14

TRANSISTOR OSCILLATORS
(Sinusoidal)

We have studied that an amplifier produces an output signal whose waveform is similar to the input signal, of course, the power level is generally high. The amplifier is an energy converter *i.e.*, it takes energy from supply source and converts into a.c. signal at signal frequency. When there is no input signal, there is no energy conversion *i.e.*, there is no output signal. On the other hand, oscillator does not require an external input source and produces an output signal so long as d.c. power source is connected. Hence the *oscillator may be defined as a circuit which generates an a.c. output signal without any externally applied input signal or a circuit which converts d.c. energy into a.c. energy at very high frequency.* The output waveform may be sine, square, saw tooth or pulse shapes.

Now the question is that an alternator (a.c. machine) can serve the purpose of an oscillator or not. The answer is no. The reason is that usually an alternator generates frequencies upto 1000 Hz. To generate higher frequencies, there are so many practical difficulties. For example, either the number of poles has to be made large of the speed of rotation of armature has to be made extremely high. Both these factors are impracticable. Hence the alternator can not serve the purpose. So we have to depend on electronic circuit.

The theory of oscillations developed in vacuum tube oscillators is equally valid for transistor oscillators. The network associated with the transistor determines the frequency of oscillations, while characteristics of transistor with circuit determine the conditions of oscillations. The oscillators may be classified as :

(i) sinusoidal oscillators, and
(ii) relaxation oscillators.

Sinusoidal oscillators are those oscillators which operate on the linear portion of the characteristics, whereas the relaxation oscillators operate on non-linear region of its characteristics. They are again classified as :

(i) feedback type,
(ii) negative resistance type.

In feedback oscillators, part of the output is fed back to the input in proper phase and magnitude. In negative resistance oscillators, the transistor provides the negative resistance which cancels the positive resistance of the associated circuit and thus provides for oscillations.

The essential components of an oscillator are :

(i) Tank circuit. The tank circuit consists of an inductance coil in parallel with a capacitor. The frequency of oscillations in the circuit depends upon the values of inductance and capacitance. This is given by

$$f = \left\{\frac{1}{2\pi\sqrt{LC}}\right\}$$

where L is the inductance of induction coil and C is the capacitance of the capacitor.

(ii) Transistor amplifier. A block diagram of transistor amplifier is shown in fig. (1). It receives d.c. power from the battery and changes it into a.c. power for supplying it to the tank circuit. The

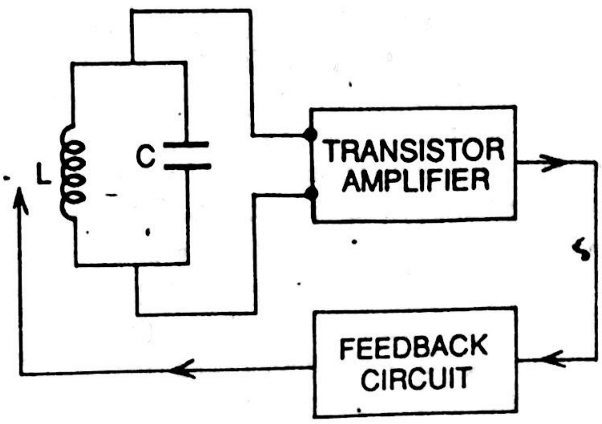

Fig. 1.

oscillations of tank circuit are fed to the transistor amplifier which are amplified due to transistor amplifying action.

(iii) Feedback circuit. The feedback circuit supplies a part of the output energy to tank circuit in correct phase to overcome the losses occurring in tank circuit and the balance is radiated out in the form of electromagnetic waves. The feedback circuit provides a positive feedback.

FEEDBACK OSCILLATORS

14·1. TUNED COLLECTOR OSCILLATORS :

Fig. (2) shows the circuit of tuned collector oscillator in C-E configuration. It contains a tuned circuit L_1C_1 in the collector and hence the name. The combination L_1 and C_1 forms the oscillatory circuit to set the frequency of oscillation. Here C_1 is variable capacitor and L_1 forms the primary winding of a step down transformer. A feedback coil L_2 in the base circuit is magnetically coupled (with mutual inductance M) to the tank circuit coil L_1. The inductance L_2 forms the secondary of the transformer. Since transistor is connected in C-E configuration, it provides a phase shift of 180° between its input and output circuits. Another phase shift of 180° is provided by transformer. In this way a phase shift of 360° appears between output and input voltages resulting in a positive feedback. The d.c. operating point for the transistor is established by supply voltage V_{cc}, emitter resistance R_e and potential divider arrangement consisting of resistors R_1 and R_2. The capacitor C_2 connected in base circuit provides low reactance path to the oscillations. Capacitor C_e is the emitter bypass capacitor so that resistor R_e has no effect on a.c. operation of the circuit.

