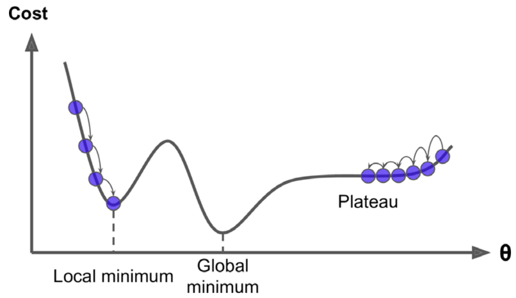


VQA challenges

- Classical optimization
  - Barren plateaus
    [McClean et al, Nat. Comm.'18]
  - Local minima
    → Training NP-hard
    [Bittel & **MK**, PRL'21]
  - Finding hyperparameters
    is QCMA-complete
    [Bittel, Gharibian, **MK**, CCC'23]



- Measurement effort
  Most efficient method: [Gresch, **MK**, Nat. Comm.'25]
- Gate noise
  → error on objective function

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- …

Fault tolerant quantum computation (FTQC) e.g. [Gottesman, arXiv:2210.15844]

1) Encode k logical qubits into $n > k$ physical qubits
2) Measure non-logical part
   to detect and correct errors
3) Implement quantum gates so that error spreading remains under control
   (challenging!)

Complexity

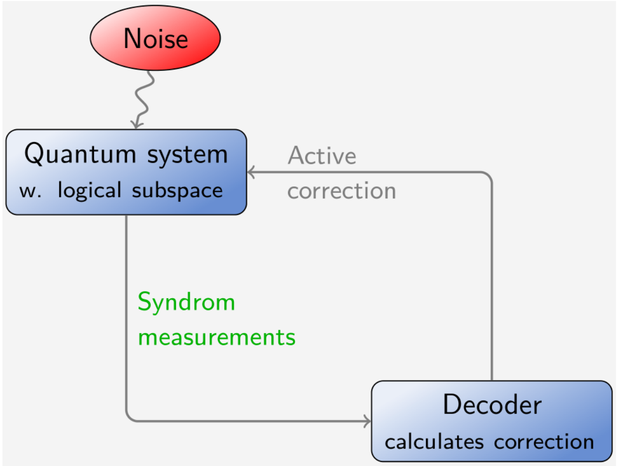
(time, #gates, #qubits)

$$\leq C\, T\, \mathrm{polylog}(T\, /\, \epsilon)$$

T: nr. logical gates

ϵ: max. error

C: constant (large)



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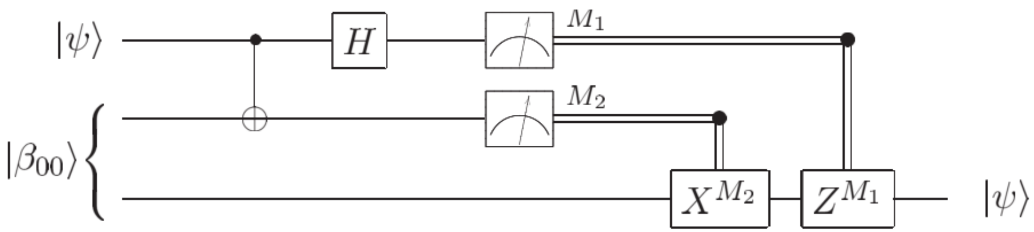

