Dynamic Hedging as Market Makers: Delta vs Delta-Gamma under the Black-Scholes Model

Aidan Richer

June 16, 2025

Abstract

The purpose of this paper is to investigate the performance of delta and delta-gamma dynamic hedging strategies within the Black-Scholes framework from the perspective of a market maker. Using geometric Brownian motion, we simulate asset price paths and implement discrete-time hedging of a written European put option. Our framework tracks the mark-to-market profit and loss (PnL) of each instrument in the portfolio, including the underlying shares, option(s), and cash holdings, at every time step. Hedging adjustments for both delta and gamma are made daily in various volatility regimes, accounting for different transaction costs and underlying asset drift, to better evaluate risk reduction performance. The results indicate that delta-gamma hedging provides stronger control over portfolio convexity and produces a more concentrated PnL distribution, whereas delta hedging results in a more dispersed distribution and is less effective at managing nonlinear risks.

1 Introduction

Modern pricing theory follows the assumption the price of a derivative is the cost of assembling a self-financing portfolio of simpler, liquid securities that replicate its payoff. The framework defined as the Black-Scholes model defines the price of an option as the cost of assembling a self-financing portfolio that replicates the pay-off of the option. The Black-Scholes approach assumes that market makers operate in a competitive environment and seek to maximize profit while hedging the risk of their option positions. To minimize portfolio variance, market makers will dynamically adjust their holdings in the underlying asset and other financial instruments, forming hedging strategies that approximate the option's behavior over time. Delta hedging, which neutralizes first-order sensitivity to changes in the underlying price, is widely used in practice but remains exposed to second-order risks—specifically, convexity or gamma risk—as market conditions evolve. Delta-gamma hedging incorporates an additional instrument, in this case an at-the-money (ATM) call option, to offset curvature risk and provide more stable protection against large movements in the underlying. This paper implements both strategies in identical market conditions to evaluate their effectiveness in reducing portfolio risk under varying conditions of volatility, drift, and transaction costs. Our simulation framework tracks the mark-to-market profit and loss (PnL) of each portfolio component at every time step, capturing changes in the values of the underlying, derivative instruments, and cash positions. We assess hedging performance by analyzing the shape and spread of resulting PnL distributions, aiming to better understand the trade-offs between precision, cost, and resilience to nonlinear market behavior.

2 Methodology

This section outlines the assumptions, models, and implementation steps to simulate delta and delta-gamma hedging strategies under the Black-Scholes framework.

2.1 Black-Scholes Framework and Assumptions

I) Underlying Asset Price Paths

We assume that the underlying asset follows a geometric Brownian motion (GBM), which is described by the differential equation:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t \tag{1}$$

In this process, the asset price at time t is S_t, the drift (annualized return) is μ, the volatility is σ, and W_t is a standard Wiener process.

II) Option Pricing

Using strike price K, we can conclude that the payoff of a European option is based on where the asset price S_t is at maturity. The payoff of a call option is given by $C = max(S_t - K, 0)$ whereas the payoff for a put option is reversed and given by $P = max(K - S_t, 0)$. The price of a European call option under the Black-Scholes formula for a non-dividend paying stock is given by:

$$C(S,t) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \tag{2}$$

The price of a European put option for a non-dividend paying stock is:

$$P(S,t) = Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1) \tag{3}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{5}$$

and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

As stated earlier, the payoff of an option is based on the relationship between the strike price K and the asset price S_t. For the holder of a put option, profit is realized when the underlying asset S decreases in value, while the holder of a call option benefits when S increases. From the perspective of the writer (seller) of the option, the payoff is the negative of the holder's payoff. That is, the writer of a call option profits when the underlying asset remains below the strike price (since the option expires worthless), and incurs a loss when the asset price rises above K. Similarly, the writer of a put option benefits when the asset price stays above the strike price, and loses money when the price falls below K.

Figure 1 illustrates the profit diagrams for European call and put options.

