

Introductory Domain Decomposition Short Course

Laurence Halpern, Felix Kwok and Martin J. Gander

DD25- Saint-John's-CANADA

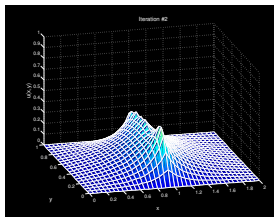
- Morning session : theory
 - Schwarz methods : zoology and methods of proof
 - Dirichlet-Neumann and Neumann-Neumann
 - Modern coarse spaces : Krylov and coarse spaces
- Afternoon session : practice, sample codes written in Matlab.

© 2010 Lyndon Keating

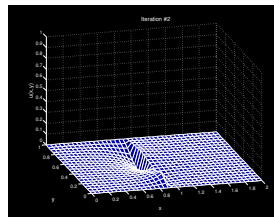
Schwarz methods-methods of proof

Laurence Halpern

DD25-Saint-John's-CANADA



$$\eta u - \Delta u = f$$

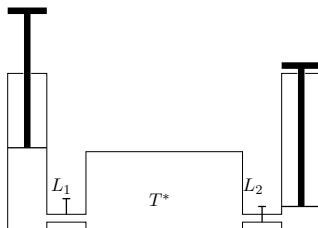
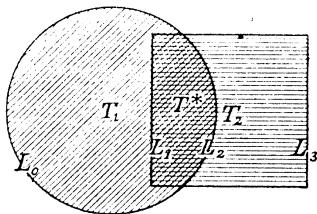


Sources

H.A. Schwarz, the original



H. A. Schwarz, *Über einen Grenzübergang durch alternierendes Verfahren*, Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, vol 15, pp. 272–286, 1870.

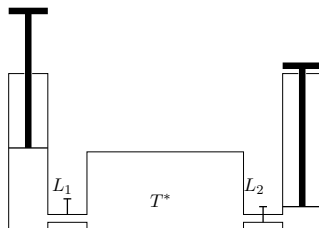
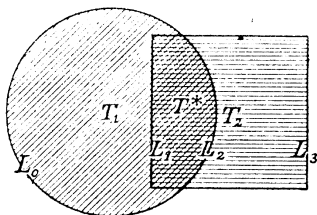


Sources

H.A. Schwarz, the original



H. A. Schwarz, *Über einen Grenzübergang durch alternierendes Verfahren*, Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, vol 15, pp. 272–286, 1870.



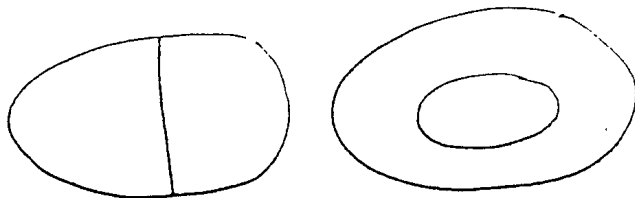
$$-\Delta u = 0 \text{ in } \Omega, \quad u = g \text{ on } \partial\Omega.$$

Sources

P.L. Lions, Extension to nonoverlap



P.-L. Lions, *On the Schwarz alternating method III : A variant for nonoverlapping subdomains*, in DD3, 1989.



$$(3) \quad -\Delta u_i^{n+1} = f \quad \text{in } \Omega_i \quad , \quad u_i^{n+1} \in H^1_\Gamma(\Omega_i)$$

$$(4) \quad \frac{\partial u_i^{n+1}}{\partial n_{ij}} + \lambda_{ij} u_i^{n+1} = \frac{\partial u_j^n}{\partial n_{ij}} + \lambda_{ij} u_j^n \quad \text{on } \gamma_{ij} \quad , \quad \forall 1 \leq j \leq m, j \neq i \quad ,$$

where n_{ij} ($= -n_{ji}$) is the unit outward normal to $\partial\Omega_i$ on γ_{ij} , and $\lambda_{ij} = \lambda_{ji} > 0$ for all $1 \leq i \neq j \leq m$.

