IBEHS 3A03

Assignment 2 – LTI System Responses and Convolution

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Introduction

In lectures we studied the concepts of impulse response, step response, and convolution. Now, we will examine and demonstrate their mathematical relationships using MATLAB. Three discrete-time LTI functions were provided for which we assume zero initial conditions. They shall be tested and observed under multiple theories relating to LTI systems, with sample respiratory and ECG signals.

Plots

I. Unit Impulse Function Response Plots

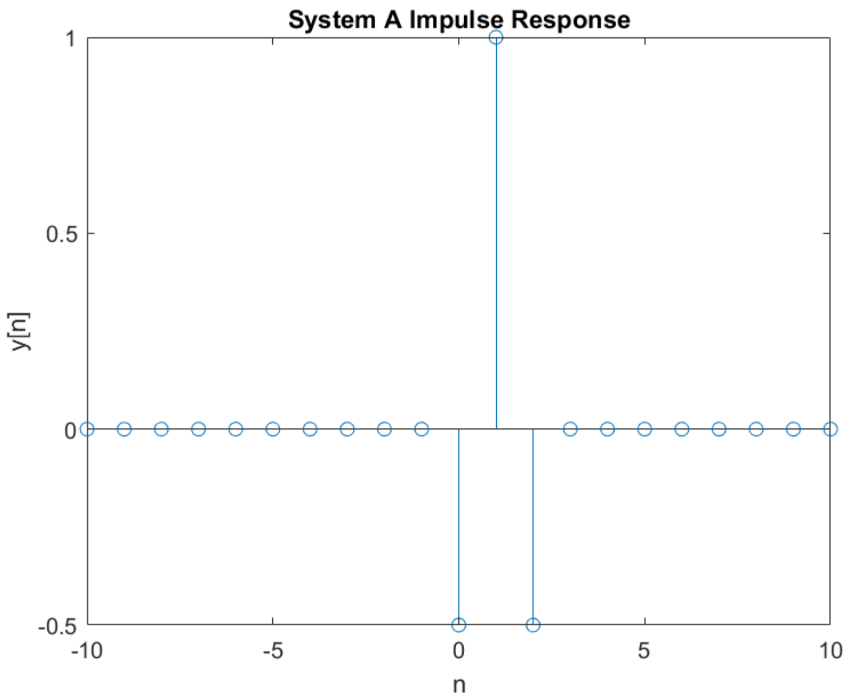

Figure 1: System A output with input of unit impulse function

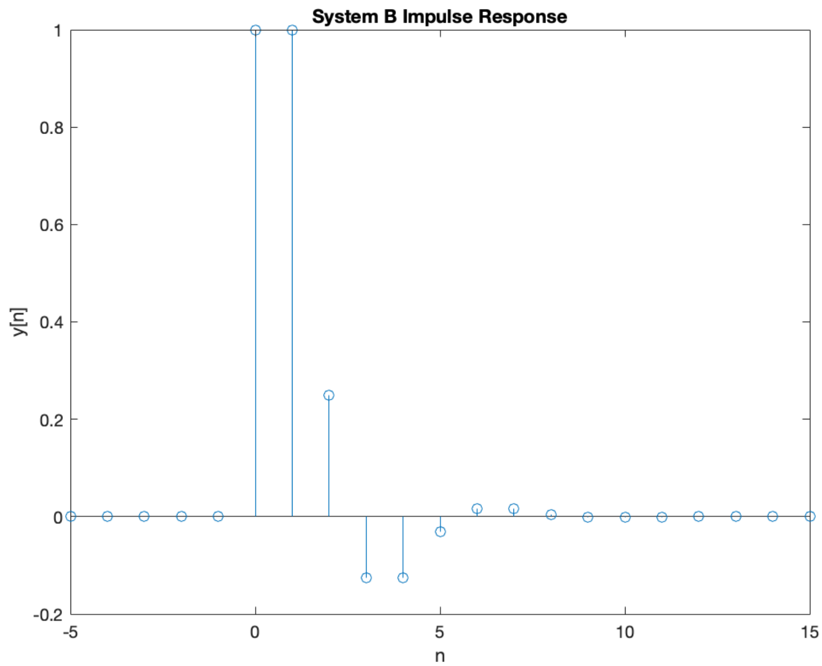

Figure 2: System B output with input of unit impulse function

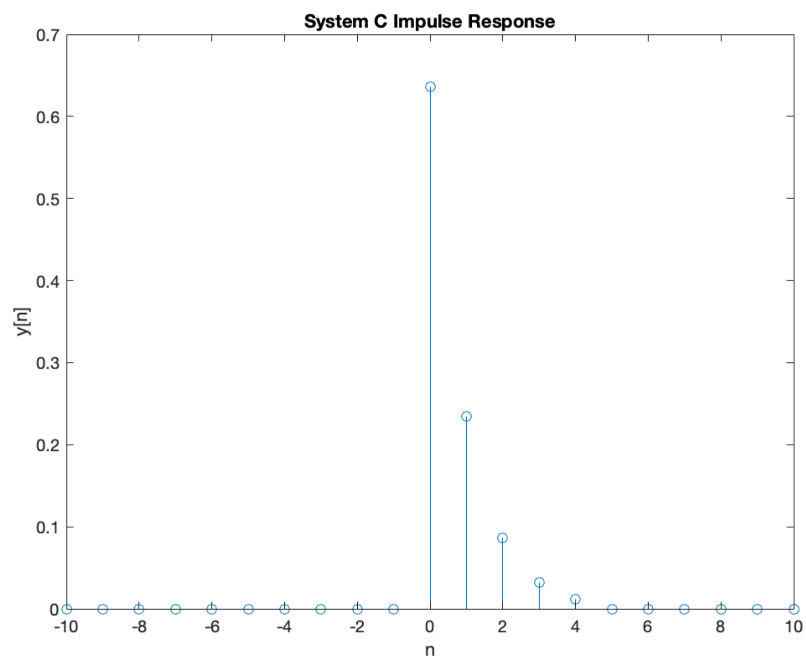

Figure 3: System C output with input of unit impulse function

II. Unit Step Function Response

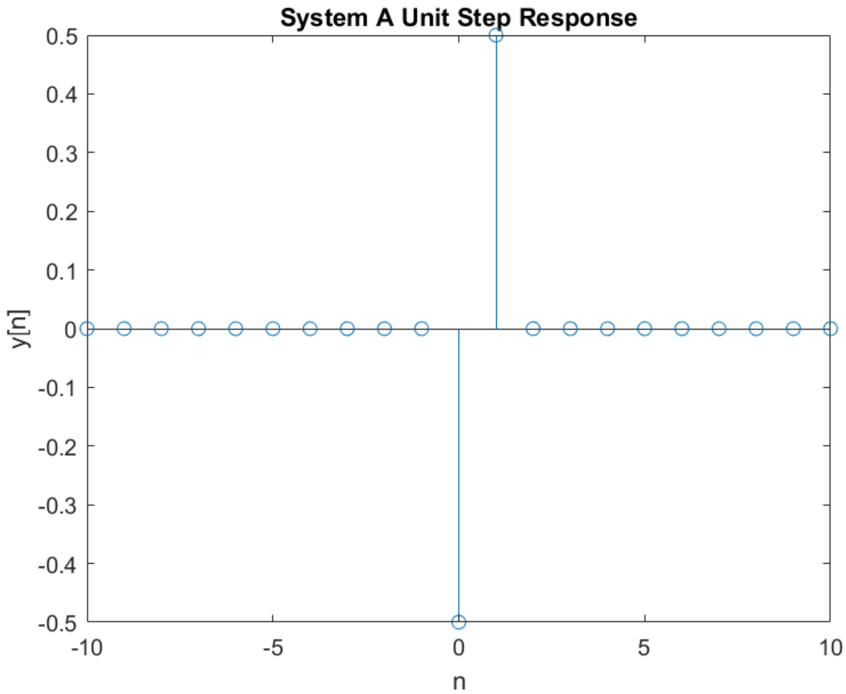

Figure 4: System A output with input of the unit step function

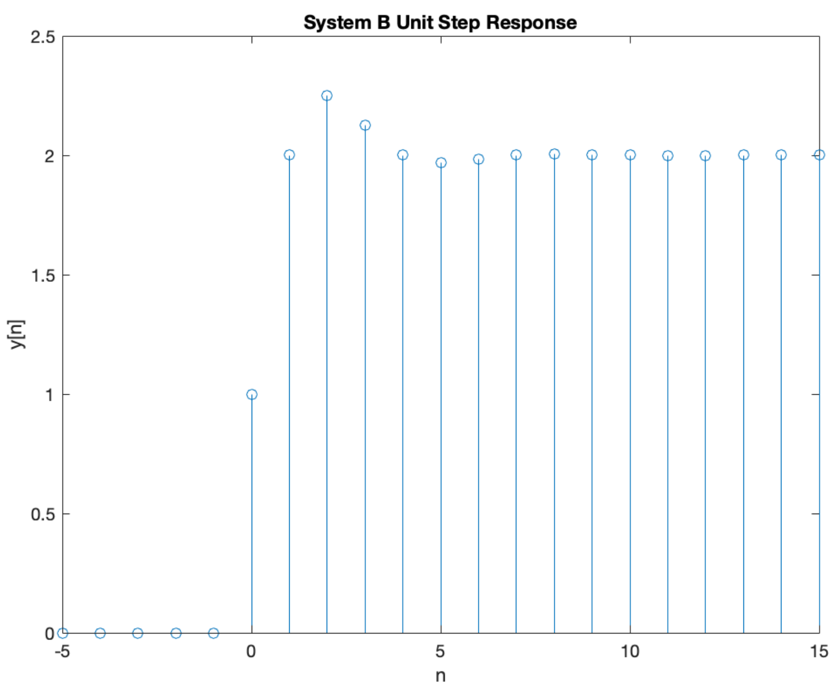

Figure 5: System B output with input of the unit step function

