Mapping real-world problems on quantum computers with the ParityQC architecture.

Stefan Rombouts
ParityQC

ARIC QUANTUM BROWN BAG SESSION - 25.06.2024

Quantum Optimization

Parity Quantum Optimization

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Quantum Optimization

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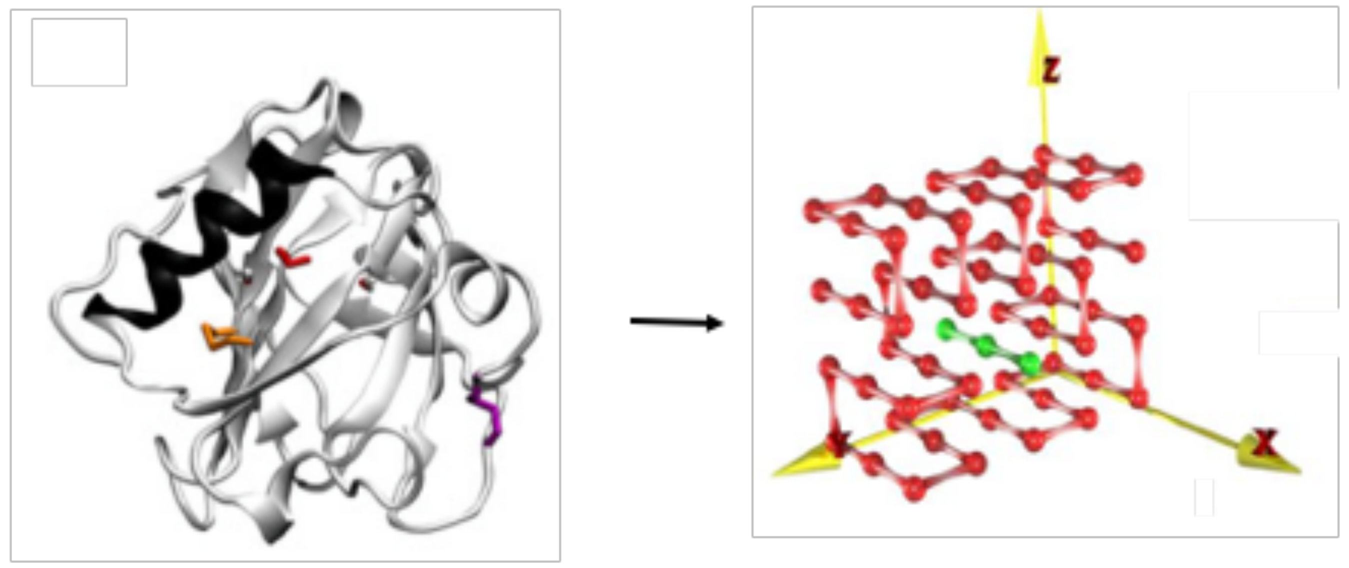
Optimization Problems

Optimization problems in science

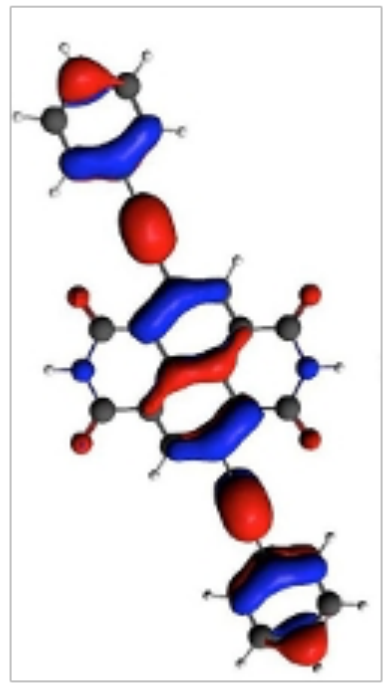
Protein folding



e.g. beta-lactoglobolin (milk protein)
Picture: Peter Bolhuis

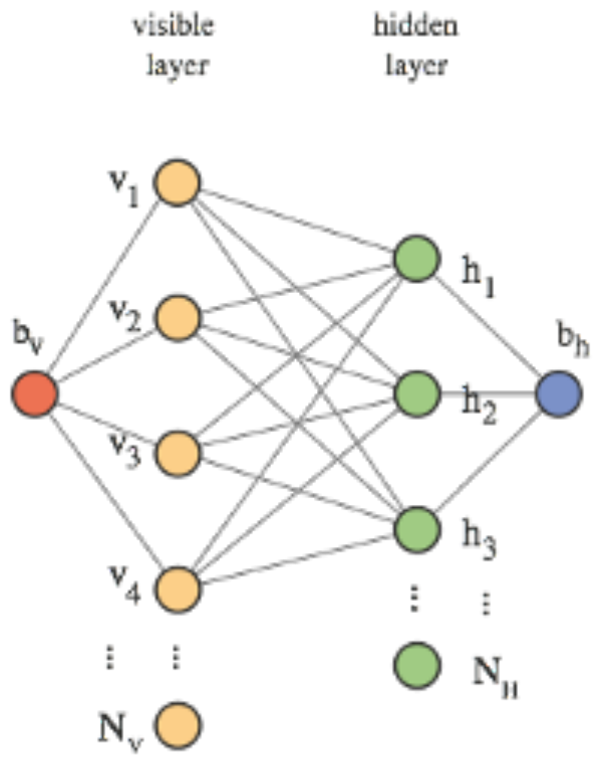
I. Coluzza, et.al. Biophys. J. (2007).

Quantum chemistry



Picture: E. Meijer, University of Amsterdam.

Machine Learning



NP-hard problems

- Logistics (Traveling Salesman)
- Optimization in production
- Factoring (RSA, …)

Minimization of cost functions

$\Updownarrow$

Finding the ground state of a quantum system

$$H = \sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Optimization Problems on Quantum Computers

Quantum computing is simple:

- Map the problem onto qubits
- Manipulate the qubits such that they give you a solution: **the ground state**

Quantum computing is hard:

- Entanglement dissipates, quantum noise takes over in ms
- Manipulating many qubits takes many control signals: lasers, cables, detectors
- You have to to better than classical computers!

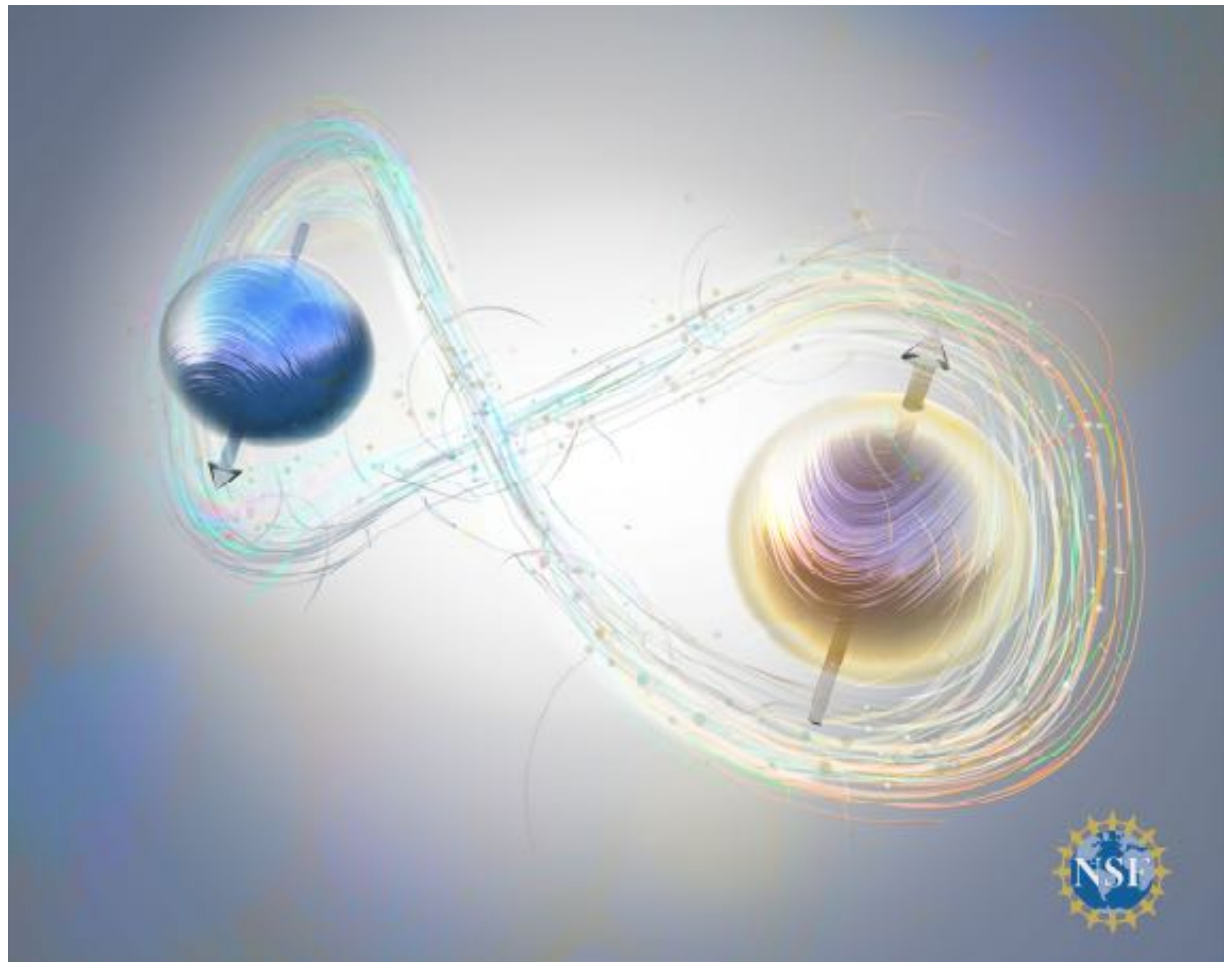
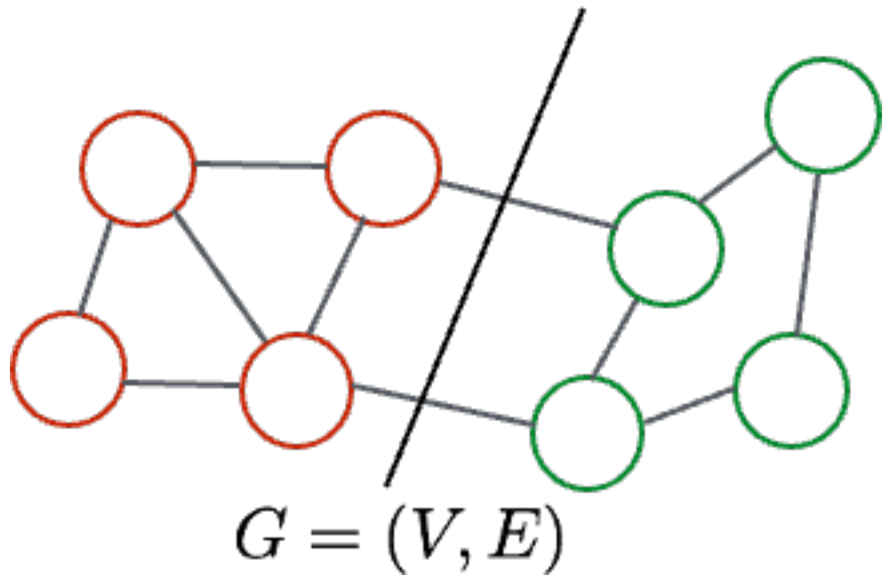


image: Nicolle R. Fuller, NSF

Quantum Optimization

Example: Graph Partitioning



Task:
Find a partitioning of the set of nodes V into two subsets of equal size, such that the number of edges connecting the subsets is minimal?