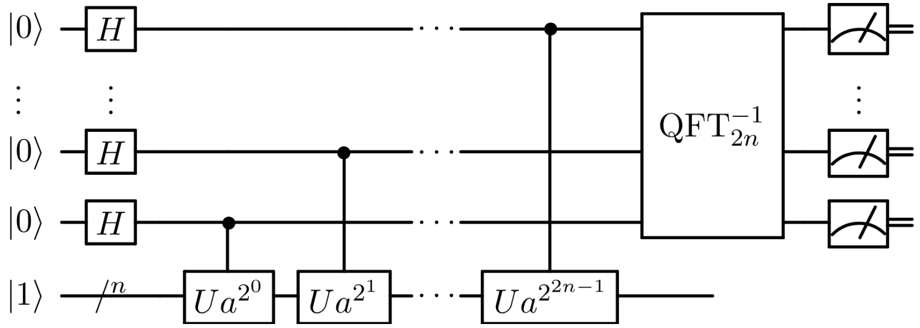

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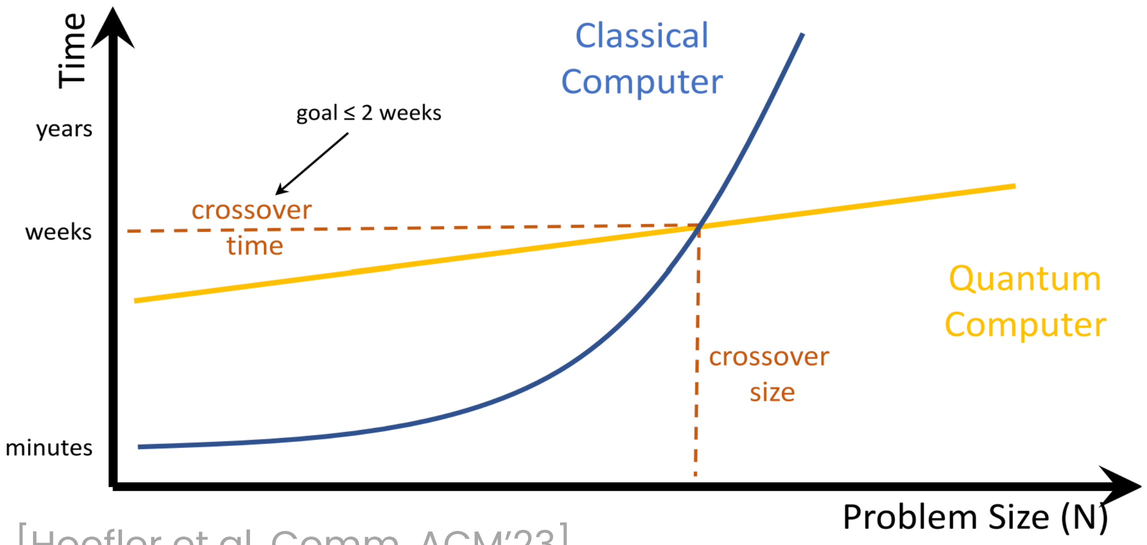
- … where the **best** classical algorithm is slow



[Hoefler et al, Comm. ACM'23]

Consensus

[Hoeffler+, Comm. ACM'23], [Babbush+, PRXQ'21]

At least cubic,
better **exponential** speed-ups are needed

Q. algorithms overview: [Dalzell+'23, arXiv:2310.03011]

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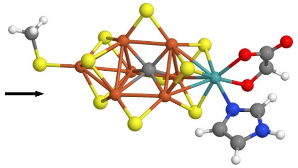
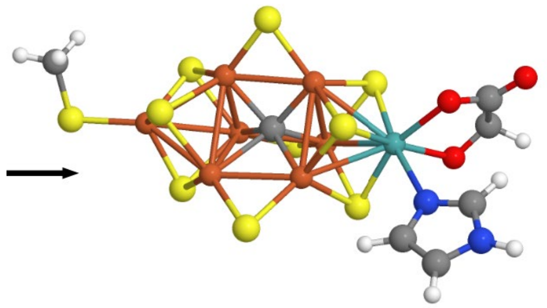
Potential applications with large end-to-end speed-ups

- Fast integer factoring [Shor '94]
- Simulation of quantum systems [Feynman'81]
  - Condensed matter
  - Quantum chemistry



Recent advancements for quantum chemistry

- FeMoco for biological nitrogen fixation (by the enzyme nitrogenase)
  iron molybdenum cofactor [Reiher+, PNAS'17]
- Improved double factorization (DF) + tensor hypercontraction (THC)
  [Caesura et al. (PsiQ+Boehringer Ingelheim)'25, arXiv:2501.06165]
- DF + THC + spectrum amplification
  [Low et al. (Google+)'25, arXiv:2502.15882]



