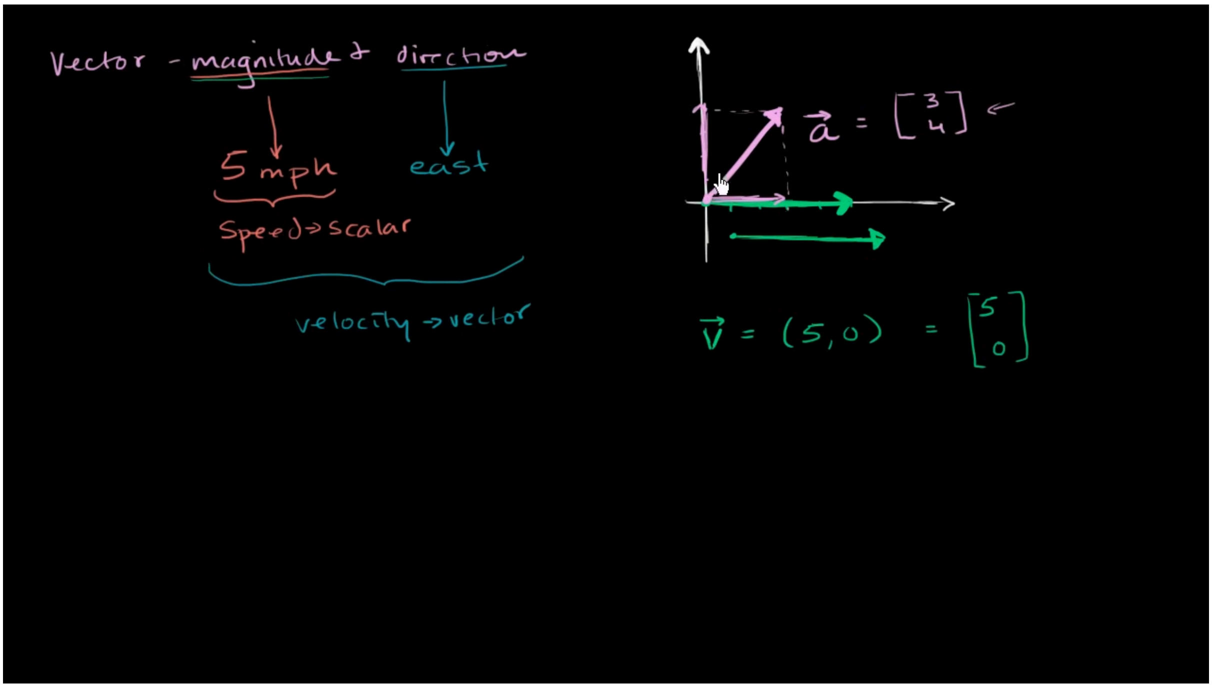
# Introduction to Vectors



Source: [Vector intro for linear algebra (video) | Khan Academy]

## 🧠 What Is a Vector?

A **vector** is a quantity that has:

- **Magnitude** (size or length) ✅
- **Direction** (where it's pointing) ✅

So, a vector = **magnitude + direction**.

---

# 🧮 Scalar vs Vector (Speed vs Velocity)

## 📏 Scalar (Speed)

- Example: "5 mph"
- Only has magnitude
- Does **not** include direction
- Called a **scalar quantity**

## 🧭 Vector (Velocity)

- Example: "5 mph east"
- Has both magnitude **and** direction
- Called a **vector quantity**

🔁 So:

- **Speed** = scalar
- **Velocity** = vector

---

# 🎯 Visualizing Vectors

## 🧾 Notation

- Vectors are often written in **bold** (in textbooks) or with an **arrow** overhead (in handwriting), like:

  → **v** or $\vec{v}$

## ✏️ Column Vector Form

A vector pointing 5 units right (east), with no vertical movement, is written as:

$$\vec{v} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

This tells us:

- 5 units in the **x-direction** (horizontal)
- 0 units in the **y-direction** (vertical)

Or in row format:

$$\vec{v} = (5, 0)$$

🧠 In 2D: the first number is horizontal (x), the second is vertical (y).

---

# 📐 Example

Shown as a diagonal arrow from origin, where:

- x (horizontal) movement = 3 units ➡️
- y (vertical) movement = 4 units ⬆️

This vector is written:

$$\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

## 📏 Magnitude of a Vector

Use the Pythagorean Theorem!

For $\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$:

$$\text{Magnitude} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

🧠 Aha! It's a **3-4-5 triangle**.

---

# 🧭 Properties of Vectors

## ✅ Equivalent Vectors

Two vectors are **equivalent** if:

- They have the **same magnitude**
- They point in the **same direction**

📍 **Doesn't matter where the arrow starts!**

---

# 🧮 Vectors in Higher Dimensions

While we can draw vectors in 2D and 3D...

- 🧠 Our brains can't visualize 4D, 5D, or 20D well.
- But ✨ linear algebra lets us **work with them algebraically**.

Hence, using vector notation like $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$, etc., is **powerful and scalable** to more dimensions.

---

# ✅ Key Takeaways

| Concept | Description |
|---|---|
| Vector | A quantity with both **magnitude** and **direction** |
| Scalar | A quantity with only **magnitude** (e.g. speed) |
| Velocity | A **vector** version of speed (includes direction) |
| Notation | Vectors as tuples $(x, y)$ or columns $\begin{bmatrix} x \\ y \end{bmatrix}$ |
| Vector Length | Calculated with Pythagoras: $\sqrt{x^2 + y^2}$ |
| Equivalent Vectors | Same magnitude + direction, position irrelevant |
| Higher Dimensions | Handled symbolically using notation—beyond visual intuition |