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Fix parameters

```
tic;
Nsteps = 80;    % Number of iterations
Nplot = 101;    % Fineness of plots
T = 10*eps;    % Time delay
az = 13;        % Plot viewing angle
el = 48;        % Plot viewing height
v = 0.0;        % Camera speed
fix_axes = 0;    % Set to 1 for fixed axes
plot_range = [-0.1 1.1]; % z-axis range if fixed
% col_range = plot_range; % Fixed colour range
% col_range = [-.7 1.1];
col_range = 'manual'; % Varying colour range
rec = 0;        % record on/off
xg = linspace(-1,1,Nplot);
yg = linspace(-1,1,Nplot);
[XX,YY] = meshgrid(xg,yg);
ZZ = NaN(size(XX));

[indx_up,indx_up]=find(XX>=0);
indx_up = unique(indx_up);
indy_up = unique(indy_up);

[indx_lo,indx_lo]=find(YY<=0);
indx_lo = unique(indx_lo);
indy_lo = unique(indy_lo);

indices_all=find(XX>=0|YY<=0);
ZZ(indices_all)=1;
Du_dy=(ZZ(1:Nplot-2,:)-ZZ(3:Nplot,:));
Du_dx=(ZZ(:,3:Nplot)-ZZ(:,1:Nplot-2));
diffy_ind=~isnan(Du_dy);
diffx_ind=~isnan(Du_dx);
```

Export geometry

```
load('upper_rectangle')
load('lower_rectangle')
```

Define the problem

```
a = 1;
c = 1;          % Coeffs in PDE
f = 1;          % RHS of PDE
% f = '1+x.^2+12*(y+1)';
% f = '1./(x.^2+(y+1/2).^2)-1./((x-1/2).^2+y.^2)';

% initguess=@(x,y) 1;
% initguess = @(x,y) 0; % u0~1 @ (0,0)
% initguess = @(x,y) -1-sin(22*x).*sin(10*y);
initguess = @(x,y) 1-sin(22*x).*sin(10*y);
% initguess = @(x,y) 0.0+(min(x.^2+(y+.5).^2,(x-.5).^2+y.^2)<.005);
% initguess = @(x,y) (min(x.^2+(y+.5).^2,(x-.5).^2+y.^2)>.05);
% initguess = @(x,y) -3+4*(x.^2+(y+1/2).^2>.001);

% Why the solution doesn't converge for the following two initial functions??
% initguess = @(x,y) 1-.001./(x.^2+(y+1/2).^2).^(1/3)-.001./((x-1/2).^2+y.^2).^(1/3);

% initguess = @(x,y) 10-7.*(abs(log(-x.^2-(y+1/2).^2))).^0.4999;
```

Generate the mesh

```
errvals_inf= zeros(4,Nsteps);
errvals_H1=zeros(4,Nsteps);

for Nmesh=25:25:100;

meshgenfun = '1./(x.^2+y.^2)'; % function for adaptive mesh generation
[~,p_up,e_up,t_up]=adaptmesh(g_up,b_up,c,a,meshgenfun,'Ngen', Nmesh);
[~,p_lo,e_lo,t_lo]=adaptmesh(g_lo,b_lo,c,a,meshgenfun,'Ngen', Nmesh);
% Generate the mesh for the first rectangle
% [p_up,e_up,t_up]=initmesh(g_up);
% [p_up,e_up,t_up]=refinemesh(g_up,p_up,e_up,t_up);
% [p_up,e_up,t_up]=refinemesh(g_up,p_up,e_up,t_up);
% [p_up,e_up,t_up]=refinemesh(g_up,p_up,e_up,t_up);
% figure(1);
% pdemesh(p_up,e_up,t_up);
% xlim ([-1.5,1.5]);
% axis equal;
% % Generate the mesh for the second rectangle
```

```

% [p_lo,e_lo,t_lo]=initmesh(g_lo);
% [p_lo,e_lo,t_lo]=refinemesh(g_lo,p_lo,e_lo,t_lo);
% [p_lo,e_lo,t_lo]=refinemesh(g_lo,p_lo,e_lo,t_lo);
% % [p_lo,e_lo,t_lo]=refinemesh(g_lo,p_lo,e_lo,t_lo);
% figure(2);
% pdemesh(p_lo,e_lo,t_lo);
% xlim ([-1.5,1.5]);
% axis equal;

