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1)

a)

$f(x,y) = ?$

$P = (-1,-1)$

$T_2(x,y) = 17 + 3x + 7y + x^2 + xy + 3y^2$

Como $f(x,y)$ es de clase C_2 $f(-1,-1) = T_2(-1,-1)$

$f(-1,-1) = T_2(-1,1) = 17 + 3\cdot(-1) + 7\cdot(-1) + (-1)^2 + (-1)(-1) + 3\cdot(-1)^2$
$= 17 - 3 - 7 + 1 + 1 + 3$
$= 22 - 10$
$\boxed{= 12}$

$f_x(x,y) = T_{2_x}(x,y) = 3 + 2x + y$
$f_x(-1,-1) = T_{2_x}(-1,-1) = 3 - 2 - 1$
$\boxed{= 0}$

$f_y(x,y) = T_{2_y}(x,y) = 7 + x + 6y$
$f_y(-1,-1) = T_{2_y}(-1,-1) = 7 - 1 - 6$
$\boxed{= 0}$

$f_{xx}(x,y) = T_{2_{xx}}(x,y) = 2$
$f_{xx}(-1,-1) = T_{2_{xx}}(-1,-1) \boxed{= 2}$

$f_{xy}(x,y) = T_{2_{xy}}(x,y) = 1$
$f_{xy}(-1,-1) = T_{2_{xy}}(-1,-1) \boxed{= 1} = f_{yx}(-1,-1) = T_{2_{yx}}(-1,-1)$

$f_{yy}(x,y) = T_{2_{yy}}(x,y) = 6$
$f_{yy}(x,y) = T_{2_{yy}}(x,y) \boxed{= 6}$

Miro si el punto $(-1,-1)$ es un punto critico de $f(x,y)$

$$\nabla f(x_0,y_0) = 0$$

$$\nabla f(-1,1) = 0$$

Por lo tanto si es un punto critico

Miro la matriz hessiana

$$Hf = \begin{bmatrix} f_{xx}(x_0,y_0) & f_{xy}(x_0,y_0) \\ f_{yx}(x_0,y_0) & f_{yy}(x_0,y_0) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$$

$$\left.\begin{array}{l} \det(Hf) = 11 > 0 \\ f_{xx}(-1,-1) = 2 > 0 \end{array}\right\} \text{Es un Minimo Local}$$

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1)

b)

Sabemos

$$\lim_{(x,y)\to(x_0,y_0)} \frac{f(x,y) - T_1(x,y)}{\|(x,y)-(-1,-1)\|} = \frac{R_1(x,y)}{\|(x,y)-(-1,-1)\|} = 0$$

$$f(x,y) - T_2(x,y) = R_1(x,y)$$

$$T_1 = 12 + 0(x+1) + 0.(y+1)$$
$$T_1 = 12$$

$$\lim_{(x,y)\to(-1,-1)} \frac{f(x,y) - 12}{\|(x,y)-(-1,-1)}$$

Reemplazamos

$$f(x,y) - 12 = R_1(-1,-1)$$

Por lo tanto el limite queda como

$$\lim_{(x,y)\to(-1,-1)} \frac{R_1(-1,-1)}{\|(x,y)-(-1,-1)}$$

y eso sabemos que tiende a 0

Por lo tanto

$$\lim_{(x,y)\to(-1,-1)} \frac{f(x,y) - 12}{\|(x,y)-(-1,-1)\|} \boxed{= 0}$$

2)