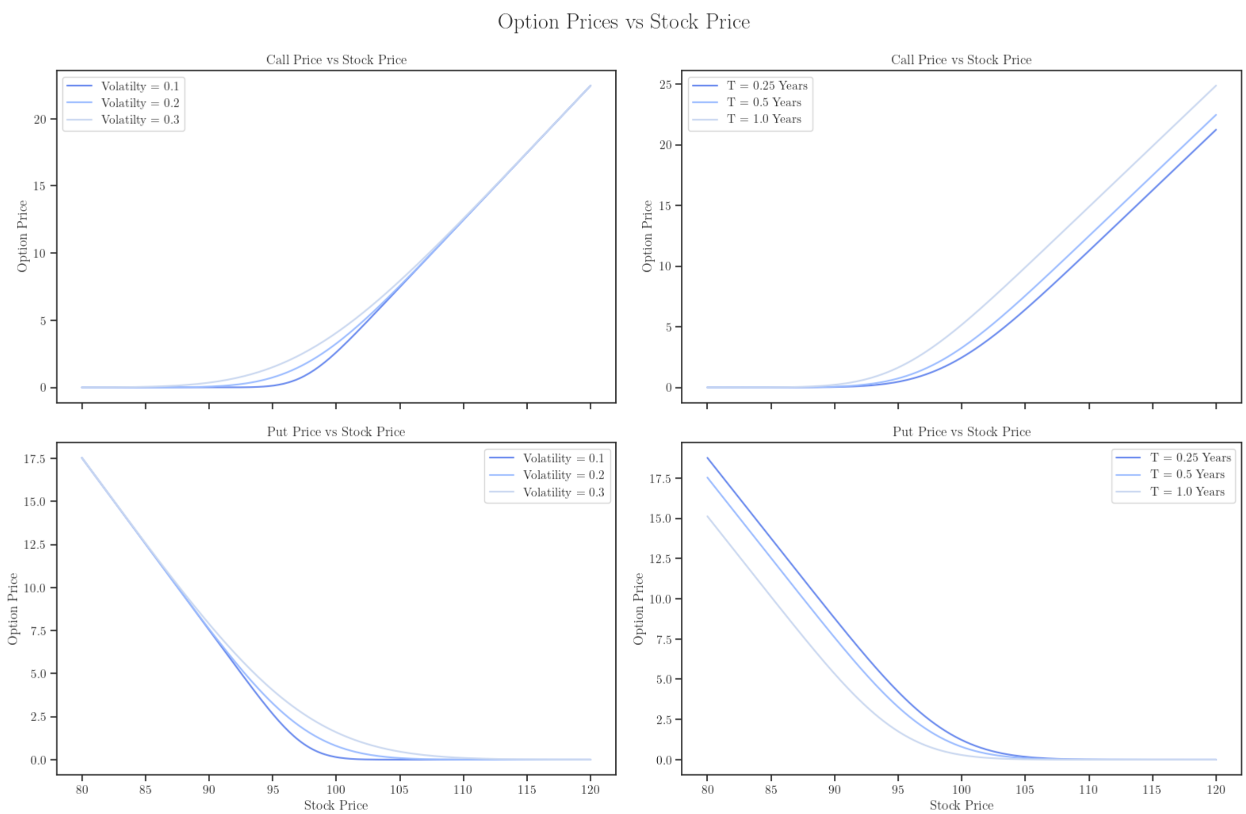


Figure 1: Profit diagrams for European call and put options – *Holders POV*.

III) Delta

Delta Δ is a primary Greek used in option price and hedging, as it measures the sensitivity of an option price to changes in the price of the underlying asset. Mathematically, delta is the first derivative of the option price with respect to the underlying asset price:

$$\Delta = \frac{\partial V}{\partial S}$$

where V is the option price and S is the current price of the underlying asset. Delta plays a crucial role in hedging, particularly in the construction of a delta-neutral portfolio. By holding $-\Delta$

shares of the underlying asset for each written option, the market maker can offset small changes in the price of the asset, making the portfolio value relatively insensitive to minor fluctuations in S.

IV) Gamma

Gamma Γ is the second-order Greek that measures the rate of change of delta with respect to the underlying asset price. It reflects the curvature of the option's value with respect to changes in the underlying and is defined as:

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S}$$

Under the Black-Scholes model, the gamma of both European call and put options is given by the same formula:

$$\Gamma = \frac{\phi(d_1)}{S\sigma\sqrt{T-t}}$$

Where $\phi(\cdot)$ represents the probability density function of the normal distribution. Gamma is particularly important in dynamic hedging strategies as it accounts for second-order risk. While delta hedging aims to neutralize first-order risk, it assumes that delta remains constant between rebalancing intervals. However, in reality, delta changes as the underlying asset price moves — especially rapidly or by large amounts. This is where gamma risk arises. A portfolio with high gamma is more sensitive to movements in the underlying, which means delta can change quickly and frequently. To reduce this convexity risk, market makers may engage in delta-gamma hedging, which involves using a second option to offset gamma exposure. By doing so, the overall portfolio becomes less sensitive to both small and large price movements, offering better stability under volatile conditions. As illustrated in the graphs below, delta and gamma behaviours vary significantly with moneyness, expiration, and volatility. As expiration approaches and the option is near the strike, delta becomes more sensitive to changes in the price of the underlying, which causes gamma to spike sharply. Higher volatility also smooths the delta curve and broadens the gamma peak. This broader gamma peak means the option remains highly sensitive to price changes reflecting greater uncertainty (volatility) about where the option's moneyness will finish.

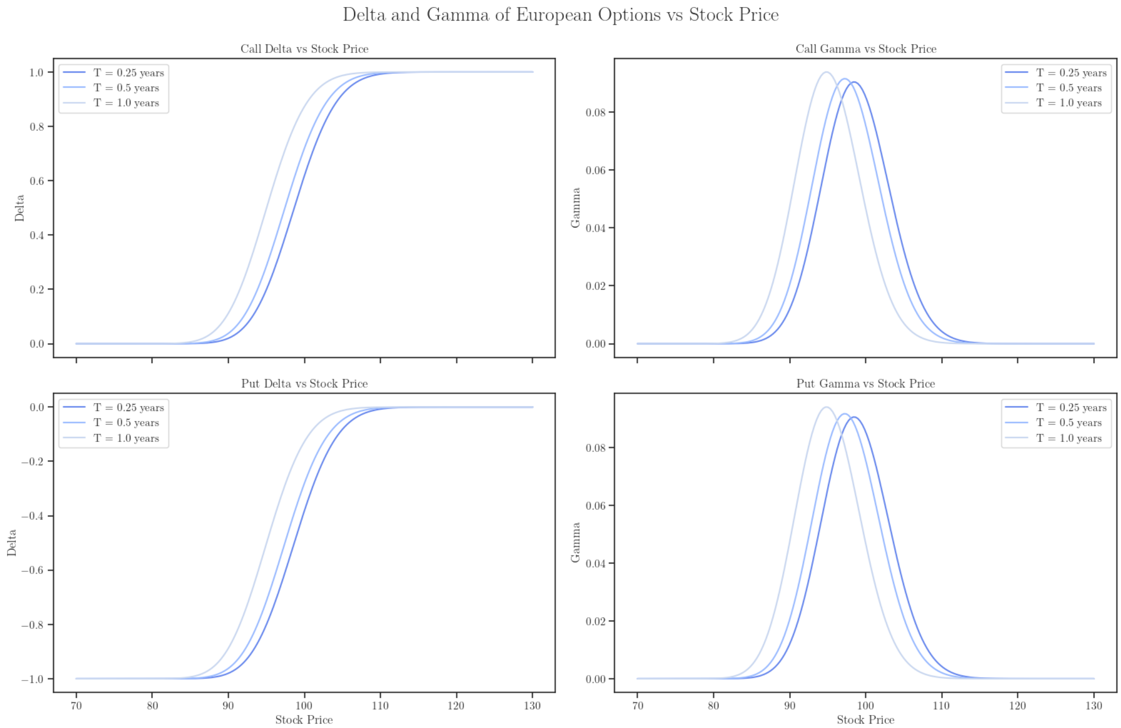


Figure 2: Delta and Gamma under different days to expiration.