A. Lucas, *Frontiers in Physics* **2**, 00005 (2014).

Quantum Optimization

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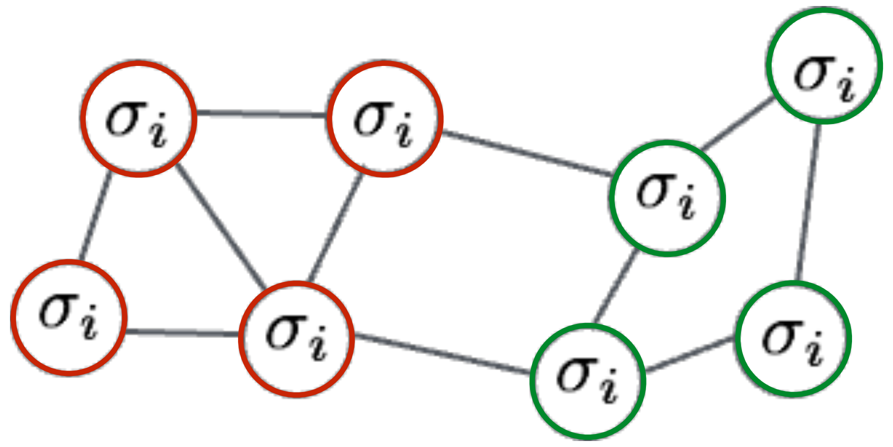

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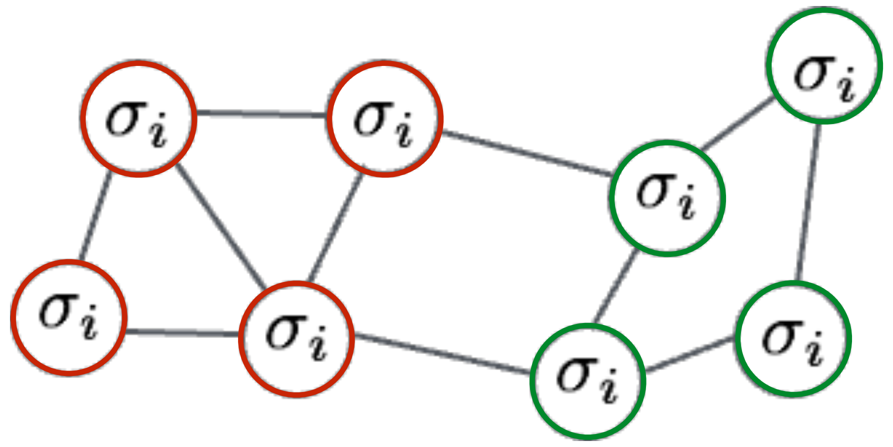

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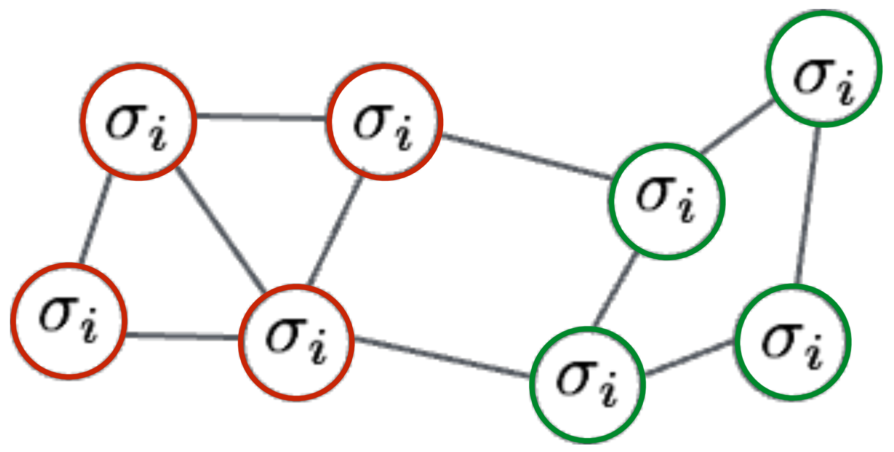

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Quantum Optimization

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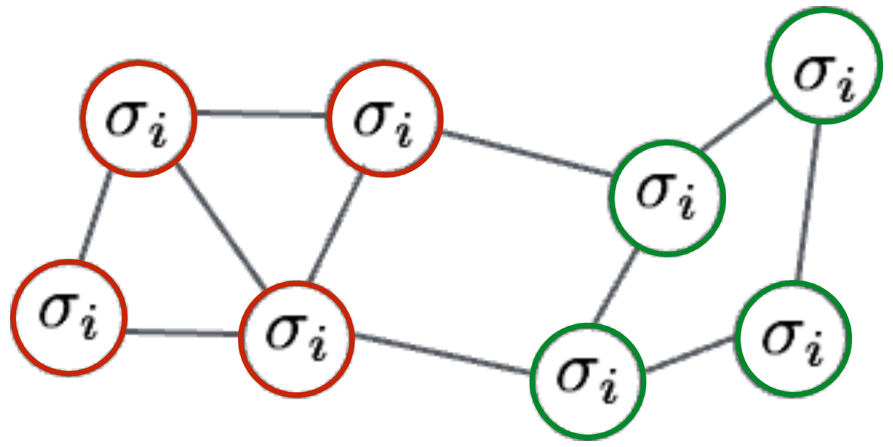

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Generic form: Spin-Glas Hamiltonian

$$\mathcal{H} = \sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

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Adiabatic Quantum Computing

$$H(t) = \boxed{\left(1 - \frac{t}{T}\right) H(0)} + \boxed{\frac{t}{T} H_p}$$

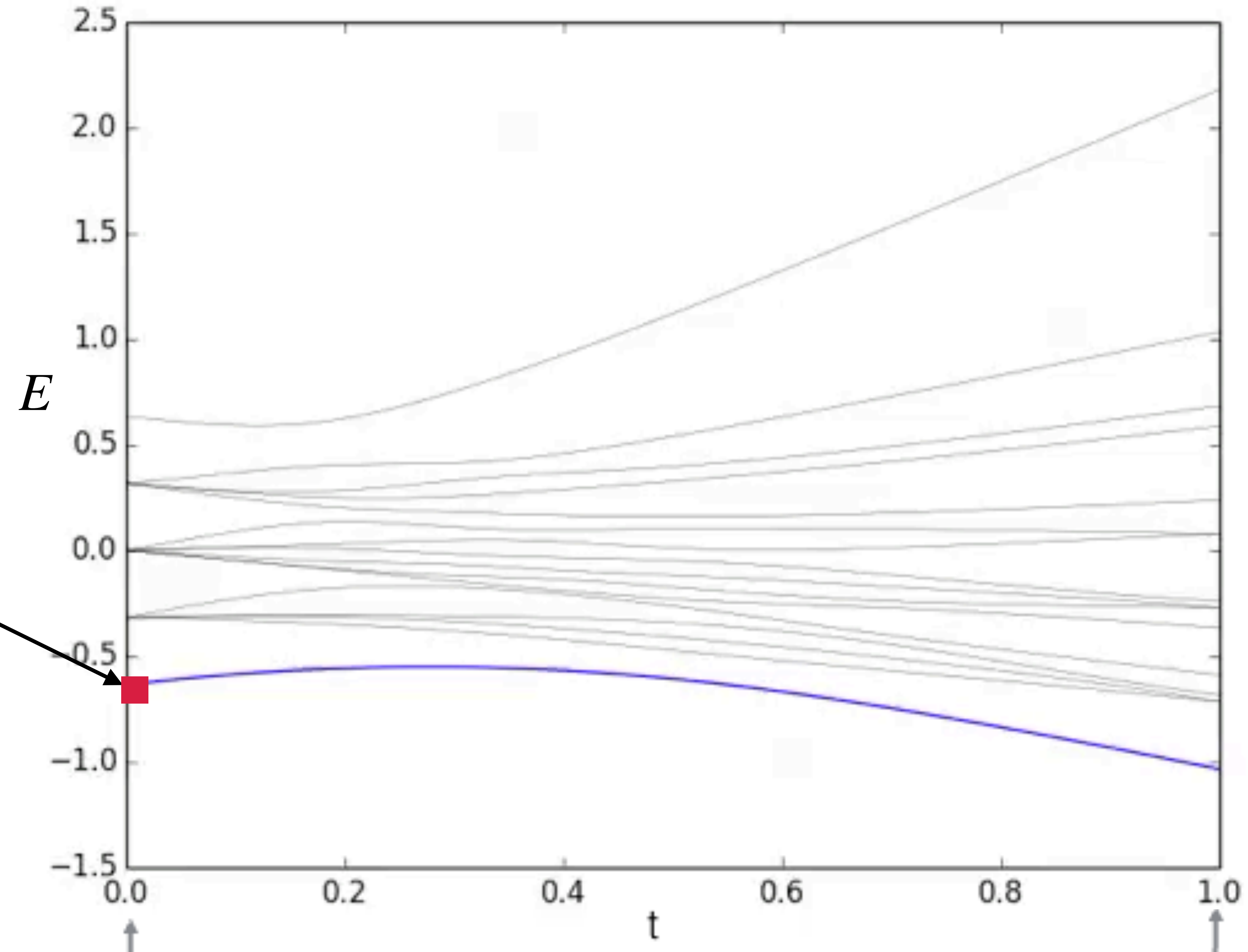
Trivial initial Hamiltonian   Problem-Hamiltonian

1. Prepare the system in $|\psi_0\rangle$, the ground state of H(0)

2. Evolve the system adiabatically ("slowly enough")

$$|\psi(t)\rangle = e^{-i \int_0^t H(t) dt} |\psi_0\rangle$$

3. The quantum adiabatic theorem implies that one will end up in the ground state of H_p

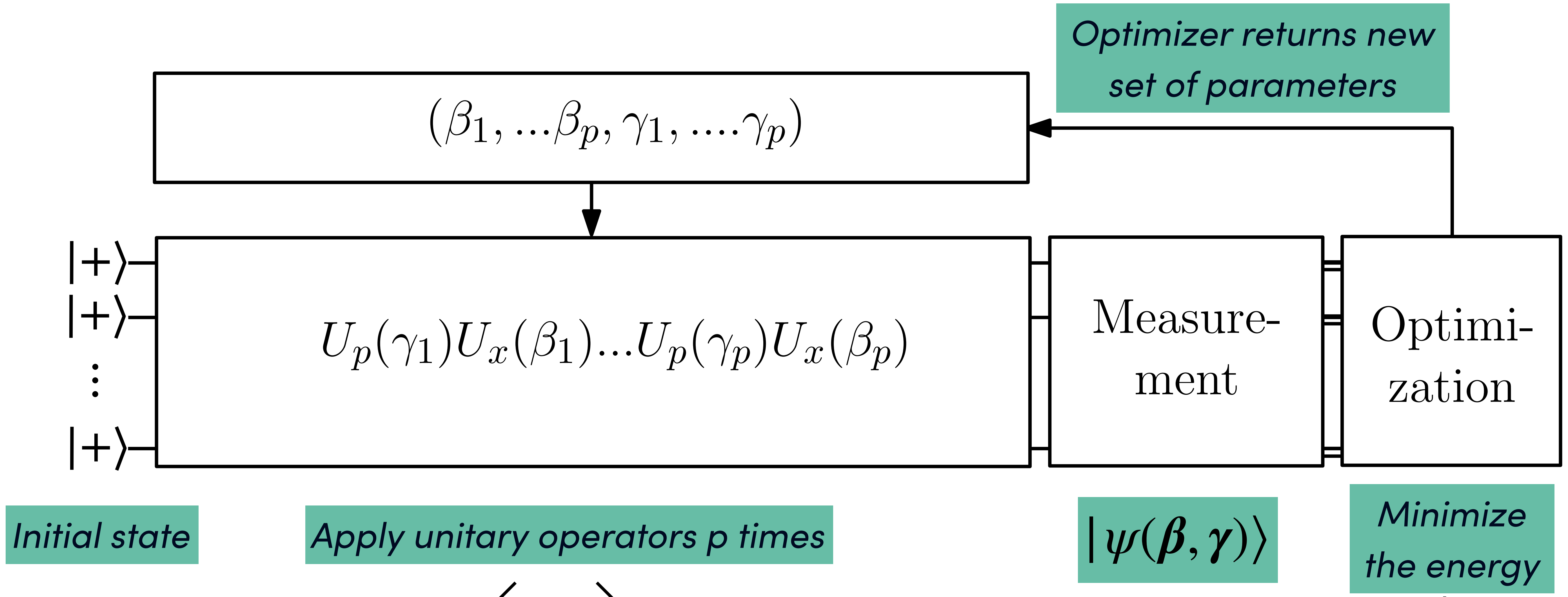


$$H_0 = \sum_i^N \sigma_x^{(i)}$$

$$H_p = \sum_{i<j} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Quantum Approximate Optimization Algorithm (QAOA)

E. Farhi, J. Goldstone, and S. Gutmann, arXiv:1411.4028 (2014)



$$E(\boldsymbol{\beta},\boldsymbol{\gamma}) = \langle\psi(\boldsymbol{\beta},\boldsymbol{\gamma})|H_p|\psi(\boldsymbol{\beta},\boldsymbol{\gamma})\rangle$$

$$U_x(\beta) = \prod_{i=1}^{n} e^{-i\beta\sigma_x^{(i)}}$$

Driver term

$$U_p(\gamma) = e^{-i\gamma H_p}$$

Problem Hamiltonian

$$H_P = \sum_{i=1}^{N}\sum_{j=1}^{i-1} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

Repeat loop till convergence criteria or a certain amount of steps is reached

$p \rightarrow \infty$
Exact solution

Shallow circuit depth

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QUANTUM ECOSYSTEM

Obstacles to Quantum Advantage

Scalability

Current approaches are not scalabe. The bigger the system gets, the more error prone it is.

Qubit Control

The technological bottleneck is not the amount of qubits, but how to control them.

Algorithm

Complexity of the problem is encoded in gates between qubits.