Year	Innovation	FeMoco-54 [37]			FeMoco-76 [38]		
		Qubits	Toffolis	Reference	Qubits	Toffolis	Reference
2017	First resource estimate by Trotterization [37]	111	5.0×10^{13}	[37]	-	-	-
2019	Qubitization of Single-Factorization [17]	3320	9.5×10^{10}	[7]	3628	1.2×10^{11}	[7]
2020	Qubitization of Double-Factorization (DF) [9]	3600	2.3×10^{10}	[9]	6404	5.3×10^{10}	[7]
2020	Tensor-Hyper-Contraction (THC) [7]	2142	5.3×10^{9}	[7]	2196	3.2×10^{10}	[7]
2024	Symmetry compression of DF [39]	1994	2.6×10^{9}	[39]	-	-	-
2025	Symmetry compression of THC [8]	-	-	-	1512	4.3×10^{9}	[8]
This work	Spectrum amplification & DFTHC	1137	3.41×10^{8}		1459	9.99×10^{8}	
Improvement of this work over [39] and [8][a]		1.8×	7.0×		1.0 ×	4.3 ×	

[Low et al. (Google+)'25]

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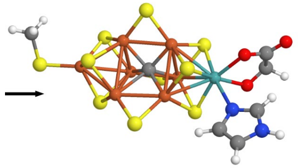

- Linear system solving [HHL'09] ?
  Challenges
  - control of the condition number
  - I/O: output as quantum state vector

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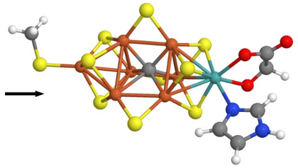


- Linear system solving [HHL'09] ?
- Differential equations ?
- Machine learning ? [rigorous scaling unclear]
- Big data ? [I/O bottleneck]
- Combinatorial optimization ? [NP intermedia problem (instances) interesting?]

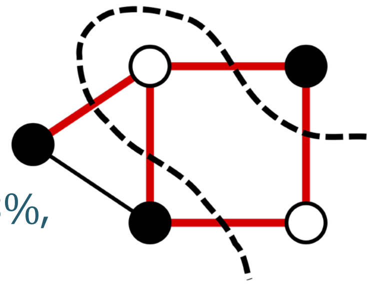
Recent advancements on combinatorial optimization

Generally accepted hypothesis

Quantum computers cannot solve NP-hard problems in polynomial time

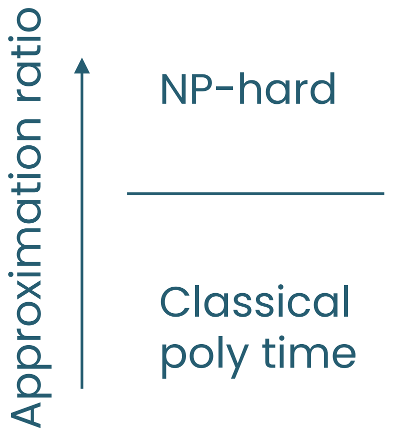
What about approximate solutions?

Example: MaxCut problem



Efficiently solvable up to up to approximation ratio $\sim 88\%$, NP-hard above (assuming UGC)

Interesting read: [https://scottaaronson.blog/?p=8375]



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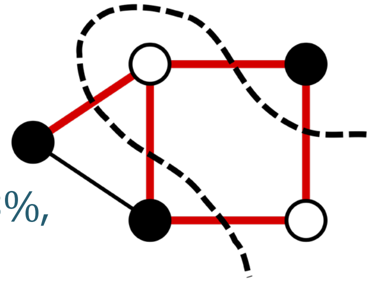

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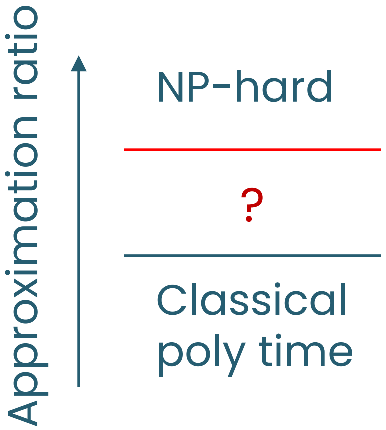
Good news