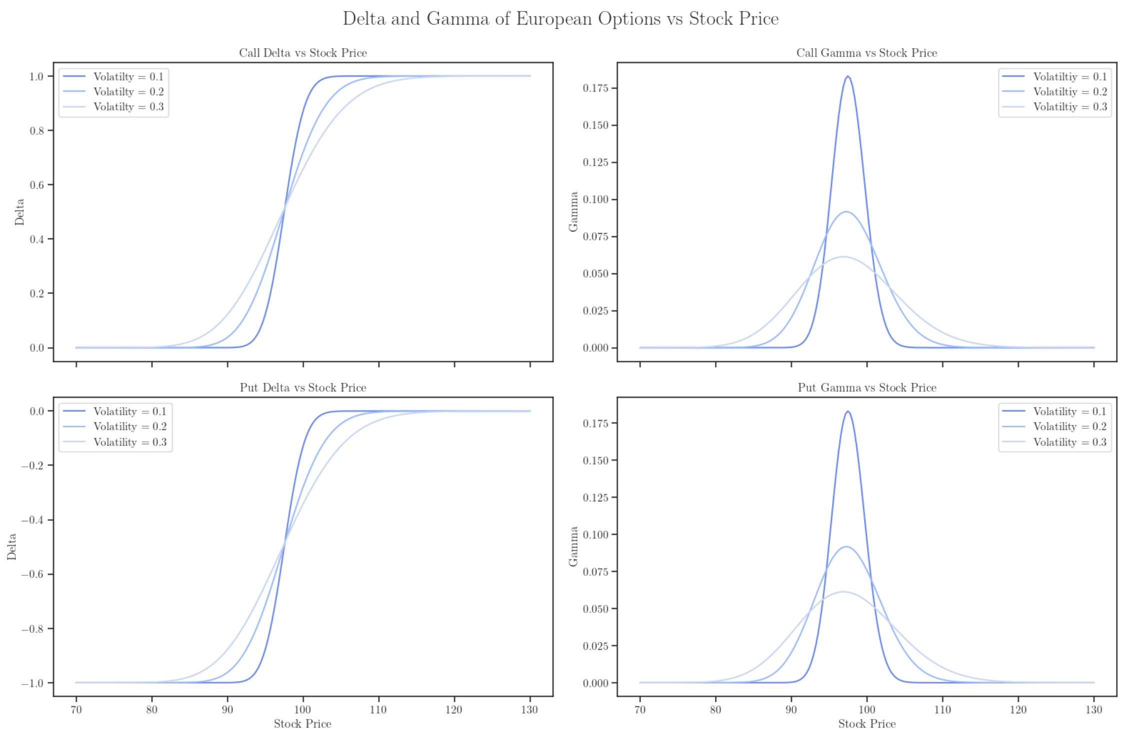


Figure 3: Delta and Gamma under different volatilities.

2.2 Hedging Process Overview

In this section, we will outline the intuition behind the dynamic delta and delta-gamma hedging simulation. Let us start with an in-depth explanation of both the delta, and delta-gamma dynamic hedging strategies.

I) Delta Hedging Overview

First, delta hedging involves neutralizing portfolio delta by offsetting the delta of the option position by either buying or selling shares of the underlying asset. A put option has delta in the range of $-1 \leq \Delta \leq 0$, while a call option has delta in the range of $0 \leq \Delta \leq 1$. In the case of the market maker in our simulation, we have written a put option that has negative delta. Thus, we are short negative delta which puts us in a position of long delta. Therefore, in order to hedge our long delta exposure, we need to short Δ shares of the underlying asset S. Essentially, we are aiming to construct a replicating portfolio of this option position through a combination of shorting shares and lending the cash proceeds at the risk-free rate. Put simply, we aim to maintain a neutral PnL throughout our simulation through proper dynamic delta hedging.

II) Delta-Gamma Hedging Overview

While delta hedging aims to neutralize the first-order sensitivity of an option position to changes in the underlying asset price, it does not account for higher-order risks—most notably, gamma risk. As stated earlier, Gamma (Γ) measures the rate of change of delta with respect to the underlying price, and is relevant when the price of the underlying asset fluctuates significantly. The gamma of both call and put options is in the range of $0 \leq \Gamma \leq 1$. A portfolio that is delta-neutral at time t may quickly become non-neutral as the asset price moves, requiring frequent rebalancing. To mitigate this second-order risk, delta-gamma hedging involves constructing a portfolio that is simultaneously neutral in both delta and gamma. This is typically achieved by introducing an additional option into the portfolio—in this case a call option with a different maturity and same strike K. For example, in our case where the market maker has written a put option, we are exposed to long delta and short gamma, and in turn we have to short Δ of the total portfolio and buy instruments with positive gamma exposure (e.g., long call options) to achieve total neutrality. To solve for the number of call options to buy at time t_0, it is simply the ratio of the gammas of the put and call option:

$$\alpha_0 = \frac{\Gamma_{put_0}}{\Gamma_{call_0}}$$

Once we have purchased α_0 call options, we then recalculate the total portfolio Δ at time t_0, which is equal to:

$$\Delta_{port_0} = -\Delta_{put_0} + \alpha_0 \cdot \Delta_{call_0}$$

In turn, we then short Δ_{port_0} shares of the underlying to offset our long delta exposure.

