Regression Analysis on Asset Returns

By John Lee

Linear Regression Use Cases

- Understand relationship between a stock's historical prices and various factors like market indexes, interest rates, trading volumes, or company-specific financial indicators.

- Quantify the impact of risk factors to understand risks associated with investments.

- Attribute fundamental performances such as revenue, profits, or sales, to various factors like marketing expenditure, pricing strategies, market conditions, or macroeconomic indicators.

- Assess correlations between asset pairs to help with activities like hedging and pairs trading.

Multiple Linear Regression

The multiple linear regression model defines a linear functional relationship between one continuous outcome variable and MULTIPLE input variables that takes on the following form:

$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \ldots + \beta_n x_{tn} + \epsilon_t$, where

y_t represents the observed outcome at time t
β_0 represents the y-intercept, or y value when $x = 0$
β_{tn} represents the slope of the n-th explanatory variable at time t
ϵ_t represents the error term, which accounts for the total deviations from the linear hyperplane at time t

Multiple Linear Regression

- In reality, multiple regression is more frequently used than simple regression because a dependent variable is rarely explained by only one variable. For instance, it takes more than just the oil price to explain the inflation in the economy.
- Similar to the simple linear regression, we can use the OLS or MLE methods to estimate the hyperplane that best fit the data points.
- Since multiple regression is more complex than simple regression, it possesses lower bias and higher variance.

Predicting Russell 2000 Returns

We compiled a list of features for building our linear model for predicting the performance of a the Russell 2000 ETF (Ticker: IWM). These features include ETFs considering Environmental, Social and Governance (ESG) factors as well as economic indicators.

Technology Breakthrough	**Social Change**	**Urbanization**	**Climate Change**	**Global Wealth**
• IBLC • IRBO • IHAK	• IDNA • IWFH • BMED	• IFRA • IGF • EMIF	• ICLN • IDRV • IVEG	• CNYA

Predicting Russell 2000 Returns

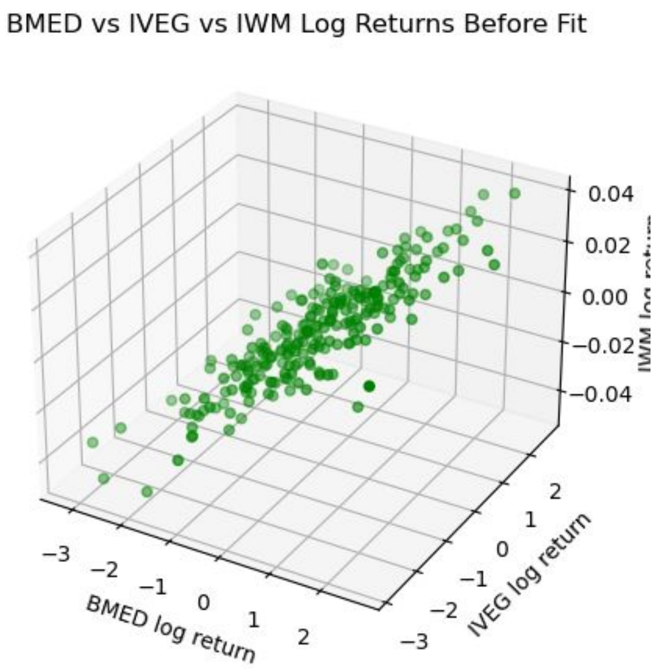
- ETF data comes from *yfinance*; economics data comes from *pandas_datareader*.
- We gather these data dating between 2022-06-02 and 2023-06-31, a total of 248 days.

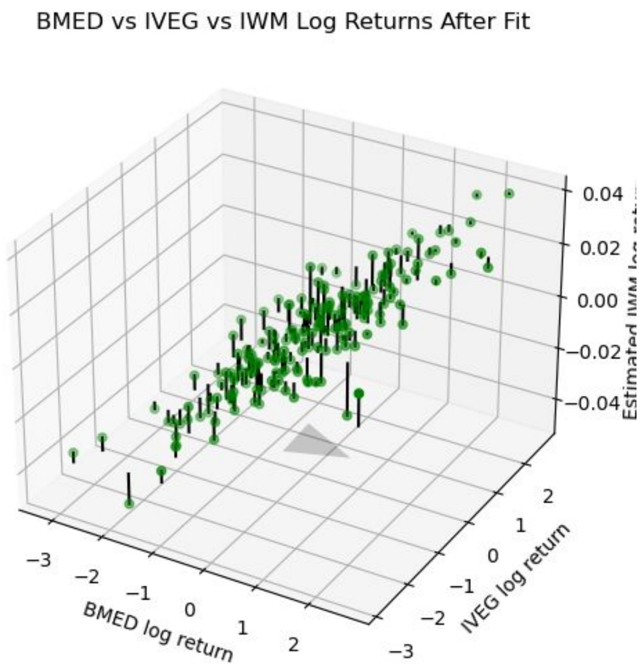
```
DatetimeIndex: 248 entries, 2022-06-02 to 2023-05-31
Data columns (total 17 columns):
 #   Column                   Non-Null Count  Dtype
---  ------                   --------------  -----
 0   BMED                     248 non-null    float64
 1   CNYA                     248 non-null    float64
 2   EMIF                     248 non-null    float64
 3   IBLC                     248 non-null    float64
 4   ICLN                     248 non-null    float64
 5   IDNA                     248 non-null    float64
 6   IDRV                     248 non-null    float64
 7   IFRA                     248 non-null    float64
 8   IGF                      248 non-null    float64
 9   IHAK                     248 non-null    float64
 10  IRBO                     248 non-null    float64
 11  IVEG                     248 non-null    float64
 12  IWFH                     248 non-null    float64
 13  IWM                      248 non-null    float64
 14  volatiliity_index        248 non-null    float64
 15  option_adjusted_spread   248 non-null    float64
 16  inflation_rate           248 non-null    float64
dtypes: float64(17)
```

