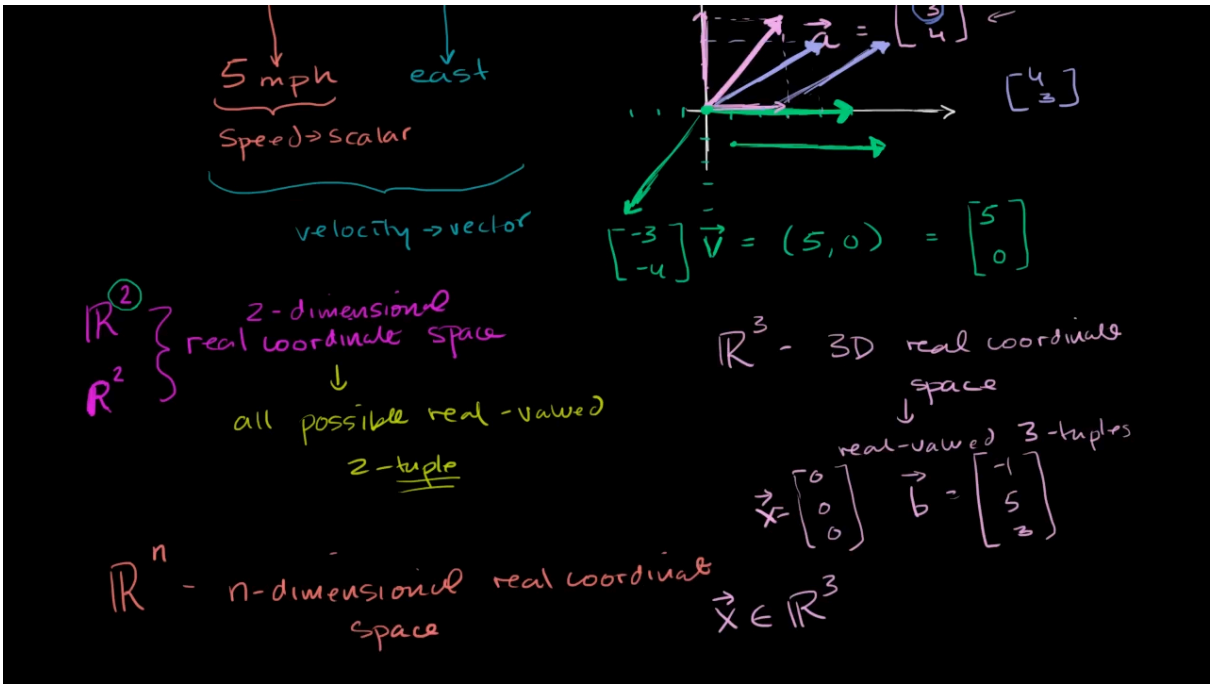
Real Coordinates Spaces



Source: Real coordinate spaces (video) | Vectors | Khan Academy

📊 Coordinate Spaces

$\mathbb{R}^2$ – 2D Real Coordinate Space

- **Meaning**: All possible real-valued 2-tuples
- **2-tuple**: Ordered list of 2 real numbers, e.g., `(3, 4)` or `(-3, -4)`
- **Visual**: Standard XY plane (horizontal & vertical axes)

- **Examples**:
  - $\vec{a} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$
  - $\vec{b} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$
  - Zero vector: $\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- **Important**: Order *matters*! $(3, 4) \neq (4, 3)$

$\mathbb{R}^3$ – 3D Real Coordinate Space

- **Meaning**: All possible real-valued 3-tuples
- **3-tuple**: Ordered list of 3 real numbers
- **Examples**:
  - $\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^3$
  - $\vec{b} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix} \in \mathbb{R}^3$

🛑 What **is Not** in $\mathbb{R}^3$?

- $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ – not a 3-tuple → belongs to $\mathbb{R}^2$
- $\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}$ – has imaginary part → **not real-valued**

🌌 Generalizing to Higher Dimensions

$\mathbb{R}^n$ – n-Dimensional Real Coordinate Space

- **Definition**: The set of all real-valued `n-tuples`
  - Each vector has `n` components, all real numbers
- **Examples**:
  - $\vec{x} \in \mathbb{R}^4$: 4 real values
  - $\vec{z} \in \mathbb{R}^{100}$: 100-dimensional vector 💀

🧙‍♂️ Visualization Note

- You can **visualize** up to $\mathbb{R}^3$ easily
- For $n > 3$: you can still **represent mathematically** even though visualization gets impossible

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📦 Key Definitions

Term	Definition	Example
Tuple	Ordered list of numbers	(x, y), (x, y, z), etc.
2-Tuple / 3-Tuple	Tuple with 2 / 3 elements	$(4, 3)$, $(-1, 5, 3)$
$\mathbb{R}^n$	Real coordinate space with `n` dimensions	$\mathbb{R}^2$, $\mathbb{R}^3$, etc.
$\vec{v} \in \mathbb{R}^n$	Vector `v` belongs to n-dimensional real space	$\vec{b} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix} \in \mathbb{R}^3$

🗝️ Key Takeaways

Concept	Summary
$\mathbb{R}^n$	All real-valued n-tuples; each point is a vector
Dimensions	$\mathbb{R}^2$ is 2D, $\mathbb{R}^3$ is 3D, and so on
Real-valued	All components are **real numbers** (no imaginary parts)
Visualization limits	We visualize up to 3D, but math allows any nD!