- Decoded Quantum Interferometry (DQI):
  an approximation algorithm for the max-XORSAT problem
  with exponential speed-up
  in comparison to best known classical algorithms.
  [Jordan et al. (Google+), arXiv:2408.08292, QIP'25 plenary]
- Quartic quantum speedups
  for the Planted Noisy kXOR problem (crypto related)
  [Schmidhuber et al. (Google), PRX'25+SODA'25+QIP'25]



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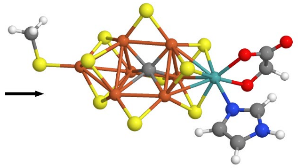

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→ Existence of super-quadratic speed-up often unclear

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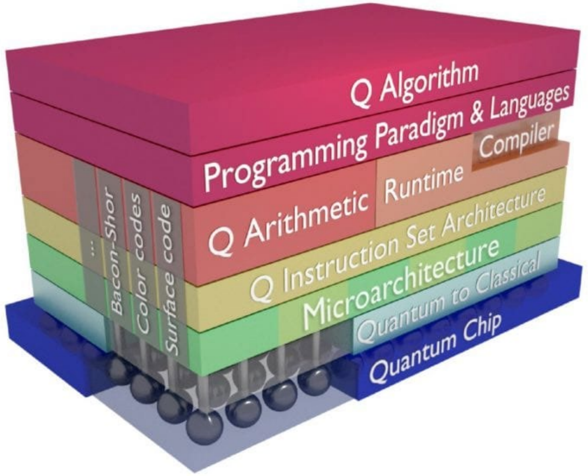
3) Implementation (whole stack)

optimization of the whole QC stack

- Gate design & compiling
- Noise characterization
- Error mitigation
- …



QuTech

Noise characterization

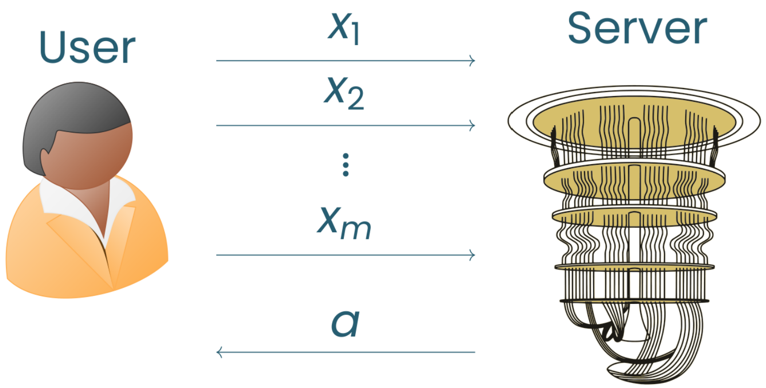
Does a quantum computer work as desired?

Quantum system characterization a.k.a. QCVV

- Obtain information from measurements
  in a platform-independent way

Goals

- Building trust
- Benchmarking
- Error characterization
  - Components (e.g. gates)
  - Interplay of components
- Application:
  Error mitigation /software calibration