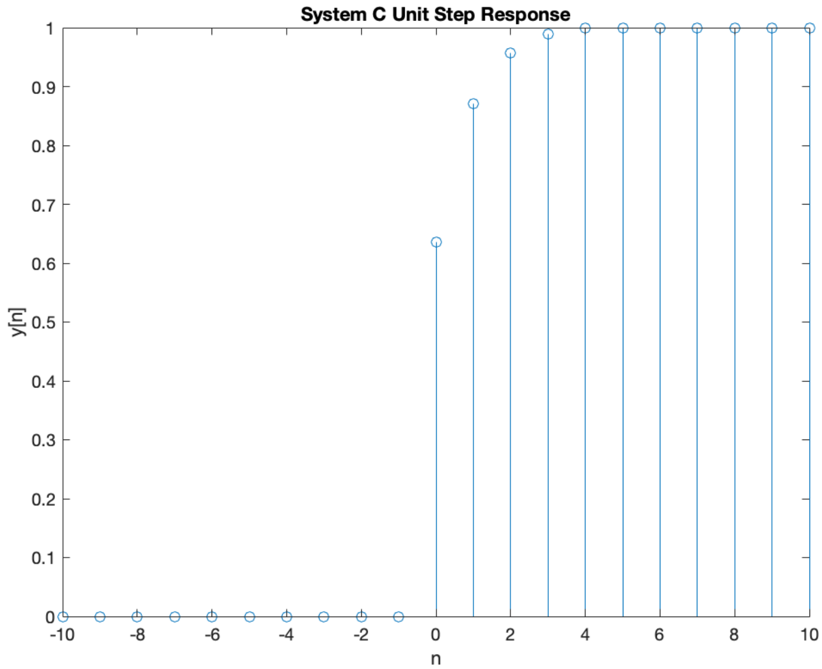

Figure 6: System C output with input of the unit step function

III. Cumulative Impulse Response vs. Step Function Response

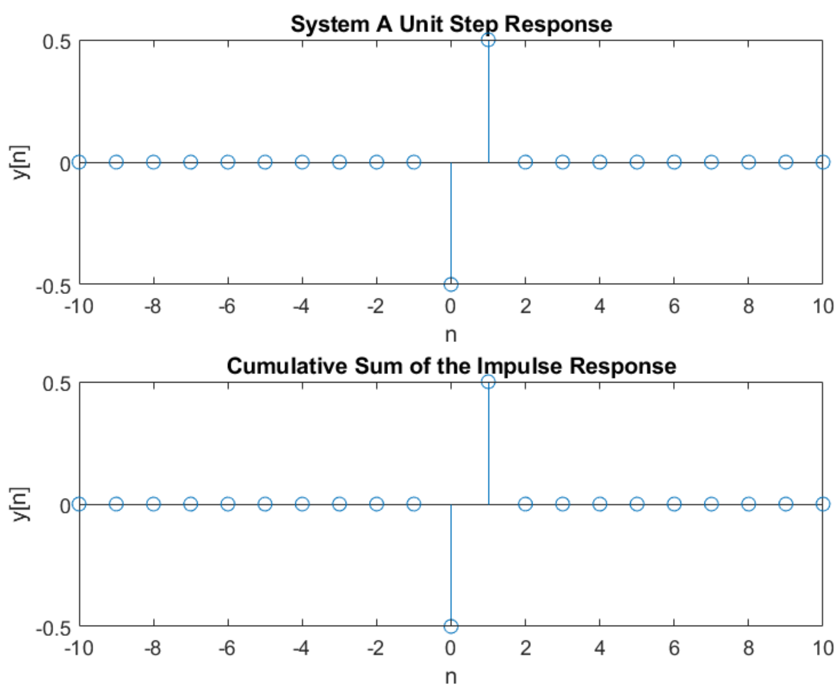

Figure 7: Demonstration that the cumulative sum of the unit impulse response is equal to the output of the unit step function of System A

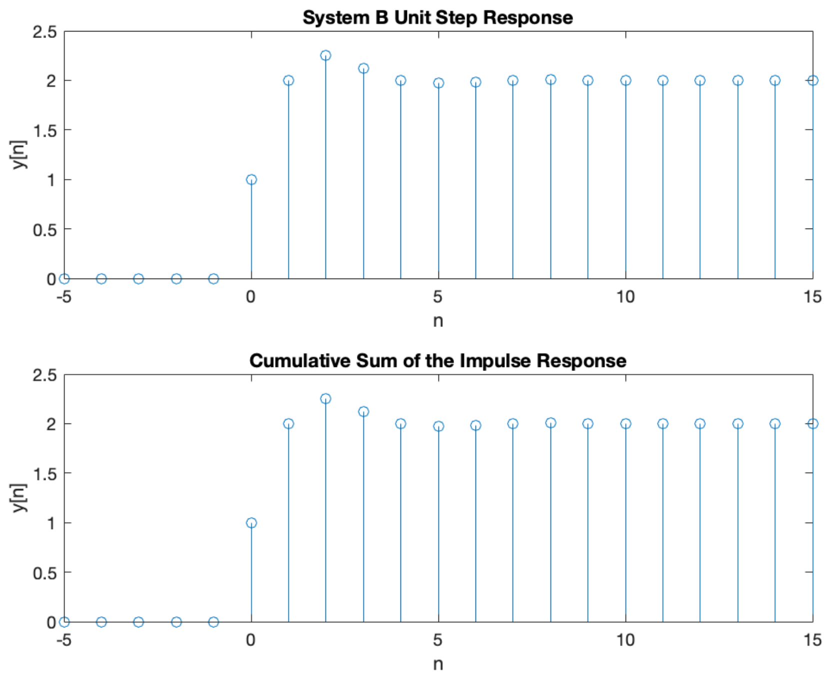

Figure 8: Demonstration that the cumulative sum of the unit impulse response is equal to the output of the unit step function of System B