2.3 Hedging Algorithms

In this section, we will derive the algorithms used in both simulations using the following variables:

- Δ : delta
- Γ : gamma
- ϕ : contract multiplier (represents 100 shares of the underlying)
- ζ : price of call option
- α : number of rebalancing call options (gamma ratio)
- η : number of rebalancing shares
- Ψ : total portfolio cost
- Θ : overnight PnL
- λ_s : stock transaction fee (% of entire share transaction)
- λ_d : option transaction fee (% of entire option transaction)
- r : risk-free rate

I) Delta Hedging

To derive the algorithm for delta hedging, we will adjust our delta position in the option at every time step represented as $(0, \Delta t, 2\Delta t, ..., N\Delta t)$. Now, suppose that we are a market maker who has written a put option that has premium of $P_{N\Delta t}$ at time $N\Delta t$. We have to hedge our long delta exposure and measure overnight PnL including transaction costs throughout the simulation. Here is how the algorithm works:

- **At t = 0:** We receive the premium of the written put option as P_0 and pay transaction cost of $\lambda_d \cdot |P_0|$ for executing the trade. To hedge our long delta exposure, we need to short Δ_0 shares of underlying asset S by selling $\eta_0 = \Delta_0 \cdot S_0 \cdot \phi$ shares and pay $\lambda_s \cdot |\eta_0|$ in transaction costs. Thus, to find total portfolio cost at $t = 0$, we compute $\Psi_0 = -P_0 - \eta_0 + \lambda_s \cdot |\eta_0| + \lambda_d \cdot |P_0|$. Portfolio cost should be a negative value because we are receiving funds in exchange for the written put and selling shares.
- **At t = Δt:** We can now collect interest on our cash proceeds of $|\Psi_0|$ growing at the risk-free rate r. We then have to rebalance our delta exposure, so we sell $\eta_{\Delta t} = (\Delta_{\Delta t} - \Delta_0) \cdot \phi \cdot S_{\Delta t}$ and pay transaction costs of $\lambda_s \cdot |\eta_{\Delta t}|$. To calculate total portfolio cost we have $\Psi_{\Delta t} = \Psi_0 - \eta_{\Delta t} + \lambda_s \cdot |\eta_{\Delta t}|$. Since one day has passed, we then calculate our mark-to-market PnL of our option and shares. The mark-to-market PnL for the written option is computed as

$(P_0 - P_{\Delta t}) \cdot \phi$, and the shares as $\eta_0 \cdot (S_{\Delta t} - S_0)$. The interest earned on the portfolio is calculated as $|\Psi_0| \cdot (e^{r \cdot \frac{1}{365}} - 1)$. Thus, finding overnight PnL at $t = \Delta t$ is computed as $\Theta_{\Delta t} = \eta_0 \cdot (S_{\Delta t} - S_0) + (P_0 - P_{\Delta t}) \cdot \phi + |\Psi_0| \cdot (e^{r \cdot \frac{1}{365}} - 1)$.

- **At t =** $N\Delta t$**:** We are at the final step of the simulation. We have portfolio value of $|\Psi_{N\Delta t-1}|$ growing at the risk-free rate r. To rebalance delta, we sell $\eta_{N\Delta t} = (\Delta_{N\Delta t} - \Delta_{N\Delta t-1}) \cdot \phi \cdot S_{N\Delta t}$ and pay transaction costs of $\lambda_s \cdot |\eta_{N\Delta t}|$. Total portfolio cost is computed as $\Psi_{N\Delta t} = \Psi_{N\Delta t-1} - \eta_{N\Delta t} + \lambda_s \cdot |\eta_{N\Delta t}|$. We then calculate the mark-to-market PnL for the written option as $(P_{N\Delta t-1} - P_{N\Delta t}) \cdot \phi$ and the shares as $\eta_{N\Delta t-1} \cdot (S_{N\Delta t} - S_{N\Delta t-1})$. The interest earned on the portfolio is calculated as $|\Psi_{N\Delta t-1}| \cdot (e^{r \cdot \frac{1}{365}} - 1)$. Thus, finding overnight PnL at $t = N\Delta t$ is computed as $\Theta_{N\Delta t} = \eta_{N\Delta t-1} \cdot (S_{N\Delta t} - S_{N\Delta t-1}) + (P_{N\Delta t-1} - P_{N\Delta t}) \cdot \phi + |\Psi_{N\Delta t-1}| \cdot (e^{r \cdot \frac{1}{365}} - 1)$.

II) Delta-Gamma Hedging

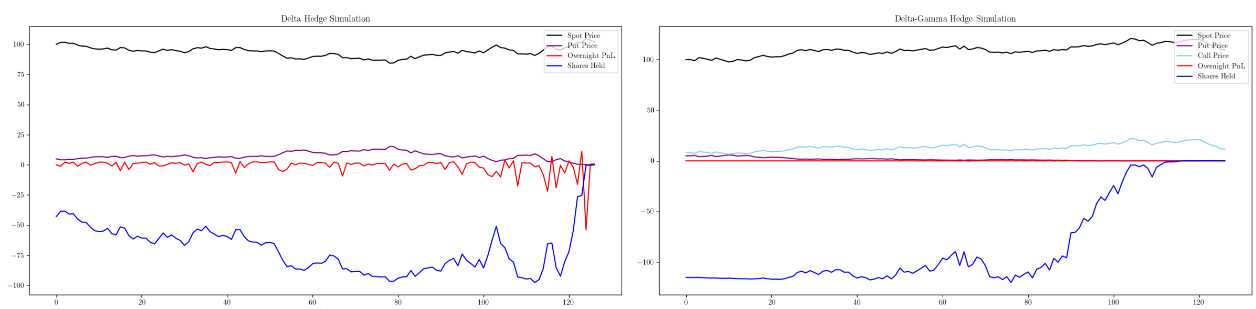
To derive the algorithm for delta-gamma hedging, we will adjust our delta and gamma position at every time step represented as $(0, \Delta t, 2\Delta t, ..., N\Delta t)$. Now, as simulated before we are a market maker who has written a put option that has premium of $P_{N\Delta t}$ at time $N\Delta t$. The nature of this position is long delta and short gamma, so we will have to dynamically neutralize our delta and gamma exposure at each time step and measure overnight PnL including transaction costs. Here is how the algorithm works:

- **At t = 0:** We receive the premium of the written put option as P_0 and pay transaction cost of $\lambda_d \cdot |P_0|$ for executing the trade. We then compute the ratio of gammas which gives us the number of calls to buy given as $\alpha_0 = \frac{\Gamma_{put_0}}{\Gamma_{call_0}}$. On $t = 0$ we must buy the exact gamma ratio of calls as $\alpha_0 \cdot \zeta_0$ and pay transaction costs of $\lambda_d \cdot |\alpha_0 \cdot \zeta_0|$ for executing the trade. We then calculate total portfolio delta now including the delta of the call option. This is given as $\Delta_{port_0} = -\Delta_{put_0} + (\alpha_0 \cdot \Delta_{call_0})$. Next, we sell Δ_{port_0} shares of the underlying asset as $\eta_0 = \Delta_{port_0} \cdot S_0 \cdot \phi$ and pay transaction costs of $\lambda_s \cdot |\eta_0|$. Thus, to find portfolio cost at $t = 0$, we calculate $\Psi_0 = -P_0 - \eta_0 + \lambda_s \cdot |\eta_0| + \alpha_0 \cdot \zeta_0 + \lambda_d \cdot |\alpha_0 \cdot \zeta_0|$. Again, portfolio cost should be a negative value because we are receiving funds in exchange for the written put and selling shares.