The Parity QC Architecture

ONE CHIP FOR EVERY OPTIMIZATION PROBLEM

- Optimization problems are encoded in local fields only
- Qubit-qubit interactions are independent from the optimization problem

SCALABILITY THROUGH PARALLELIZABILITY

- Parallelizable gates enable constant depth algorithms
- Eliminating crosstalk by executing parallelizable gates

THE ARCHITECTURE INFLUENCES WHOLE QC STACK

- Hardware Design
- Algorithms

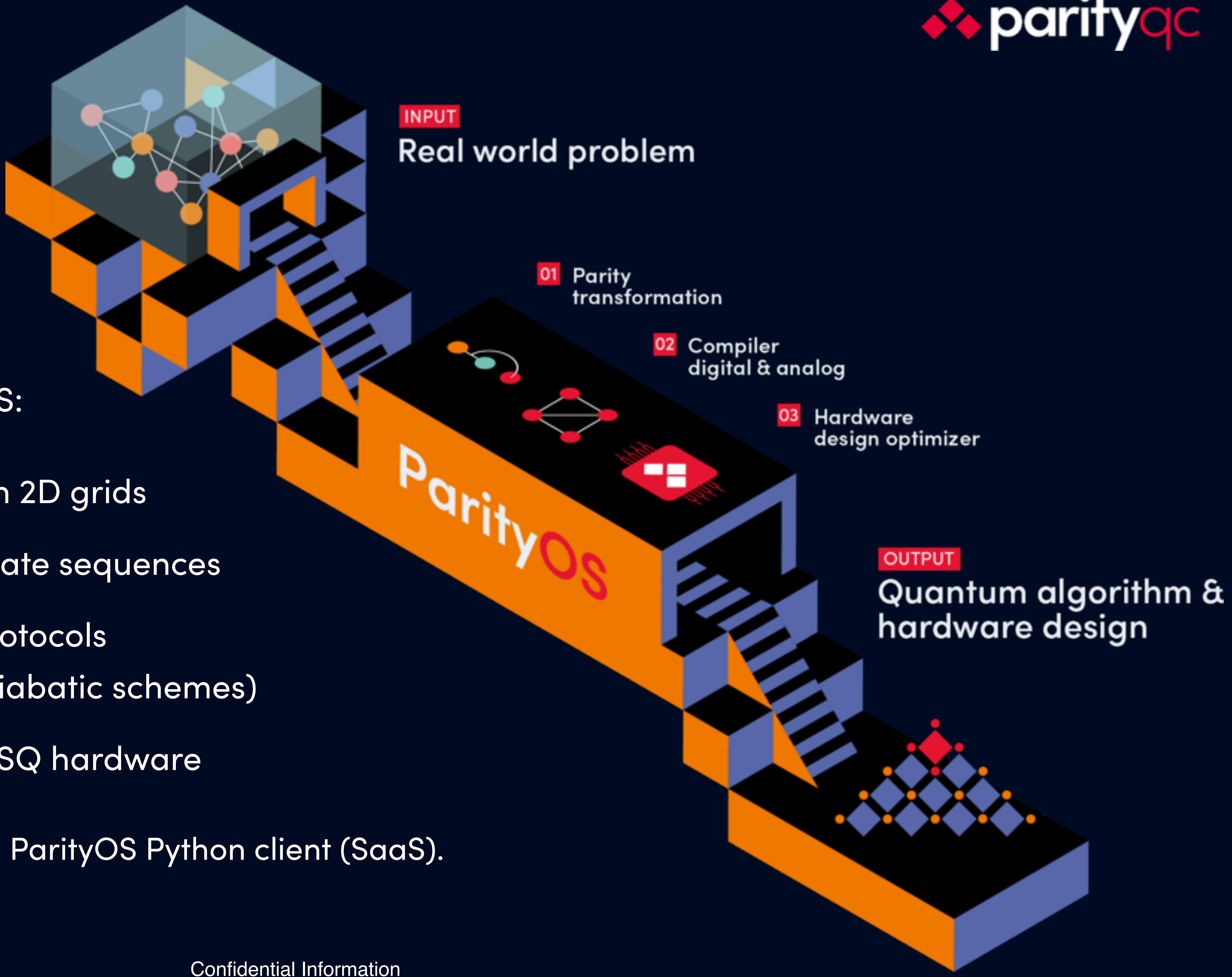
ParityOS

FROM PROBLEM TO HARDWARE

Advantages of the architecture are delivered automatically via ParityOS:

- All interactions are short range on 2D grids
- Natural parallelization of CNOT gate sequences
- Advanced quantum annealing protocols (inhomogeneous driving, non-adiabatic schemes)
- QAOA with error mitigation on NISQ hardware

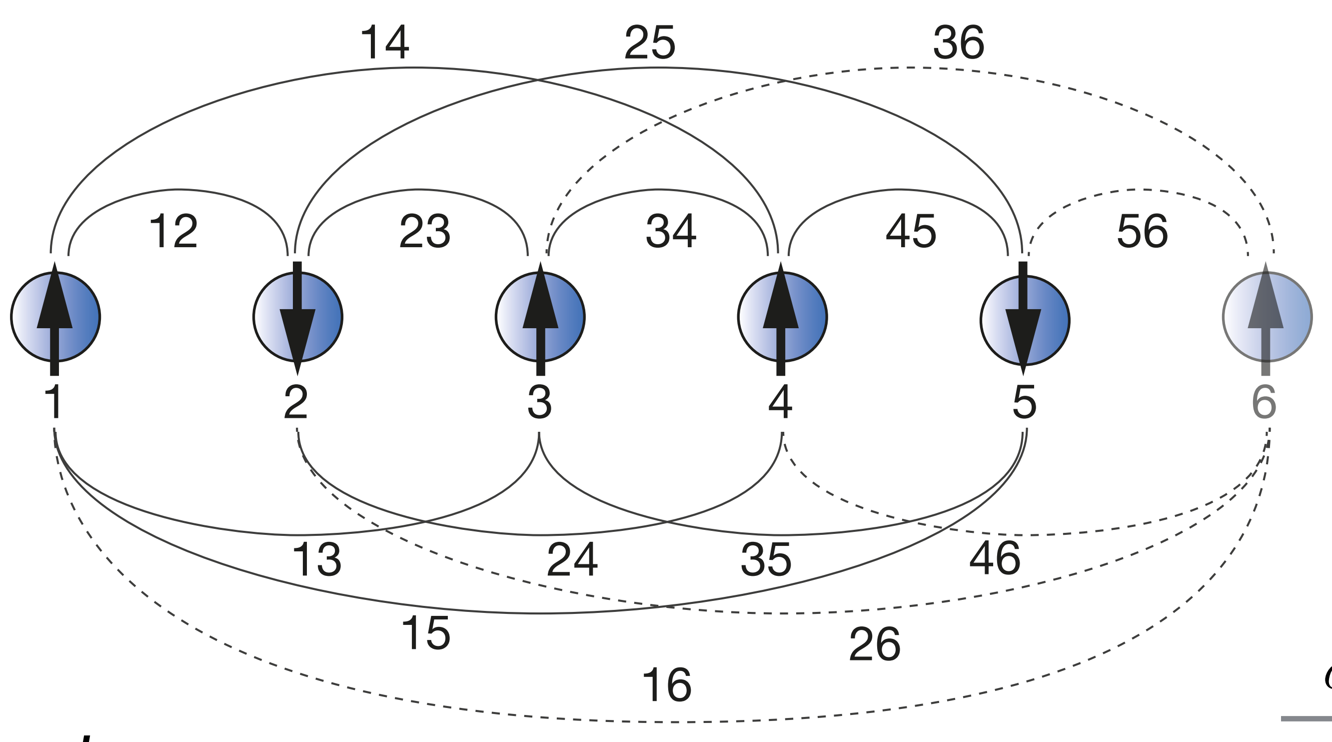
Easy access via the Cloud through the ParityOS Python client (SaaS).



PARITY QUANTUM OPTIMIZATION

The Parity Transformation

W. Lechner., P. Hauke, and P. Zoller. *Sci. Adv.* ***1**, e1500838* (2015).

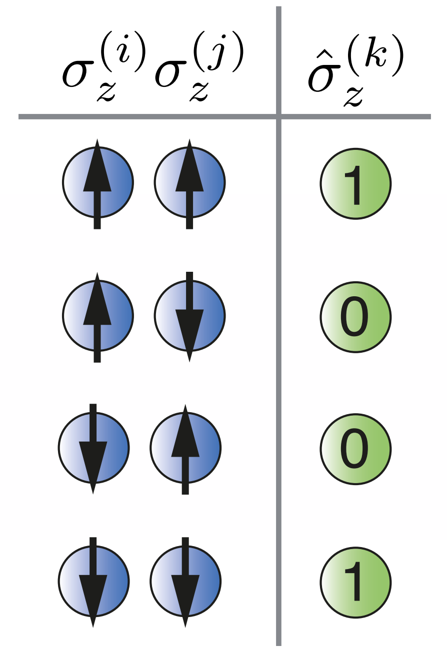


$$K = \frac{N(N-1)}{2}$$ *degrees of freedom*

$K - N + 1$ *constraints*

N *degrees of freedom*

$$H = \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_{z,L}^{(i)} \sigma_{z,L}^{(j)}$$



$$H = \sum_{i=1}^{K} J_i \sigma_z^{(i)}$$

$$C_l = -c_l \sigma_z^{(l_1)} \sigma_z^{(l_2)} \sigma_z^{(l_3)} \sigma_z^{(l_4)}$$

- NP-hard optimization problems can be formulated as Ising Spin-glasses.

- This formulation requires high connectivity.

What are these constraints?

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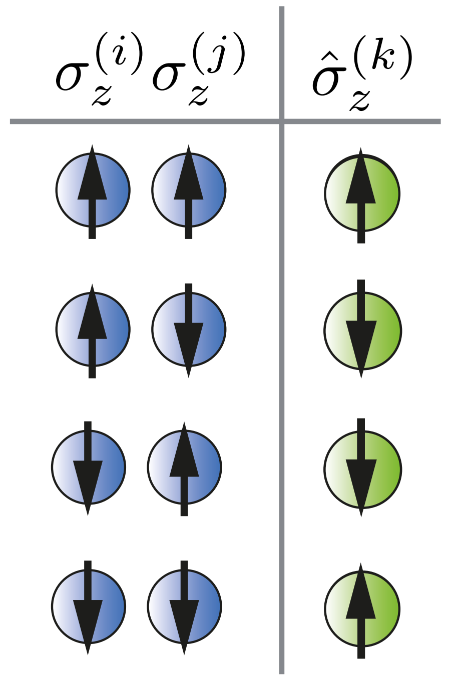
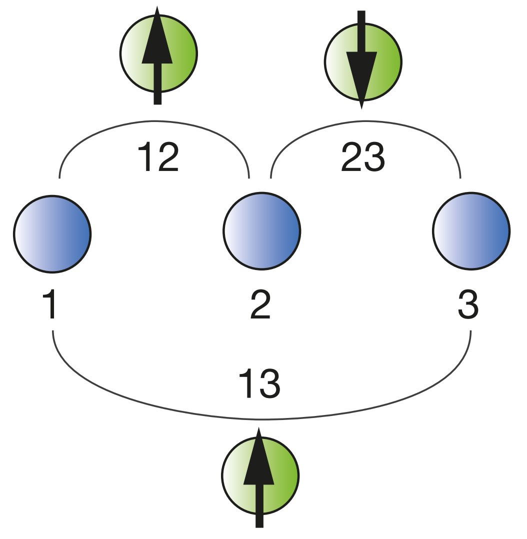
Conditions on closed loops