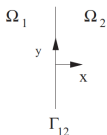
Sources

Optimized Schwarz methods



Caroline Japhet. *Méthode de décomposition de domaine et conditions aux limites artificielles en mécanique des fluides : Méthode Optimisée d'Ordre 2*. PhD thesis, Université Paris 13, 1998.

Le domaine de calcul $\Omega = \mathbb{R}^2$ est découpé en 2 sous-domaines, sans recouvrement, $\Omega_1 = \mathbb{R}^- \times \mathbb{R}$ et $\Omega_2 = \mathbb{R}^+ \times \mathbb{R}$ (voir figure 3.1).



$$\rho(k, c_2, c_3) = \left(\frac{\lambda^-(k) - \lambda_{ap}^-(k)}{\lambda^+(k) - \lambda_{ap}^-(k)} \right)^2$$

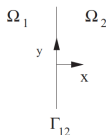
Sources

Optimized Schwarz methods



Caroline Japhet. *Méthode de décomposition de domaine et conditions aux limites artificielles en mécanique des fluides : Méthode Optimisée d'Ordre 2*. PhD thesis, Université Paris 13, 1998.

Le domaine de calcul $\Omega = \mathbb{R}^2$ est découpé en 2 sous-domaines, sans recouvrement, $\Omega_1 = \mathbb{R}^- \times \mathbb{R}$ et $\Omega_2 = \mathbb{R}^+ \times \mathbb{R}$ (voir figure 3.1).

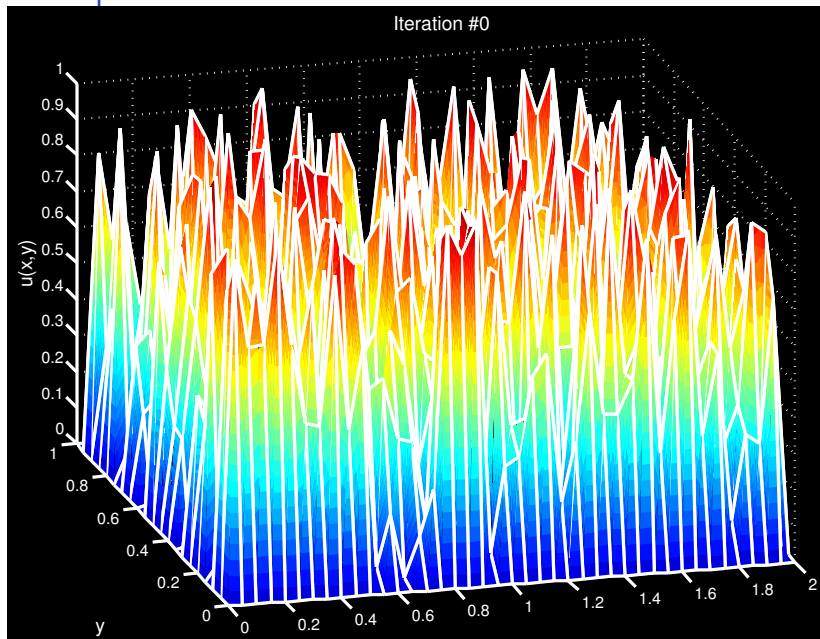


$$\rho(k, c_2, c_3) = \left(\frac{\lambda^-(k) - \lambda_{ap}^-(k)}{\lambda^+(k) - \lambda_{ap}^-(k)} \right)^2$$