- **At t =** Δt**:** We can now collect interest on our cash proceeds of $|\Psi_0|$ growing at the risk-free rate r. We then recompute the ratio of gammas as $\alpha_{\Delta t} = \frac{\Gamma_{put_{\Delta t}}}{\Gamma_{call_{\Delta t}}}$. Now, to rebalance our calls according to the gamma ratio at $t = \Delta t$, we purchase additional call options valued at $\zeta_{\Delta t} \cdot (\alpha_{\Delta t} - \alpha_0)$ and pay the transaction costs of $\lambda_d \cdot |\zeta_{\Delta t} \cdot (\alpha_{\Delta t} - \alpha_0)|$ for the execution of the trade. We then recalculate total portfolio delta including the call options purchased. This is given as $\Delta_{port_{\Delta t}} = -\Delta_{put_{\Delta t}} + (\alpha_{\Delta t} \cdot \Delta_{call_{\Delta t}})$. Next, to hedge our delta exposure we sell $\Delta_{port_{\Delta t}}$ shares of the underlying asset as $\eta_{\Delta t} = \Delta_{port_{\Delta t}} \cdot S_{\Delta t} \cdot \phi$ and pay transaction costs of $\lambda_s \cdot |\eta_{\Delta t}|$. To find total portfolio cost, we calculate $\Psi_{\Delta t} = \Psi_0 + \zeta_{\Delta t} \cdot (\alpha_{\Delta t} - \alpha_0) + \lambda_d \cdot |\zeta_{\Delta t} \cdot (\alpha_{\Delta t} - \alpha_0)| - \eta_{\Delta t} + \lambda_s \cdot |\eta_{\Delta t}|$. Now, we can calculate our mark-to-market PnL for our portfolio components. For our call option PnL we do $\alpha_{\Delta t}(\zeta_{\Delta t} - \zeta_0) \cdot \phi$, for the put option we do $(P_0 - P_{\Delta t}) \cdot \phi$, and for our shares we do $\eta_{\Delta t} \cdot (S_{\Delta t} - S_0)$. Interest earned on the portfolio is calculated as $|\Psi_0| \cdot (e^{r \cdot \frac{1}{365}} - 1)$. Finally, we find overnight PnL at $t = \Delta t$ is computed as $\Theta_{\Delta t} = \alpha_{\Delta t} \cdot (\zeta_{\Delta t} - \zeta_0) \cdot \phi + (P_0 - P_{\Delta t}) \cdot \phi + \eta_{\Delta t} \cdot (S_{\Delta t} - S_0) + |\Psi_0| \cdot (e^{r \cdot \frac{1}{365}} - 1)$.

- **At t = $N\Delta t$:** First, we recompute our gamma ratio once again to determine how many calls to hold as $\alpha_{N\Delta t} = \frac{\Gamma_{put_{N\Delta t}}}{\Gamma_{call_{N\Delta t}}}$. To hedge our gamma exposure we rebalance according to our new gamma ratio and buy $\zeta_{N\Delta t} \cdot (\alpha_{N\Delta t} - \alpha_{N\Delta t-1})$ calls and pay transaction costs of $\lambda_d \cdot |\zeta_{N\Delta t} \cdot (\alpha_{N\Delta t} - \alpha_{N\Delta t-1})|$ for the execution of the trade. Now we can recalculate total portfolio delta including the call options as $\Delta_{port_{N\Delta t}} = -\Delta_{put_{N\Delta t}} + (\alpha_{N\Delta t} \cdot \Delta_{call_{N\Delta t}})$. Next, we hedge our total portfolio delta exposure by selling $\Delta_{port_{N\Delta t}}$ shares computed as $\eta_{N\Delta t} = \Delta_{port_{N\Delta t}} \cdot S_{N\Delta t} \cdot \phi$ and pay transaction costs of $\lambda_s \cdot |\eta_{N\Delta t}|$. To find total portfolio cost, we calculate $\Psi_{N\Delta t} = \Psi_{N\Delta t-1} + \zeta_{N\Delta t} \cdot (\alpha_{N\Delta t} - \alpha_{N\Delta t-1}) + \lambda_d \cdot |\zeta_{N\Delta t} \cdot (\alpha_{N\Delta t} - \alpha_{N\Delta t-1})| - \eta_{N\Delta t} + \lambda_s \cdot |\eta_{N\Delta t}|$. Now, we can calculate our mark-to-market PnL for each portfolio component. We calculate for the call options as $\alpha_{N\Delta t} \cdot (\zeta_{N\Delta t} - \zeta_{N\Delta t-1}) \cdot \phi$, for the put option as $(P_{N\Delta t-1} - P_{N\Delta t}) \cdot \phi$, and for our shares we do $\eta_{N\Delta t} \cdot (S_{N\Delta t} - S_{N\Delta t-1})$. Interest earned on the portfolio is calculated as $|\Psi_{N\Delta t-1}| \cdot (e^{r \cdot \frac{1}{365}} - 1)$. Thus, finally we compute overnight PnL as $\Theta_{N\Delta t} = \alpha_{N\Delta t} \cdot (\zeta_{N\Delta t} - \zeta_{N\Delta t-1}) \cdot \phi + (P_{N\Delta t-1} - P_{N\Delta t}) \cdot \phi + \eta_{N\Delta t} \cdot (S_{N\Delta t} - S_{N\Delta t-1}) + |\Phi_{N\Delta t-1}| \cdot (e^{r \cdot \frac{1}{365}} - 1)$.

