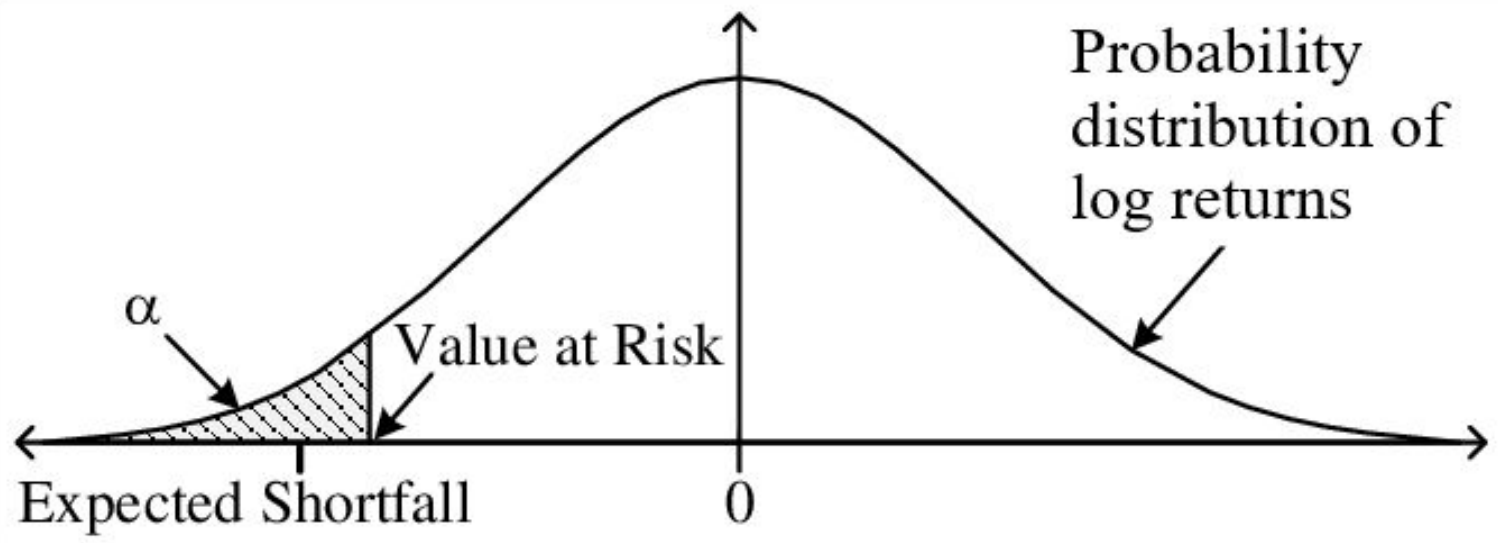
Risk Profiling

By John Lee

Risk Metrics

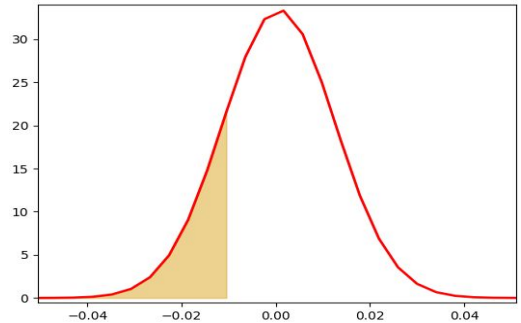
- Ideas is to help quantify the amount of capital needed for covering a loss in a portfolio.
- Also helps traders set up inversely correlated stock trades to offset losing trades.
- The area under the probability distribution curve represents the probability for the log returns to fall within the range of covered values.
- The total area under the curve is always one since it represents a probability value.

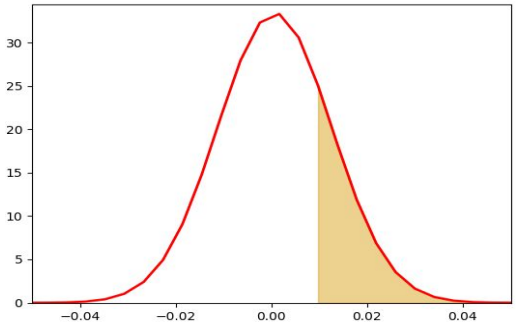
Risk Metrics



Risk Metrics

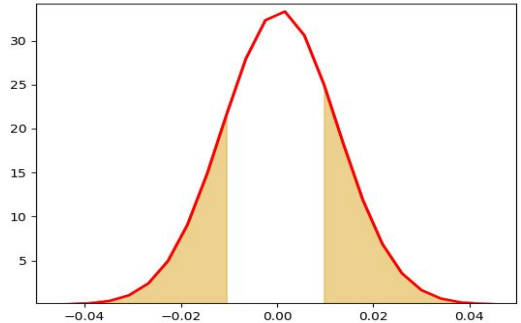
Probability that log return is lower than -0.01.

