

## EFM Week 6 Assignment

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In this assignment, we have just one objective: to test the CAPM. We've also included some simple NumPy questions at the start to get your warmed up. If you have any questions, feel free to email me (zzwang@uchicago.edu) or Allan (zhangallan@uchicago.edu) or Jason (zphang@uchicago.edu).

### 1 NUMPY EXERCISES

#### 1. Bernoulli Distribution

- a) We want to generate the results of 10,000 coin flips, where  $H = 1$  and  $T = 0$ . Generate a NumPy array of 10,000 random numbers, using **numpy.random.random**. This produces a random draw of real numbers between 0 and 1.
- b) Produce a corresponding sequence of 1's (heads) and 0's (tails) by checking the each element of the array is larger than  $p = 0.5$ . This is NumPy, so this can be done in a single vector operation! (Your result should be a boolean array of length 10,000).
- c) Calculate the mean and variance of your draws using the relevant NumPy functions. What we have just done is 10,000 draws from something called a Bernoulli distribution with parameter  $p = 0.5$ , which gives 1 with probability  $p$  and 0 with probability  $p$ . This distribution has a theoretical mean and variance of:

$$\text{Mean} = p$$

$$\text{Variance} = p(1 - p)$$

Do you get approximately correct mean and variance numbers?

- d) Now repeat the above exercise using  $p = 0.1$ ,  $p = 0.3$  and  $p = 0.9$ . Are your numbers still in line with the above formulae?

## 2. Normal Distribution

- a) Now we want to draw  $n = 10,000$  standard normal numbers. Do the same as the above, except using **numpy.random.randn**.
- b) Calculate the mean and variance. Since this is a standard normal distribution, it should have mean 0 and variance 1.
- c) From statistics, we know that

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

Where  $X$  is our random variable, and  $a$  and  $b$  are constants.

Now, choose your favorite  $a$  and  $b$  (say,  $a = 3.14$  and  $b = 2014$ ). Multiply your existing array of normal draws by  $a$  and add  $b$ , and see if your mean and variance change as predicted.

## 2 TESTING THE CAPM

**Note:** If you are short on time, I would prefer that you spent your time thinking about parts 4 and 5, rather than parts 2 and 3. Thus, if you wish to use it, I have attached python code for a function that takes in a ticker as a string and returns a list of monthly returns, which should greatly speed up parts 2 and 3.

1. Skim through the Quandl Python guide (found here: "<https://www.quandl.com/help/python>") and download the Quandl Python library.
2. Get monthly adjusted closing prices of (1) the S&P 500 index and (2) all of the stocks in the DJIA 30 (with the exception of Visa) from Jan 1, 2007 to December 31, 2013.

For your convenience, I have listed all of the tickers (minus VISA) here:

```
stock_list = ["AXP", "BA", "CAT", "CSCO", "CVX", "DD", "DIS", "GE", "GS", "HD", "IBM",
              "INTC", "JNJ", "JPM", "KO", "MCD", "MMM", "MRK", "MSFT", "NKE", "PFE", "PG", "T", "TRV",
              "UNH", "UTX", "VZ", "WMT", "XOM", "INDEX_GSPC"]
```

3. Write a function that takes these adjusted closing prices and gives us the monthly returns of the S&P 500 index as well as each of the 29 specified members of the DJIA 30 over this time period.

4. For each stock  $i$ , calculate the  $\beta^{i,M}$  and the expected returns of the stock. Recall that  $\beta^{i,M}$  of a stock is given by:

$$\beta^{i,M} = \frac{\sigma_{i,M}}{\sigma_M^2},$$

where we use S&P 500 returns (INDEX\_GSPC) to represent the market.

5. Plot each stock by plotting  $\beta^{i,M}$  on the x-axis and expected return on the y-axis. What would CAPM expect this graph to look like? What does it actually look like?