📘

Vector Examples

Source: Vector examples (video) | Vectors | Khan Academy

📌 Vector Basics

🧭 Introduction to Vectors

- **Vector** = Ordered list of numbers. In $\mathbb{R}^n$, each element is a real number.
- Example: In $\mathbb{R}^2$, a vector looks like $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where $x_1, x_2 \in \mathbb{R}$.
- **Rn** is the space of all such n-tuples.

  Like a plane (for $\mathbb{R}^2$) or a line (for $\mathbb{R}^1$).

> 💡 Think of $\mathbb{R}^2$ as an infinite flat canvas, where each point is a vector!

🎯 Vector Representation (Position-Independent)

📍 Standard Position

- Convention: place vectors starting at origin (0, 0).
- Helps simplify drawing and interpretation.

🌍 Arbitrary Position

- You can start a vector **anywhere**, not just at origin.
- The **shape + direction** define a vector, not its starting point.
- Infinite valid drawings of vectors!

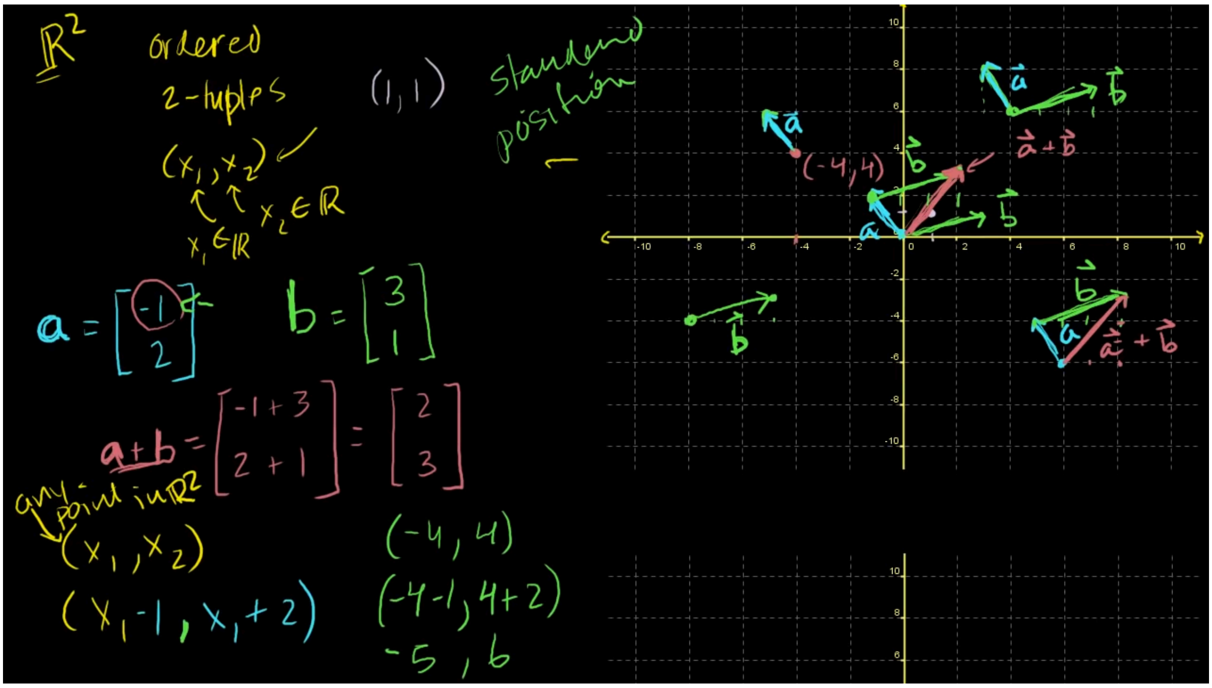
📈 **Visual:**

- Let's say vector $\vec{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.
- Start at $(-4, 4)$. Add $\vec{a}$'s components:
  - $4 - 1 = -5$
  - $4 + 2 = 6$
- So draw an arrow from $(-4, 4)$ to $(-5, 6)$.

> ↗️ A vector is like a direction arrow in space — you can slide it around, and it's still the same vector.

🔢 Basic Operations with Vectors

➕ 1. Vector Addition



Let:

- $\vec{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$
- $\vec{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

