

# Theoretical Questions:

1.

- a.  $\{f : [T2 \rightarrow T3], g : [T1 \rightarrow T2], a : \text{Number}\} \vdash (f (g a)) : T3 \rightarrow \text{true}$

Explanation – we apply f that accepts T2 and return T3, the operand of f is the return of the apply of g with a – g gets T1 and return T2, we assume a (which is a number) is T1, thus g return T2 and f return T3, to conclude the application return T3 as required.

- b.  $\{f : [T1 \rightarrow [T2 \rightarrow \text{Boolean}]], x : T1, y : T2\} \vdash (f x y) : \text{Boolean} \rightarrow \text{false}$

Explanation – Notice! There is one parenthesis missing, we only apply f and not the lambda that returns from  $f(T2 \rightarrow \text{Boolean})$  thus, instead of getting Boolean at the end, we returns a closure.

- c.  $\{f : [T1 \times T2 \rightarrow T3], y : T2\} \vdash (\text{lambda } (x) (f x y)) : [T1 \rightarrow T3] \rightarrow \text{true}$

Explanation – the lambda expression type is the variables – x which is free at the time, and the last expression in the body of the lambda – which is the appliance of f – T3. In f we send x as T1 so in conclusion the type is  $T1 \rightarrow T3$ .

- d.  $\{f : [T2 \rightarrow T1], x : T1, y : T3\} \vdash (f x) : T1 \rightarrow \text{true}$

Explanation – we operate f with x – T1 and f expects T2, so we can infer that T1 equals to T2, , then f return T1 as written.

2. 1.

- a.  $(\text{inter number boolean}) \rightarrow \text{never}$

- b.  $(\text{inter any string}) \rightarrow \text{string}$

- c.  $(\text{union any never}) \rightarrow \text{any}$

- d.  $(\text{diff } (\text{union number string}) \text{ string}) \rightarrow \text{number}$

- e.  $(\text{diff string } (\text{union number string})) \rightarrow \text{never}$

- f.  $(\text{inter } (\text{union boolean number}) (\text{union boolean } (\text{diff string never}))) \rightarrow \text{boolean}$

2. 2.  $[a] \rightarrow \text{is? Boolean}$

$[b] \rightarrow \text{is? Boolean}$

$[c] \rightarrow (\text{isBoolean } z)$

3.  $(\text{union string union}(\text{Boolean number}))$