$$f(x,y) = (x-1)(x+y)$$
$$= x^2 + xy - x - y$$

$$f_x(x,y) = 2x + y - 1$$

$$f_y(x,y) = x - 1$$

Busco los puntos críticos en el Interior

Es un punto crítico cuando hace al $\nabla f(x_0,y_0) = 0$

$$\nabla f = (2x + y - 1,\ x - 1)$$

$$\begin{cases} 2x + y - 1 = 0 \\ x - 1 = 0 \end{cases}$$

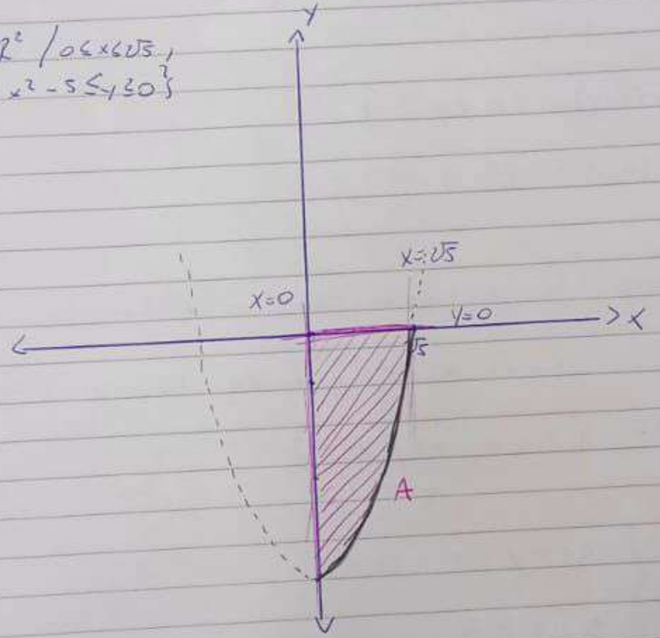
Si $x = 1$

$$2x + y - 1 = 0$$
$$2 \cdot 1 + y - 1 = 0$$
$$2 + y - 1 = 0$$
$$y + 1 = 0$$
$$\boxed{y = -1}$$

$P_1 = (1, -1)$

Ahora Analizo el borde

$A = \{(x,y) \in \mathbb{R}^2 \ / \ 0 \leq x \leq \sqrt{5},\ x^2 - 5 \leq y \leq 0\}$



→ ANULO

Miro Lagrange

$\nabla f = (2x + y - 1,\ x - 1)$

Tomo como $g(x,y)$ la restriccion

$\nabla g(x,y) = (2x, 0)$

Separo el borde en 3 partes: $\begin{cases} x = 0 \\ y = 0 \\ y = x^2 - 5 \end{cases}$

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⑨

Miro la recta $x=0$ de la forma $(0,y)$

Miro $f(0,y) = -y$

$\nabla f(x,y) = (0,-1)$

$P_2(0,-1)$

Miro la recta $y=0$ de la forma $(x,0)$

Miro $f(x,0) = x^2 - x$

$\nabla f(x,y) = (2x, 0)$

$2x = 0$

$\boxed{x=0}$

$P_3 = (0,0)$

Miro la recta $x=\sqrt{5}$ de la forma $(\sqrt{5},y)$

$f(\sqrt{5},y) = 5 + \sqrt{5}y - \sqrt{5} - y$

$\nabla f(x,y) = (0, \sqrt{5}-1)$

ANULO

$\sqrt{5}-1 = 0$

Abs!

$\boxed{\frac{\pi}{6} = \varphi}$

Y por ultimo miro la curva x^2-5 de la forma (x, x^2-5)