🧮 Compute

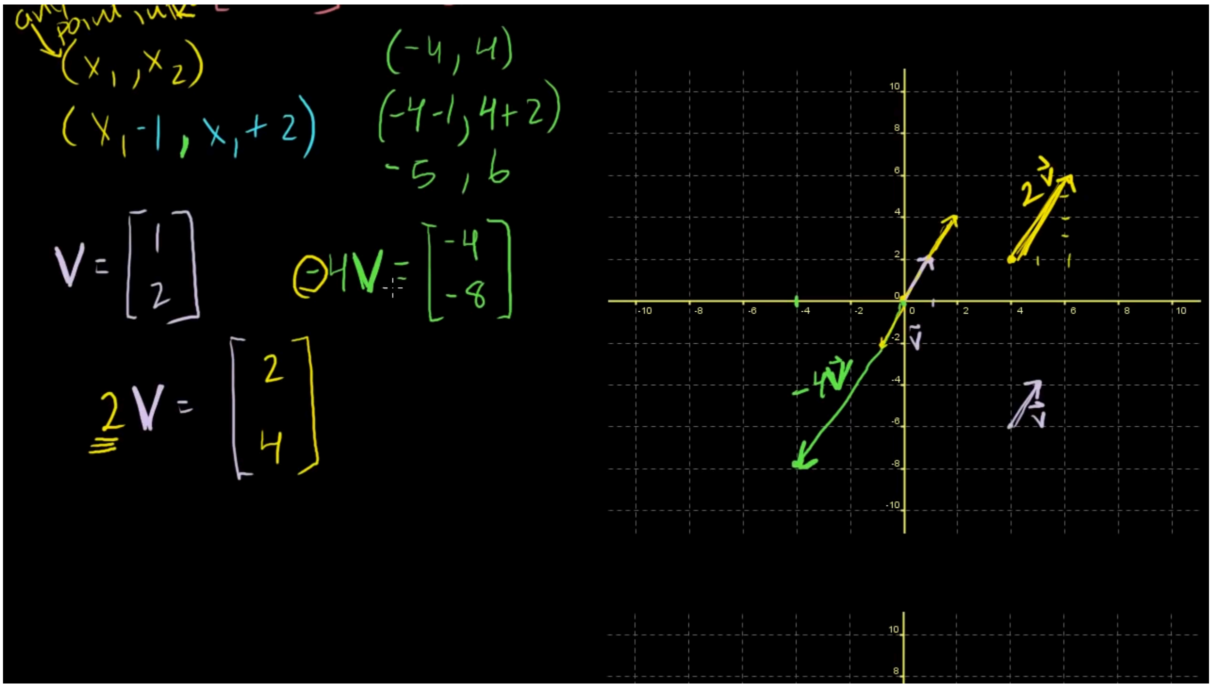
$$\vec{a} + \vec{b} = \begin{bmatrix} -1+3 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

🧠 Visual Interpretation

- **Head-to-tail** rule: Place tail of $\vec{b}$ at head (tip) of $\vec{a}$, then draw the resulting arrow.
- **Resultant vector** connects the tail of $\vec{a}$ to the head of $\vec{b}$.

> Vector addition = “walking the path” of one vector and then the other. 🎯

✖️ 2. Scalar Multiplication



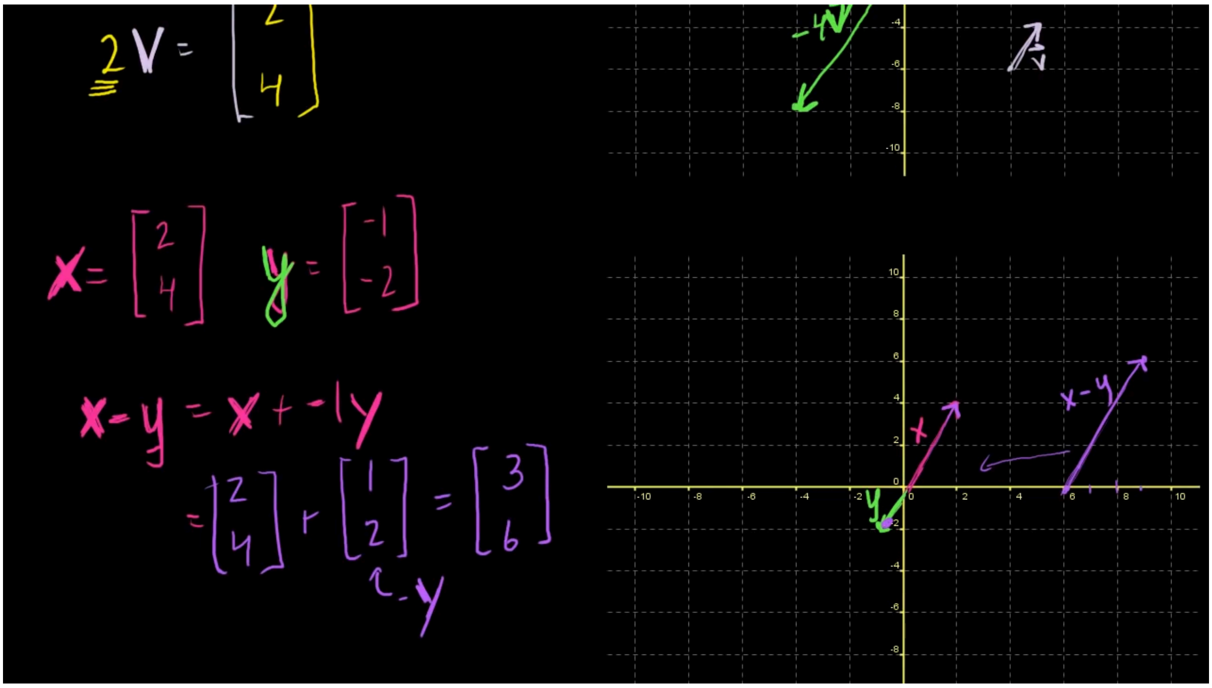
Let $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