Multiple Linear Regression Fit

Our model has two independent variables, so its predictive formula looks something like this:

$$\text{IWM } \hat{\text{log}} \text{ return} = \hat{\beta_0} + \hat{\beta_1} \cdot (\text{BMED log return}) + \hat{\beta_2} \cdot (\text{IVEG log return})$$





Fitting a Multiple Linear Regression

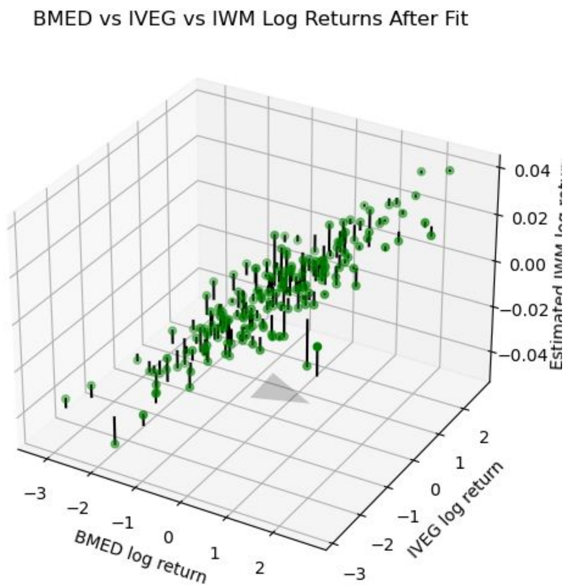
- This is a multiple linear regression with two independent variables
- Using OLS, we find that $\beta_0 = -0.0003$, $\beta_1 = 0.0077$, and $\beta_2 = 0.0081$ using OLS
- We fit the estimated coefficients into the regression model and arrived with :

$\hat{y_t} = -0.0003 + 0.0077x_{t1} + 0.0081x_{t2}$, where $\hat{y_t}$ denotes the predicted value at time t,.

where $\hat{y_t}$ denotes the predicted IWM log return at time t,

x_{t1} denotes BMED log return at time t,

x_{t2} denotes IVEG log return at time t.



OLS Regression Result Summary

What can you say about this multiple linear regression model?

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    IWM   R-squared:                       0.874
Model:                            OLS   Adj. R-squared:                  0.873
Method:                 Least Squares   F-statistic:                     649.3
Date:                Wed, 07 Jun 2023   Prob (F-statistic):           7.01e-85
Time:                        12:17:05   Log-Likelihood:                 719.95
No. Observations:                 190   AIC:                            -1434.
Df Residuals:                     187   BIC:                            -1424.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0003      0.000     -0.723      0.471      -0.001       0.001
BMED           0.0077      0.001     13.699      0.000       0.007       0.009
IVEG           0.0081      0.001     14.378      0.000       0.007       0.009
==============================================================================
Omnibus:                        6.279   Durbin-Watson:                   1.952
Prob(Omnibus):                  0.043   Jarque-Bera (JB):                8.704
Skew:                           0.168   Prob(JB):                       0.0129
Kurtosis:                       3.993   Cond. No.                         2.36
==============================================================================
```

OLS Regression Result Summary

- **T-test:** p-value < 0.05, rejects null hypothesis and conclude that BMED and IVEG log returns are able to explain the variance of IWM log returns.

- **F-test:** p-value < 0.05, rejects the hypothesis and conclude that at least BMED or IVEG log returns can explain the variance of IWM log returns.

OLS Regression Result Summary

- **Jarque Bera tests:** p-value < 0.05, so rejects the null hypothesis and conclude that the model residuals are normally distributed.

- **Durbin-Watson test:** < 2 so conclude that there is a positive relationship between the model residuals at adjacent time periods.

- **Condition number:** < 30 so conclude that the model residuals do not seem to have significant multicollinearity, hence implying more stable and reliable model.

- **AIC & BIC:** a number alone is meaningless. We need to compare them to the other models.

OLS Regression Result Summary

We fit the multiple linear regression model on the test data.

```
R-squared:  0.8496716652941945
Adjusted R-squared:  0.8429904059739364
MAE:  0.004589376026359801 5
RMSE:  0.005849070813181978
```

- **R-squared:** the model explains 84.97% of the variation in IWM's log returns, without adjusting for penalty in feature numbers.

- **Adjusted R-squared:** the model explains 82.30% of the variation in IWM's log returns, adjusting for penalty in feature numbers.

- **MAE:** On average, the total absolute distance of the predicted log returns for IWM from their actual values is 0.0046.

- **RMSE:** The square root of the average of squared differences between the predicted IWM log returns and their actual values is 0.0058.

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