ALGEBRAIC GEOMETRY — EXERCISE SHEET 3
DUE ON 4/12/2023

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Exercise 0.1. Show that if $X \to S$ is a finite type morphism to a noetherian scheme S, then X is noetherian.

Exercise 0.2. Show that closed immersions are local on the target.

Exercise 0.3. Given an injective integral ring homomorphism $A_0 \hookrightarrow A$, prove that A is a field if and only if A_0 is a field.

Exercise 0.4. Let $\mathbb{F}$ be a field. Let $\phi : A \to B$ be a homomorphism of finitely generated $\mathbb{F}$-algebras. Show that $f_\phi : \operatorname{Spec} B \to \operatorname{Spec} A$ maps closed points to closed points.

Exercise 0.5. Construct a $\mathbb{C}$-scheme $X \subset \mathbb{A}^2_{\mathbb{C}}$ such that $X_{\text{red}} = \operatorname{Spec} \mathbb{C}[x, y]/(y - x^2)$ and X is nonreduced at the point $(x-1, y-1)$ only.

Exercise 0.6. Show that any irreducible quadric $Q \subset \mathbb{P}^{n+1}_{\mathbb{C}}$ is birational to $\mathbb{P}^n_{\mathbb{C}}$.