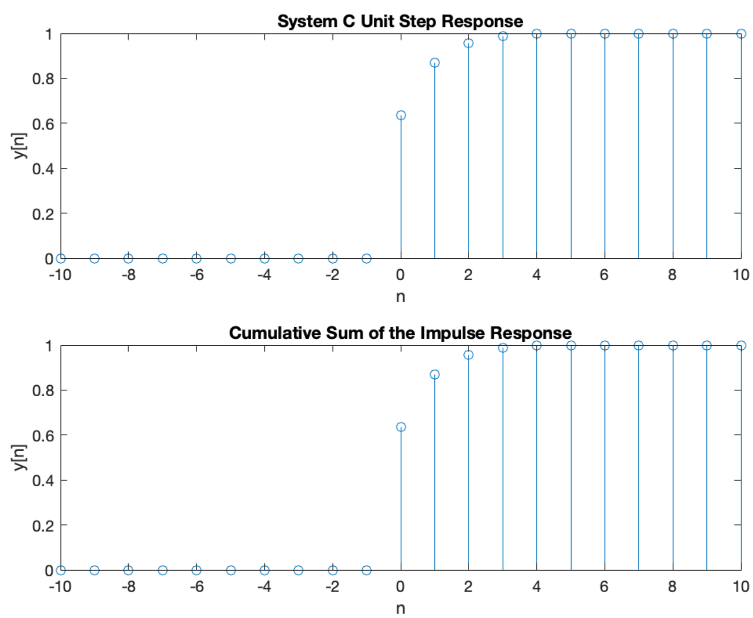

Figure 9: Demonstration that the cumulative sum of the unit impulse response is equal to the output of the unit step function of System C

IV. Step Response First Difference vs. Impulse Response

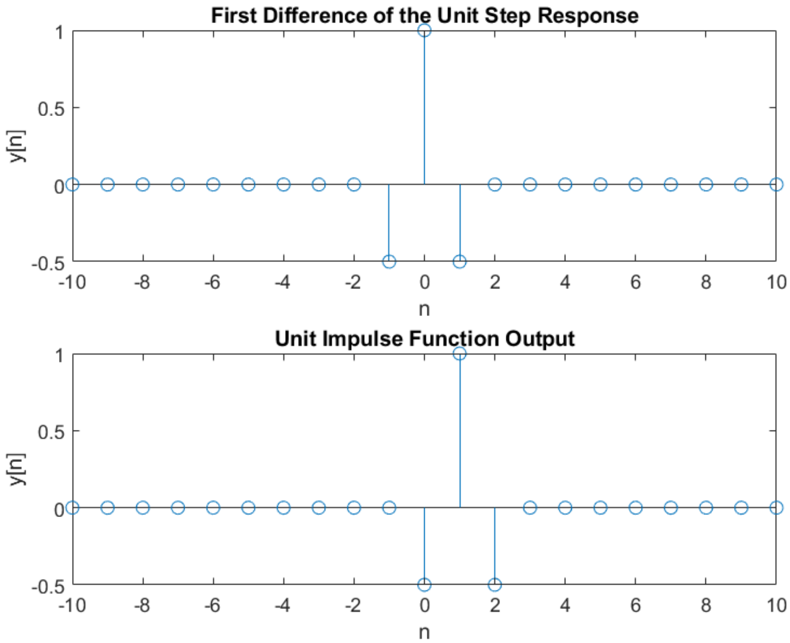

Figure 10: Demonstration that the first difference of the unit step response is equal to the unit impulse output of System A

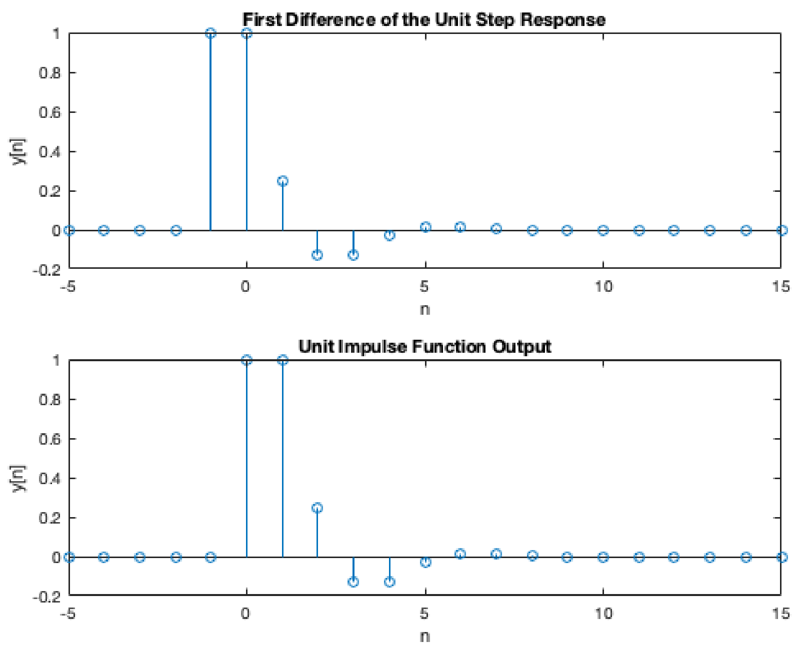

Figure 11: Demonstration that the first difference of the unit step response is equal to the unit impulse output of System B