$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
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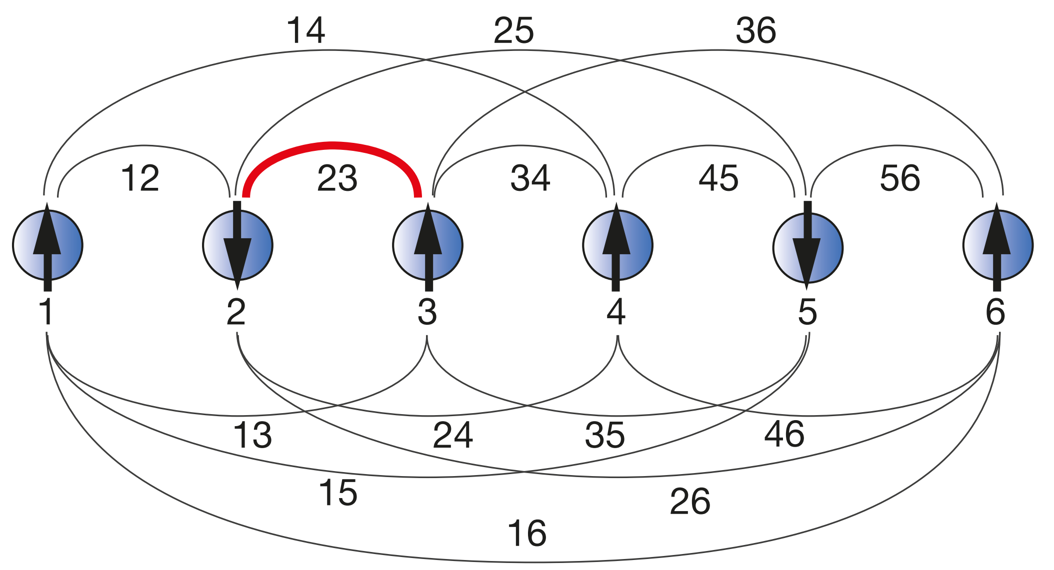
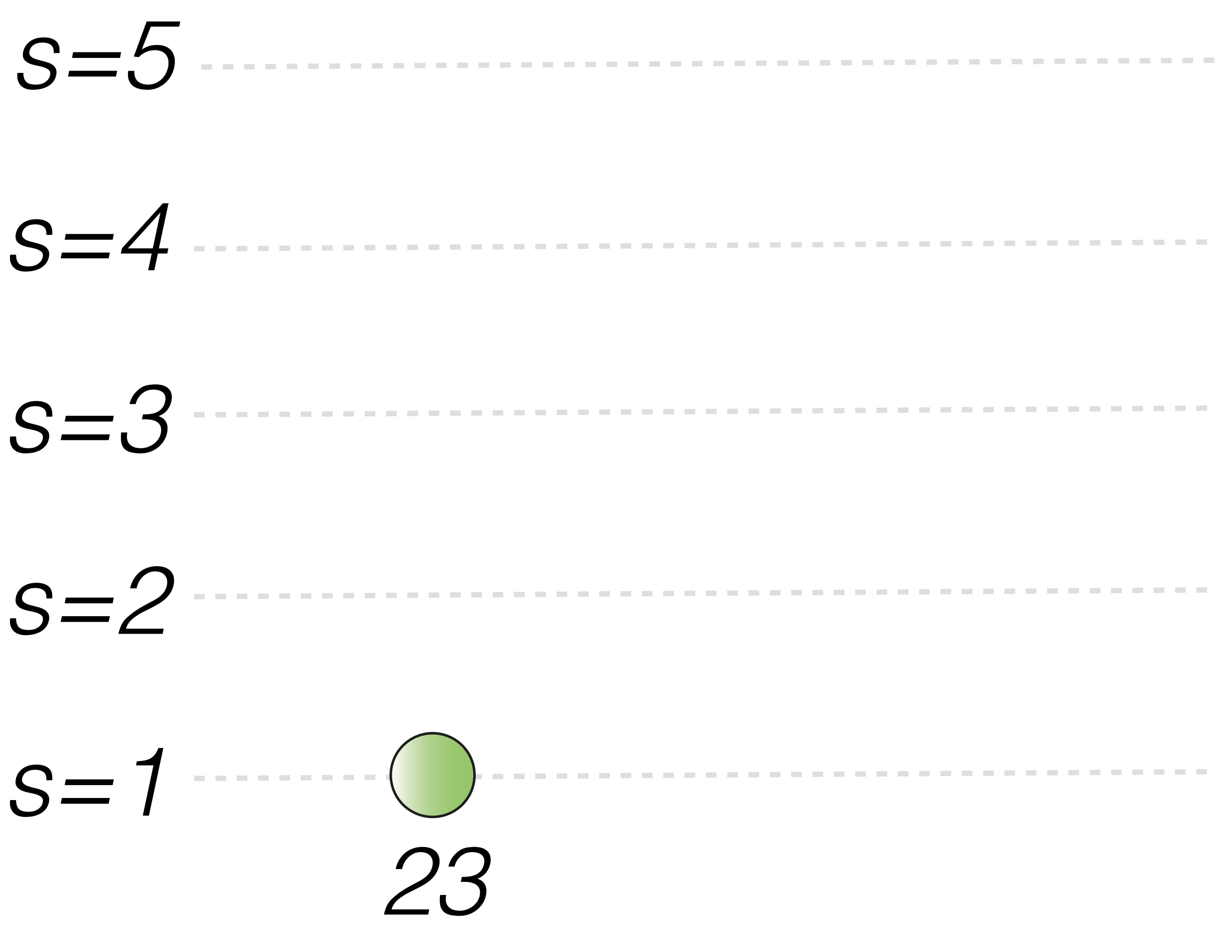
1	1	1	0	0	0
1	1	0	0	1	1
	…			…	
			0	0	1





In **each closed loop**, the number of **spins down** has to be an **even** number or 0.

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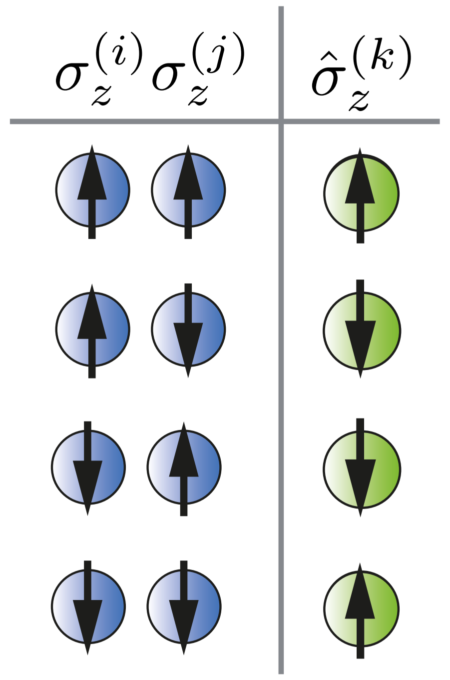

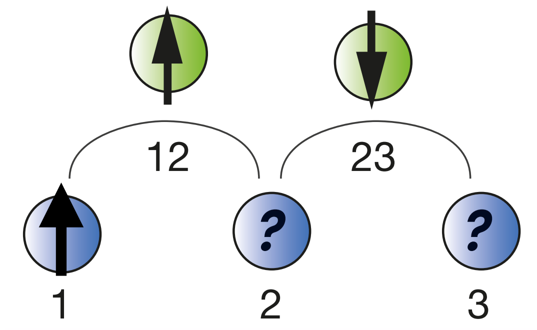

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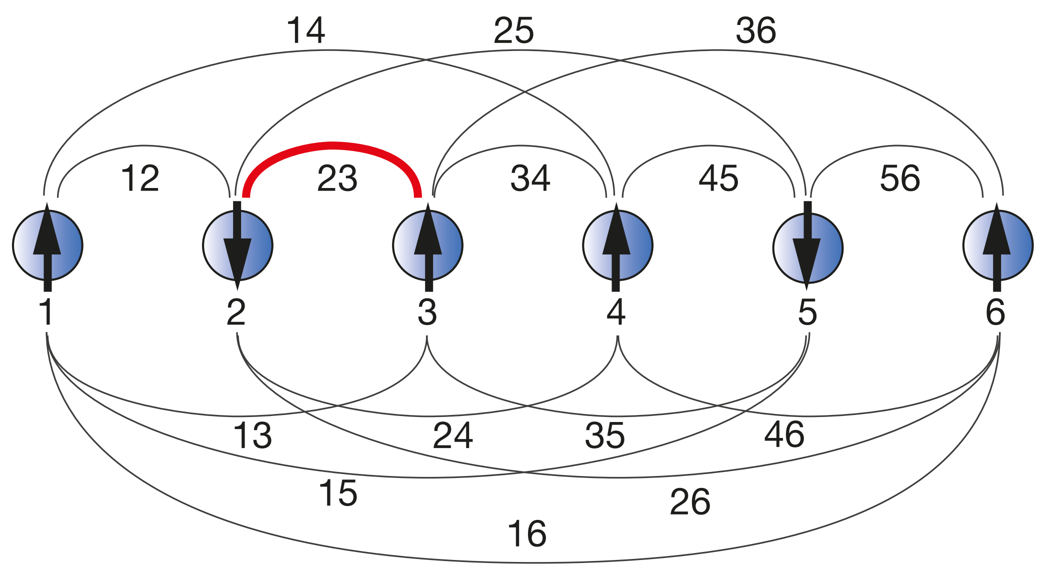

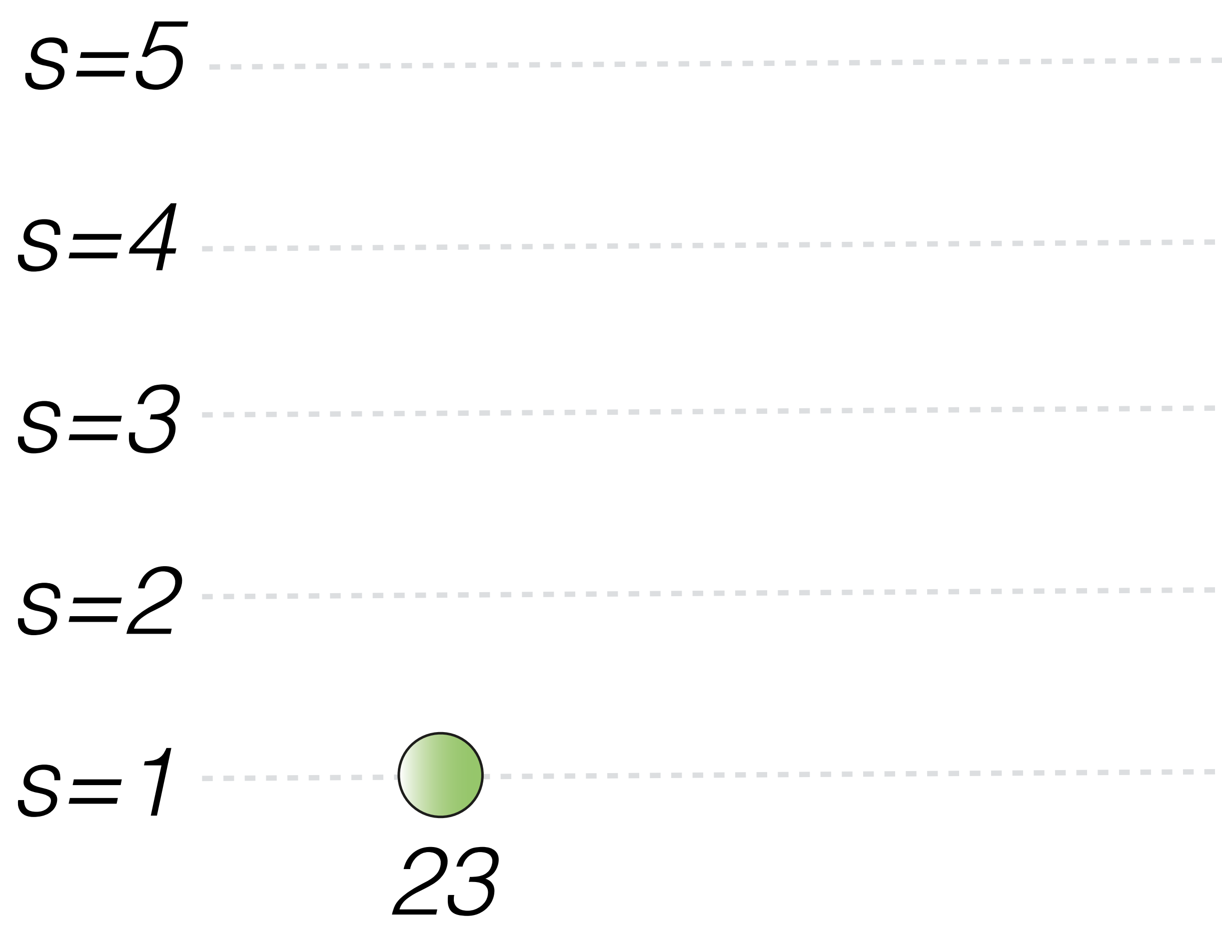

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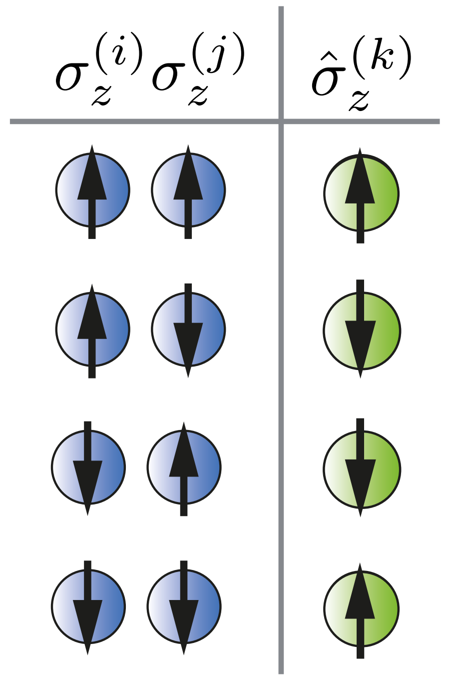