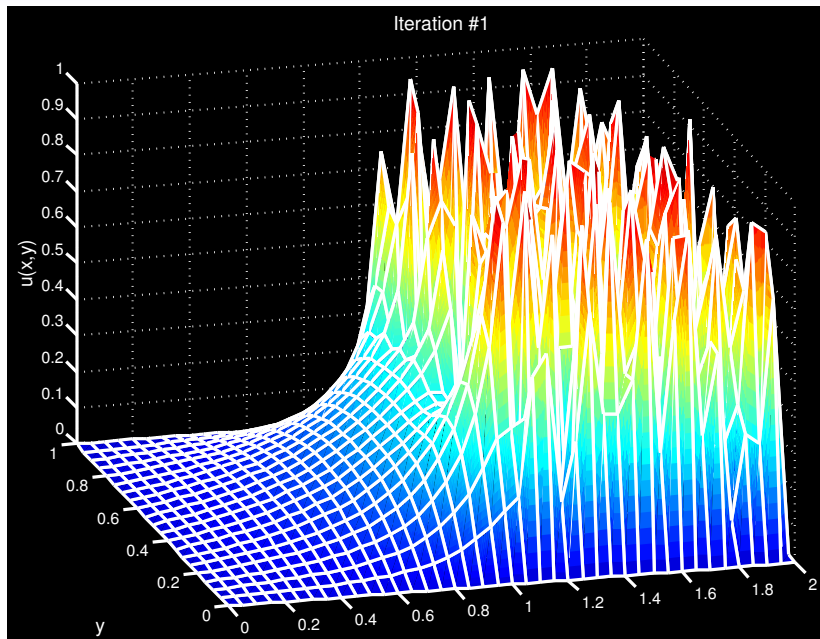


Martin J. Gander, *Optimized Schwarz methods*. SIAM Journal on Numerical Analysis, 44(2), pp. 699–731, 2006.