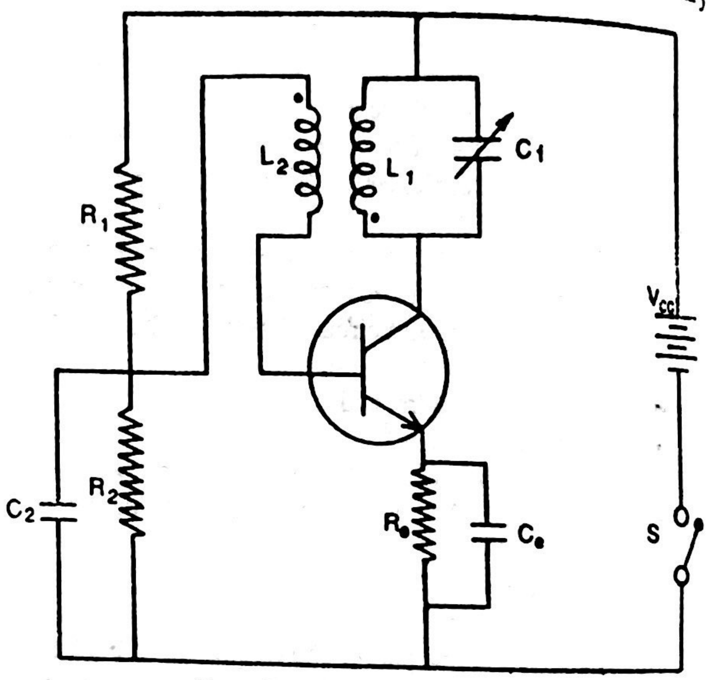

Fig. 2. Tuned collector oscillator.

Circuit operation : When the switch S is closed, collector current starts increasing and charges the capacitor C_1. When capacitor C_1 is fully charged, it discharges through coil L_1 setting up natural oscillations in the tank circuit. These natural oscillations induce a small e.m.f. into L_2 by mutual induction. The value of e.m.f. depends upon the number of turns in L_2 and coupling between L_1 and L_2. The voltage across L_2 is applied between base and emitter and thus causes corresponding variation in base-current. The variations in I_b are amplified β times and appear in collector circuit. A part of the amplified energy is used in overcoming the losses occurring in tank circuit and the balance is radiated out in the form of electromagnetic waves.

Analysis. Fig. (3) shows the equivalent circuit of a tuned collector oscillator using *CE* hybrid model. Here it is assumed that $h_{re} = 0$.

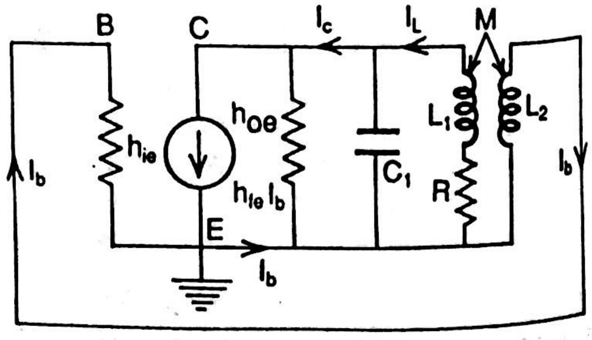

Fig. 3. Equivalent circuit of tuned collector oscillator using CE hybrid model.

The voltage gain of amplifying circuit without feedback is given by

$$A_{ve} = \frac{-h_{fe} Z_L}{h_{ie} + \Delta h\, Z_L} \qquad \ldots (1)$$

where $\Delta h = h_{ie}\, h_{oe} - h_{re}\, h_{fe}$ and Z_L is the equivalent impedance of the capacitive reactance $1/\omega\, C_1$ and impedance $(R + j\omega\, L_1)$ in parallel.