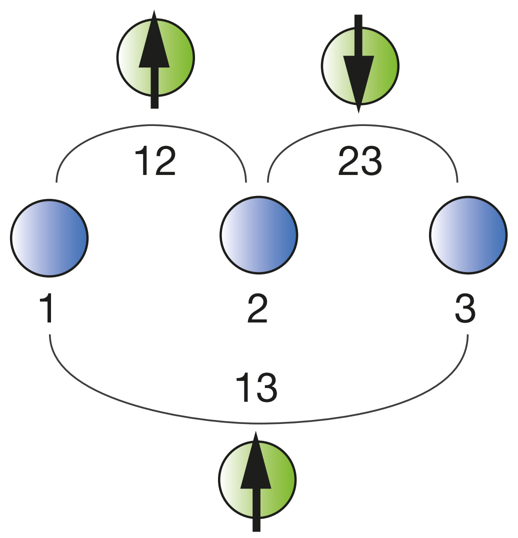

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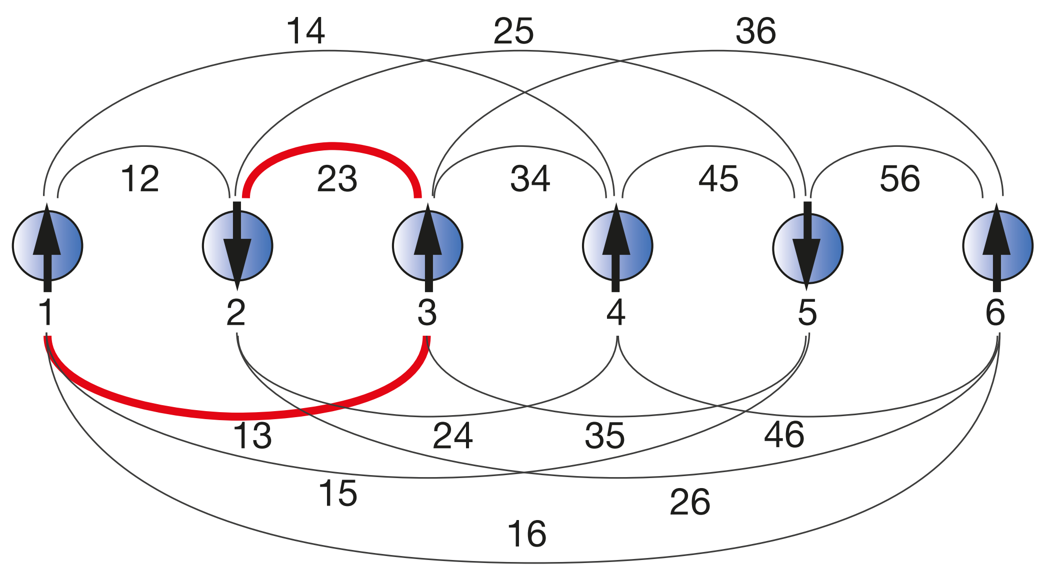

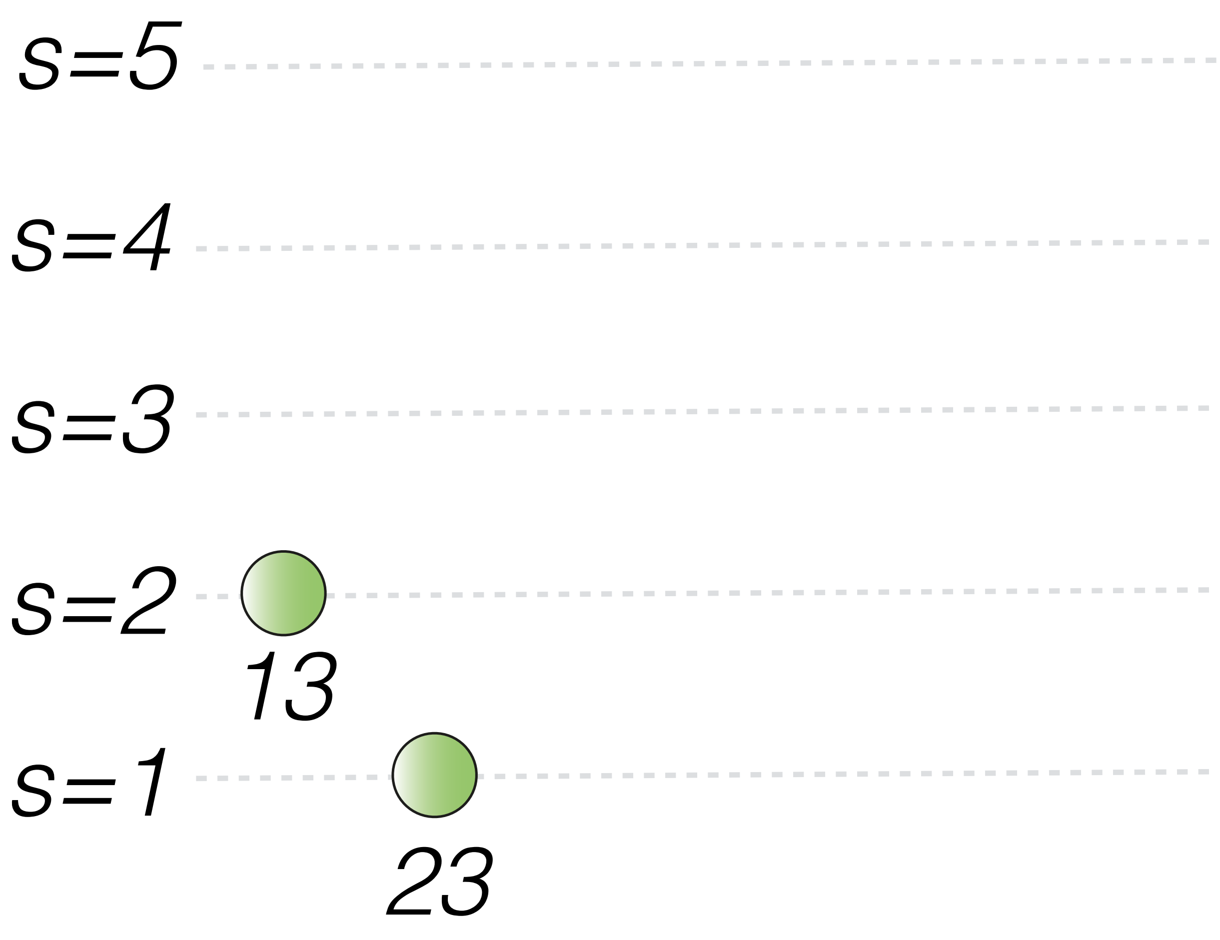

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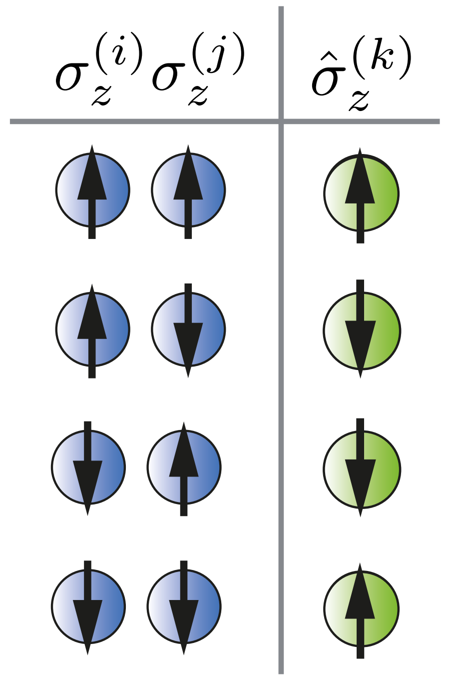

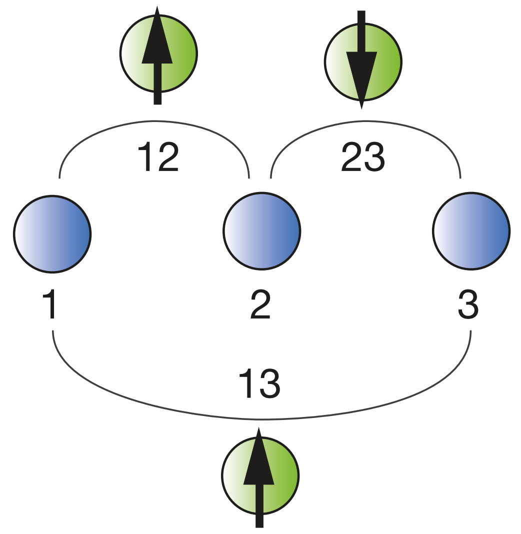

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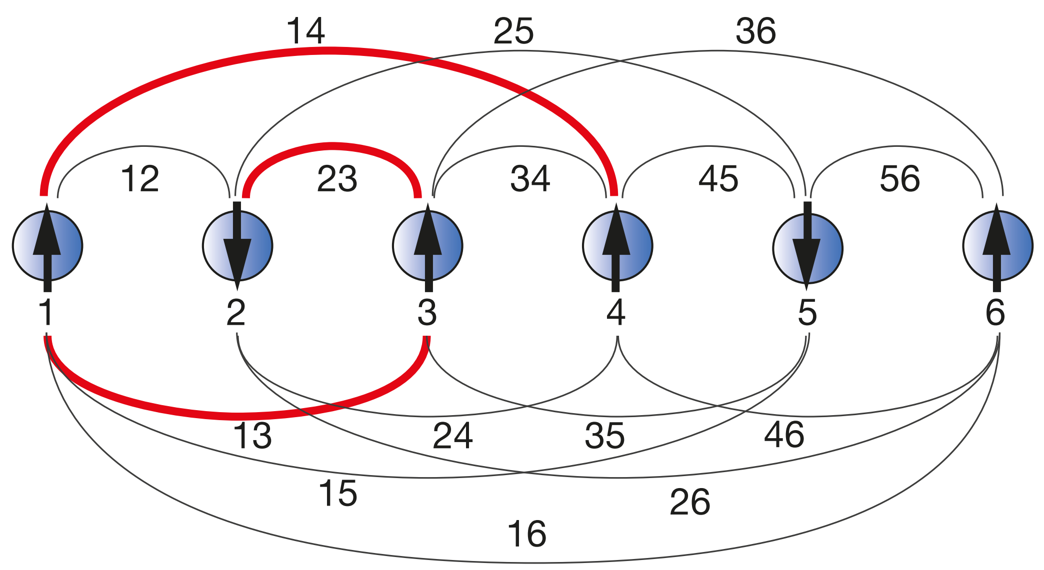

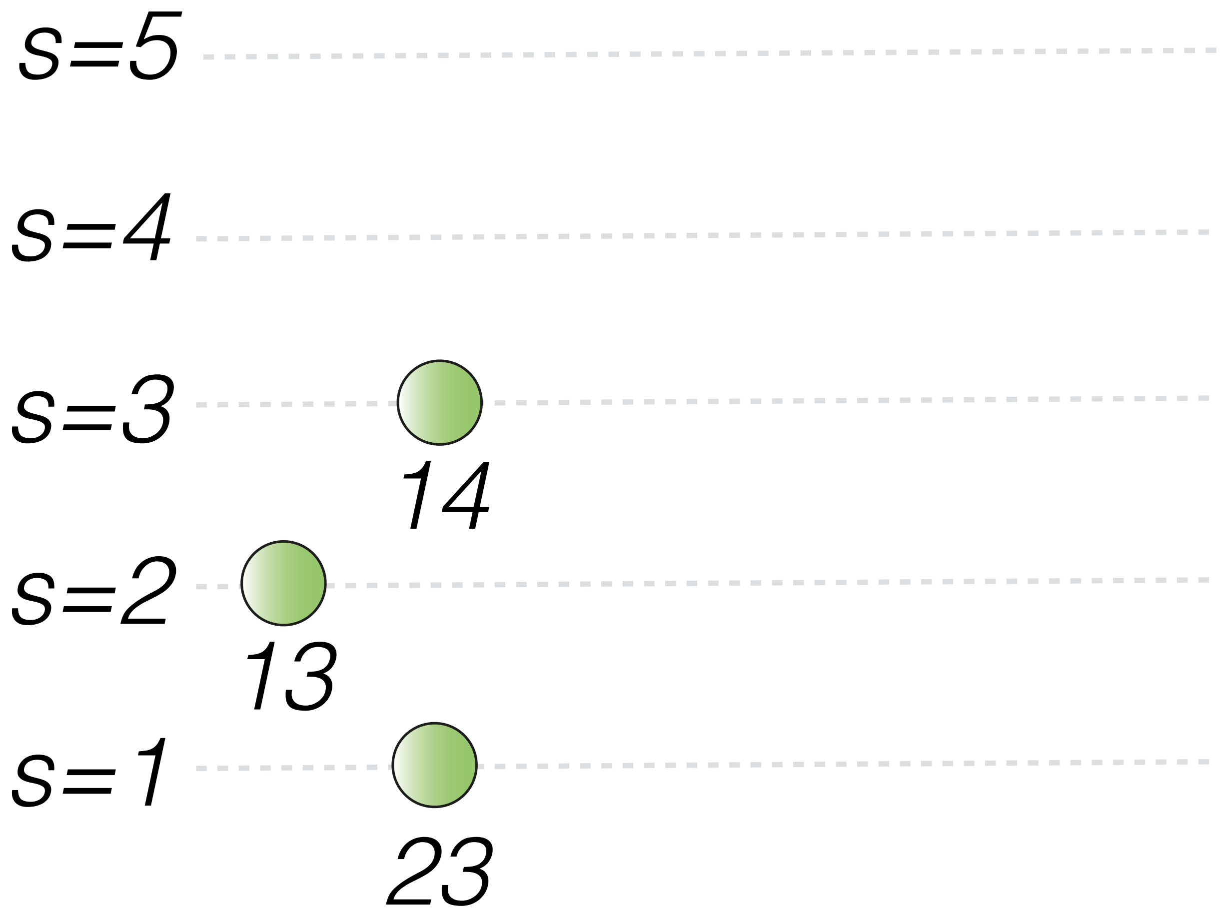

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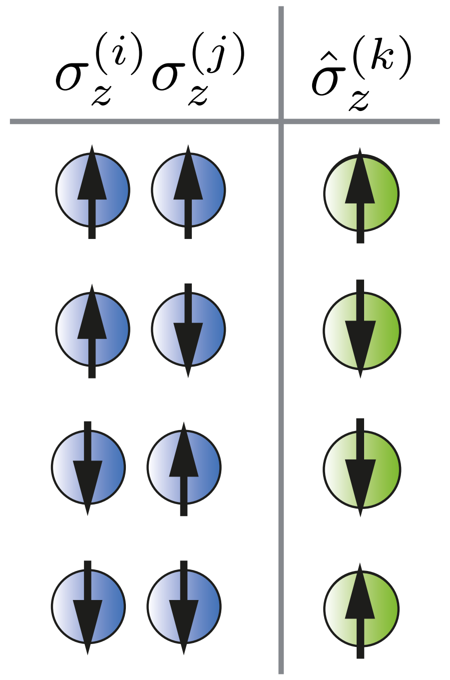

