EFM Fall 2014, Week 5: Portfolio Theory

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Introduction

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- But portfolio theory is very relevant in that it models how agents optimize
- Can apply to: 1) Investors, 2) Firm investment decisions

Early Beginnings and Assumptions

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- 3) All agents have perfect information
- Do you think these assumptions always hold?

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- Stop me at any point if I lose you

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- $\sigma_{i,j}$ represents the covariance of the return on any two assets

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- Note the magnitude doesn't say too much, but we can normalize covariance to see how much they change linearly with each other (this is called the correlation coefficient)

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- $\sigma^2 = Var[R] = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} x_i x_j$ - The variance of the portfolio must be the weighted average of the variances of each asset and how it varies with every other asset

Full Maximization Problem

$$\min_{\{x_i\}_{i=1}^{n}} \sigma^2 - A\mu$$

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- $A \in [0, \infty)$ = Risk Aversion Index. To see what this means: What happens if A = 0? What happens if A = ∞?

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- $A \in [0, \infty)$ = Risk Aversion Index. To see what this means: What happens if A = 0? What happens if A = ∞?
- Because of the set up of the maximization problem, the resulting portfolio is often called a Mean-Variance Efficient Portfolio.

Maximization Solution: Graphical

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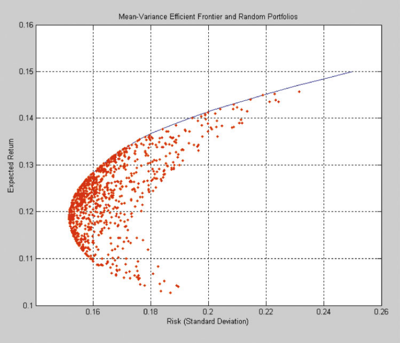
- Once you go above 3 assets, people get lazy and just represent the resulting portfolios graphically, with something like this:



Maximization Solution: Analytical

But being at UChicago we're not really satisfied with that, so we want an analytical solution:

$$\sigma^2 = \frac{Z\mu^2 - 2X\mu + Y}{YZ - X^2}$$

$$X = \sum_1^n \sum_1^n \sigma_{i,j}^T \mu_j$$

$$Y = \sum_1^n \sum_1^n \sigma_{i,j}^T \mu_j \mu_i$$

$$Z = \sum_1^n \sum_1^n \sigma_{i,j}^T$$

So you can see that σ is parabolic in μ, and you get a hyperbola when you flip the axes. For a detailed derivation of the above, see Merton 1972. And as for where A went, I was too lazy in tracking exactly where it goes in these equations, but it's just constant so whatever.

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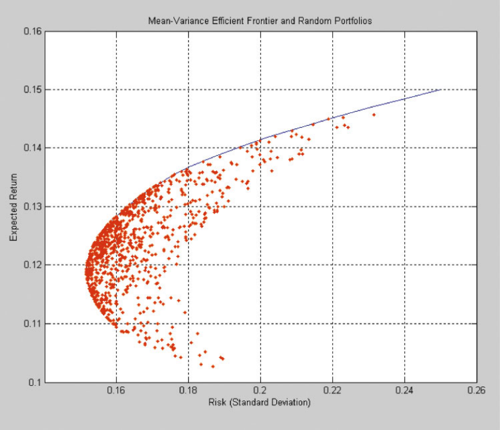
Which Portfolio Would You Pick?

Going back to the previous graph (called the Efficient Frontier), which portfolio would you pick?:



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- Or you could just note that clearly the top left of this graph is the best, and the bottom right is the worst.
- But without appealing to the investor's utility function, can we figure out what portfolio he would ideally choose?

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- If you're unconvinced by 2), think of the "every other risky asset" as a portfolio of risky assets that have already been maximized over.

Finding the New Optimal Portfolio

Our portfolio with the above conditions is:

$$\mu = (1-x)\mu_f + x\mu_r = \mu_f + x(\mu_r - \mu_f)$$
$$\sigma^2 = (1-x)^2\sigma_{f,f} + 2x(1-x)\sigma_{f,r} + x^2\sigma_{r,r} = x^2\sigma_r^2$$

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Where f denotes the risk-free asset, r denotes the risky asset, and x is the share of the portfolio given to the risky asset. Now we can solve for x in the second equation and substitute:

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Interpretation? What happens when $\sigma = 0$? If the investor wants to maximize expected returns, what should he do?

Solving the New Maximization Problem

Now let's maximize our portfolio for the previous problem. The two asset example from before is really trivial, so let's look at the general set up:

$$\max_{\{x_i\}_{i=1}^{n}} \frac{\mu_r - \mu_f}{\sigma_r}$$

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Why don't we maximize over $\mu = \mu_f + \frac{\mu_r - \mu_f}{\sigma_r}\sigma$?

Results

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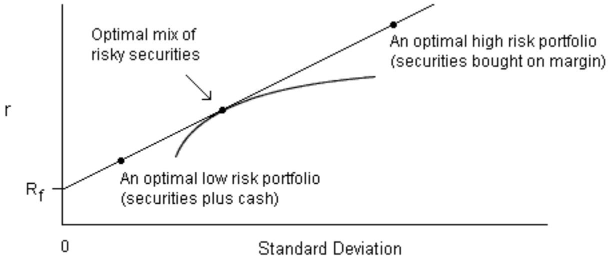

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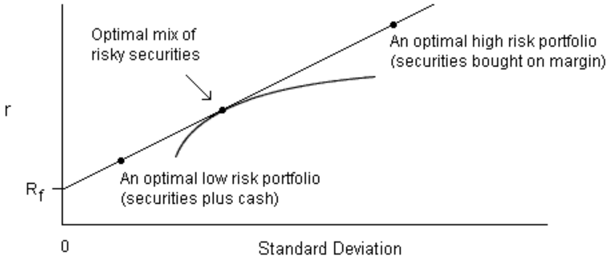

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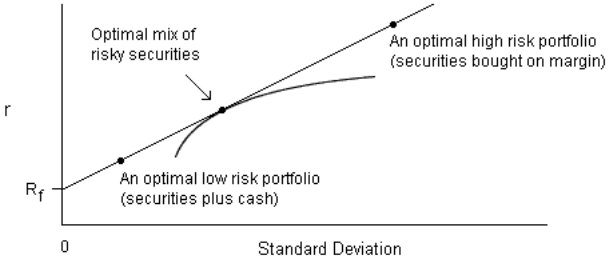

- The line is called the **Capital Allocation Line**. What does the line represent?
- The fact that you can mix portfolios to get to any other point on the line is a non-rigorous proof of the **Two Mutual Fund Theorem**.

Slight Detour: The Sharpe Ratio

The slope of the Capital Allocation Line, which we found before was: $\frac{\mu_r - \mu_f}{\sigma_r}$ is generally known as the **Sharpe Ratio**, and is an important measure of how strong a portfolio is.

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- The elephant in the room: What would Sharpe ratios look like following a market readjustment (big or small?). As an extreme example, what happens to the ratio after the crash of 2008?

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- In general: $2n + \binom{n}{2}$ pieces of information.

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- Getting covariance data in particular is really, really hard, and the models we will tackle later will attempt to solve this issue.

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- Behavioral Issues/Preference Shocks
- Equilibrium Changes and "Follow the Leader" Problems

Next Week

To-Do

- Practical assignment for next week! You know the theory and the concepts, so apply them using Python
- If you get stuck, feel free to e-mail Jason/Allan with questions

Next Week

- We make our first stab at eliminating the need for covariances in our data and to finally sink our teeth into building a good portfolio.
- Primary topic will be the **Capital Asset Pricing Model (CAPM)**