Ej 1:

$$A_n = \begin{pmatrix} \frac{1}{n} & 0 & \cdots & n^2 \\ 0 & \frac{1}{n} & \cdots & 0 \\ & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{n} \end{pmatrix} \in \mathbb{R}^{n \times n}$$

a) $\|A_n\|_\infty = \frac{1}{n} + n^2$

$$\mathrm{Cond}_\infty(A_n) \geq \frac{\|A_n\|_\infty}{\|A_n - B_n\|_\infty}$$ B_n singular.

$$B_n = \begin{pmatrix} 0 & \cdots & n^2 \\ & \ddots & \\ & & 0 \end{pmatrix} \qquad A_n - B_n = \frac{1}{n} I$$

$$\|A_n - B_n\|_\infty = \frac{1}{n}$$

$$\frac{\frac{1}{n} + n^2}{\frac{1}{n}} = 1 + n^3 \longrightarrow +\infty.$$

b) $\mathrm{Cond}_2(A_n)$

$$\geq \frac{1}{n} \mathrm{Cond}_\infty(A_n)$$

$$\|x\|_2 \leq \sqrt{n} \|x\|_\infty$$

$$\|x\|_\infty \leq \|x\|_2$$

$$\Rightarrow \frac{\|Ax\|_2}{\|x\|_2} \leq \frac{\sqrt{n}\|Ax\|_\infty}{\|x\|_\infty}$$

$$\frac{\|Ax\|_\infty}{\|x\|_\infty} \leq \frac{\sqrt{n}\|Ax\|_2}{\|x\|_2}$$

Ej 2

$$\begin{array}{c} \begin{array}{cccc} B & LG & T & NP \end{array} \\ P = \begin{pmatrix} a & 0.5 & 0 & 0.5 \\ b & 0.5 & 0 & 0 \\ d & 0 & 1 & 0 \\ c & 0 & 0 & 0.5 \end{pmatrix} \end{array}$$

$a+b+c+d = 1$

Además

$$P \cdot \frac{1}{8}\begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

$3a + 1 + 0.5 = 3 \qquad 3a = 1.5 = \frac{3}{2} \quad a = \frac{1}{2}$

$3b + 1 = 2 \qquad b = 1/3$

$3d + 2 = 2 \qquad d = 0$

$3c + 0.5 = 1 \qquad c = 1/6$

$$\Rightarrow P = \begin{pmatrix} 1/2 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/6 & 0 & 0 & 1/2 \end{pmatrix}$$

$$\lambda I - P = \begin{pmatrix} \lambda - 1/2 & -1/2 & 0 & -1/2 \\ -1/3 & \lambda - 1/2 & 0 & 0 \\ 0 & 0 & \lambda - 1 & 0 \\ -1/6 & 0 & 0 & \lambda - \frac{1}{2} \end{pmatrix}$$

$$\Rightarrow \chi_A(\lambda) = (\lambda - 1) \cdot \det \begin{pmatrix} \lambda - 1/2 & -1/2 & -1/2 \\ -1/3 & \lambda - 1/2 & 0 \\ -1/6 & 0 & \lambda - 1/2 \end{pmatrix}$$

$$= (\lambda - 1) \cdot \left[\frac{1}{3} \cdot \left(-\frac{1}{2} (\lambda - 1/2) \right) + (\lambda - 1/2) \left[\left(\lambda - \frac{1}{2}\right)^2 - \frac{1}{12} \right] \right]$$

$$= (\lambda - 1) \left(\lambda - \frac{1}{2}\right) \left[\underbrace{-\frac{1}{6} + \left(\lambda - \frac{1}{2}\right)^2 - \frac{1}{12}}_{} \right]$$

$$\left(\lambda - \frac{1}{2}\right)^2 = \frac{3}{12} = \frac{1}{4}$$

$$\lambda = 1 \text{ ó } \lambda = 0$$

a)

-1 no es autovalor $\Rightarrow$ existe P^{∞}.