```

```

Number of triangles: 172
Number of triangles: 178
Number of triangles: 197
Number of triangles: 201
Number of triangles: 213
Number of triangles: 217
Number of triangles: 229
Number of triangles: 233
Number of triangles: 245
Number of triangles: 249
Number of triangles: 261
Number of triangles: 265
Number of triangles: 277
Number of triangles: 281
Number of triangles: 293
Number of triangles: 297
Number of triangles: 309
Number of triangles: 313
Number of triangles: 325
Number of triangles: 329
Number of triangles: 341
Number of triangles: 345
Number of triangles: 357
Number of triangles: 361
Number of triangles: 373
Number of triangles: 377

```

Maximum number of refinement passes obtained.

```

Number of triangles: 168
Number of triangles: 174
Number of triangles: 192
Number of triangles: 196
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```

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Number of triangles: 376

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Number of triangles: 361

Number of triangles: 373
Number of triangles: 377
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Number of triangles: 393
Number of triangles: 405
Number of triangles: 409
Number of triangles: 421
Number of triangles: 425
Number of triangles: 437
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Number of triangles: 453
Number of triangles: 457
Number of triangles: 469
Number of triangles: 473
Number of triangles: 485
Number of triangles: 489
Number of triangles: 501
Number of triangles: 505
Number of triangles: 517
Number of triangles: 521
Number of triangles: 533
Number of triangles: 537
Number of triangles: 549
Number of triangles: 553
Number of triangles: 565
Number of triangles: 569
Number of triangles: 581

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Number of triangles: 468
Number of triangles: 472
Number of triangles: 484
Number of triangles: 488
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Number of triangles: 504
Number of triangles: 516
Number of triangles: 520
Number of triangles: 532
Number of triangles: 536
Number of triangles: 548
Number of triangles: 552
Number of triangles: 564
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Number of triangles: 580

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Number of triangles: 581
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Number of triangles: 597
Number of triangles: 601

Number of triangles: 613
Number of triangles: 617
Number of triangles: 629
Number of triangles: 633
Number of triangles: 645
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Number of triangles: 693
Number of triangles: 697
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Number of triangles: 725
Number of triangles: 729
Number of triangles: 741
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Number of triangles: 761
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Number of triangles: 792
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Number of triangles: 808
Number of triangles: 820
Number of triangles: 824
Number of triangles: 836
Number of triangles: 840
Number of triangles: 852
Number of triangles: 856
Number of triangles: 868
Number of triangles: 872
Number of triangles: 884
Number of triangles: 888
Number of triangles: 900
Number of triangles: 904
Number of triangles: 916
Number of triangles: 920
Number of triangles: 932
Number of triangles: 936
Number of triangles: 1094
Number of triangles: 1098
Number of triangles: 1110
Number of triangles: 1114
Number of triangles: 1122

```

Maximum number of refinement passes obtained.

Obtain relative matrices

```

[K_up,M_up,F_up,Q_up,G_up,H_up,R_up]=asempde(b_up,p_up,e_up,t_up,c,a,f);
[K_lo,M_lo,F_lo,Q_lo,G_lo,H_lo,R_lo]=asempde(b_lo,p_lo,e_lo,t_lo,c,a,f);

```

Assign true solution

```

u_up_true = ones(1,length(p_up));
u_lo_true = ones(1,length(p_lo));