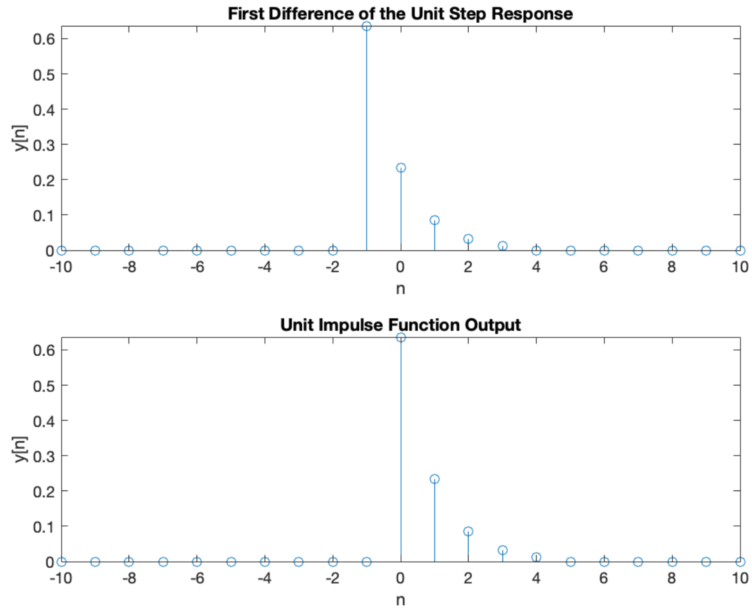

Figure 12: Demonstration that the first difference of the unit step response is equal to the unit impulse output of System C