The negative sign in eq. (1) indicates a phase change of 180° introduced by the amplifier.

Here
$$\frac{1}{Z_L} = \frac{1}{(R + j\,\omega L_1)} + \frac{1}{1/j\,\omega C_1} = \frac{1}{(R + j\,\omega L_1)} + j\,\omega C_1$$

$$= \frac{1 + j\,\omega C_1 \times (R + j\,\omega L_1)}{(R + j\,\omega L_1)} = \frac{1 + j\,\omega C_1 R - \omega^2 L_1 C_1}{(R + j\,\omega L_1)}$$

$$\therefore \qquad Z_L = \frac{1 + j\,\omega L_1}{(R - \omega^2 L_1 C_1) + j\,\omega C_1 R} \qquad \ldots (2)$$

According to Barkhausen criterion for sustained oscillations

$$|A_{ve}\beta| = 1 \quad \text{or} \quad |\beta| = \frac{1}{|A_{ve}|} \qquad \ldots (3)$$

where $\beta = \dfrac{\text{Voltage induced in the secondary}}{\text{Voltage across the primary coil}} = \dfrac{-j\,\omega M\, I_L}{(R + j\,\omega L_1)\, I_L}$

or $\beta' = \dfrac{-j\,\omega M}{(R + j\,\omega L_1)}$ (negative sign shows a phase change of 180° by transformer) ... (4)

Substituting the values of β and A_v from eqs. (4) and (1) in eq. (3), we get

$$\frac{j\,\omega M}{(R + j\,\omega L_1)} = \frac{h_{ie} + \Delta h\, Z_L}{h_{fe} Z_L} = \frac{h_{ie}}{h_{fe} Z_L} + \frac{\Delta h}{h_{fe}}$$

or
$$\frac{j\,\omega M\, h_{fe}}{(R + j\,\omega L_1)} = \frac{h_{ie}}{Z_L} + \Delta h$$

Substituting the value of Z_L, we get

$$\frac{j\,\omega M\, h_{fe}}{(R + j\,\omega L_1)} = \frac{h_{ie}\left\{(1 - \omega^2 L_1 C_1) + j\,\omega C_1 R\right\}}{(R + j\,\omega L_1)} + \Delta h$$

or $\quad j\,\omega M\, h_{fe} = h_{ie}\,(1 - \omega^2 L_1\, C_1) + j\omega\, C_1\, R\, h_{ie} + \Delta h\,(R + j\,\omega\, L_1)$

or $\quad j\,\omega M\, h_{fe} = h_{ie} - h_{ie}\,\omega^2 L_1\, C_1 + j\,\omega\, C_1\, R\, h_{ie} + \Delta h\,.\,R + j\,\omega\, L_1\,\Delta h$

or $\quad j\,\omega M\, h_{fe} = (h_{ie} - h_{ie}\,\omega^2 L_1\, C_1 + \Delta h.R) + j\,\omega\,(C_1\, R\, h_{ie} + L_1\,\Delta h)$

or $\quad (h_{ie} - h_{ie}\,\omega^2 L_1\, C_1 + \Delta h.R) + j\,\omega\,(C_1\, R\, h_{ie} + L_1\,\Delta h - M\, h_{fe}) = 0 \qquad \ldots (5)$

The real and imaginary parts of eq. (5) must separately be zero. Equating the real part to zero, we get the frequency of oscillations and equating the imaginary part to zero, we get the *condition for sustained oscillations.*

Equating the real part to zero, we get

$$h_{ie} - h_{ie}\,\omega^2 L_1\, C_1 + \Delta h\,.\,R = 0$$

or $$1 - \omega^2 L_1 C_1 + (\Delta h . R / h_{ie}) = 0$$

or $$\omega^2 L_1 C_1 = 1 + (\Delta h . R / h_{ie})$$

$$\omega = \frac{1}{\sqrt{(L_1 C_1)}} \cdot \sqrt{\left(1 + \frac{R . \Delta h}{h_{ie}}\right)}$$

The frequency of oscillations is given by

$$f = \frac{1}{2\pi \sqrt{L_1 C_1}} \cdot \sqrt{\left(1 + \frac{R . \Delta h}{h_{ie}}\right)} \quad \dots (6)$$

Since Δh and R are small and h_{ie} is large, hence the frequency of oscillations is given by

$$f = \frac{1}{2\pi \sqrt{L_1 C_1}} \quad \dots (7)$$

Equating the imaginary part to zero, we have

$$C_1 R h_{ie} + L_1 \Delta h - M h_{fe} = 0$$

or $$C_1 R h_{ie} + L_1 \Delta h = M h_{fe}$$

$\therefore$ $$M = \frac{C_1 R h_{ie} + L_1 \Delta h}{h_{fe}} \quad \dots (8)$$

Equation (8) gives the minimum value of the mutual inductance which is necessary so that the circuit may oscillate.