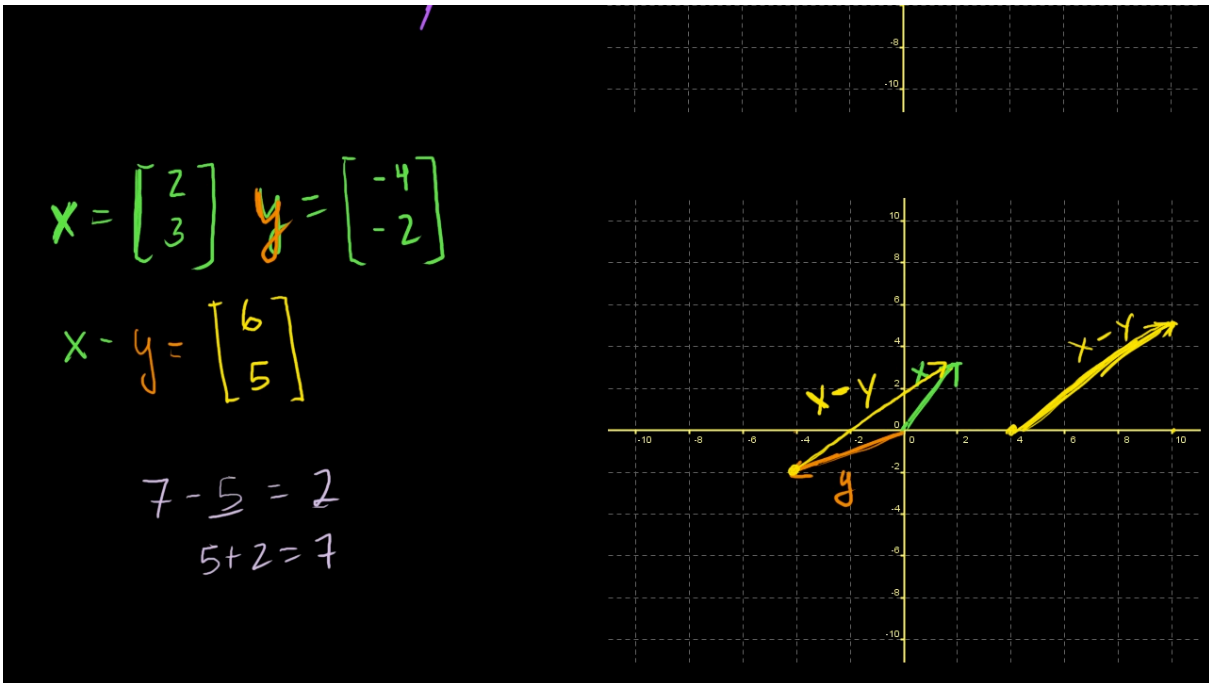
🧮 Examples

- $2\vec{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$: Twice as long, same direction
- $-4\vec{v} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$: 4× longer, opposite direction 🌪️

> 🧠 Scalar multiplication = stretching or flipping the vector (+ still along the same line).

➖ 3. Vector Subtraction





Let:

- $\vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$
- $\vec{y} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

🧮 Compute

$$\vec{x} - \vec{y} = \vec{x} + (-1)\vec{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

🧠 Visual Interpretation

- Subtraction = "how do I get from $\vec{y}$ to $\vec{x}$?"
- Or: subtract head-to-head, connect tips.
- If vectors are collinear, subtraction still follows same rules—**direction & length tell the difference**.

> 📏 Subtracting vectors = finding the difference arrow between them.

🚀 Higher-Dimensional Vectors (R³, R⁴...)

Let:

- $\vec{a} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ 3 \end{bmatrix}$
- $\vec{b} = \begin{bmatrix} 4 \\ -2 \\ 0 \\ 5 \end{bmatrix}$

🧮 Compute

$$4\vec{a} - 2\vec{b} = \begin{bmatrix} 0 \\ -4 \\ 8 \\ 12 \end{bmatrix} - \begin{bmatrix} 8 \\ -4 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 8 \\ 2 \end{bmatrix}$$

- No drawing possible 😵‍💫, but math is same as R^2!

📚 Key Takeaways

Concept	Description
Vector	Direction + magnitude (can slide anywhere!)
Standard position	Vector starting at (0,0) for simplicity
Addition	Add coordinates element-wise, visualize head-to-tail
Scalar multiplication	Stretch or flip vector
Subtraction	Like adding the negative vector, shows difference
High-Dimensional Vectors	Treated the same as 2D vectors—just can't draw them 😅

🧠 Visual Memory Hook

> Imagine vectors as arrows shot across space — their tail is where they start, and their tip points where they're going.
>
> Scaling is like pulling the arrow back harder 💥, and subtraction is like **reversing the arrow** 🔄.