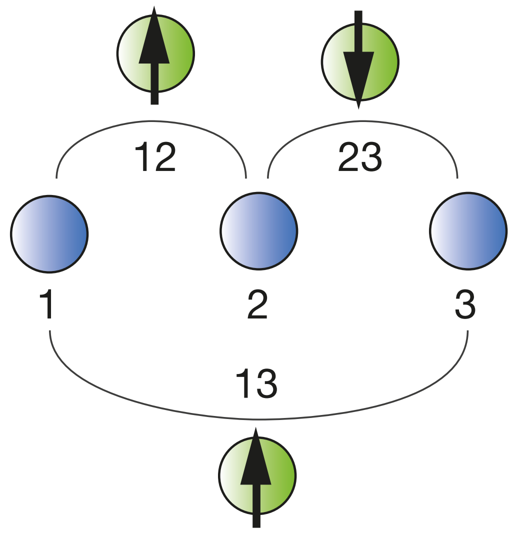

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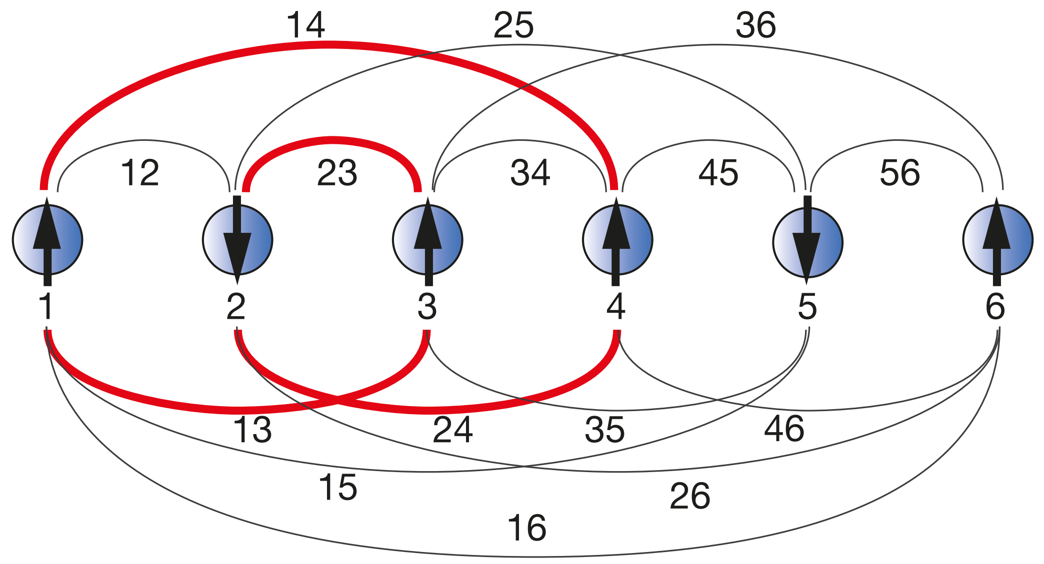

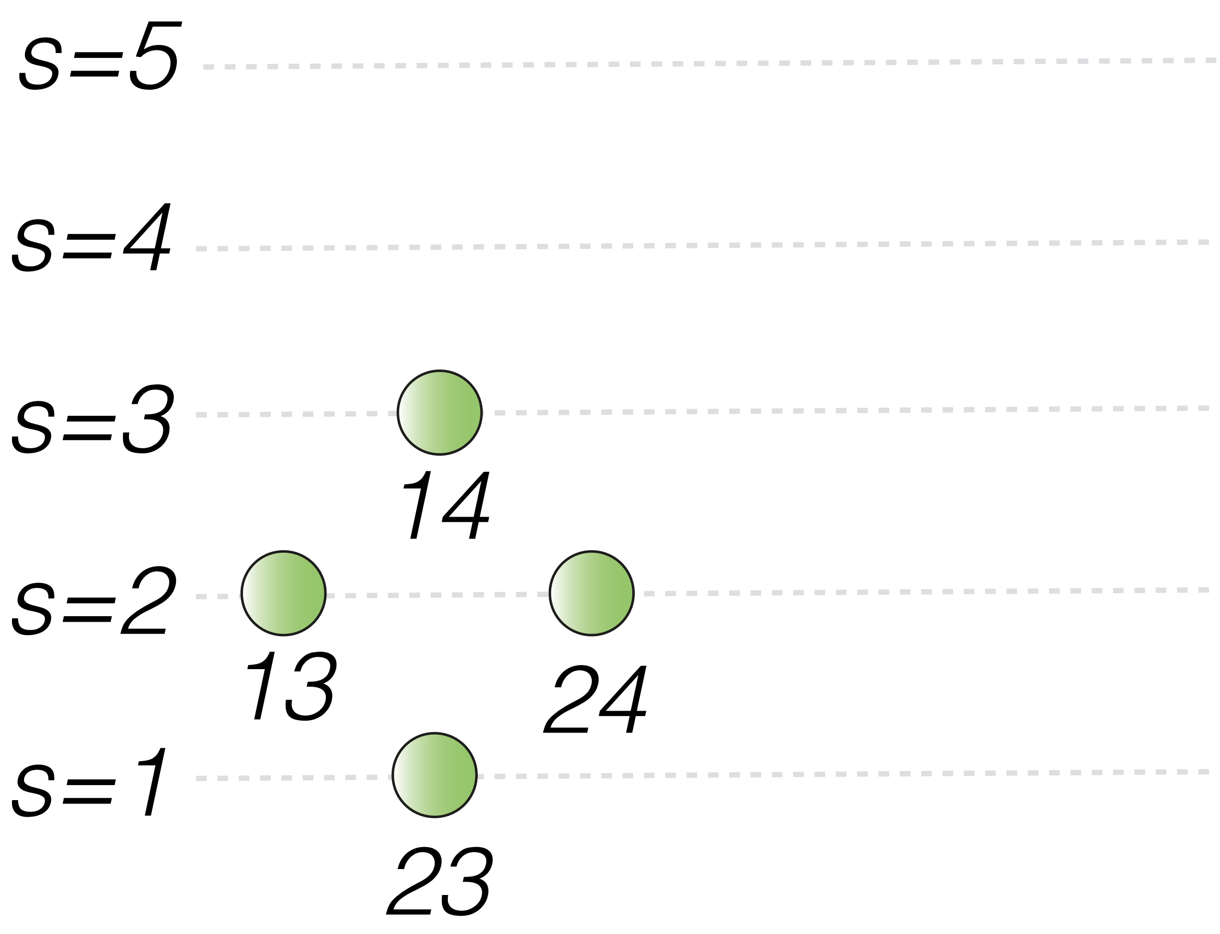

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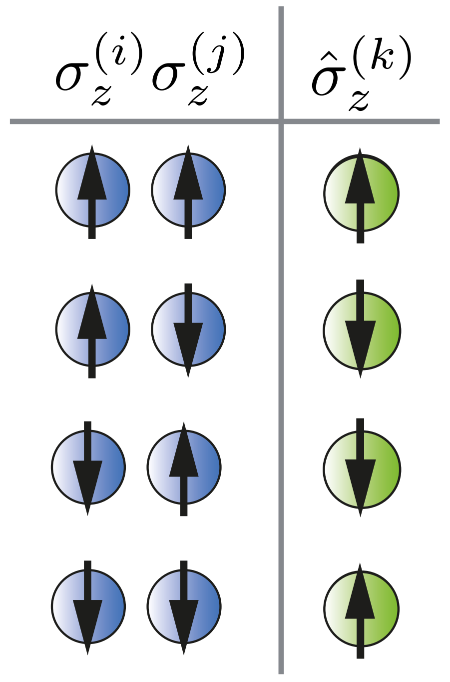

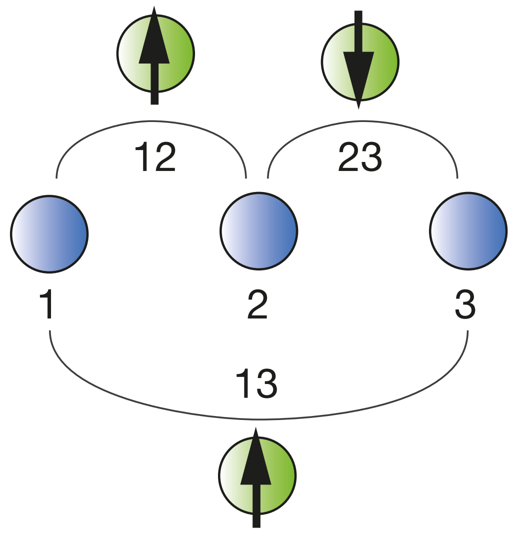

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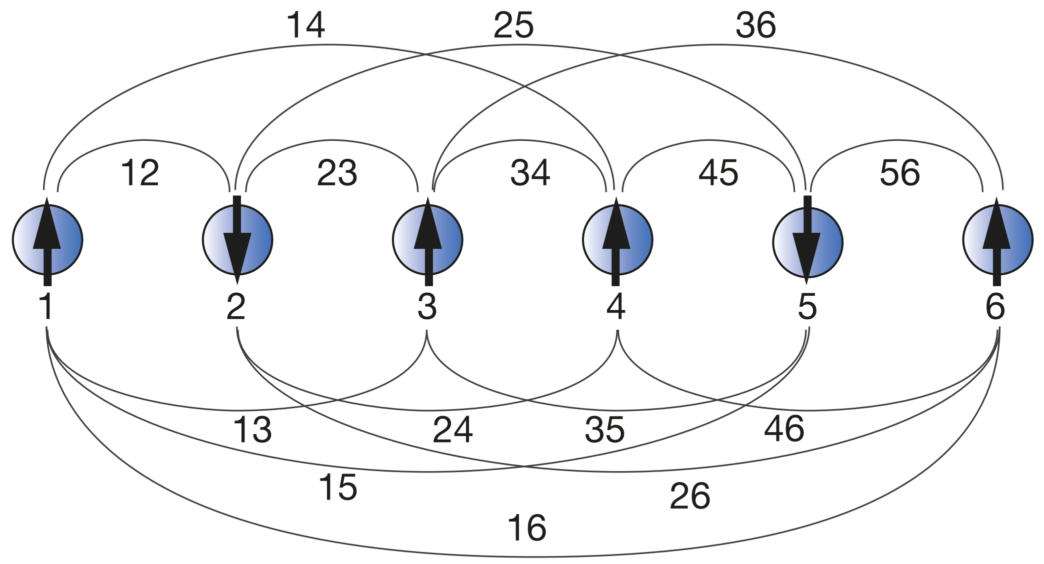