b) $$v^{(0)} = \begin{pmatrix} 300 \\ 100 \\ 0 \\ 300 \end{pmatrix}$$

Autovectores:

$\lambda = 1$

$$\begin{pmatrix} 1/2 & -1/2 & 0 & -1/2 \\ -1/3 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/6 & 0 & 0 & 1/2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & -1 \\ -2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{2} \end{pmatrix}$$

$x_2 = 2x_4$

$x_1 = x_2 + x_4 = 3x_4$

$(3x_4, 2x_4, x_3, x_4)$

$$E_1 = \langle (3,2,0,1), (0,0,1,0) \rangle$$

$\lambda = 0$

$$\begin{pmatrix} -1/2 & -1/2 & 0 & -1/2 \\ -1/3 & -1/2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1/6 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{2} \end{pmatrix}$$

$x_3 = 0, \quad x_2 = 2x_4$

$x_1 = -x_2 - x_4 = -3x_4$

$(-3x_4, 2x_4, 0, x_4)$

$$E_0 = \langle (-3,2,0,1) \rangle$$

$\lambda = 1/2$

$$\begin{pmatrix} 0 & -1/2 & 0 & -1/2 \\ -1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ -1/6 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$x_3 = x_1 = 0 \quad x_2 = -x_4 \quad \langle (0,1,0,-1) \rangle = E_{1/2}.$

$$\left(\begin{array}{cccc|c} 3 & 0 & -3 & 0 & 3 \\ 2 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 2 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -5 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & -1 \\ 0 & 0 & 0 & 3 & -5 \end{array}\right)$$

$\alpha_4 = -5/3$

$4\alpha_3 + \alpha_4 = -1 \quad \alpha_3 = 1/6$

$\alpha_2 = 0$

$\alpha_1 = \alpha_3 + 1 = 7/6$

Todo $\times$ 100:

$$\Rightarrow \begin{pmatrix} 300 \\ 100 \\ 0 \\ 300 \end{pmatrix} = \frac{700}{6} \cdot \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \frac{100}{6} \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \frac{500}{3} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow P^{10} \begin{pmatrix} 300 \\ 100 \\ 0 \\ 300 \end{pmatrix} = \frac{700}{6} \cdot \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \frac{100}{6} \cdot 0 \cdot \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \frac{500}{3} \left(\frac{1}{2}\right)^{10} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

NP a los 20 min $\rightarrow \frac{700}{6} + \frac{500}{3} \cdot \left(\frac{1}{2}\right)^{10}$

3) $A = \begin{pmatrix} 4 & \alpha+2 & 2 \\ \alpha^2 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

$\alpha^2 = \alpha + 2 \iff \alpha^2 - \alpha - 2 = 0 \iff \alpha = \frac{1 \pm \sqrt{1+4 \cdot 2}}{2}$

$\alpha = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$

$\alpha = 2$

$A = \begin{pmatrix} 4 & 4 & 2 \\ 4 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ $\det A = 0 \Rightarrow \lambda = 0$ es autovalor.

$\alpha = -1$

$A = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 2 \\ 0 & -6 & -3 \\ 0 & -15 & -6 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 0 & 5 & 2 \end{pmatrix}$ $\lambda = 0$ no es autovalor.

$\alpha = 2$ $\quad A = \begin{pmatrix} 4 & 4 & 2 \\ 4 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

$$(\lambda I - A) = \begin{pmatrix} \lambda-4 & -4 & -2 \\ -4 & \lambda-4 & -2 \\ -2 & -2 & \lambda-1 \end{pmatrix}$$

$$\det(\lambda I - A) = (\lambda-4)\left[(\lambda-4)(\lambda-1)-4\right]$$
$$+4\left(-4(\lambda-1)-4\right) - 2\left(8+2(\lambda-4)\right)$$
$$= (\lambda-4)(\lambda^2-5\lambda) + 4(-4\lambda) - 4\lambda$$
$$= \lambda\left[\underbrace{(\lambda-4)(\lambda-5)-16-4}_{\lambda^2-9\lambda}\right]$$
$$= \lambda^2(\lambda-9)$$

