

# Poisson project for InFoMM C++ training

Consider the 1D Poisson equation

$$\frac{d^2y}{dx^2} + f(x) = 0, \quad 0 \leq x \leq 2\pi,$$

where  $f(x) = 2 \cos(x) \exp(-x)$ , and where we are given the Dirichlet boundary conditions  $y(0) = 0$ ,  $y(2\pi) = 0$ . The closed form solution to this BVP is  $y(x) = \sin(x) \exp(-x)$ . Discretization of the partial second derivative operator using central differences and step length  $h = 2\pi/(n-1)$  leads to the system of equations  $Ay = b$  where

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & & & \vdots \\ 0 & -1 & 2 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ \vdots & & & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ h^2 f(h) \\ h^2 f(2h) \\ \vdots \\ h^2 f((n-2)h) \\ 0 \end{bmatrix}.$$

*Note: the first and last rows are different due to the boundary conditions.*

1. Create a size  $n$  vector  $x$  containing evenly spaced values from 0 to  $2\pi$ . Use any C++ linear algebra library you wish, but you might want to consider the [Eigen](#) library. Create a sparse matrix  $A$  and the corresponding source vector  $b$  and solve the equation  $Ay = b$  to find  $y$ . Print out the error between your solution and the closed form solution and see how this varies with  $n$ .
2. Create STL vectors (`std::vector<>`) representing the diagonals of  $A$  and  $b$ . Use the [Thomas Algorithm](#) to obtain a solution to  $Ay = b$ . Use as many STL algorithms you can to implement your algorithm.
3. Using the Boost timer library, compare the speed of 1 and 2 and a Matlab solution for varying  $n$ .