$$\text{Sean } M, N, P \in M_{3x3}(\mathbb{Z}), M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, N = \begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix}, P = \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix}$$

$$\text{Por demostrar } \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \left(\begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix} \right) = \left(\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \right) \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \left(\begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix} \right) = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j\alpha + k\epsilon + l\xi & j\beta + k\eta + l\omega & j\gamma + k\theta + l\delta \\ m\alpha + n\epsilon + o\xi & m\beta + n\eta + o\omega & m\gamma + n\theta + o\delta \\ q\alpha + p\epsilon + r\xi & q\beta + p\eta + r\omega & q\gamma + p\theta + r\delta \end{pmatrix}$$

$$\begin{pmatrix} a(j\alpha + k\epsilon + l\xi) + b(m\alpha + n\epsilon + o\xi) + c(q\alpha + p\epsilon + r\xi) & a(j\beta + k\eta + l\omega) + b(m\beta + n\eta + o\omega) + c(q\beta + p\eta + r\omega) & a(j\gamma + k\theta + l\delta) + b(m\gamma + n\theta + o\delta) + c(q\gamma + p\theta + r\delta) \\ d(j\alpha + k\epsilon + l\xi) + e(m\alpha + n\epsilon + o\xi) + f(q\alpha + p\epsilon + r\xi) & d(j\beta + k\eta + l\omega) + e(m\beta + n\eta + o\omega) + f(q\beta + p\eta + r\omega) & d(j\gamma + k\theta + l\delta) + e(m\gamma + n\theta + o\delta) + f(q\gamma + p\theta + r\delta) \\ g(j\alpha + k\epsilon + l\xi) + h(m\alpha + n\epsilon + o\xi) + i(q\alpha + p\epsilon + r\xi) & g(j\beta + k\eta + l\omega) + h(m\beta + n\eta + o\omega) + i(q\beta + p\eta + r\omega) & g(j\gamma + k\theta + l\delta) + h(m\gamma + n\theta + o\delta) + i(q\gamma + p\theta + r\delta) \end{pmatrix}$$

$$\begin{pmatrix} aj\alpha + ak\epsilon + al\xi + bm\alpha + bn\epsilon + bo\xi + cq\alpha + cp\epsilon + cr\xi & aj\beta + ak\eta + al\omega + bm\beta + bn\eta + bo\omega + cq\beta + cp\eta + cr\omega & aj\gamma + ak\theta + al\delta + bm\gamma + bn\theta + bo\delta + cq\gamma + cp\theta + cr\delta \\ dj\alpha + dk\epsilon + dl\xi + em\alpha + en\epsilon + eo\xi + fq\alpha + fp\epsilon + fr\xi & dj\beta + dk\eta + dl\omega + em\beta + en\eta + eo\omega) + fq\beta + fp\eta + fr\omega) & dj\gamma + dk\theta + dl\delta + em\gamma + en\theta + eo\delta) + fq\gamma + fp\theta + fr\delta \\ gj\alpha + gk\epsilon + gl\xi + hm\alpha + hn\epsilon + ho\xi) + iq\alpha + ip\epsilon + ir\xi & gj\beta + gk\eta + gl\omega + hm\beta + hn\eta + ho\omega + iq\beta + ip\eta + ir\omega & gj\gamma + gk\theta + gl\delta + hm\gamma + hn\theta + ho\delta + iq\gamma + ip\theta + ir\delta \end{pmatrix}^{(1)}$$

Por otra parte veamos que

$$\left(\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \right) \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix} = \begin{pmatrix} aj + bm + cq & ak + bn + cp & al + bo + cr \\ dj + em + fq & dk + en + fp & dl + eo + fr \\ gj + hm + iq & gk + hn + ip & gl + ho + ir \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix}$$

$$\begin{pmatrix} (aj + bm + cq)\alpha + (ak + bn + cp)\epsilon + (al + bo + cr)\xi & (aj + bm + cq)\beta + (ak + bn + cp)\eta + (al + bo + cr)\omega & (aj + bm + cq)\gamma + (ak + bn + cp)\theta + (al + bo + cr)\delta \\ (dj + em + fq)\alpha + (dk + en + fp)\epsilon + (dl + eo + fr)\xi & (dj + em + fq)\beta + (dk + en + fp)\eta + (dl + eo + fr)\omega & (dj + em + fq)\gamma + (dk + en + fp)\theta + (dl + eo + fr)\delta \\ (gj + hm + iq)\alpha + (gk + hn + ip)\epsilon + (gl + ho + ir)\xi & (gj + hm + iq)\beta + (gk + hn + ip)\eta + (gl + ho + ir)\omega & (gj + hm + iq)\gamma + (gk + hn + ip)\theta + (gl + ho + ir)\delta \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} aj\alpha + bm\alpha + cq\alpha + ak\epsilon + bn\epsilon + cp\epsilon + al\xi + bo\xi + cr\xi & aj\beta + bm\beta + cq\beta + ak\eta + bn\eta + cp\eta + al\omega + bo\omega + cr\omega & aj\gamma + bm\gamma + cq\gamma + ak\theta + bn\theta + cp\theta + al\delta + bo\delta + cr\delta \\ dj\alpha + em\alpha + fq\alpha + dk\epsilon + en\epsilon + fp\epsilon + dl\xi + eo\xi + fr\xi & dj\beta + em\beta + fq\beta + dk\eta + en\eta + fp\eta + dl\omega + eo\omega + fr\omega & dj\gamma + em\gamma + fq\gamma + dk\theta + en\theta + fp\theta + dl\delta + eo\delta + fr\delta \\ gj\alpha + hm\alpha + iq\alpha + gk\epsilon + hn\epsilon + ip\epsilon + gl\xi + ho\xi + ir\xi & gj\beta + hm\beta + iq\beta + gk\eta + hn\eta + ip\eta + gl\omega + ho\omega + ir\omega & gj\gamma + hm\gamma + iq\gamma + gk\theta + hn\theta + ip\theta + gl\delta + ho\delta + ir\delta \end{pmatrix}^{(2)}$$

Finalmente, notemos que (1) y (2) son iguales pues los coeficientes pertenecen a $\mathbb{Z}$ y ahí, la suma es conmutativa por lo que

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \left(\begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix} \right) = \left(\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j & k & l \\ m & n & o \\ q & p & r \end{pmatrix} \right) \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \eta & \theta \\ \xi & \omega & \delta \end{pmatrix}$$

$$\therefore \cdot \text{ es asociativo en } M_{3x3}(\mathbb{Z})$$