```

Iterations

```
counter = 0;
if rec == 1
    Vid = VideoWriter('Temp_video', 'MPEG-4');
    Vid.FrameRate = 3;
    Vid.Quality = 100;
    open(Vid);
end
ZZ(XX>=0) = initguess(XX(XX>=0),YY(XX>=0));
ZZ(YY<=0) = initguess(XX(YY<=0),YY(YY<=0));
figure(3);
my_plot_new(XX,YY,ZZ,az,el,v, plot_range, col_range,0,fix_axes);

if rec == 1
    frame = getframe(gcf);
    writeVideo(Vid,frame);
end
pause

% Note that pind_up2 represents the indices of the mesh points at the boundary in the
% upper domain
[pind_up1,pind_up2]=find(H_up);
% Remove the problematic point
ind=find((abs(p_up(1,pind_up2) -0)<1e-10) & (abs(p_up(2,pind_up2) -0)<1e-10));
pind_up1(ind)=[];
pind_up2(ind)=[];
% Update the Dirichlet condition matrix H_up
H_up=H_up(pind_up1,:);

[pind_lo1,pind_lo2]=find(H_lo);
% Remove the problematic point
ind=find((abs(p_lo(1,pind_lo2) -0)<1e-10) & (abs(p_lo(2,pind_lo2) -0)<1e-10));
pind_lo1(ind)=[];
pind_lo2(ind)=[];
% Update the Dirichlet condition matrix H_lo
H_lo=H_lo(pind_lo1,:);

bnew_up=zeros(length(pind_up2),1);
bnew_lo=zeros(length(pind_lo2),1);

for step = 1:Nsteps;
    if step == 1
        bfun_up = @(x,y) initguess(x,y);
```

```

        else
            bfun_up = @(x,y) tri2grid(p_lo,t_lo,u_lo,x,y);
        end
% Update boundary conditions
for i=1:length(pind_up2);
    bnew_up(i)=bfun_up(p_up(1,pind_up2(i)),p_up(2,pind_up2(i)));
end
% Update the Dirichlet condition vector R_up
R_up = H_up(:,pind_up2)*bnew_up;

% Solve the pde in the upper domain
u_up=asempde(K_up,M_up,F_up,Q_up,G_up,H_up,R_up);

% Plot the solution
ZZ(XX>=0) = tri2grid(p_up,t_up,u_up,xg(indx_up),yg(indy_up));
figure(3);
my_plot_new(XX,YY,ZZ,az,el,v, plot_range, col_range, counter,fix_axes);

if rec == 1
    frame = getframe(gcf);
    writeVideo(Vid,frame);
end

pause(T)
counter = counter+1;

bfun_lo = @(x,y) tri2grid(p_up,t_up,u_up,x,y);

% Update the boundary conditions
for i=1:length(pind_lo2);
    bnew_lo(i)=bfun_lo(p_lo(1,pind_lo2(i)),p_lo(2,pind_lo2(i)));
end

% Update the Dirichlet condition vector
R_lo = H_lo(:,pind_lo2)*bnew_lo;

% Solve the pde in the lower domain
u_lo=asempde(K_lo,M_lo,F_lo,Q_lo,G_lo,H_lo,R_lo);

%Plot the solution
ZZ(YY<=0) = tri2grid(p_lo,t_lo,u_lo,xg(indx_lo),yg(indy_lo));
figure(3)
my_plot_new(XX,YY,ZZ,az,el, v, plot_range, col_range, counter,fix_axes);

if rec == 1
    frame = getframe(gcf);