$$f(x, x^2-5) = x^2 + x(x^2-5) - (x^2-5) - x$$

$$= \cancel{x^2} + x^3 - 5x - \cancel{x^2} + 5 - x$$

$$= x^3 - 6x + 5$$

$$\nabla f(x, y) = (3x^2 - 6, 0)$$

$$3x^2 - 6 = 0$$

$$3x^2 = 6$$

$$x^2 = \frac{6}{3}$$

$$x^2 = 2$$

$$\boxed{x = \sqrt{2}}$$

$$P_4 = (\sqrt{2}, 0)$$

Resumen delos puntos encontrados

$P_1 = (1, -1)$

$P_2 = (0, -1)$

$P_3 = (0, 0)$

$P_4 = (\sqrt{2}, 0)$

Evaluo esos puntos en $f(x,y)$

$f(1,-1) = 1^2 + 1.(-1) - 1 - (-1)$

$= 1 + (-1) - 1 + 1$

$\boxed{= 0}$ -> Minimo Absoluto

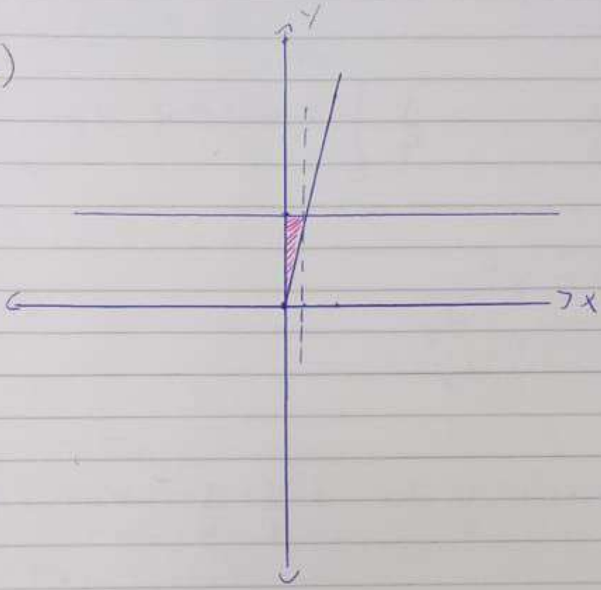
$f(0,-1) \boxed{= 1}$ -> Maximo Absoluto

$f(0,0) \boxed{= 0}$ -> Minimo Absoluto

$f(\sqrt{2},0) = 2 - \sqrt{2}$

3)

a)



$0 \le x \le \frac{\sqrt{\pi}}{4}$

$4x \le y \le \sqrt{\pi}$

Como no se puede integrar, cambio los limites de Integracion

$0 \le 4x \le y$

$\boxed{0 \le x \le \frac{y}{4}}$

$\boxed{0 \le y \le \sqrt{\pi}}$

Por Fubini puedo invertir el orden de integraci

$$\int_0^{\sqrt{\pi}}\int_0^{\frac{y}{4}} \sin(y^2)\, dx\, dy$$

Ahora si hay que una integral facil de calcular

3)

$\frac{d}{dx}\cos(x) = -\sin(x)$

$$\sin(y^2)\int_0^{\frac{y}{4}} dx$$

$$\int_0^{\sqrt{\pi}} \sin(y^2)\cdot\frac{y}{4} = \frac{1}{4}\int_0^{\sqrt{\pi}} \sin(y^2)\cdot y\, dy$$

$$u = y^2$$
$$du = 2y\, dy$$
$$\frac{du}{2} = y\, dy$$

$$\frac{1}{4}\int_0^{\sqrt{\pi}} \sin(u)\cdot\frac{1}{2}\, du = \frac{1}{4}\cdot\frac{1}{2}\int_0^{\sqrt{\pi}} \sin(u)\, du$$

$$\frac{1}{8}\int_0^{\sqrt{\pi}} \sin(u)\, du = \frac{1}{8}\cdot(-\cos(u))\Big|_0^{\sqrt{\pi}}$$

$$\frac{-\cos(y^2)}{8}\Big|_0^{\sqrt{\pi}} = \left(-\frac{\cos(\sqrt{\pi}^2)}{8}\right) - \left(-\frac{\cos(0^2)}{8}\right)$$

$$\frac{1}{8} + \frac{1}{8} = \frac{2}{8} \boxed{= \frac{1}{4}}$$

$\sqrt{}^{2}\ dV$

$, z \geq \frac{3\sqrt{3}}{2} \}$

3)

b)

$x > 0 \quad , \quad x = 4$

$y > 0 \quad , \quad y = \sqrt{x}$

$z > 0 \quad , \quad z = 2 - y$

$0 \leq x \leq 4$

$0 \leq y \leq \sqrt{x}$

$0 \leq z \leq 2 - y$

$$\int_0^4 \int_0^{\sqrt{x}} \int_0^{2-y} 1 \, dz \, dy \, dx$$

$$\int_0^{2-y} 1 \, dz = 2 - y$$

$$\int_0^{\sqrt{x}} 2 - y \, dy = 2\int_0^{\sqrt{x}} dy - \int_0^{\sqrt{x}} y \, dy =$$

$$2y - \frac{y^2}{2} \Big|_0^{\sqrt{x}} = 2\sqrt{x} - \frac{\sqrt{x}^2}{2} = 2\sqrt{x} - \frac{x}{2}$$