V. Electrocardiogram (ECG) Signal Input

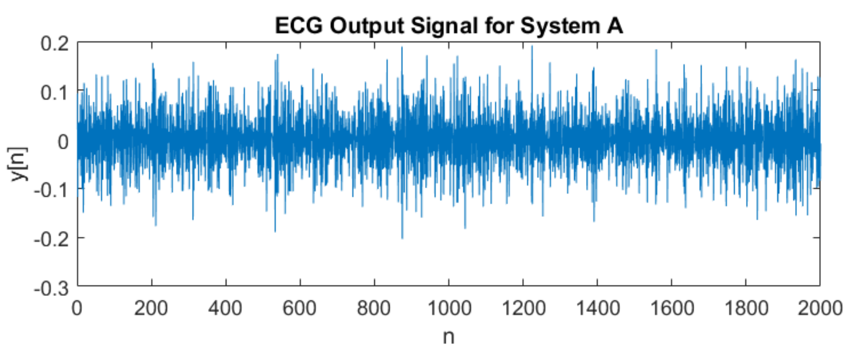

Figure 13: ECG output for system A

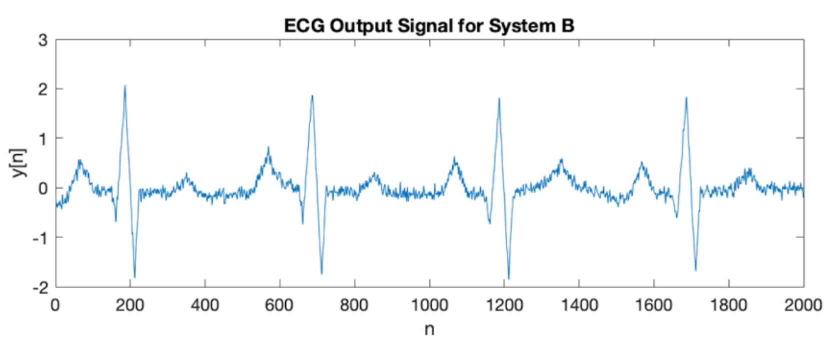

Figure 14: ECG output for system B

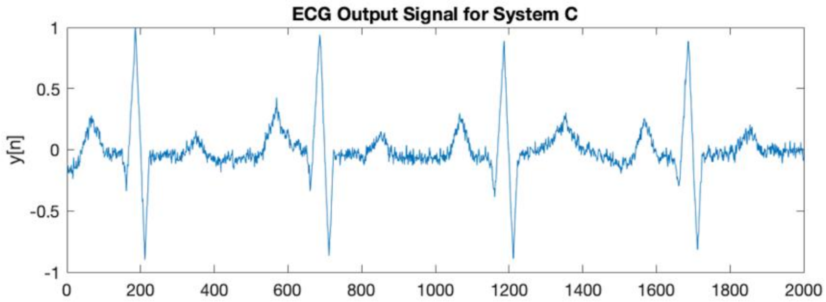

Figure 15: ECG output for system C

VI. Respiratory Signal Input

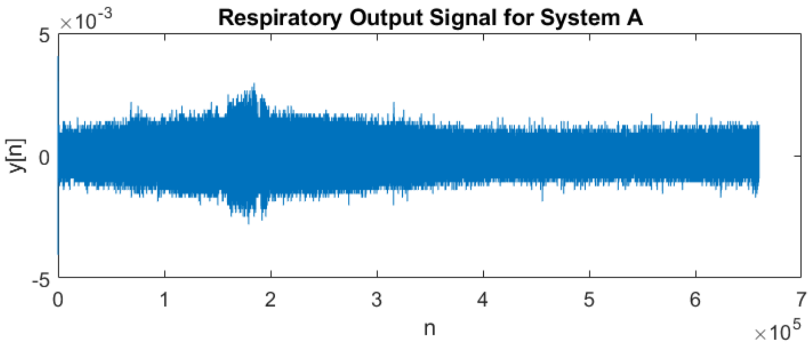

Figure 16: Respiratory output for System A

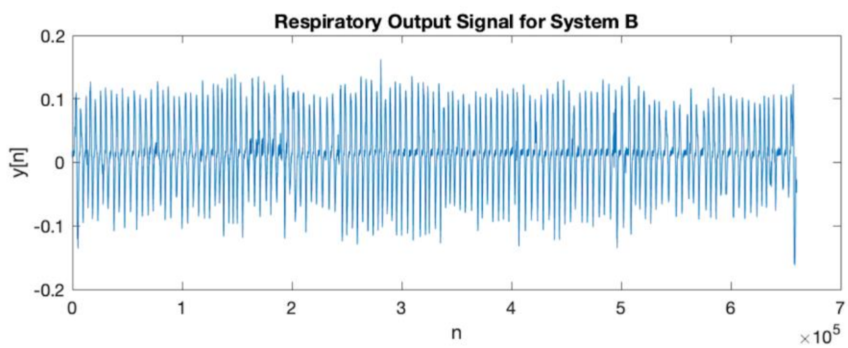

Figure 17: Respiratory output for System B

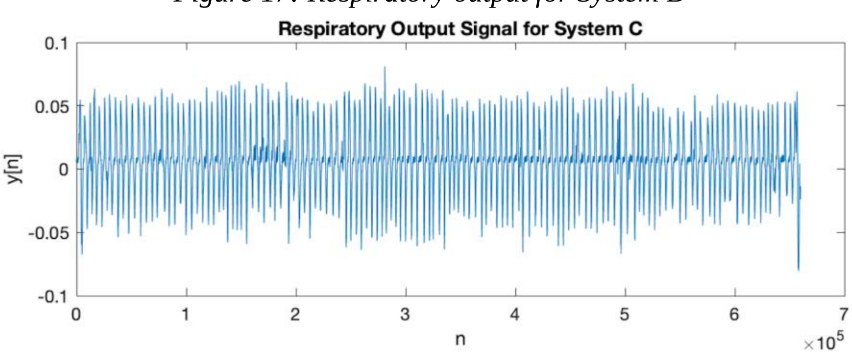

Figure 18: Respiratory output for System C

VII. Direct Sample Output to Impulse Convolution

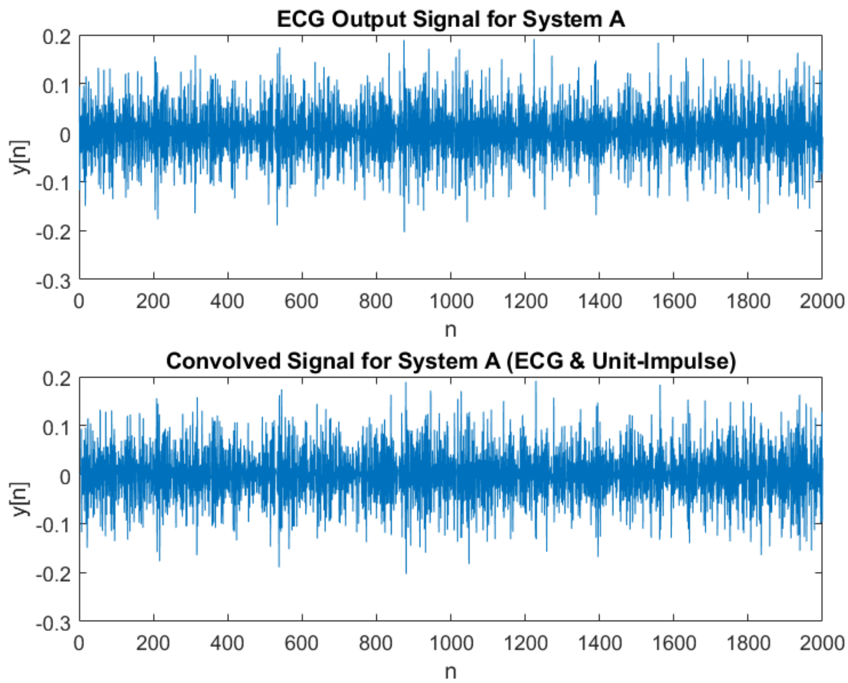

Figure 19: Demonstration for System A that the output of the ECG signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