autoval 0 (mult 2) y 9

$\lambda = 0$ $\quad E_0 = \langle (1,0,-2), (1,-1,0) \rangle$

$\lambda = 9$ $\quad \begin{matrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{matrix} \sim \begin{matrix} 1 & 1 & -4 \\ 0 & -9 & 18 \\ \cancel{0} & \cancel{-9} & \cancel{18} \end{matrix}$

$$x_1+x_2-4x_3=0 \qquad -x_2+2x_3=0$$

$$x_1-2x_3=0 \quad \Leftarrow \quad x_2=2x_3$$

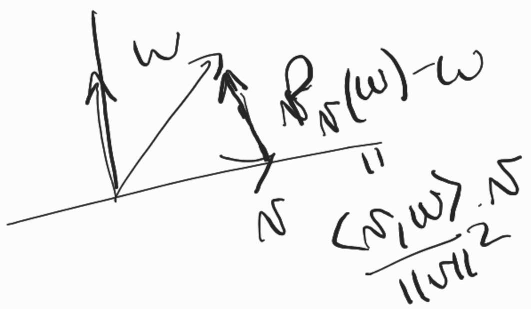
$$x_1=2x_3$$

$$E_9=\langle(2,2,1)\rangle \rightsquigarrow v_3=\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$$

Ortogonalizo en E_0.

$$\tilde{v}_1=(1,0,-2) \qquad v_1=\left(\frac{1}{\sqrt{5}},0,\frac{-2}{\sqrt{5}}\right)=\left(\frac{1}{5}\sqrt{5},0,\frac{-2}{5}\sqrt{5}\right)$$

$$\tilde{v}_2=\underbrace{(1,-1,0)}_{w}-\underbrace{\langle(1,0,-2),(1,-1,0)\rangle\frac{(1,0,-2)}{5}}_{P_v(w)}$$



$$=(1,-1,0)-\frac{(1,0,-2)}{5}$$

$$\frac{16}{25}+1+\frac{4}{25}=\frac{45}{25}=\frac{9}{5} \qquad =\left(\frac{4}{5},-1,\frac{2}{5}\right)$$

$$v_2=\left(\frac{4}{5},-1,\frac{2}{5}\right)\Big/\frac{3}{\sqrt{5}}=\left(\frac{4\sqrt{5}}{3},-\frac{5}{3}\sqrt{5},\frac{2}{3}\sqrt{3}\right)$$

Recu 2do C 2022

Ej: $0 < \varepsilon < 1$ $\quad A_\varepsilon = \begin{pmatrix} 1+\varepsilon & 1 & 1-\varepsilon \\ \varepsilon & \varepsilon & 0 \\ \varepsilon-1 & 0 & -1 \end{pmatrix}$

a) $\varepsilon = 10^{-3}$ $\quad Ax = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ con elim. 2 dígitos y redondeo

$= 0.001$

$fl(1+10^{-3}) = fl(1.001) = 1$

$fl(1-\varepsilon) = fl(0.999) = 1$

$fl(\varepsilon-1) = fl(-0.999) = -1$

$fl(10^{-3}) = 0.1 \times 10^{-2}$

$fl(1.001) = 0.10 \times 10 = 1$

$fl(A_\varepsilon) = \begin{pmatrix} 1 & 1 & 1 \\ \varepsilon & \varepsilon & 0 \\ -1 & 0 & -1 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ \varepsilon & \varepsilon & 0 & 2 \\ -1 & 0 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -\varepsilon & 2 \\ 0 & 1 & 0 & 0 \end{array}\right)$ $\rightarrow fl(2-\varepsilon) = 2$