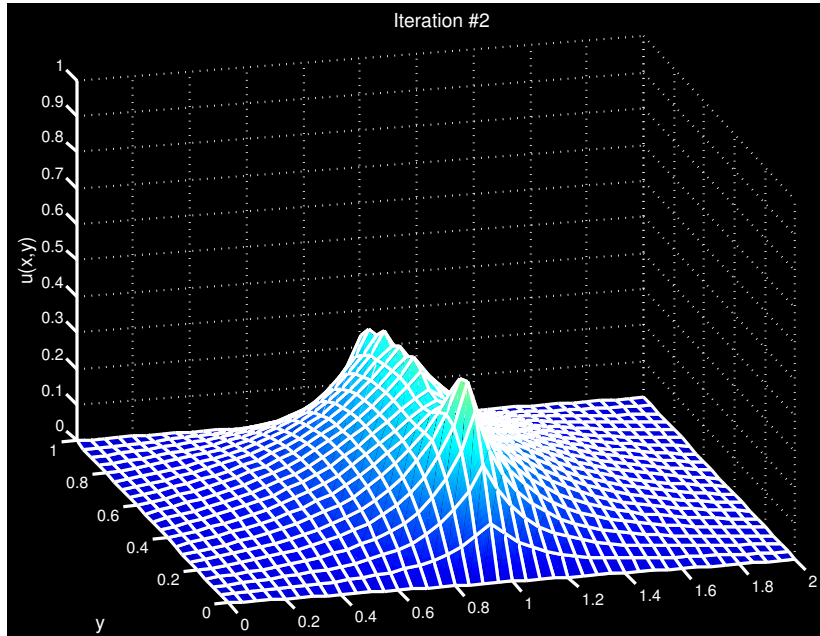
An Example



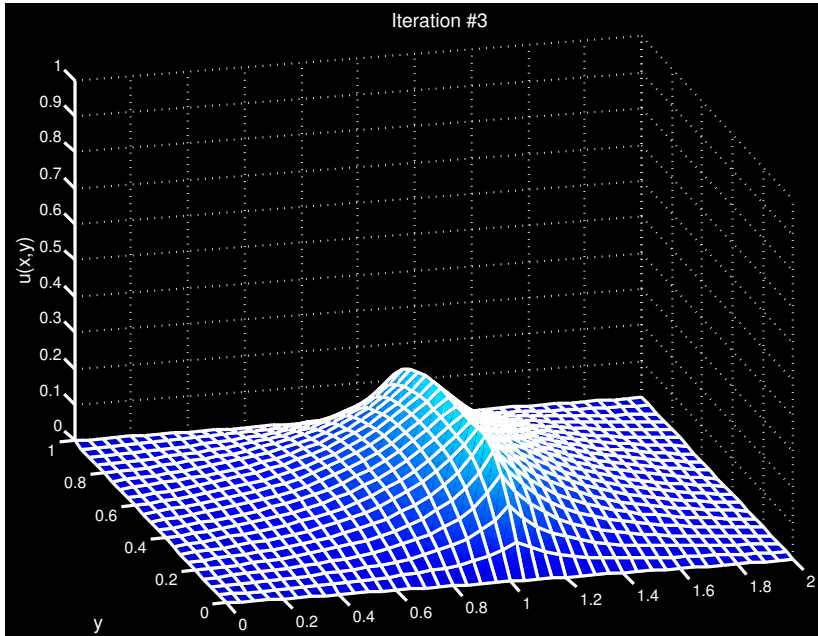
Iteration 1



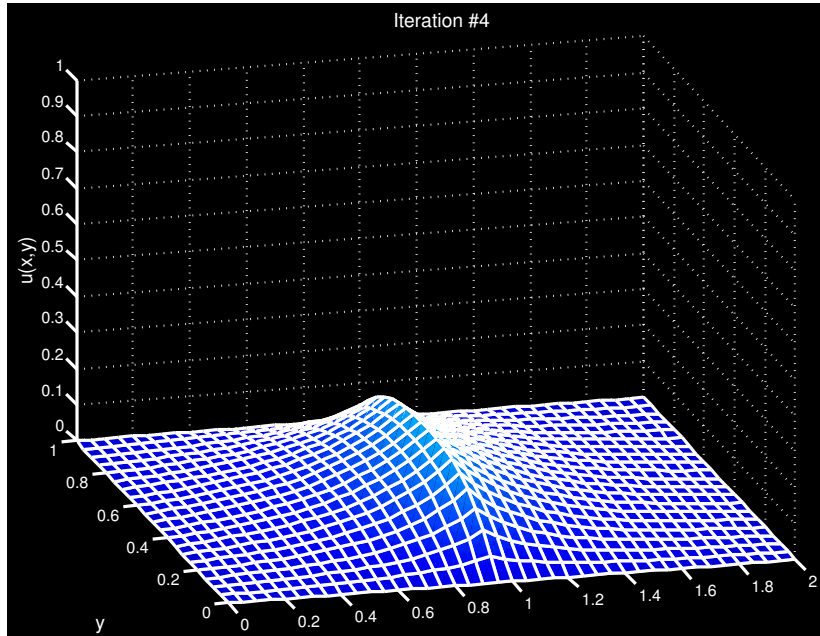
Iteration 2