```



```

        writeVideo(Vid,frame);
    end

    pause(T)

    counter = counter+1;

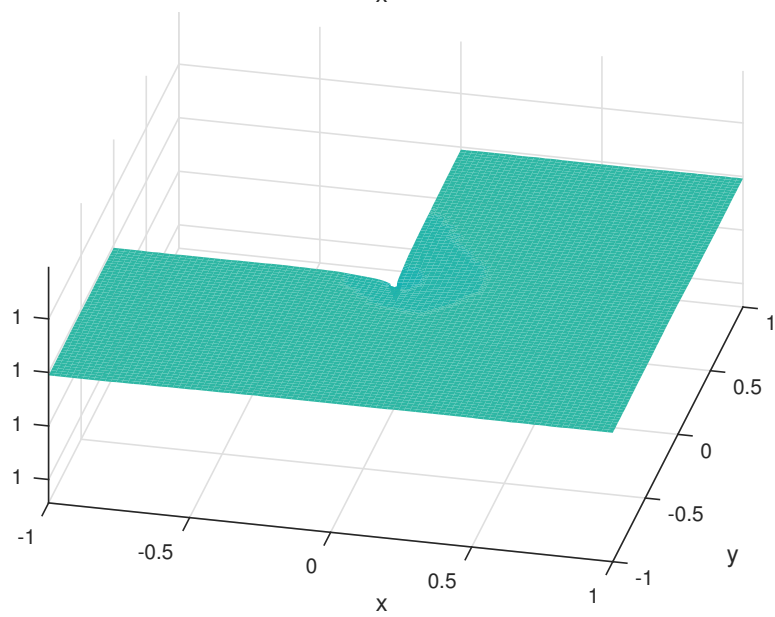
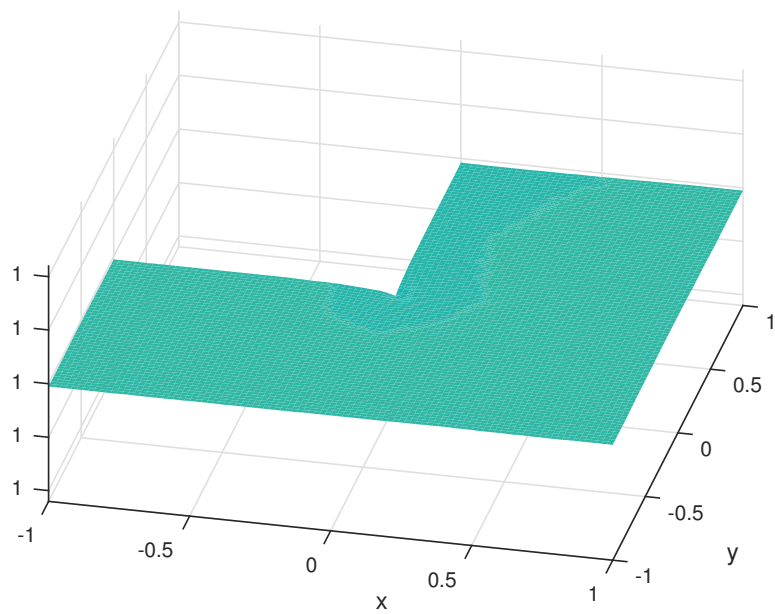
    % The error calculated with respect to infinity norm
    errvals_inf(Nmesh/25,step)=max(max(abs([u_up(:); u_lo(:)]-[u_up_true(:); u_lo_true(:)])));

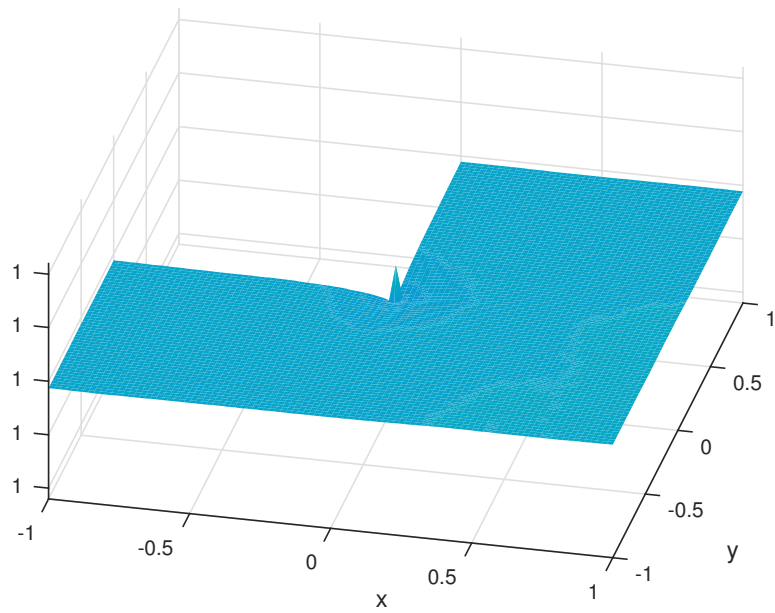
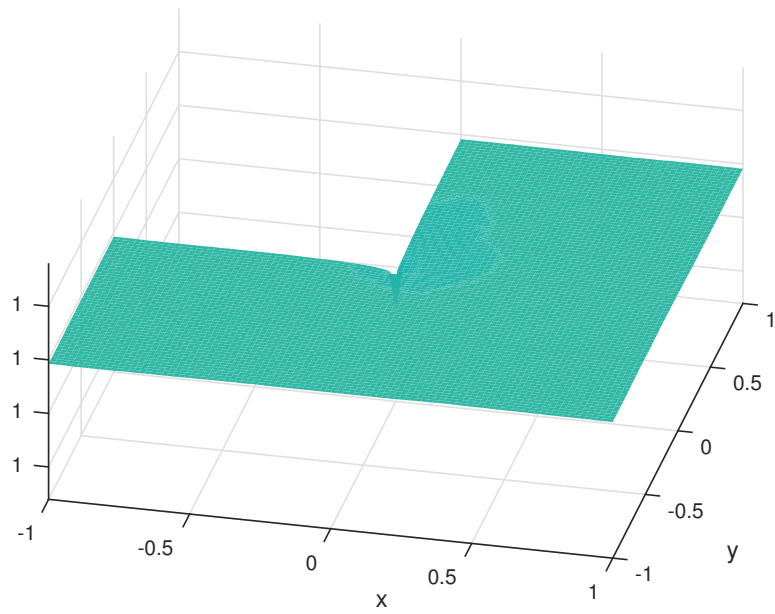
    % The error calculated with respect to H1 norm

    % The difference between the solution from ASM and the true solution
    u_diff=ZZ-ones(size(ZZ));
    % Square the difference
    u_diff_square=u_diff.*u_diff;
    % stepsize on x-axis
    dx=(1-(-1))/(Nplot-1);
    % stepsize on y-axis
    dy=(1-(-1))/(Nplot-1);
    integral_1=sum(sum (dx*dy*u_diff_square(indices_all))));

    % Approximate the derivatives by Central Difference Method
    Du_dy=(ZZ(3:Nplot,:)-ZZ(1:Nplot-2,:))/(2*dy);
    Du_dx=(ZZ(:,3:Nplot)-ZZ(:,1:Nplot-2))/(2*dx);