$x_2 = 0$ $\quad -\varepsilon \cdot x_3 = 2$

$x_3 = fl(-2/\varepsilon) = -2000$

$\frac{-2}{0.001} = -2000$

$x_1 = 1 - x_2 - x_3 = 2001$

$x = (2001, 0, 2000)$

chequeo soluciones $\left\{ \begin{array}{l} (1+\varepsilon)(2001) + (1-\varepsilon)(-2000) = 1 + \varepsilon \cdot 4001 \simeq 5 \text{ lejos de } 1 \\ \varepsilon \cdot 2001 + \varepsilon \cdot 0 \simeq 2 \\ (\varepsilon - 1) \cdot 2001 - 2000 = 1 + \varepsilon \cdot 2001 \simeq 3 \text{ lejos de } -1. \end{array} \right.$

B_ε sing

$$\mathrm{Cond}_\infty (A_\varepsilon) \geq \frac{\|A_\varepsilon\|_\infty}{\|A_\varepsilon - B_\varepsilon\|_\infty} = \frac{3}{2\varepsilon} \xrightarrow[\varepsilon \to 0]{} +\infty$$

$$\|A_\varepsilon\|_\infty = \max \{ 1 + \cancel{\varepsilon} + 1 + \underbrace{1 \cancel{\varepsilon}}_{>0}, 2\varepsilon, 1 - \varepsilon + 1 \}$$

$$3, 2\varepsilon, 2 - \varepsilon$$

$$\|A_\varepsilon\|_\infty = 3$$

$$B_\varepsilon = \begin{pmatrix} 1+\varepsilon & 1 & 1-\varepsilon \\ 0 & 0 & 0 \\ \varepsilon - 1 & 0 & -1 \end{pmatrix}$$

$$A_\varepsilon - B_\varepsilon = \begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\|A_\varepsilon - B_\varepsilon\|_\infty = 2\varepsilon$$

1^{er} P $\quad 1^{\text{er}}$ C 2022

Ej 1: $f: \mathbb{R}^4 \to \mathbb{R}^4$ proy. ortog en

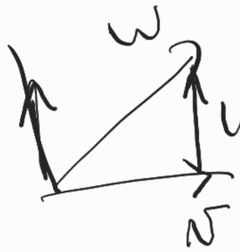
$S = \langle (1,1,1,1), (1,2,1,0) \rangle$.

$T = \{ x_1 - x_2 + x_3 = 0 \quad x_2 = x_4 \}$

$W = \langle (2,0,-2,0), (-2,1,0,1) (2,1,-4,1) \rangle$

a) $S = \langle \overbrace{(1,1,1,1)}^{v_1}, (0,1,0,-1) \rangle$

$v_2 = (1,2,1,0) - \cancel{4} \cdot \dfrac{(1,1,1,1)}{\cancel{4}} = (0,1,0,-1)$



$w - \underbrace{P_v(w)}_{\frac{\langle v, w \rangle}{\|v\|^2} \cdot v}$

$$f(x) = P_S(x) = P_{(1,1,1,1)}(x) + P_{(0,1,0,-1)}(x)$$

$$= \frac{\langle (1,1,1,1), x \rangle}{4} \cdot (1,1,1,1)$$

$$+ \frac{\langle (0,1,0,-1), x \rangle}{2} (0,1,0,-1)$$

$$= (x_1 + x_2 + x_3 + x_4) \left(\tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4} \right)$$

$$+ (x_2 - x_4)\left(0, \tfrac{1}{2}, 0, -\tfrac{1}{2}\right)$$

$$= \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{x_1+x_2+x_3+x_4}{4} + \frac{x_2}{2} - \frac{x_4}{2}, \frac{x_1+x_2+x_3+x_4}{4},\right.$$

$$\left.\frac{x_1+x_2+x_3+x_4}{4} - \frac{x_2}{2} + \frac{x_4}{2}\right)$$

$$[P_S]_E = \begin{pmatrix} 1/\sqrt{4} & 0 \\ 1/\sqrt{4} & 1/\sqrt{2} \\ 1/\sqrt{4} & 0 \\ 1/\sqrt{4} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} \\ 0 & 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