Below are sample paths generated from the algorithms derived above for both the simulations *Delta Hedging* and *Delta-Gamma Hedging*.



(a) Delta Hedge

(b) Delta-Gamma Hedge

Figure 4: Sample paths generated from hedging algorithms.

3 Analysis

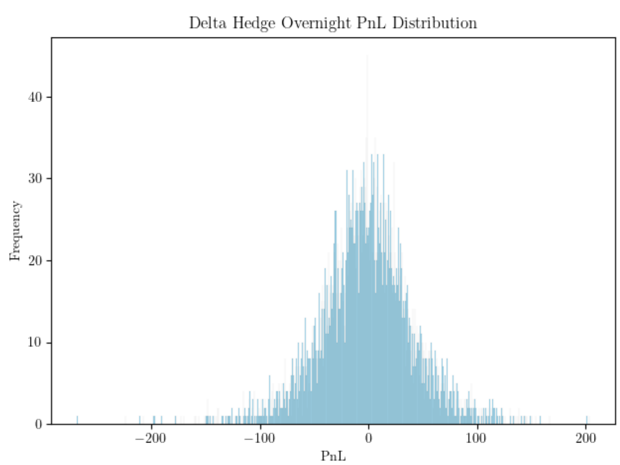
The simulation for our analysis utilizes a base case for the model parameters, and then introduces scenario analysis. The goal is to evaluate the effectiveness of each strategy under different regimes, including volatility, drift, and transaction costs. For each simulation path, we compute the mean, standard deviation, 95% VaR, CVar, skewness, and the 98% empirical quantile range as performance measures of overnight PnL. We define the 98% empirical quantile range as $Q_{99} - Q_1$, which measures the spread of the central 98% of simulated outcomes. Suppose a market maker has sold an ATM put option representing 100 shares of the underlying asset S, here is our base case assumptions:

Table 1: Base Case Assumptions Used in Simulation

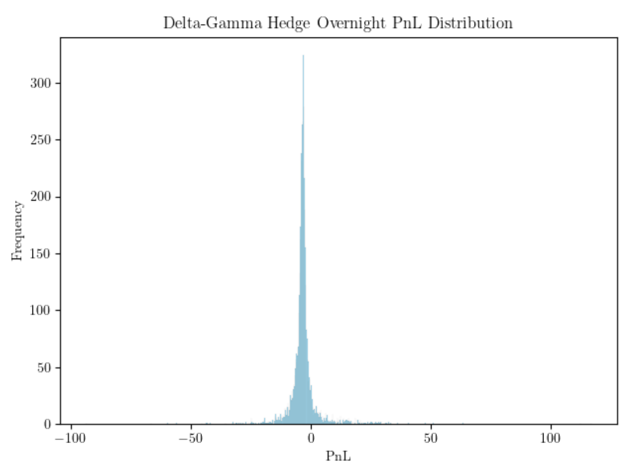
Parameter	**Value**
Initial stock price	$S_0 = 100$
Strike price	$K = 100$
Volatility	$\sigma = 0.20$
Drift	$\mu = 0.10$
Risk-free rate	$r = 0.03$
Put Option maturity	$T_{put} = 0.50$
Call Option maturity	$T_{call} = 0.75$
Equity transaction cost	$\lambda_s = 0.5\%$
Option transaction cost	$\lambda_d = 1.0\%$

Table 2: Scenario Analysis Variations

Scenario	**Value(s)**
Volatility Regimes	$\sigma = 0.10, 0.20, 0.30, 0.40$
Drift Variations	$\mu = 0.10, 0.20, 0.30, 0.40$
Transaction Costs	$\lambda_s, \lambda_d =$ No cost, 0.5%, 1.0%, 5.0%



(a) Base Case Delta Hedge PnL Distribution



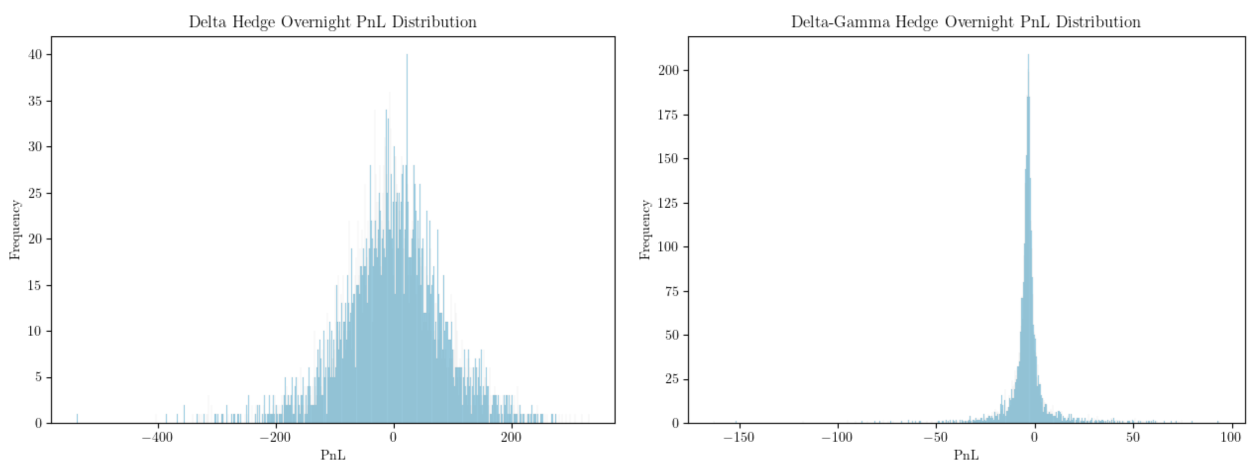
(b) Base Case Delta-Gamma Hedge PnL Distribution

Figure 5: Base case overnight PnL distributions.