$$\int_0^4 2\sqrt{x} - \frac{x}{2} \, dx$$

$$2\int_0^4 x^{1/2} - \frac{1}{2}\int_0^4 x \, dx$$

$$\left(2\cdot\frac{2x^{3/2}}{3}\right) - \left(\frac{1}{2}\cdot\frac{x^2}{2}\right)\Bigg|_0^4 = \frac{4x^{3/2}}{3} - \frac{x^2}{4}\Bigg|_0^4$$

$$\frac{4\cdot 4^{3/2}}{3} - \frac{4^2}{4} =$$

$$\boxed{\frac{4}{3}\cdot 4^{3/2} - 4} = \boxed{\frac{20}{3}}$$

1) $\iiint_W \frac{1}{(x^2+y^2+z^2)^2}\, dV$

$W = \left\{ x^2+y^2+z^2 \le 9 \quad ; z \geq \frac{3\sqrt{3}}{2} \right\}$

Coordenadas esfericas

$$\begin{cases} x = \rho \sin(\varphi)\cos(\theta) \\ y = \rho \operatorname{sen}(\varphi)\sin(\theta) \\ z = \rho\cos(\varphi) \end{cases}$$

$\rho^2 = x^2+y^2+z^2$

$J\Phi = |\rho^2 \sin(\varphi)| \qquad \varphi \in [0, \pi]$

Paso a coordenadas esfericas

$x^2+y^2+z^2 = 9$

$\rho^2 = 9$

$\rho = \sqrt{9}$

$\boxed{\rho = 3}$

$z = \rho\cos(\varphi)$

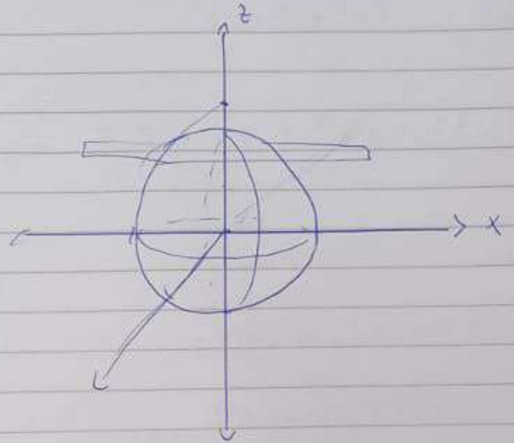
$\frac{3\sqrt{3}}{2} = 3\cos(\varphi)$

$\frac{3\sqrt{3}}{6} = \cos(\varphi)$

$\boxed{\frac{\pi}{6} = \varphi}$

4. (2 puntos) Sea W

$$\iiint \frac{1}{\rho^4} \cdot |\rho^2 \sin(\varphi)| \; dV$$



$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 3$

$0 \leq \varphi \leq \frac{\pi}{6}$

$$\int_0^{2\pi}\int_0^{3}\int_0^{\frac{\pi}{6}} \frac{\cancel{\rho}\,\sin(\varphi)}{\rho^2} \; d\varphi \, d\rho \, d\theta$$

$$\frac{1}{\rho^2}\int_0^{\frac{\pi}{6}} \operatorname{sen}(\varphi) \; d\rho = \left(\frac{1}{\rho^2} \cdot \left(-\cos(\varphi)\right)\right)\Bigg|_0^{\frac{\pi}{6}}$$

$$-\frac{\cos\left(\frac{\pi}{6}\right)}{\rho^2} + \frac{\cos(0)}{\rho^2} = -\frac{\cos\left(\frac{\pi}{6}\right)}{\rho^2} + \frac{1}{\rho^2} = \frac{-\cos\left(\frac{\pi}{6}\right) - 1}{\rho^2}$$

$$2\pi \int_0^3 \frac{-\cos(\frac{\pi}{6})-1}{\rho^2}\, d\rho$$

$$2\pi \cdot -\cos(\frac{\pi}{6})-1 \int_0^3 \frac{1}{\rho^2}\, d\rho$$

$$2\pi \cdot -\cos(\frac{\pi}{6})-1 \cdot \ln(\rho^2)\Big|_0^3$$

$$\left(-2\pi\cos(\frac{\pi}{6})-1)\cdot \ln(9)\right)$$