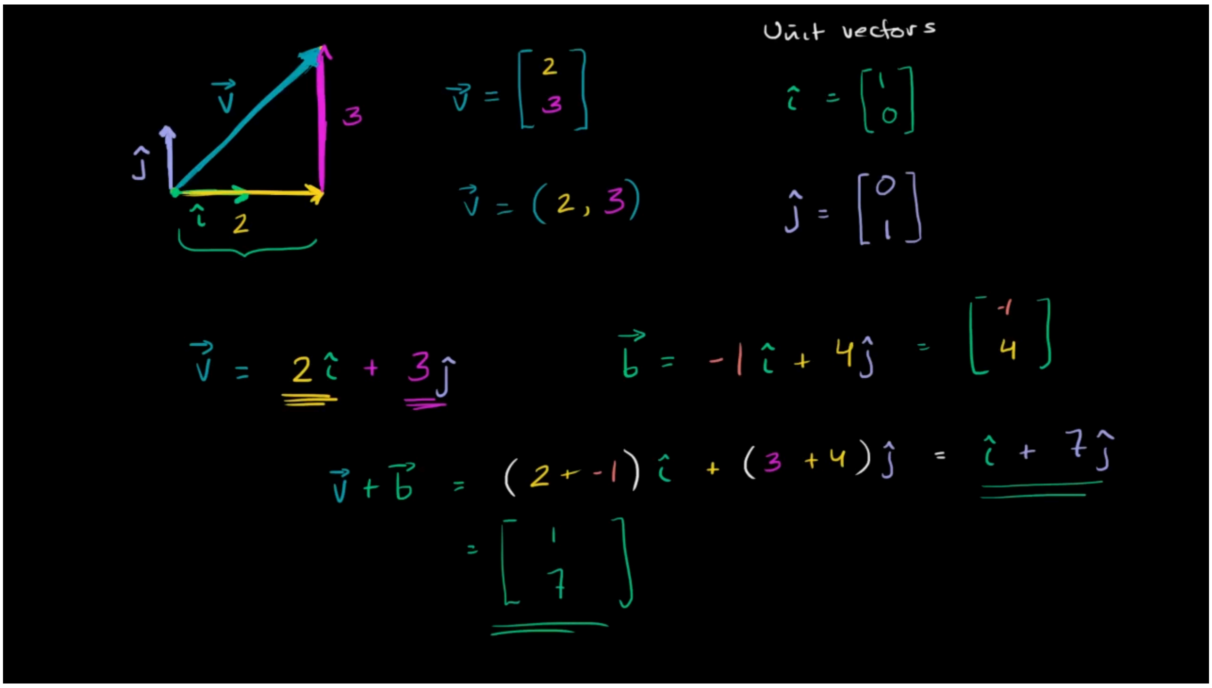
Unit Vector Notation



Source: [Unit vectors intro (video) | Vectors | Khan Academy]

Vector Representation

What is a Vector?

A **vector** is a quantity with both:

- **Magnitude** (how much)
- **Direction** (which way)

We can **visualize** it as an arrow 🏹:

- **Length of the arrow** = magnitude
- **Arrow direction** = vector direction

💡 Example

A vector **v** that moves:

- 2 units right ⏩ (horizontal)
- 3 units up ⏫ (vertical)

Can be written as:

- **Column vector:**

  $$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

- **Tuple notation:**

  $$\vec{v} = (2, 3)$$

🧩 Unit Vectors: The Lego Bricks of Vector World

🧭 Definition

Unit vectors are the "building blocks" of direction. They have:

- **Length = 1**
- Point in a coordinate direction (horizontal or vertical)

💡 Notation

We use a **hat** (^) to denote unit vectors.

🎯 2D Unit Vectors

Name	Symbol	Vector Form	Meaning
Horizontal unit	$\hat{\imath}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	Right (x-direction)
Vertical unit	$\hat{\jmath}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	Up (y-direction)

🧮 Expressing Vectors with Unit Vectors

🧠 Idea

Any 2D vector can be written as a **linear combination** of $\hat{\imath}$ and $\hat{\jmath}$.

✏️ Example

Given $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, we write:

$$\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$$

This means:

- Move 2 units in the $\hat{\imath}$ direction ➡️
- Move 3 units in the $\hat{\jmath}$ direction ⬆️

📦 Think of this like assembling a custom vector using pre-made blocks:

🧱 **2 blocks of $\hat{\imath}$** + 🧱 **3 blocks of $\hat{\jmath}$**

🧮 Vector Addition with Unit Vectors

🎯 Given

- $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$
- $\vec{b} = -1\hat{\imath} + 4\hat{\jmath}$

➕ Adding

$\vec{v} + \vec{b} = (2 + (-1))\hat{\imath} + (3 + 4)\hat{\jmath} = \hat{\imath} + 7\hat{\jmath}$

Or as a column vector:

$$\vec{v} + \vec{b} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

🔍 Geometric Intuition

You're stacking arrows tip-to-tail:

- First move like vector $\vec{v} \longrightarrow$ ➡️↗️
- Then from that end, move like vector $\vec{b} \longrightarrow$ ⬅️⬆️
- Resulting vector connects the start to the final point 💥

🔑 Key Takeaways

Concept	Description
Vector	A quantity with magnitude + direction
Unit Vectors	Basic direction vectors: $\hat{i} = [1, 0]$, $\hat{j} = [0, 1]$
Vector Representation	Can use tuples, columns, or unit vector notation
Vector Addition	Add corresponding components (horizontal with horizontal, etc.)
Vector Construction	Any vector in 2D is just a sum of scaled unit vectors (a linear combination)

🎯 Final Thoughts

Unit vectors are like the X and Y axes' personal agents—they define **pure direction**. Think of $\hat{i}$ and $\hat{j}$ like the **north-south and east-west tiles** of a vector GPS system. Every vector is just a clever remix of them 🔀✨.

Once you're comfy with this idea, scaling to **3D** is just adding a third buddy: $\hat{k}$ for the Z-axis ⬆️📈.