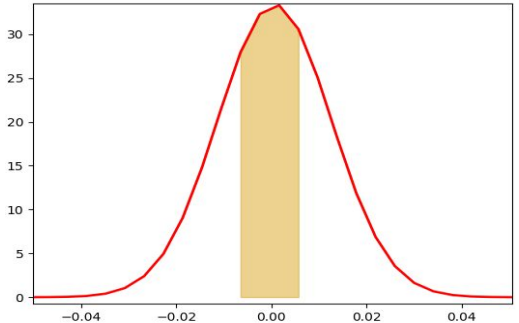




Probability that log return is larger than 0.01.

Probability that log return is either lower than -0.01 or higher than 0.01.





Probability that log return is between -0.01 and 0.01.

Value-at-Risk (VaR)

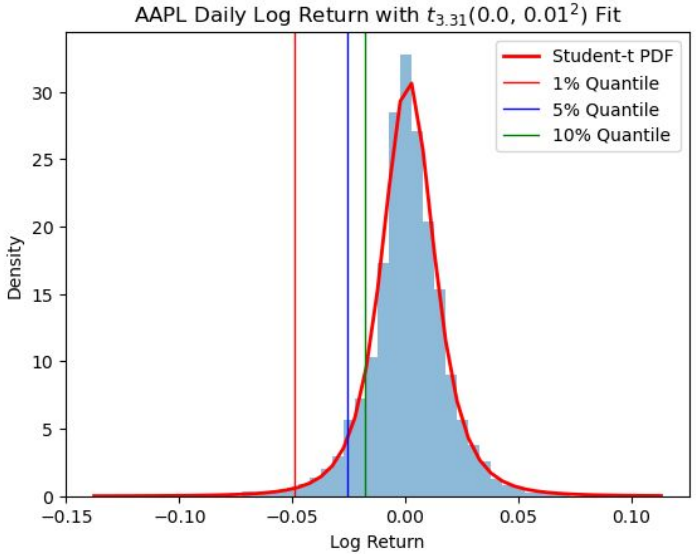
Risk measure over the same time frame as the log return data, daily in this case.

- The area under the left tail of the distribution curve represents the probability of stock losses.
- VaR = dollar value of minimum loss; rVaR = proportion of minimum loss.
- “A portfolio has a α% probability of losing at least rVaR*100% of capital in the next time period.”
- Usually, we look at VaR at $\alpha = 1\%$, $\alpha = 5\%$, and $\alpha = 10\%$ significance levels.

Value-at-Risk (VaR)

Step 1: Find log returns cutoffs (quantiles) which the left-tail probabilities are 1%, 5%, and 10%.

Step 2: Visualize the log return cutoff points on the best-fitted distribution curve.



Value-at-Risk (VaR)

Step 3: Take the exponential of the quantiles, subtract 1, then negate the result to get to rVaR (proportion of capital loss).

$$1\%\,\text{rVaR} = -\left(e^{-0.049149} - 1\right) \approx 4.8\%$$

- There is a 1% probability that AAPL will incur at least a 4.8% loss in the next trading day.

$$5\%\,\text{rVaR} = -\left(e^{-0.025840} - 1\right) \approx 2.55\%$$

- There is a 5% probability that AAPL will incur at least a 2.55% loss in the next trading day.

$$10\%\,\text{rVaR} = -\left(e^{-0.017845} - 1\right) \approx 1.77\%$$

- There is a 10% probability that AAPL will incur at least a 1.77% loss in the next trading day.

Value-at-Risk (VaR)

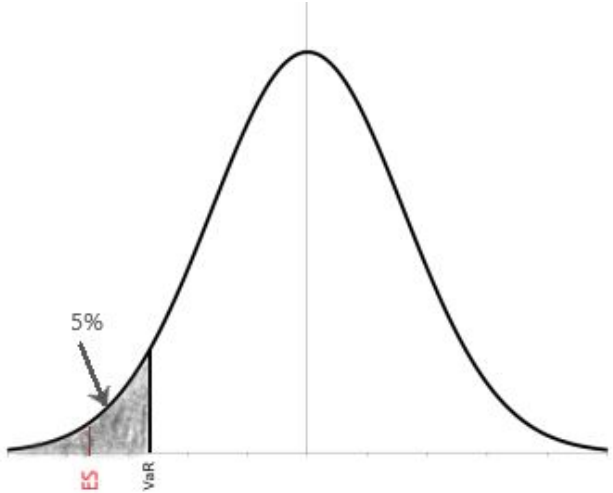
Step 4: Output and interpret rVaR values. Typically, we reference rVaR at the 5% significance level

	Significance Level	Quantile	rVaR
0	1%	-0.049149	0.047960
1	5%	-0.025840	0.025509
2	10%	-0.017845	0.017687

There is a 5% probability that AAPL log return will lose at least 0.025509 * 100 = 2.55% in the next trading day. This means that if we buy $1000 worth of AAPL shares, there is a 5% probability that we will lose at least $25.5 in the next trading day.