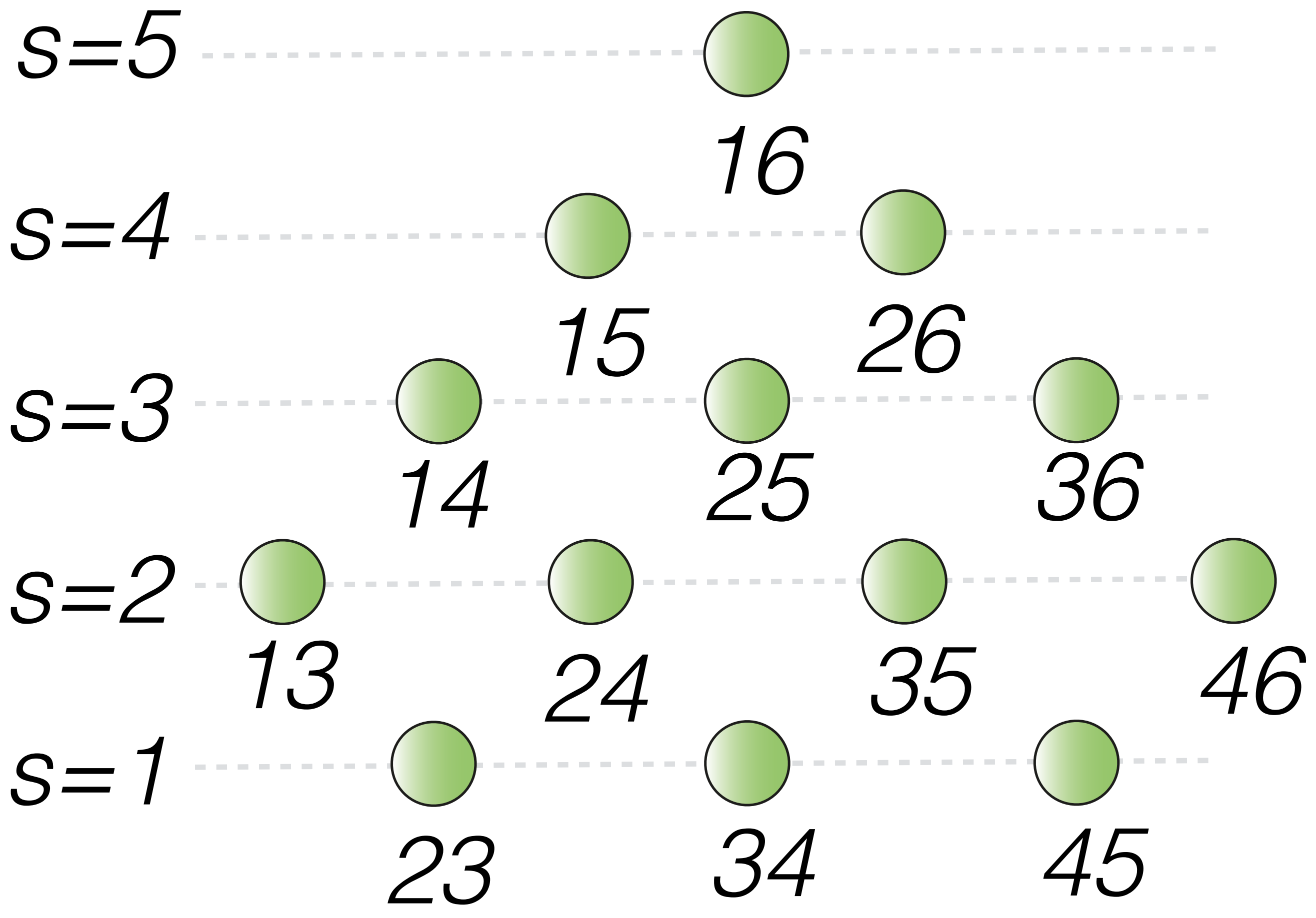

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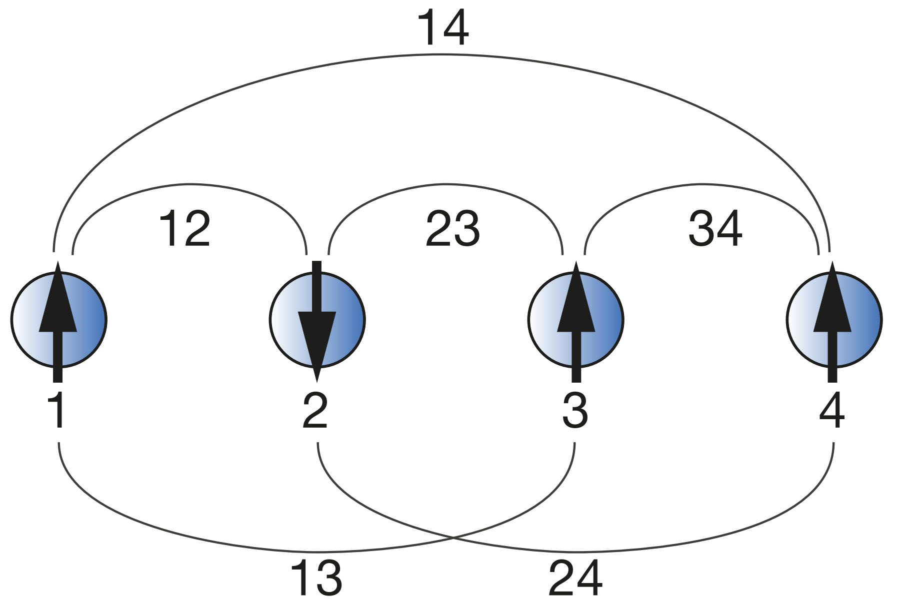
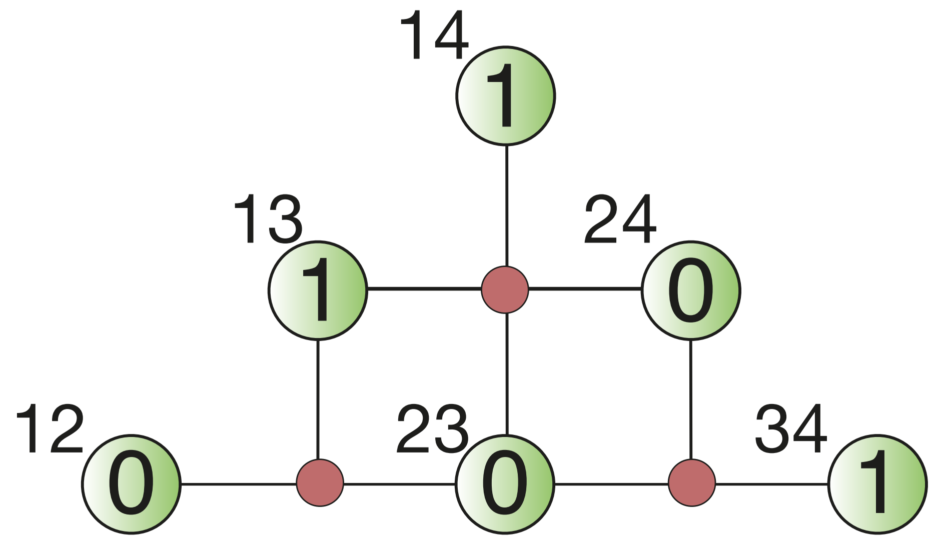
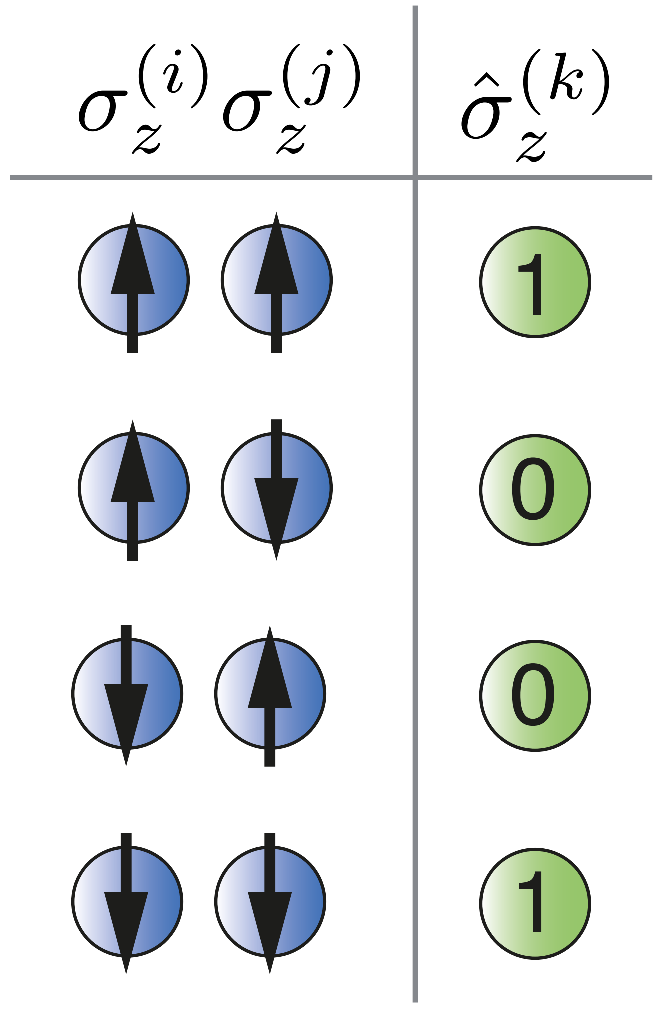
LHZ Architecture: Summary

W. Lechner., P. Hauke, and P. Zoller. *Sci. Adv.* ***1****, e1500838* (2015).







$$H_P = \sum_{i=1}^{N}\sum_{j=1}^{i-1} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

$$H_{\mathrm{LHZ}} = \boxed{\sum_{i=1}^{K} J_i\sigma_z^{(i)}} + \boxed{\sum_{l=1}^{K-N+1} - c_l\sigma_z^{(l_1)}\sigma_z^{(l_2)}\sigma_z^{(l_3)}[\sigma_z^{(l_4)}]}$$

- The LHZ architecture maps an all-to-all connected spin model to a spin model with only **quasi-local** interactions.
- The physical qubits encode the **parity** of the logical qubits.
- **No long-range interactions** but only local 3- or 4-body couplings are necessary.
- The parity architecture requires nearest-neighbour interactions on a square lattice, **regardless of the qubit platform.**

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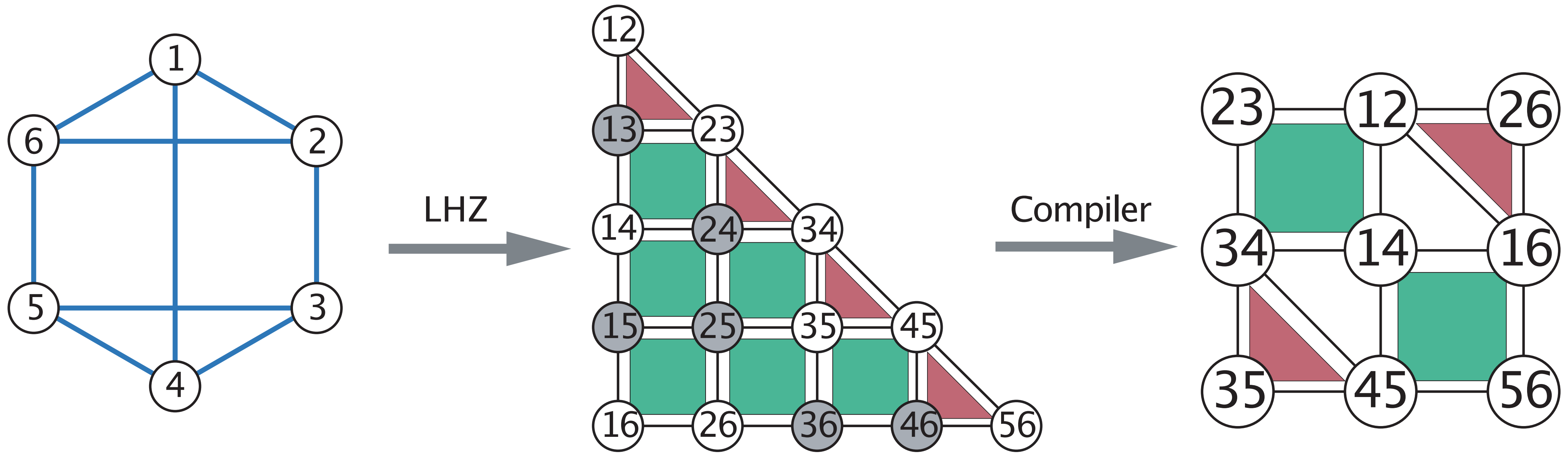
Parity Compilation

