Return Probability & Statistics

By John Lee

Probability Density Function (pdf)

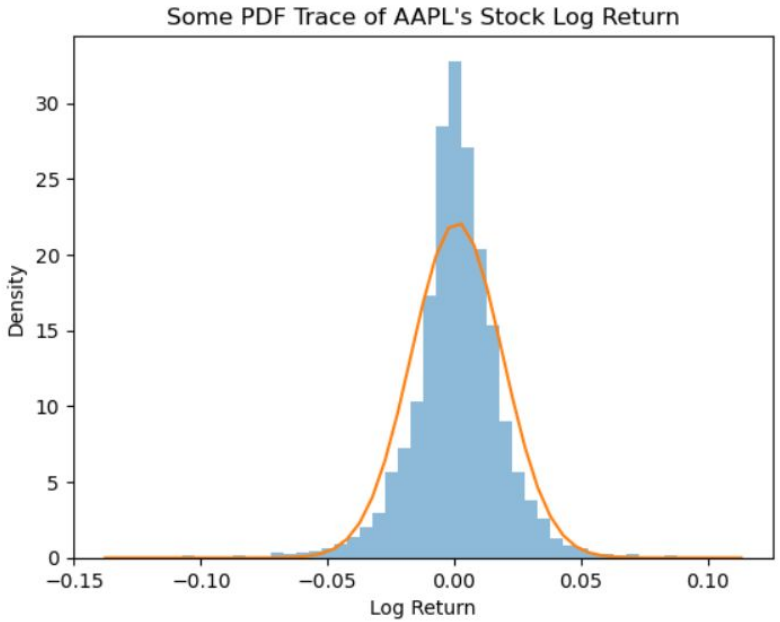
Definition:
A mathematical function that gives the probabilities of the variable, X over all possible outcomes
The pdf of a discrete / continuous random variable is denoted by $f(x)$.

Probability Density Function (pdf)

Example:
Suppose that we propose a random function to trace the the distribution of AAPL's log returns. This funciton is the pdf and takes on the form

$$f(x) = \frac{1}{\sqrt{2\pi(0.018)^2}} e^{\frac{-(x-0.001)^2}{2(0.018)^2}}, \quad -\infty < x < \infty$$

- This function takes in a value and outputs the probability of achieving that value
- We recognize this function as a $N(0.001, 0.018)$ distribution

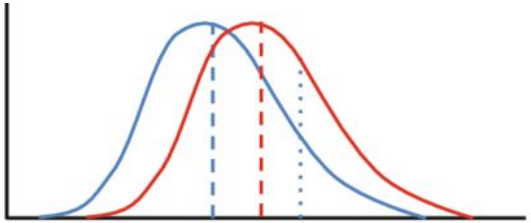
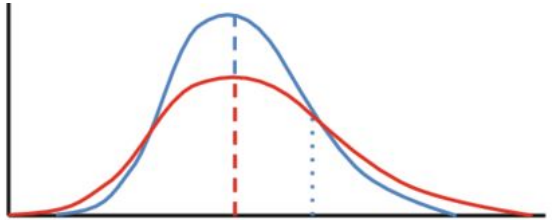


Quantiles

Definition:

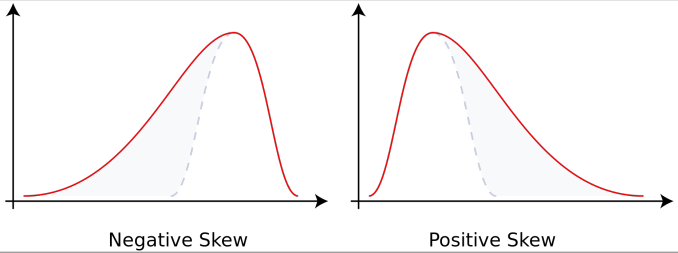
- Cut points dividing the range of probability distribution into continuous intervals with equal probabilities
- For a random variable , X, with cdf , F, the quantile funciton is an inverse of F. We would denote it as F^{-1}.