Example 1. *The resonant circuit of a tuned collector transistor oscillator has a frequency of* 5 MHz. *If the value of capacitance is increased by* 50%, *calculate the new resonant frequency.*

We know that the resonant frequency is given by

$$f = \frac{1}{2\pi \sqrt{(LC)}}$$

In first case, $$5 \times 10^6 = \frac{1}{2\pi \sqrt{(LC)}} \quad \dots (1)$$

In second case, $$f_0 = \frac{1}{2\pi \sqrt{L (1 \cdot 5C)}} \quad \dots (2)$$

($\because$ when capacity is increased by 50%, the new capacitance becomes $1 \cdot 5C$)

Dividing eq. (2) by eq. (1), we get

$$\frac{f_0}{5 \times 10^6} = \frac{1}{\sqrt{1 \cdot 5}} \quad \text{or} \quad f_0 = \frac{5 \times 10^6}{\sqrt{(1 \cdot 5)}}$$

$\therefore$ $$f_0 = \mathbf{4 \cdot 08\ MHz.}$$

Example 2. *A tuned collector oscillator employs a tranformer whose primary inductance is* 10 mH. *The capacitor connected across the primary has a capacitance of* 100 pF. *The d.c. resistance of the primary coil is* 10 *ohm and the transistor used has* $h_{ie} = 1\ k\Omega$, $h_{re} = 10^{-4}$, $h_{fe} = 50$ *and* $h_{oe} = 10^{-4}$ A/V. *Find the frequency of ocillations and the mutual inductance between the primary and secondary coils required to sustain the ocillations.*

We know that $$f = \frac{1}{2\pi \sqrt{(L_1 C_1)}}$$

$\therefore$ $$f = \frac{1}{2\pi \sqrt{(10 \times 10^{-3})(100 \times 10^{-12})}}$$

$$= 1 \cdot 592 \times 10^5 \text{ Hz} = \mathbf{159 \cdot 2\ kHz}$$

Here $\Delta h = h_{ie}\, h_{oe} - h_{fe}\, h_{re}$

$$= 1 \times 10^3 \times 10^{-4} - 50 \times 10^{-4} = 0{\cdot}095$$

Further,

$$M = \frac{C_1 R\, h_{ie} + L_1 \Delta h}{h_{fe}}$$

$$\therefore \quad M = \frac{\left(100 \times 10^{-12}\right)(10)\left(1 \times 10^{3}\right) + \left(1 \times 10^{-3}\right)(0{\cdot}095)}{50}$$

$$= 19{\cdot}02 \times 10^{-6}\ \text{H} = \mathbf{19{\cdot}02\ \mu H}$$

14·2. TUNED EMITTER OSCILLATOR :

Figure 4 (a and b) shows the tuned emitter oscillator and its equivalent circuit.

In the analysis, the effect of bias circuit is ignored. The collector resistance being high and the resistance of coil L_2 being low, their effect are neglected.

The loop potential equations for meshes (1) and (2) are

$$I\,(r + j\,\omega L_1 + 1/j\,\omega C)$$
$$-I_e / j\,\omega C \pm j\,\omega M\,\alpha\, I_e = 0 \quad \ldots (1)$$

and $I_e\,(r_c + r_b + 1/j\,\omega C)$

$$-1/j\,\omega C - \alpha\, I_e\, r_b = 0. \quad \ldots (2)$$

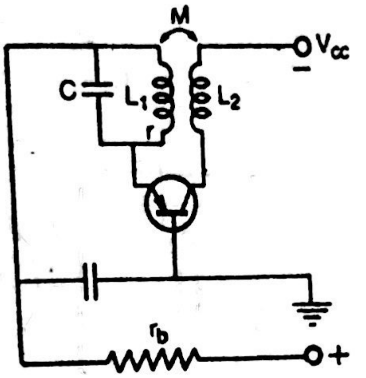

Fig. 4. (a) Tuned emitter oscillator.