    % Integrate on the y-direction
    integrand_y=Du_dy.*Du_dy*dx*dy;
    % Integrate on the x-direction
    integrand_x=Du_dx.*Du_dx*dx*dy;
    integrals=[integral_1,sum(sum(integrand_y(diffy_ind))),sum(sum(integrand_x(diffx_ind)))]';
    errvals_H1(Nmesh/25,step)=sqrt(sum(integrals));
end

```





```

end

if rec == 1
    close(Vid);
end

```

Plot of $|u_n - u_{\text{true}}|$ versus Nsteps

```
figure(4)
nn = 1:Nsteps;
semilogy(nn,[errvals_inf(1,:);errvals_inf(2,:);errvals_inf(3,:);errvals_inf(4,:)]);
legend('Nmesh=25','Nmesh=50','Nmesh=75','Nmesh=100')
xlabel('Number of Iterations'); % x-axis label
ylabel('Log(errvals_{inf})') ; % y-axis label

figure(5)
nn = 1:Nsteps;
semilogy(nn,[errvals_H1(1,:);errvals_H1(2,:);errvals_H1(3,:);errvals_H1(4,:)]);
legend('Nmesh=25','Nmesh=50','Nmesh=75','Nmesh=100');
xlabel('Number of Iterations'); % x-axis label
ylabel('Log(errvals_{H1})'); % y-axis label
toc
```

Elapsed time is 252.425862 seconds.