e.g. , x_1: Hadamard gate H_1

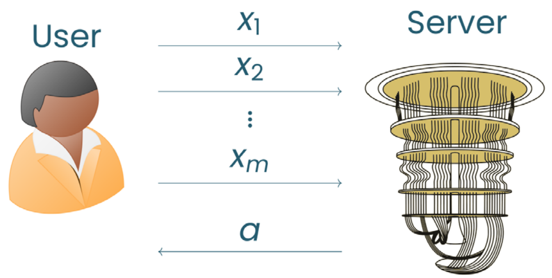
Components

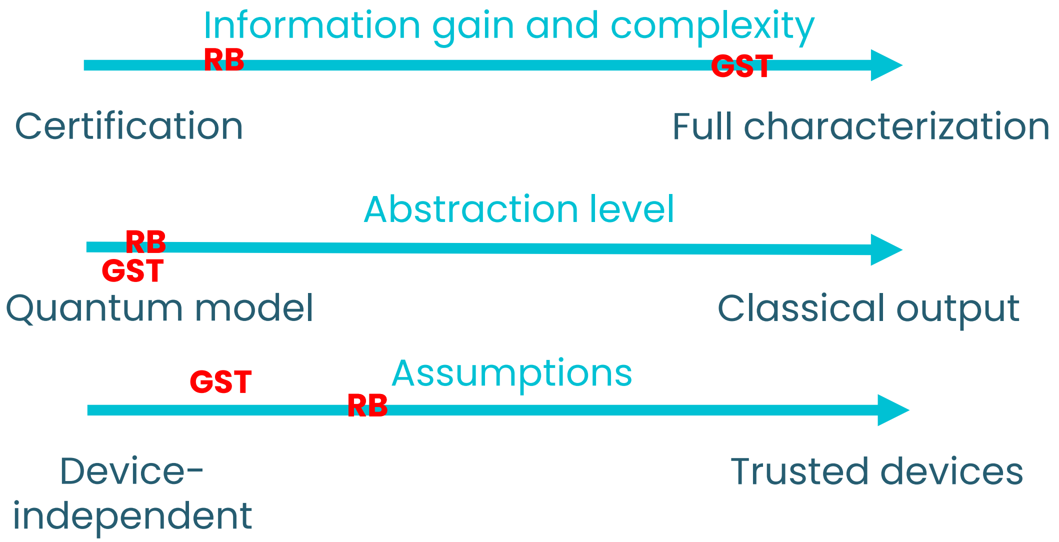
Interplay

Noise characterization

Example protocols

- Quantum state or process tomography
- Gate set tomography (GST)
  [Merkel et al'13; Blume-Kohout+'13;
  Nielsen+, Quantum'21; Brieger, Roth, **MK**, PRXQ'23; …]
- Direct fidelity estimation
  [Flammia & Liu + Silva+, PRL'11]
- Direct certification methods
  [Pallister+, PRL'18;…]
- Randomized benchmarking (RB)
  [Emerson+'05; Knill+'08; Danker+'09, Magesan+'11; …
  Helsen+, PRXQ'22; Heinrich, **MK**, Roth, QIP'23; …]
- Gate set certification
  [Nöller, …, **MK**, QCTiP'25]
- Shadow estimation for quantum states
  [Huang+, Nat. Phys.'20; Brieger,…,**MK**, PRL'25]
- Pauli channel estimation
  [Flammia, Wallman'20; Wagner, Kampermann, Bruß, **MK**, Q.'22+PRL'23;
  v.d.Berg+, Nat.Phys.'23; Chen+QIP'25, …]
- …





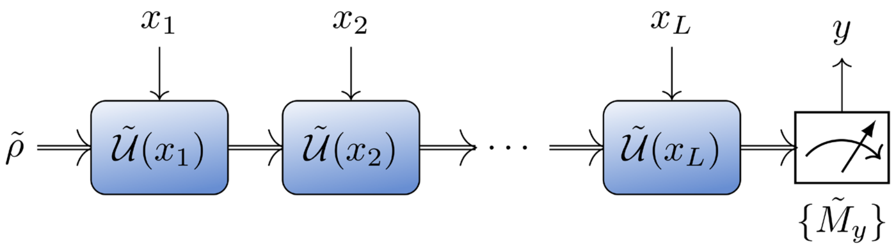
Gate set tomography (GST)