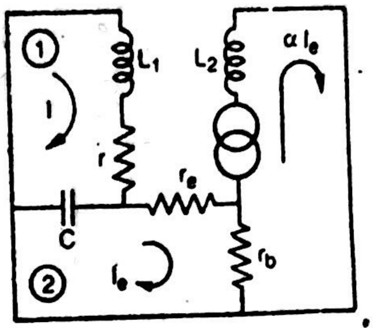

Fig. 4. (b) Equivalent circuit.

Putting $R = r_e + r_b\,(1 - \alpha)$, in equation (2), we have

$$I\,(r + j\,\omega L_1 + 1/j\,\omega C) = I_e\,[1/j\,\omega C \pm j\,\omega M\,\alpha] \quad \ldots (3)$$

and

$$1/j\,\omega C = I_e\,[R + 1/j\,\omega C]. \quad \ldots (4)$$

From equations (3) and (4), we have

$$\frac{1}{j\,\omega C}\left(\frac{1}{j\,\omega C} \pm j\,\omega M\,\alpha\right) = \left(R + \frac{1}{j\,\omega C}\right)\left(r + j\,\omega L_1 + \frac{1}{j\,\omega C}\right)$$

Equating real parts :

$$-\frac{1}{\omega^2 C^2} \pm \frac{M\,\alpha}{C} = R\,r + \frac{L_1}{C} - \frac{1}{\omega^2 C^2}.$$

Solving it, we have $M = \pm\,[(CRr/\alpha) + (L_1/\alpha)]$.

This is the maintenance condition and $\omega^2 = 1/(L_1 C)$.

Equating the imaginary parts,

$$-\frac{r}{\omega C} + \omega L_1 R - \frac{R}{\omega C} = 0\,.$$

$$\therefore \quad \omega^2 L_1 C R = R + r$$

or

$$f = \frac{1}{2\pi \sqrt{L_1 C}} \sqrt{\left(1 + \frac{r}{R}\right)}$$

This is the generated frequency.

14·3. TUNED BASE OSCILLATOR :

Fig. (5) shows a tuned base oscillator using a transistor in CE configuration. The resistors R_1 and R_2 provide the d.c. bias of the circuit. The parallel combination of $R_e - C_e$ in the emitter circuit is the stabilizing circuit. C_2 is the blocking capacitor. L_1-C_1 is a tuned circuit which is connected to the base of the transistor. The mutually coupled coils L_1 and L_2 form the primary and secondary of an RF

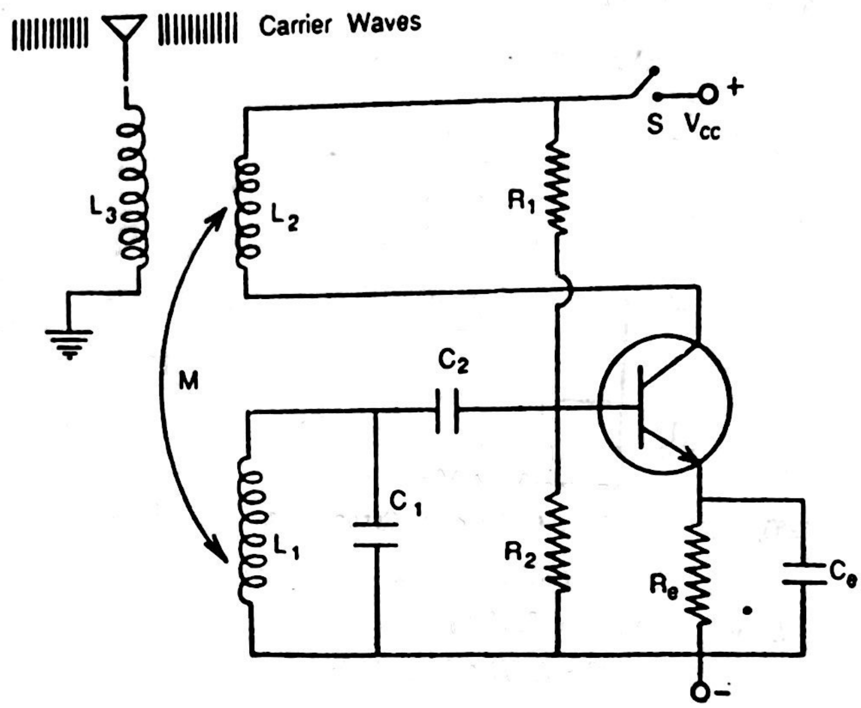

Fig. 5. Tuned base oscillator.