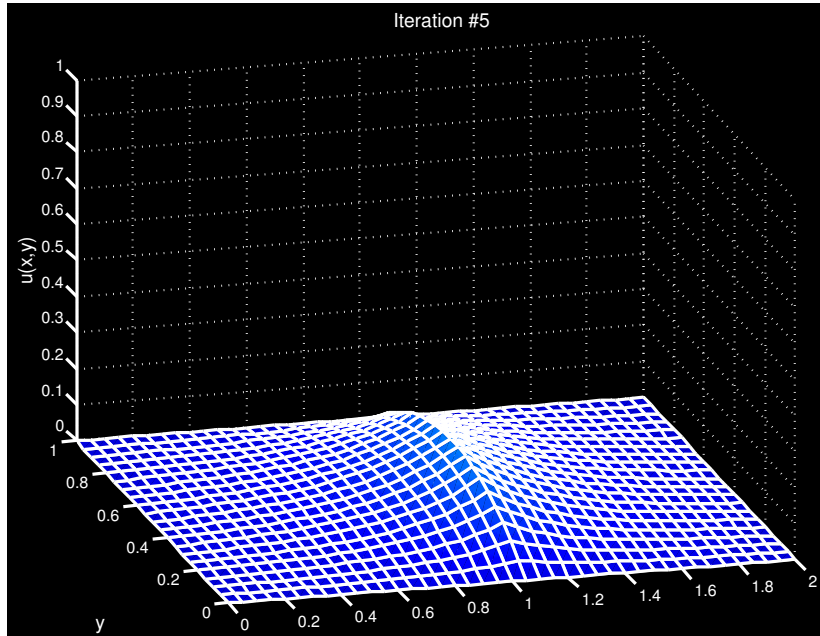
Iteration 3



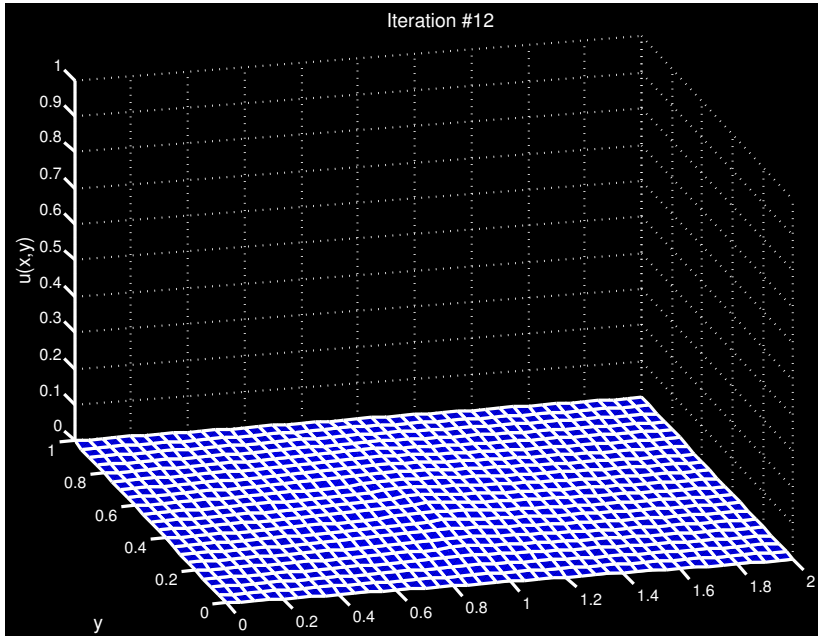
Iteration 4



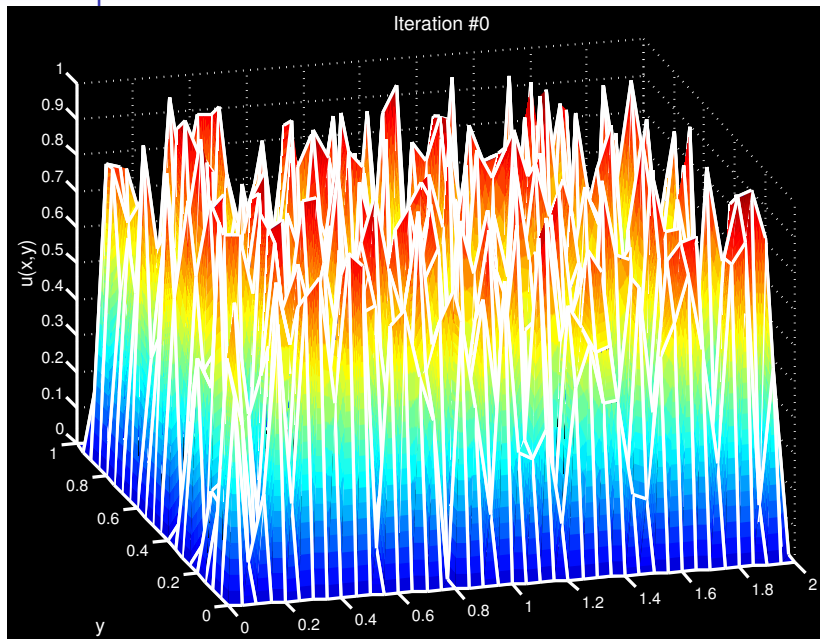
Iteration 5



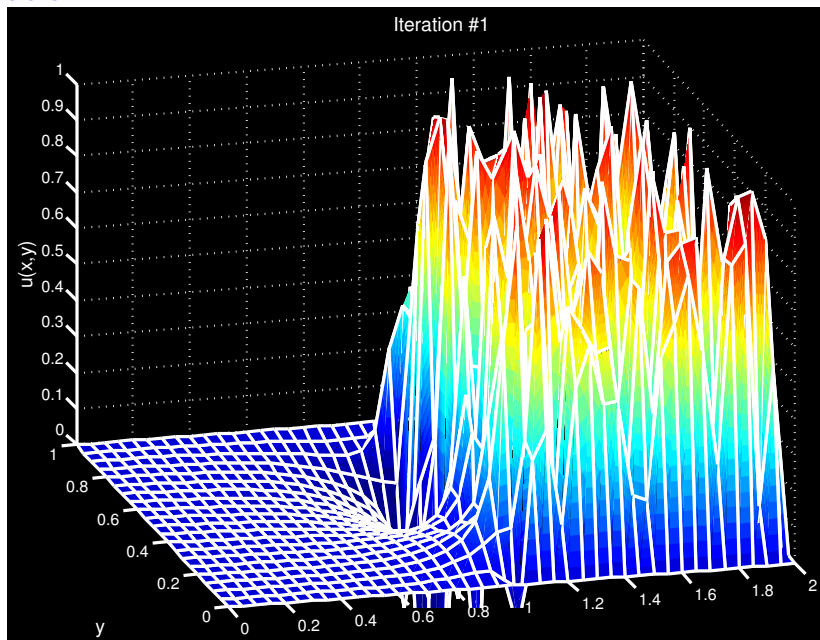