3.1: Volatility Regimes

Parameter: σ	Delta Hedge				Delta-Gamma Hedge			
	0.10	**0.20**	**0.30**	**0.40**	**0.10**	**0.20**	**0.30**	**0.40**
Mean	-2.210	-1.263	-2.682	-0.763	-3.170	-3.129	-3.420	-3.212
Std	20.246	43.980	65.476	88.222	3.236	6.166	10.078	12.615
95% VaR	-35.911	-74.361	-109.729	-140.372	-6.136	-9.714	-13.804	-17.113
CVaR	-48.867	-105.040	-147.157	-194.448	-10.171	-16.250	-25.705	-30.317
Skewness	-0.065	-0.329	-0.046	0.065	-0.718	0.460	-0.885	0.376
98% EQR	106.637	231.256	329.712	456.635	19.687	39.527	64.562	85.450

Table 3: Volatility Scenario Analysis – Delta vs Delta-Gamma.



(a) Delta Hedge

(b) Delta-Gamma Hedge

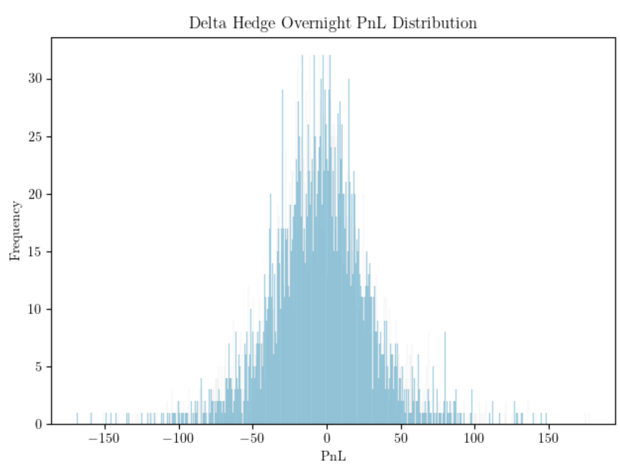
Figure 6: Overnight PnL distributions: $\sigma = 0.40$.

As illustrated in the table above, the performance of delta and delta-gamma hedging strategies varies significantly across different volatility regimes. As we increase the volatility parameter in our simulation, the mean overnight PnL remains negative, but the standard deviations widen. However, you can see that Delta-gamma hedging significantly outperforms delta hedging. The standard deviations grow at a smaller rate, yielding a more concentrated distribution. Again, delta-gamma hedging outperforms by consistently producing significantly smaller 95% VaR and CVaR values. As volatility increases, the delta hedging strategy becomes extremely vulnerable to rare but severe losses. Whereas the delta-gamma hedging contains this tail risk much more effectively and reduces the likelihood of severe losses due to the second-order risk mitigation. Specifically, at $\sigma = 0.40$, the 95% VaR is just -17.11 and CVar is -30.32 – which is over 6x better than Delta hedging and is a reduction of approximately 83% in CVaR. The skewness of the delta hedging strategy remains relatively close to zero with large 98% EQR values, indicating a wide but symmetric distribution of PnL. In contrast, delta-gamma hedging shifts from negative to slightly positive skewness, with much lower 98% EQR values, indicating a much tighter distribution of PnL. These results conclude the consistency of delta-gamma hedging in high-volatility environments , offering a much tighter control on portfolio convexity.

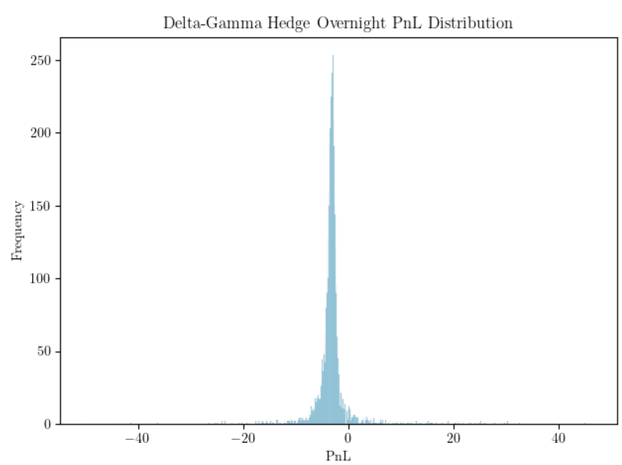
3.2: Drift Variations

Parameter: μ	**Delta Hedge**				**Delta-Gamma Hedge**			
	0.10	**0.20**	**0.30**	**0.40**	**0.10**	**0.20**	**0.30**	**0.40**
Mean	-3.338	-3.247	-4.526	-6.006	-3.215	-3.141	-3.219	-3.136
Std	43.060	41.653	38.405	35.294	6.273	5.826	5.658	4.727
95% VaR	-72.431	-71.196	-66.863	-63.455	-10.051	-9.072	-8.086	-7.080
CVaR	-105.096	-98.939	-92.029	-89.183	-16.981	-15.373	-14.333	-12.021
Skewness	-0.228	-0.159	-0.103	-0.200	0.224	1.978	1.093	2.580
98% EQR	229.487	218.283	202.180	192.693	40.502	37.898	33.799	27.966

Table 4: Drift Scenario Analysis – Delta vs Delta-Gamma.



(a) Delta Hedge



(b) Delta-Gamma Hedge

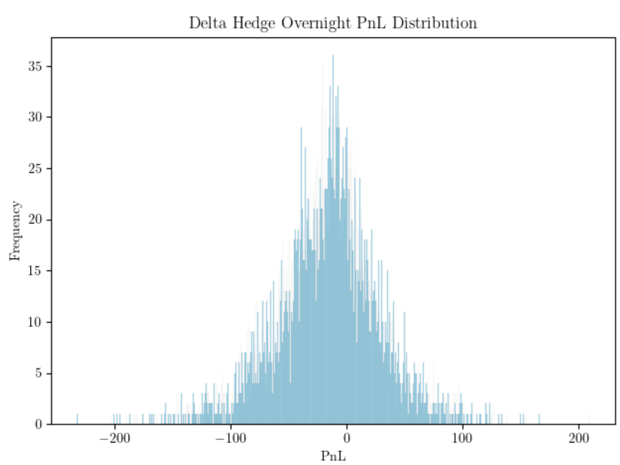
Figure 7: Overnight PnL distributions: $\mu = 0.40$.