transformer. We know that CE connected transistor provides a phase shift of 180° between input and output circuits and the transformer also provides a phase shift of 180°. In this way a total phase shift of 360° is produced which is the essential requirement for producing oscillations.

Operation : As the moment switch S is closed, collector current start increasing. Due to increase in collector current, there is an expanding magnetic field through coil L_2 which is linked with the coil L_1. So there is an induced e.m.f. (feedback voltage) in coil L_1. This feedback voltage increases the base current and thus a further increase in collector current. In this way feedback voltage helps in increasing the collector current until saturation is reached. Meanwhile, the condenser C_1 is fully charged. Now as soon as collector current ceases to increase, magnetic field of L_2 ceases to expand and there is no feedback voltage in L_1. At this stage, the condenser C_1 which has been charged fully, discharges through L_1. The discharge of the condenser C_1, decreases the emitter-base bias. As a result, the collector current decreases and hence there is a collapsing magnetic field in coil L_1. This is accompanied by an induced feed back voltage in L_2 in opposite direction to previous one. This decreases the emitter-base bias and hence the collector current. The process continues till collector current reaches its cut-off value. In this way the feedback voltage helps in decreasing the plate current. During this time, the capacitor which has lost its original charge, is now fully charged in opposite direction. When the transistor is cut-off, the capacitor begins to discharge through L_1. This is accompanied by an increase in emitter-base *i.e.*, an increase in collector current. The increase in collector current, produces an expanding magnetic field in L_2 which results in an increase induced feedback voltage in L_1. Now there is a further increase in emitter-base bias. The process continues till collector current reaches its saturation value. This cycle of operation keeps repeating so long as enough energy is supplied to meet the losses in L_1-C_1 circuit. The output can be taken out by means of a third winding L_3 magnetically coupled to L_1. The frequency of oscillation is equal to the resonant frequency L_1 C_1 circuit.

14.4 HARTLEY OSCILLATOR :

Fig. (6a) shows the circuit of a shunt fed Hartley oscillator using a transistor in C-E configuration. In this circuit the parallel combination of R_e and C_e in conjuction with R_1 and R_2 combination provides stabilized self-bias. The frequency determining network is made up of the variable capacitor C and the inductors L_1 and L_2. The coil L_1 is inductively coupled to coil L_2 and the combination forms an auto-transformer. So far as a.c. signals are concerned, one side of L_1 is connected to base via C_b and the other to emitter via ground and R_e. Similarly, one end of L_2 is connected to collector via C_c and the other to common emitter via C_e. In this way L_1 is connected in the base-emitter circuit (input circuit) and L_2 is connected in the collector-emitter circuit (output circuit). Feedback between output and input is accomplished through transformer action. The transformer introduces a phase change of 180°. The transistor also introduces a phase change of 180°. Thus total phase change becomes 360°. This makes the feedback positive which is the essential requirement for oscillations. Radio frequency choke (RFC) provides d.c. load for the collector and also keeps a.c. current out of the d.c. supply V_{cc}. The reactance of RFC is higher than L_2 and hence may be omitted from the equivalent circuit. The condenser C_c blocks d.c. and provides an a.c. path from the collector to the tank circuit. It acts as an open circuit at zero frequency. The capacitor C_b has a low reactance at the frequency of oscillations and may be omitted from the equivalent circuit.