Iteration 12



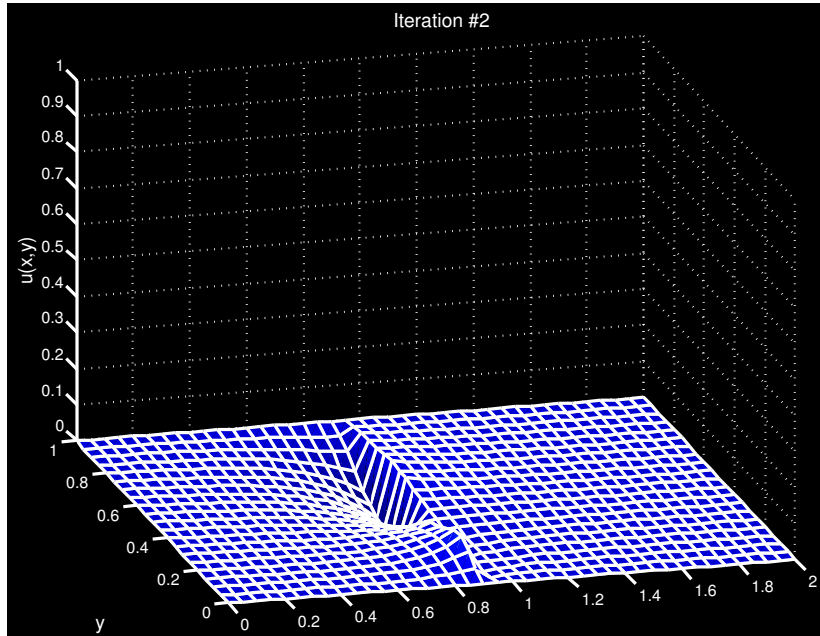
An Example



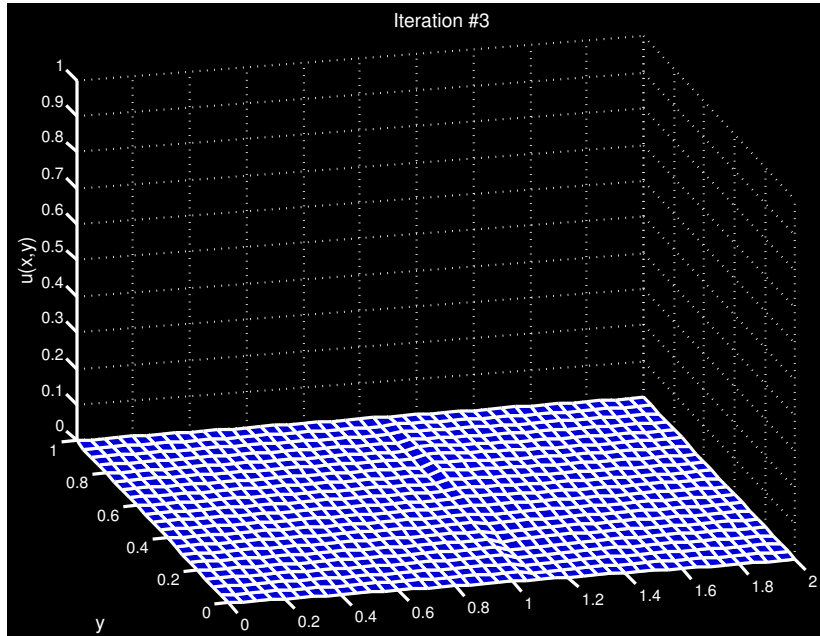
Iteration 1



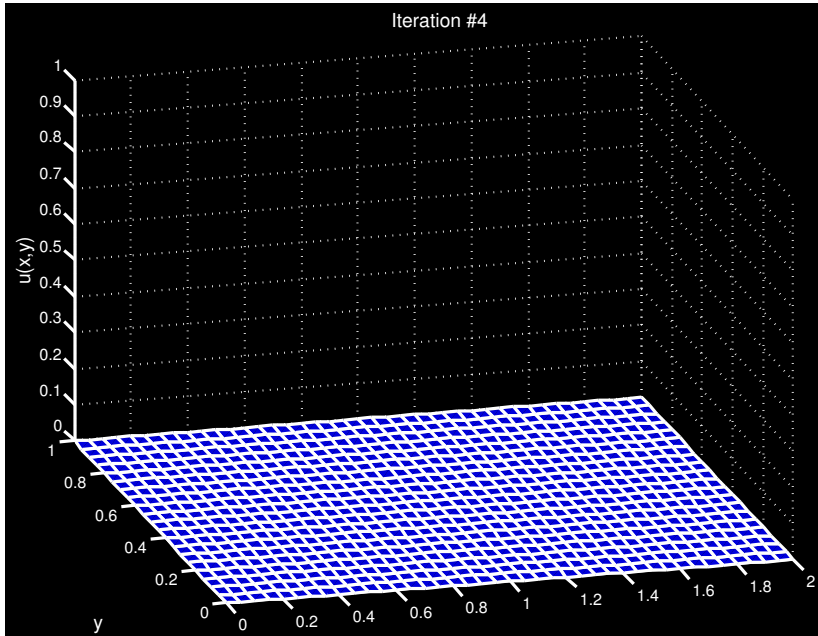
Iteration 2



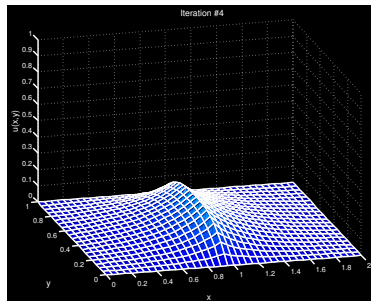
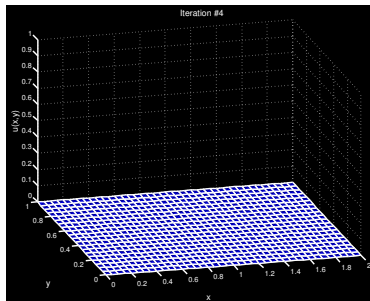
Iteration 3



Iteration 4



Comparison at Iteration 4



Same error