Queremos $g: \mathbb{R}^4 \to \mathbb{R}^4$

$g(W) = \langle (2,0,1,1)(0,-1,2,2) \rangle = U$

$g(v) = P_S(v) \ \forall v \in T$

$T \cap W = ?$

W:

$$\begin{matrix} 2 & 0 & -2 & 0 \\ -2 & 1 & 0 & 1 \\ 2 & 1 & -4 & 1 \end{matrix} \sim \begin{matrix} 2 & 0 & -2 & 0 \\ 0 & 1 & -2 & 1 \\ \cancel{0} & \cancel{1} & \cancel{-2} & \cancel{1} \end{matrix}$$

$$W = \langle (1,0,-1,0), (0,1,-2,1) \rangle$$

$$(\alpha, \beta, -\alpha-2\beta, \beta) \in T\,?$$

$$\cancel{\alpha} - \beta + \cancel{-\alpha} - 2\beta = 0 \quad \Rightarrow \beta = 0$$

$$-\beta + \beta = 0 \ \checkmark$$

$$T \cap W = \langle (1,0,-1,0) \rangle = \langle (-1,0,1,0) \rangle$$

T:

$$x_1 = x_2 - x_3 \qquad (x_2 - x_3, x_2, x_3, x_2)$$

$$x_4 = x_2 \qquad T = \langle (1,1,0,1), (-1,0,1,0) \rangle$$

Base de $T \cap W = \{(-1,0,1,0)\}$

Base de $T = \{(-1,0,1,0), (1,1,0,1)\}$

Base de $W = \{(-1,0,1,0), (0,1,-2,1)\}$

$$g(-1,0,1,0) = P_S(-1,0,1,0) =$$

$$P_S(-1,0,1,0) = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \in U = \langle (2,0,1,1) \ (0,-1,2,2) \rangle$$

$$g(1,1,0,1) = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/4 \\ 3/4 \\ 3/4 \\ 3/4 \end{pmatrix}$$

$g(0,1,-2,1) =$ elijo uno en $U = u$

$\Rightarrow g(W) = \langle u \rangle \Rightarrow \dim g(W) = 1$.

Pedían $g(W) = U$ con $\dim U = 2$

$\Rightarrow$ no se puede definir g.

b) $h: \mathbb{R}^4 \to \mathbb{R}^3$ ✱ $4 = \underbrace{\dim \operatorname{Im} h}_{\leq 3} + \underbrace{\dim \operatorname{Nu} h}_{\geq 1}$

$h = (2x_1 + 4x_2 + 2x_3, -2x_1 + 4x_2 + 2x_3, 4x_4)$

$$[h]_{EE} = \begin{pmatrix} 2 & 4 & 2 & 0 \\ -2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$h \circ f(e_1) = h\left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\right) = (2, 1, 1)$

$h \circ f(e_2) = h\left(\frac{1}{4}\ \frac{3}{4}\ \frac{1}{4}\ -\frac{1}{4}\right) = (4, 3, -1)$

$h \circ f(e_3) = h\left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\right) = (2, 1, 1)$

$h \circ f(e_4) = h\left(\frac{1}{4}\ -\frac{1}{4}\ \frac{1}{4}\ \frac{3}{4}\right) = (0, -1, 3)$

$\langle (2,1,1)\ (4,3,-1)\ \cancel{(2,1,1)}\ (0,-1,3) \rangle$

$= \langle (2,1,1), (0,-1,3) \rangle$

$$\begin{matrix} 2 & 1 & 1 \\ 0 & -1 & 3 \\ 4 & 3 & -1 \end{matrix} \sim \begin{matrix} 2 & 1 & 1 \\ 0 & -1 & 3 \\ \cancel{0} & \cancel{-1} & \cancel{3} \end{matrix}$$

$\dim \operatorname{Im} h \circ f = 2$

$h \circ f : \mathbb{R}^4 \to \mathbb{R}^3$

$\Rightarrow h \circ f$ no es epi

y por ello dim ✱ no puede ser mono