The effect of changing drift on hedging performance is more muted compared to changing volatility, but meaningful patterns still emerge. Across drift levels, delta-gamma hedging outperforms delta hedging in terms of mean overnight PnL, driven from the additional position in long call options. We see little to no change in measurements such as standard deviation, 95% VaR, and CVaR, as delta-gamma hedging is still the leading strategy. Skewness becomes a key differentiator between the strategies, with delta-gamma hedging exhibiting increasingly positive skewness as the drift parameter rises. This reflects a PnL distribution with a longer right tail signaling a higher likelihood in positive gains. In contrast, delta hedging skewness remains close to zero, and sees little to no change at all. Finally, the 98% EQR under delta-gamma hedging again produces increasingly positive results as the drift increases, indicating a tighter distribution of expected outcomes. Whereas delta hedging still displays a broader, but slightly declining, 98% EQR as drift increases. Overall, the delta-gamma hedging strategy still significantly outperforms the delta hedging strategy and delivers better risk-adjusted performance across drift regimes.

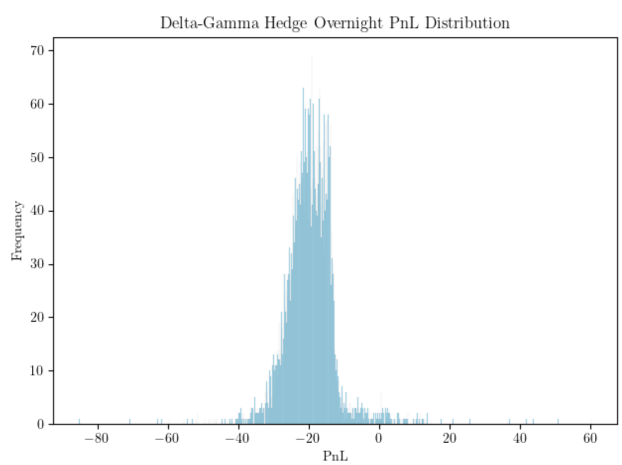
3.3: Transaction Cost Variations

Parameter: λ_d, λ_s	**Delta Hedge**				**Delta-Gamma Hedge**			
	No cost	**0.5%**	**1.0%**	**5.0%**	**No cost**	**0.5%**	**1.0%**	**5.0%**
Mean	-0.345	-1.649	-4.173	-17.656	-1.142	-3.248	-4.945	-19.430
Std	43.994	42.932	42.698	42.603	6.954	6.740	7.140	7.632
95% VaR	-69.528	-71.848	-73.283	-90.451	-8.262	-10.335	-11.904	-29.139
CVaR	-100.841	-100.664	-103.366	-118.728	-15.790	-18.133	-19.770	-35.079
Skewness	-0.146	-0.207	-0.169	-0.333	-1.383	-3.100	-0.749	0.449
98% EQR	231.130	225.340	223.493	222.406	42.261	41.553	40.491	44.034

Table 5: Transaction Cost Scenario Analysis – Delta vs Delta-Gamma.



(a) Delta Hedge



(b) Delta-Gamma Hedge

Figure 8: Overnight PnL distributions: $\lambda_s, \lambda_d = 5.0\%$.

For the final scenario analysis, we tested different transaction cost variations for both equity and option transactions in an effort to display the cost effectiveness of each strategy. Delta-gamma hedging is definitely the more costly of the two, as more hedging trades are placed in order to achieve portfolio neutrality across both delta and gamma. This leads to higher negative mean PnL, as higher transaction costs will significantly impact our portfolio cost. As illustrated in the table and graphs above, it is apparent and expected that mean PnL falls as transaction costs increase. Across both strategies, the standard deviation remains relatively stable, indicating that losses are primarily driven by the transaction costs and not volatility. Correspondingly, the 95% VaR and CVaR metrics decline significantly as costs increase, with delta-gamma producing the highest CVaR yet of -35.08 at 5% cost. This truly reflects the cost impact this strategy has, due to its high volume of rebalancing trades by introducing an additional hedging component of the portfolio. Interestingly, the skewness of the delta-gamma slightly increased as costs go up and produced a positive skewness at the highest transaction cost, where the skewness for delta hedging was relatively unchanged. Despite the decline in performance, delta-gamma hedging still produced a tighter portfolio convexity and yielded significantly lower 98% EQR. These results ultimately highlight the sensitivity of second-order hedging to cost friction and illustrate the cost effectiveness of each strategy.

4 Conclusions

Based on our analysis, we can conclude that both dynamic delta and delta-gamma hedging strategies can effectively produce stable overnight PnL, with delta-gamma yielding a stronger control of total convexity. As seen in Figure 4, overnight PnL is significantly tighter and close to zero for delta-gamma hedging, as it is for delta hedging. This enables market makers to operate in profit neutral environments, maintaining a near-perfect hedge on their positions. Moreover, when examining the distribution of returns across thousands of simulated asset price paths, delta-gamma hedging produces a significantly narrower PnL distribution with skewness often positive or close to zero. This suggests that second-order risk management can reduce exposure to large, nonlinear losses. On the other hand, delta-only hedging is effective at neutralizing first-order risk, but struggles with tail-risk management and is prone to larger losses as seen in the distributions. In exchange for this superior strategy, market makers have to bear a higher transaction cost when delta-gamma hedging. Delta-gamma hedging requires more frequent and complex adjustments, especially depending on the rebalancing interval per step (trading day). Such adjustments require adding additional portfolio components such as longer-dated call or put options depending on the market maker's position. In contrast, delta-only hedging is more practical and easier to implement, leading to much lower friction in transaction costs. Therefore, for market makers with the tools to execute more sophisticated hedging strategies, delta-gamma hedging is the superior strategy in yielding a more concentrated overnight PnL, and maintaining stronger control over portfolio convexity.