6.5/10

1.5	2.0	1.5	1.5

Optimization Methods

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Here, we are hoping a more complicate model, to applied computational techniques, or a small problem to show that we know how to solve it by hand

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1. Motivación/Introducción (2pt.)
2. Descripción de la solución/Aplicación(3ptos)
3. Resultados/Análisis de resultados (2)
4. Presentación (3)

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Abstract

This project presents a linear programming model applied to a realistic production planning scenario in the automotive industry. A fictional company, EcuAuto S.A., manufactures three types of vehicles with limited resources such as assembly time, skilled labor, and imported components. The goal is to determine the optimal number of vehicles to produce per week to maximize total profit while satisfying operational constraints. The model was solved using Azomath, yielding a weekly profit of \$382,500. The results show how linear programming can effectively support decision-making in complex manufacturing environments.

We should emphasize the importance of the large-scale problem. Those interested in this topic solve problems of larger dimensions; so it would be worthwhile to have extended it to a problem with more variables.

1 Introduction

In the automotive industry, efficient production planning plays a critical role in maintaining profitability, meeting market demand, and ensuring optimal resource utilization. Car manufacturing involves numerous constraints, including limited labor, restricted availability of imported components, and strict production schedules. Managing these constraints manually or through trial-and-error approaches often leads to inefficiencies and suboptimal outcomes.

This project focuses on the weekly production planning of EcuAuto S.A., a car manufacturing company that produces three vehicle models: a compact sedan (EC-S1), a pickup truck (EC-P1), and a family SUV (EC-X1). Each model requires different amounts of assembly time, skilled labor, and imported parts, and provides a different profit margin.

To address this problem, we develop a linear programming algorithm that determines the optimal number of units of each vehicle to produce in order to maximize total profit while satisfying all operational constraints. Linear programming is a powerful mathematical optimization technique that enables systematic decision-making in such complex production environments, providing a structured approach to allocate limited resources efficiently.

Workflow Summary

The project followed a structured methodology, outlined below:

1. **Problem Statement:** A production planning problem in the automotive industry was proposed, aiming to maximize weekly profit given resource constraints.

2. **Data Collection and Parameters:** All necessary values (profit, time, labor, parts, storage) were defined per vehicle type, along with total resource availability and production minimums.

3. **Mathematical Model Formulation:** Decision variables were introduced and used to construct a linear programming model, including an objective function and constraints.

4. **Model Implementation:** The model was entered into the eMathHelp LP solver using a simplified algebraic syntax.

5. **Computation and Solution:** The solver returned the optimal solution, including decision variables and slack values.

6. **Analysis and Interpretation:** Results were analyzed through tables, and resource utilization percentages, identifying active constraints and bottlenecks.

2 Problem Description

EcuAuto S.A. is a car manufacturing company located in Ecuador. The company produces three vehicle models:

- **EC-S1**: a compact sedan for urban use,
- **EC-P1**: a pickup truck designed for light cargo and utility,
- **EC-X1**: a family-oriented SUV for long-distance travel and comfort.

Each model differs in production cost, resource consumption, and profit margin. The objective of EcuAuto is to determine the optimal weekly production quantities of each vehicle model that will maximize its total profit. The production process is subject to the following operational constraints:

- **Assembly time:** The factory has a weekly limit of 1,200 hours available for vehicle assembly.
- **Skilled labor:** A total of 800 hours of skilled technician labor is available each week.
- **Imported parts:** Due to supply limitations, only 1,000 imported components can be used per week.
- **Storage capacity:** The factory has room to store a maximum of 70 finished vehicles per week.
- **Minimum production:** To satisfy dealership contracts, at least 10 EC-S1 sedans and 5 EC-P1 pickups must be produced weekly.

The profit and resource requirements per unit for each vehicle model are summarized in Table 1.