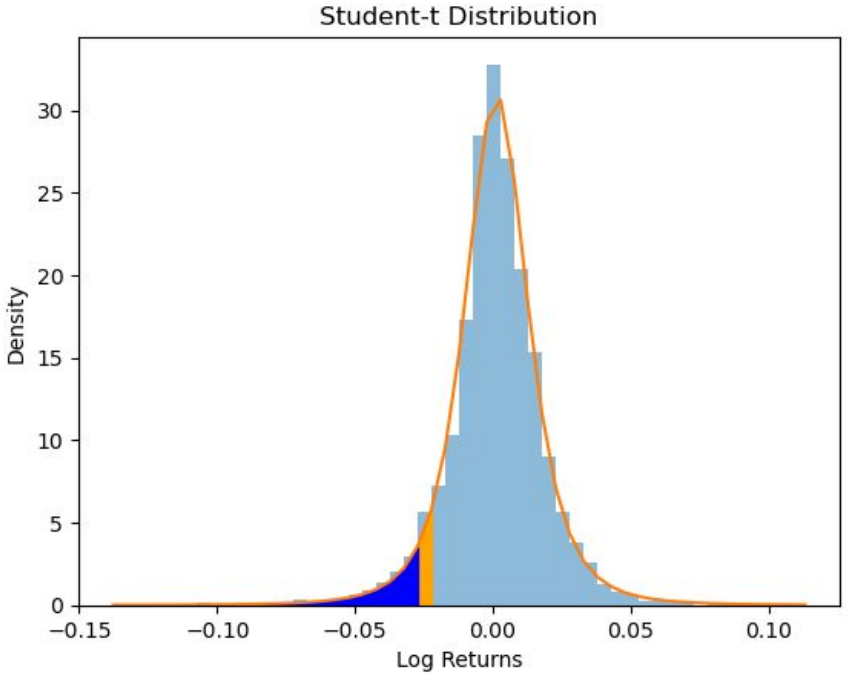
Expected Shortfall (ES)

- VaR tells us at least how much we can expect to lose, but doesn't tell us the magnitude of loss we can expect (e.g., when VaR is $15.391, actual loss can be $150.391).
- ES = dollar value of average loss, given VaR; rES = proportion of average loss, given VaR.
- "Given that the portfolio has a α% probability of losing at least rVaR*100% of capital, it is expected to lose an average of rES*100% in the next time period."



Expected Shortfall (ES)

Step 1: Use Monte Carlo simulation to generate 1 million daily losses from the selected distribution, $t_{1.74}(0, 0.01^2)$.



Expected Shortfall (ES)

Step 2: Find all the simulated losses that are above the rVaR value and then take their mean to get rES (proportion of average capital loss, given rVaR).

$1\%\,\text{rES} \approx 7.06\%$

- There is a 1% probability that AAPL will lose an average of 7.06%, given that it loses at least 4.8% loss in the next trading day.

$5\%\,\text{rES} \approx 4.07\%$

- There is a 5% probability that AAPL will lose an average of 4.07%, given that it loses at least 2.55% loss in the next trading day.

$10\%\,\text{rES} \approx 3.1\%$

- There is a 10% probability that AAPL will lose an average of 3.1%, given that it loses at least 1.77% loss in the next trading day.

Expected Shortfall (ES)

Step 3: Output and interpret rES values. Typically, we reference rES at the 5% significance level.

	Significance Level	Quantile	rVaR	rES
0	1%	-0.049149	0.047960	0.070649
1	5%	-0.025840	0.025509	0.040731
2	10%	-0.017845	0.017687	0.030952

there is a 5% probability that AAPL log return will lose an average of 0.040731 * 100 = 4.07%, given that it loses at least a 0.025509 * 100 = 2.55% in the next trading day. This means that if we buy $1000 worth of AAPL shares, there is a 5% probability that we can expect to loss an average of $40.7, given that it loses at least $25.5 in the next trading day.

Expected Shortfall (ES)

Example #1:

What is the probability that the portfolio worth $50 million will lose an average of $2.04 million in the next trading day?

	Significance Level	Quantile	rVaR	rES
0	1%	-0.049149	0.047960	0.070649
1	5%	-0.025840	0.025509	0.040731
2	10%	-0.017845	0.017687	0.030952

Expected Shortfall (ES)

Example #1 Solution:

Losing an average of $2.04 million means losing an average of $2.04 million/$50 million=4.07% of the total capital . Therefore, the probability of losing an average of 4.07% of the capital is 5%.

Expected Shortfall (ES)

Example #2:

On average, what proportion of the capital would the asset manager set aside if she suspects that there is a 3% probability for AAPL to drop at least 7.34% in the next trading day. (Hint: you may give a range of values for this)

	Significance Level	Quantile	rVaR	rES
0	1%	-0.049149	0.047960	0.070649
1	5%	-0.025840	0.025509	0.040731
2	10%	-0.017845	0.017687	0.030952

Expected Shortfall (ES)

Example #2 Solution:

Since the there is a 3% probability that AAPL's stock can lose at least somewhere between 2.55 to 4.8% in the next trading day, it is reasonable to set aside an average between 4.07% to 7.06% of the capital to prepare for the downdraw.

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