Task

Reconstruct the implemented quantum model

$$\mathcal{M} := \left(\tilde{\rho}, (\tilde{\mathcal{U}}_i), (\tilde{M}_y) \right)$$

by measuring gate sequences



Challenge

- All components unknown

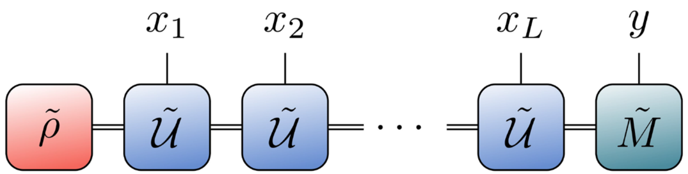
Idea

Subsample sequences $\boldsymbol{x} \in \{1, \dots, n_G\}^L$
and use structure of tensor of Born probabilities

$$\boldsymbol{p} \in \mathbb{R}^{n_E} \otimes (\mathbb{R}^{n_G})^{\otimes L}$$

$$p_{y|\boldsymbol{x}} = \mathrm{Tr}[\tilde{M}_y \, \tilde{\mathcal{U}}_{x_L} \circ \cdots \circ \tilde{\mathcal{U}}_{x_2} \circ \tilde{\mathcal{U}}_{x_1}(\tilde{\rho})]$$

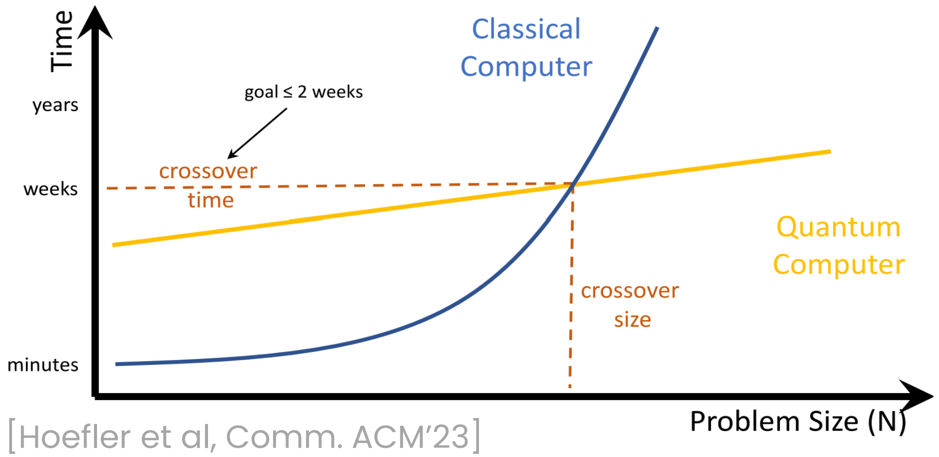
$=$



Tensor train / matrix product state (MPS)

- Tensor completion problem
- Mathematical field: Compressive sensing
- Yields state-of-the-art algorithm
  [Brieger, Roth, MK, PRX Q.'23]

Conclusion



[Hoefler et al, Comm. ACM'23]

Need an application with a separation

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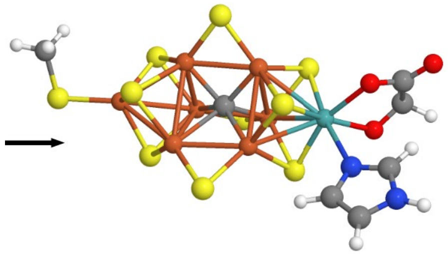
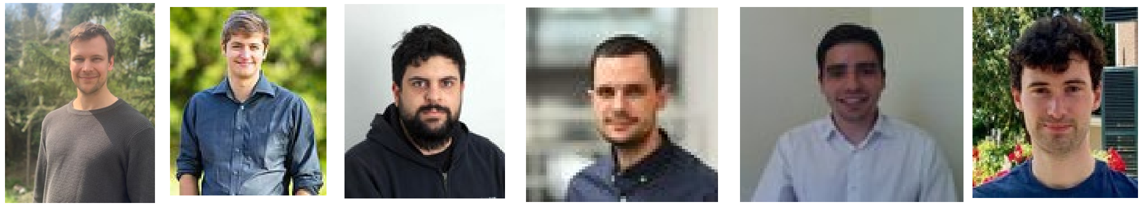
Arguable best examples

- Integer factoring
- Simulation of physical systems



Thank you for your attention!



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