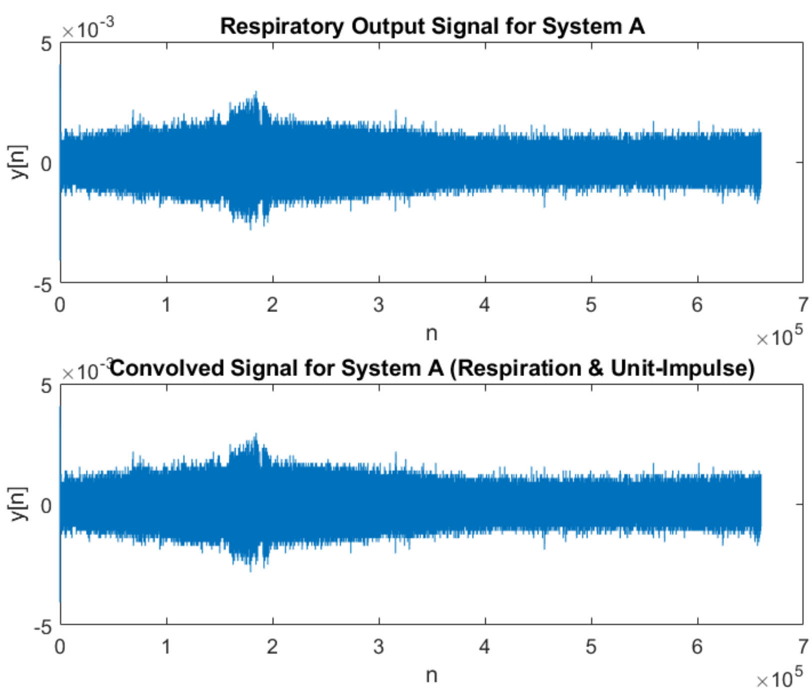

Figure 20: Demonstration for System A that the output of the Respiration signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

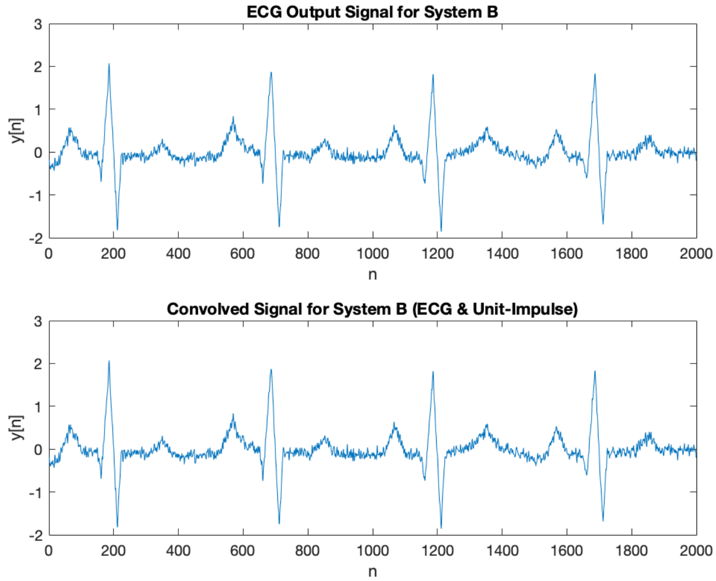

Figure 21: Demonstration for System B that the output of the ECG signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

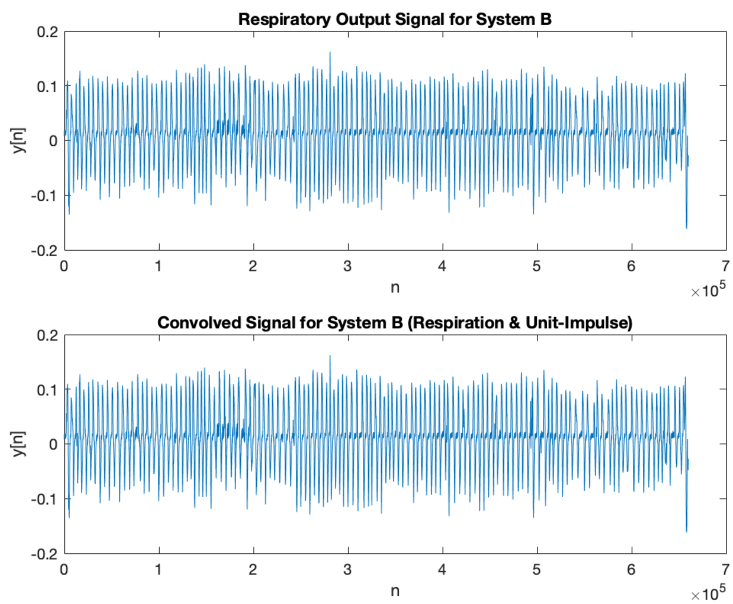

Figure 22: Demonstration for System B that the output of the Respiration signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