Probability Distribution Statistics

Mean	Standard Deviation
	
• Measures the average of a finite set of values. • Higher mean shifts the distribution curve to the right; lower mean shifts the distribution curve to the left.	• Measures on average, how far away the values are from the mean. • The higher the standard deviation, the greater the dispersion of the distributed values; the lower the standard deviation, the smaller the dispersion.

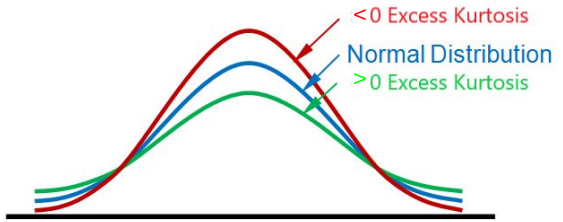
Probability Distribution Statistics

Skewness



- Measures the degree of asymmetry of the distribution curve.
- Zero skewness implies symmetry; positive skewness indicates a relatively long right tail; negative skewness indicates a relatively long left tail.

Excess Kurtosis



- Measures how much more the probability is concentrated in the center and tails of a distribution compared to a normal distribution.
- Higher excess kurtosis means heavier tail values relative to normal; lower excess kurtosis means lighter tail values relative to normal.

Summary Statistics - ADBE vs CMG Daily Log Returns

- Adobe (ADBE) and Chipotle (CMG)'s daily log returns from the past 5 years or 1259 trading days.

	mean	sd	skewness	excess kurtosis
ADBE	0.000431	0.024047	-0.575768	7.565034
CMG	0.001205	0.022763	0.439296	7.087053

What can you say about the mean, sd, skewness, and excess kurtosis of ADBE vs SMG's daily log returns?

Summary Statistics - ADBE vs CMG Daily Log Returns

	mean	sd	skewness	excess kurtosis
ADBE	0.000431	0.024047	-0.575768	7.565034
CMG	0.001205	0.022763	0.439296	7.087053

- **Mean**: the average log return for ADBE is higher, suggesting greater profit potential
- **Standard Deviation**: ADBE's sd at 0.024 and that of GMG at 0.023 are fairly similar. Hence, this means they have similar risk/volatility properties.
- **Skewness**: log returns for ADBE are left-skewed while those for CMG are right-skewed, implying more extreme down-days for ADBE and more extreme up-days for CMG
- **Excess Kurtosis**: log return tail values for ADBE and CMG are thicker than normal, but those for ADBE is thicker than CMG because its excess kurtosis of 7.57 > 7.09. This means that, in general, ADBE has a higher probability of obtaining extreme returns relative to CMG

Reward-to-Risk (RR)

- Helps investors understand the return of an investment compared to its risk.
- Measures the mean return earned per unit of risk. Risk here is a measure of the price fluctuations/ volatility.
- This helps us optimize our portfolio by maximizing return and minimizing risks.
- The larger the reward-to-risk, the higher the expected return per unit of risk; the smaller the reward-to-risk, the lower the expected return per unit of risk

Reward-to-Risk (RR)

$$\text{Reward-to-Risk Ratio} = \frac{\mu_p}{\sigma_p}, \text{ where}$$

μ_p = Expected return, or mean return of the asset or portfolio retrrns

σ_p = Volatility of return, or standard deviation of the asset or portfolio returns

Reward-to-Risk (RR) for ADBE vs CMG Daily Log Returns

	mean	sd	skewness	excess kurtosis
ADBE	0.000431	0.024047	-0.575768	7.565034
CMG	0.001205	0.022763	0.439296	7.087053

Using the Reward-to-Risk measure given the past 5 year data, which stock would you prefer?

Reward-to-Risk (RR) for ADBE vs CMG Daily Log Returns

	mean	sd	skewness	excess kurtosis
ADBE	0.000431	0.024047	-0.575768	7.565034
CMG	0.001205	0.022763	0.439296	7.087053

$$RR_{ADBE} = \frac{0.000431}{0.024047} = 0.017923$$

$$RR_{CMG} = \frac{0.001205}{0.022763} = 0.052949$$

Since CMG has a higher Reward-to-Risk compared to ADBE, this means that it can generate higher profits per unit of risk, hence, we would prefer to invest in CMG.

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