Table 1: Profit and resource requirements per vehicle model

Model	**Profit (USD)**	**Assembly (hrs)**	**Labor (hrs)**	**Imported parts**
EC-S1 (Sedan)	3,000	10	5	8
EC-P1 (Pickup)	4,500	14	8	12
EC-X1 (SUV)	6,000	18	12	15

3 Mathematical Model Formulation

Let:

- x_{sedan}: number of EC-S1 sedans produced per week
- x_{pickup}: number of EC-P1 pickup trucks produced per week
- x_{SUV}: number of EC-X1 SUVs produced per week

The objective is to maximize total profit:

$$\text{Maximize } Z = 3000x_{\text{sedan}} + 4500x_{\text{pickup}} + 6000x_{\text{SUV}}$$

Subject to the following constraints:

$$\begin{aligned}
\text{(Assembly time)} \quad & 10x_{\text{sedan}} + 14x_{\text{pickup}} + 18x_{\text{SUV}} \leq 1200 \\
\text{(Skilled labor)} \quad & 5x_{\text{sedan}} + 8x_{\text{pickup}} + 12x_{\text{SUV}} \leq 800 \\
\text{(Imported parts)} \quad & 8x_{\text{sedan}} + 12x_{\text{pickup}} + 15x_{\text{SUV}} \leq 1000 \\
\text{(Storage capacity)} \quad & x_{\text{sedan}} + x_{\text{pickup}} + x_{\text{SUV}} \leq 70 \\
\text{(Minimum demand)} \quad & x_{\text{sedan}} \geq 10 \\
& x_{\text{pickup}} \geq 5 \\
\text{(Non-negativity)} \quad & x_{\text{sedan}}, x_{\text{pickup}}, x_{\text{SUV}} \geq 0
\end{aligned}$$

4 Solution and Analysis

The mathematical model was implemented and solved using the eMathHelp optimization platform, which allows for quick prototyping and solving of linear programming problems through a simple textual interface.

The optimal solution obtained is as follows:

Table 2: Optimal solution and slack variables

Variable	**Description**	**Value**
x_1	Number of EC-S1 sedans produced	10
x_2	Number of EC-P1 pickup trucks produced	5
x_3	Number of EC-X1 SUVs produced	55
Z	Total weekly profit (USD)	382,500
S_1	Slack in assembly time (hours)	40
S_2	Slack in skilled labor (hours)	50
S_3	Slack in imported components	35
S_4	Slack in storage capacity	0
S_5	Slack in sedan minimum production	0
S_6	Slack in pickup minimum production	0

Results Discussion

The optimal production plan recommends manufacturing 10 EC-S1 sedans, 5 EC-P1 pickup trucks, and 55 EC-X1 SUVs per week. This allocation yields a maximum profit of $382,500.

The slack variables provide valuable insight into the utilization of resources:

- **Assembly time:** 40 hours remain unused, indicating moderate underutilization of this resource.
- **Skilled labor:** 50 hours of labor are still available, suggesting some flexibility in staffing.
- **Imported parts:** 35 components remain unused, indicating that component availability was not a tight constraint.
- **Storage capacity:** This constraint is fully utilized (0 slack), meaning the factory cannot store any more vehicles under the current production plan.
- **Minimum production constraints:** Both EC-S1 and EC-P1 minimum production levels are met exactly, implying that these constraints were binding and directly influenced the solution.

Table 3: Resource Utilization Summary

Resource	**Capacity**	**Used**	**Utilization (%)**
Assembly Time (hours)	1200	1160	96.7%
Skilled Labor (hours)	800	750	93.8%
Imported Components (units)	1000	965	96.5%
Storage (vehicles)	70	70	100.0%

Table 4: This table provides a clear view of how efficiently each resource is being used under the optimal production plan

Where is the solution?.
What about the slack/surplus variables.
This is important, because they tell us
if we can improve the business in some
direction, or what to do in the future

Sensitivity Discussion

If the storage constraint were relaxed (e.g., from 70 to 80 vehicles), it is likely that the model would allocate more resources to producing EC-X1 SUVs, given their higher profit margin. This change could increase overall profit while maintaining feasibility, assuming other constraints remain non-binding.

5 Conclusion

This study addressed a realistic production planning problem for a car manufacturing company using linear programming. By modeling resource constraints and profit objectives, we formulated a mathematical model and solved it using eMathHelp.

The results show that the optimal production strategy involves prioritizing the EC-X1 SUV, which offers the highest profit margin. The factory can maximize its profit to $382,500 per week by producing 10 sedans, 5 pickups, and 55 SUVs.

Slack variable analysis revealed that while assembly time, labor, and imported components are not fully exhausted, the storage capacity and minimum production requirements are binding constraints. These findings underscore the importance of storage management and demand fulfillment in maximizing profitability.

Linear programming proved to be a powerful tool in supporting strategic decisions for resource allocation in a constrained manufacturing environment.

References

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