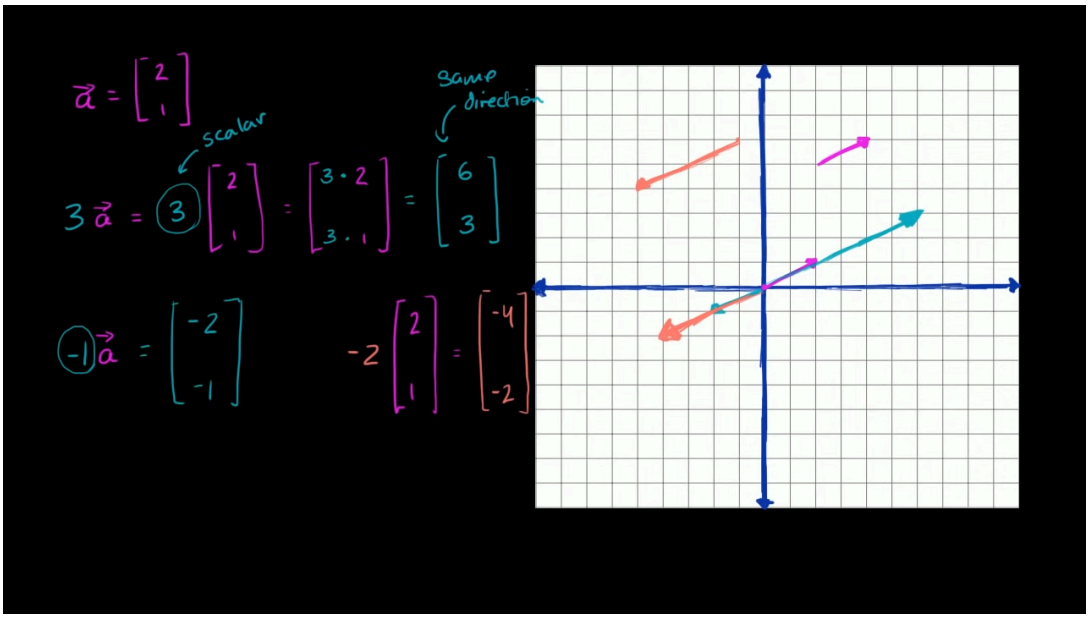
Scalar Multiplication of Vectors



Source: Multiplying a vector by a scalar (video) | Khan Academy

🟦 Multiplying a Vector by a Scalar

When we multiply a vector by a scalar (a regular number), we **scale** its magnitude while preserving or flipping its direction.

🟣 Examples

Given a 2D vector:

$$\vec{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

This means:

- Move **2 units right** (horizontal)
- Move **1 unit up** (vertical)

It's like giving directions: "Go 2 steps East, then 1 step North."

🟢 Example 1: Multiply by a Positive Scalar

Multiply by 3

$$\vec{a} = 3 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

✅ **Effect**:

- Same direction as $\vec{a}$
- Magnitude is **3× longer**

🧠 Think of this like zooming in on the original vector. You're stretching it by a factor of 3.

🔵 Example 2: Multiply by a Negative Scalar

Multiply by -1

$$\vec{a} = -1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

✅ **Effect**:

- **Flips direction**
- **Same magnitude**

🧠 This is like walking backwards the same distance you were walking forward.

🟠 Example 3: Multiply by -2

$$-2\vec{a} = -2 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

✅ **Effect**:

- **Flips direction**
- **Magnitude is 2× longer**

🧠 This is like doubling your backward stride.

✅ Key Takeaways

Original vector = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Scalar	Resulting Vector	Direction	Magnitude Effect
3	$[6, 3]$	Same	3× longer
-1	$[-2, -1]$	Opposite	Same
-2	$[-4, -2]$	Opposite	2× longer

💡 **Rule**:

$$c\vec{a} = \begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \end{bmatrix}$$

Where c is a **scalar**, and $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$