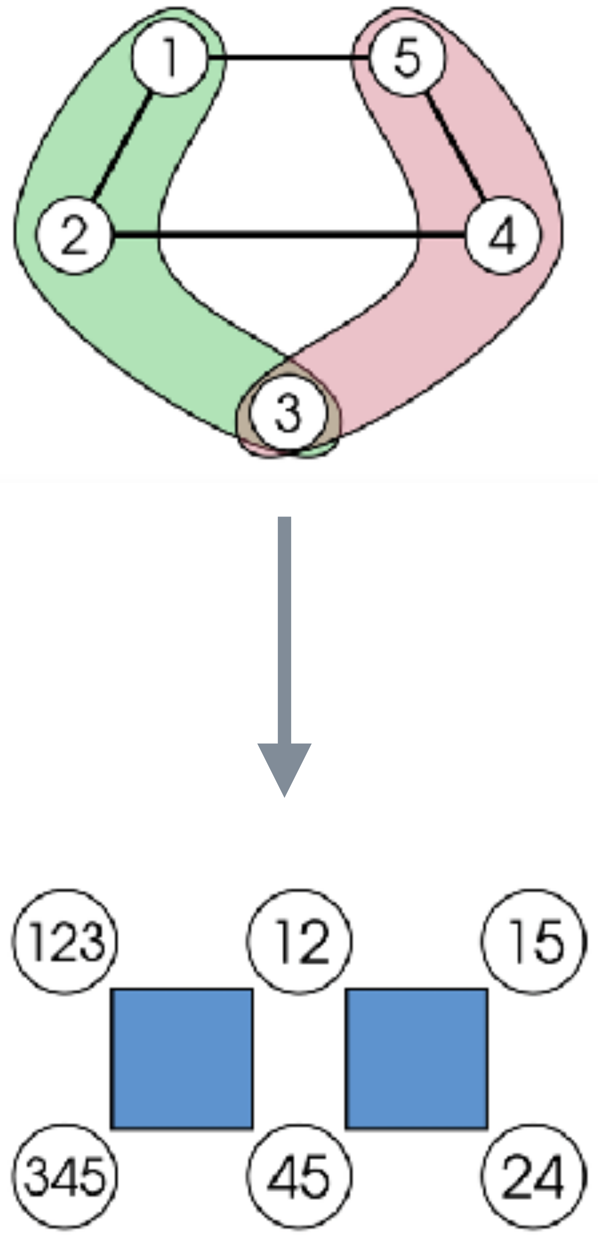
K. Ender et al., Quantum **7**, 950 (2023)
R. ter Hoeven, A. Messinger, and W. Lechner, Phys. Rev. A **108**, 042606 (2023)
R. ter Hoeven, B. E. Niehoff, S. Kale, , and W. Lechner, arXiv:2307.10626 (2023)

What if K is less than quadratic?



What if there are higher-order terms?

$$H = \sigma_z^1\sigma_z^2 + \sigma_z^2\sigma_z^4 + \sigma_z^4\sigma_z^5 + \sigma_z^1\sigma_z^5 + \sigma_z^1\sigma_z^2\sigma_z^3 = \sigma_z^3\sigma_z^4\sigma_z^5$$



Parity Quantum Approximate Optimization

W. Lechner, *IEEE Transactions on Quantum Engineering 1, 1 (2020)*

$$H = \sum_{i=1}^{K} J_i \sigma_z^{(i)} + \sum_{l=1}^{K-N+1} c_l \sigma_z^{(l_1)} \sigma_z^{(l_2)} \sigma_z^{(l_3)} \sigma_z^{(l_4)}$$

QAOA:

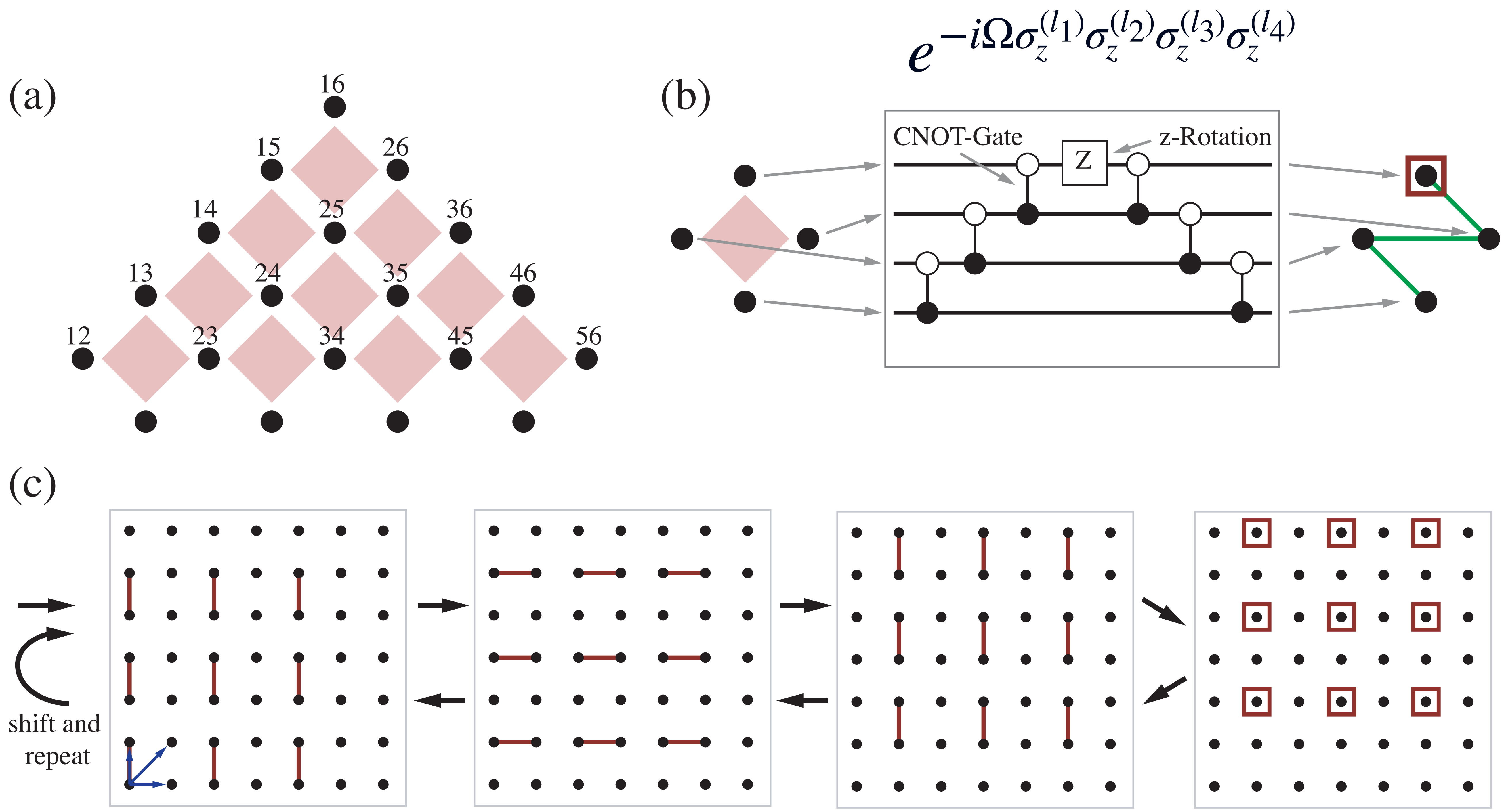
1) *Prepare quantum state*
2) *Calculate energy expectation value* $\langle H \rangle$
3) *Pass it to a classical optimizer*
4) *Update QAOA parameters*
5) *If stopping criterion not reached, go to step 1*

$$|\psi\rangle = \prod_{j=1}^{p} U_X(\beta_j) U_C(\Omega_j) U_Z(\gamma_j) |+\rangle$$

$$U_Z(\gamma) = e^{-i\gamma J_i \sigma_z^{(i)}}$$

$$U_X(\beta) = e^{-i\beta \sigma_x^{(i)}}$$

$$U_C(\Omega) = e^{-i\Omega \sigma_z^{(l_1)} \sigma_z^{(l_2)} \sigma_z^{(l_3)} \sigma_z^{(l_4)}}$$



We can perform a QAOA *layer* in **constant circuit depth**, independent of the system size!

Improvement: J. Unger, A. Messinger, B. E. Niehoff, MF, and W. Lechner, arXiv:2211:1128 (2022).

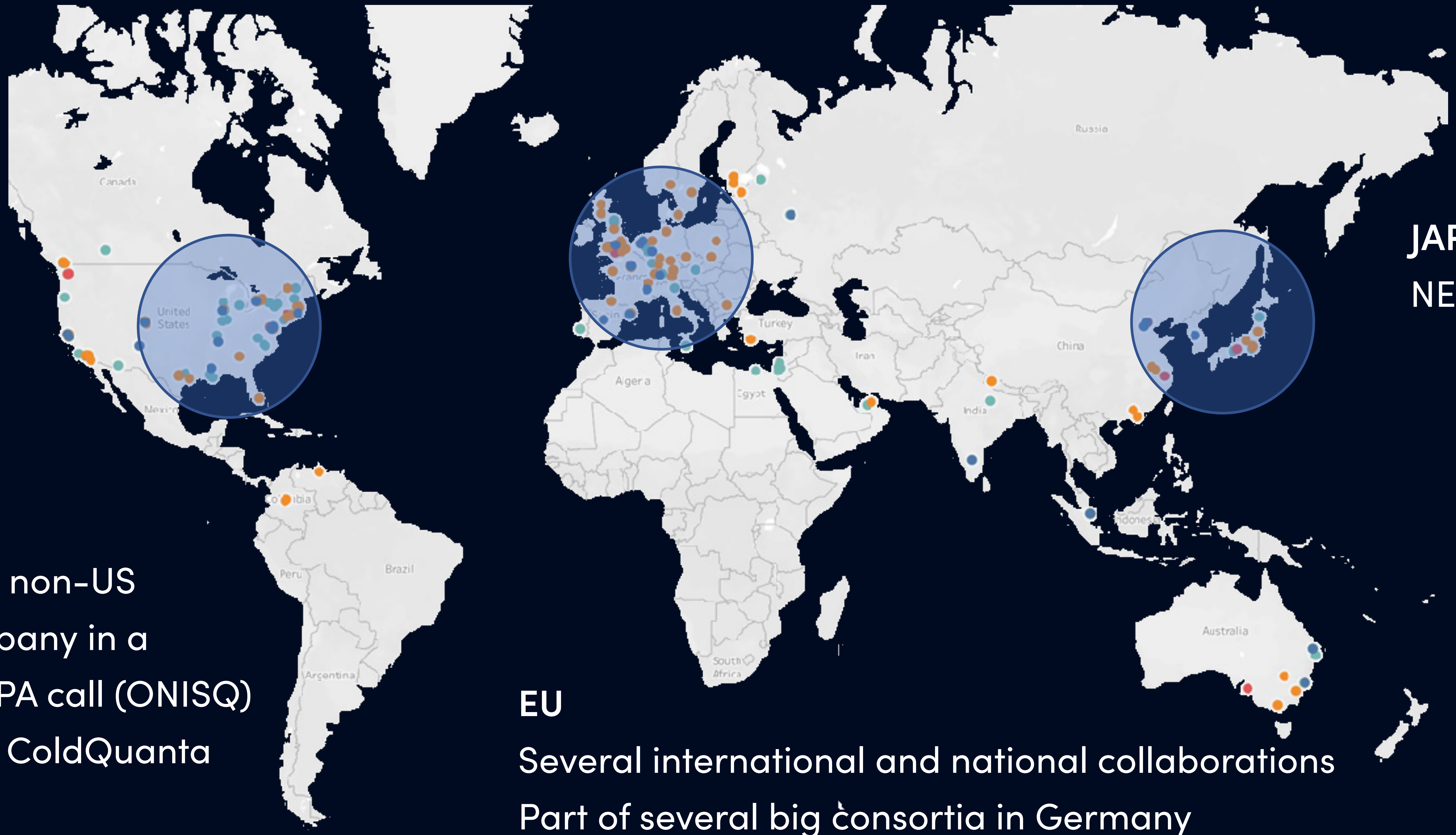
Quantum Optimization

Parity Quantum Computing

ParityQC - The Company

parityqc

Collaborations & Customers Worldwide



JAPAN
NEC as first customer

USA
Only non-US company in a DARPA call (ONISQ)
With ColdQuanta

EU
Several international and national collaborations
Part of several big consortia in Germany
Part of QuIC, QBN, Fraunhofer Advisory Board
Awarded DLR contract for Ion trap quantum computer

Already built and running on different hardware platforms:

- 3 continents
- Digital and analog
- Superconducting (KPO) and atom-based

Company


Magdalena Hauser Wolfgang Lechner

Co-Founder, Co-CEOs

Hermann Hauser

Investor

Co-Founder of Acorn and ARM

Co-Founder Amadeus Capital

100+ DeepTech investments

- Founded in 2020
- Spin-off from the University of Innsbruck and the Austrian Academy of Sciences
- Based in Austria and Germany
- Worldwide patented fundamental patent portfolio

ParityQC team

35 PhDs, Post-Docs, quantum computing engineers from all around the world

Research Group

17 PhDs and Post-Docs working full-time on fundamental research around the ParityQC architecture

The technical team

OS

- Software developers
- Quantum Engineers

Compiler

- Computer Scientists
- Programmers
- Mathematicians

Co-Design

- (Quantum) physicists (theory)
- Mathematicians

Hardware Integration

- (Quantum) physicists (theory-affine hardware or theory)

Use-Cases

- Mathematicians
- Computer scientists
- Quantum optimization experts

JOIN US

ParityQC Team

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