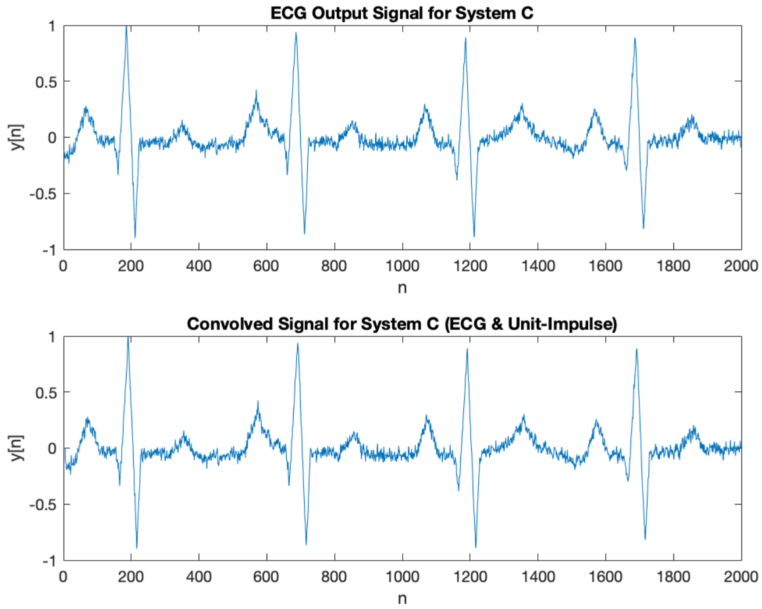

Figure 23: Demonstration for System C that the output of the ECG signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

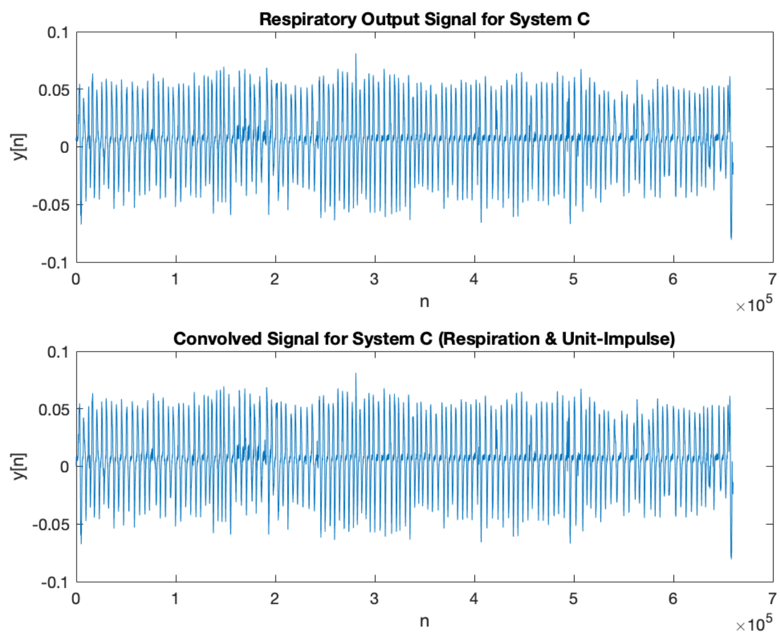

Figure 24: Demonstration for System C that the output of the Respiration signal is equal to the convolution of the input signals with the impulse response h[n] of the system computed previously

Impulse Response Systems

A finite impulse response, as the name suggests, is an impulse response that is finite in length and will become zero in time. In contrast, an infinite impulse response will continue indefinitely.

Based on Figures 1, 2, and 3, Systems A, B, and C may all look to be finite impulse responses. However, after looking at the output values found at arbitrary large indices, it can be concluded that this is not the case. The non-zero value outputs for system A end at 3, thus it reaches 0 at 3. And the non-zero value outputs for system C end at 5, thus reaching 0 at 5. Conversely, system B does not reach 0 even when testing values over 1000.

Through testing of the output at large indices, it is demonstrated that system A and C are finite impulse responses (FIR) while system B is an infinite impulse response (IIR).

Discussion of Part III

The relationship between the unit step and the unit impulse can be given by the equivalence of the unit step function output and the cumulative sum of the unit impulse response. Based on a visual comparison of the plots in Figures 7, 8, and 9, the unit step function output is equal to the cumulative sum of the unit impulse response for all systems. These results match the theory and confirm the relationship between the unit step and unit impulse.

Discussion of Part IV

The relationship between the unit impulse and unit step can also be stated by the equivalence of the unit impulse function output and the first difference of the unit step function response. This theory is matched by systems A, B, and C as seen in Figures 10, 11, and 12. Since this conformation was made by visual comparison, it is important to note that the first difference outputs have a shift of 1 to the right since it must account for the 1 index that is lost through the calculation of the first difference. Since this is seen in the aforementioned figures, these results match the theory and confirm the relationship between the unit impulse and unit step.

Discussion of Part VII

For an LTI system with zero initial conditions, this impulse response completely and sufficiently defines the system.

It is known that the system output of an input signal can be found by convolving the input signal with the unit impulse response. This theory is demonstrated by the property: $y[n] = x[n]*h[n]$ where $h[n]$ is the unit impulse response and $x[n]$ is the input system. As a result of the characteristics of an LTI system — namely the definition of the impulse response, time invariance, and the scaling and additivity properties of linearity — the unit impulse can be scaled and convolved with the input signal to produce the system output instead of directly passing the signal through the system itself.

From the visual comparison of the plots in Figures 19 to 14, all systems demonstrate that the outputs for the ECG and respiration signals are equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I. As such, the results match this theory and confirm its applicability to these systems.