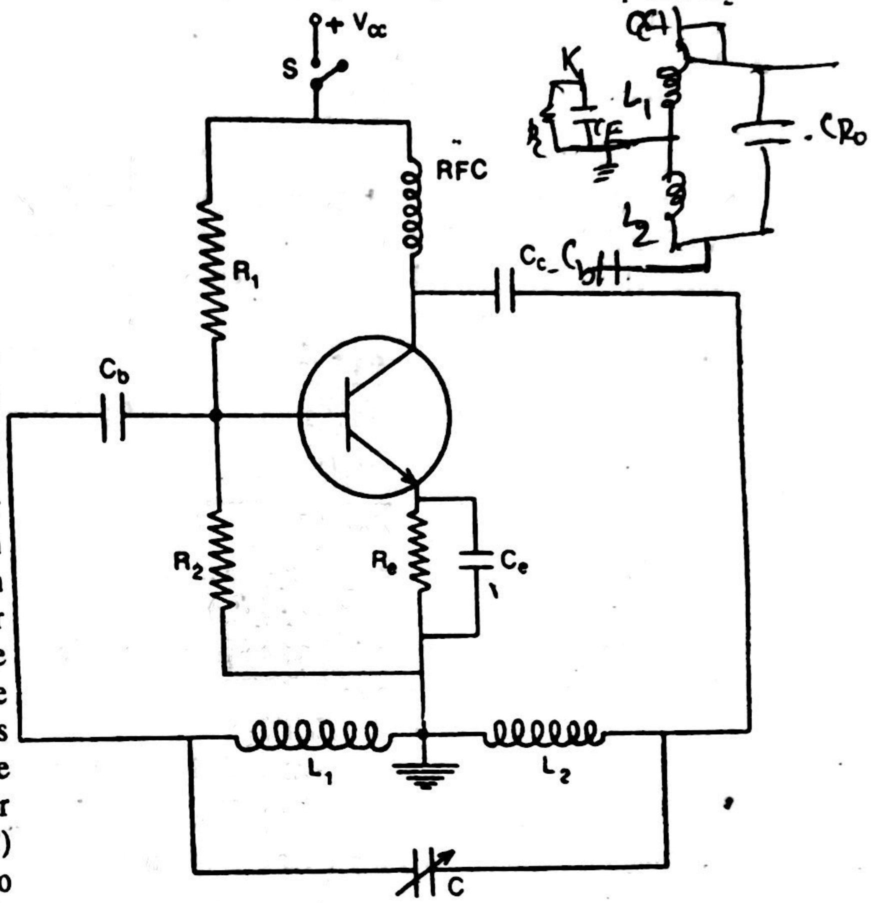

Fig. 6. (a) Shunt-fed Hartley oscillator.

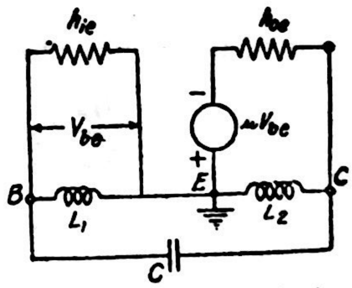

Fig. 6. (b) Equivalent circuit of Hartley oscillator.

Circuit operation. When the switch S is closed, collector current starts rising and charges the capacitor. When capacitor C is fully charged, it discharges through coils L_1 and L_2. Now damped harmonic oscillations are set up in the tank circuit. The oscillations across L_1 are applied to the input circuit (base emitter junction) and appear in the amplified form in the output circuit (collector circuit). Feedback of energy from collector-emitter circuit to the base-emitter circuit is accomplished by means of mutual inductance between L_1 and L_2. In this way energy is continuously supplied to the tank circuit to overcome the losses occurring in it. So continuous undamped output is obtained.

General theory of Hartley Oscillator :

A complete theory of Hartley oscillator using a bipolar transistor is complicated and hence we make the following assumptions :

(i) The feedback source of e.m.f. $h_{re} V_o$ is omitted because h_{re} (reverse voltage ratio) of the transistor is negligible.

(ii) The output admittance h_{oe} of the transistor is very small *i.e.*, the output resistance $(1/h_{oe})$ of the transistor in parallel with inductance L_2 is very large. Hence $(1/h_{oe})$ is omitted.

(iii) The inductive and capacitive reactances are represented by Z_1, Z_2 and Z_3.

(iv) The input terminals are taken as 1 and 2 while the output terminals are taken as 2 and 3.

With these assumptions, the general equivalent circuit is shown in fig. (7). The impedance Z_1 and input resistance h_{ie} are in parallel and hence the equivalent impedance Z'_1 is given by

$$Z'_1 = Z_1 \parallel h_{ie} \quad \text{... (1)}$$

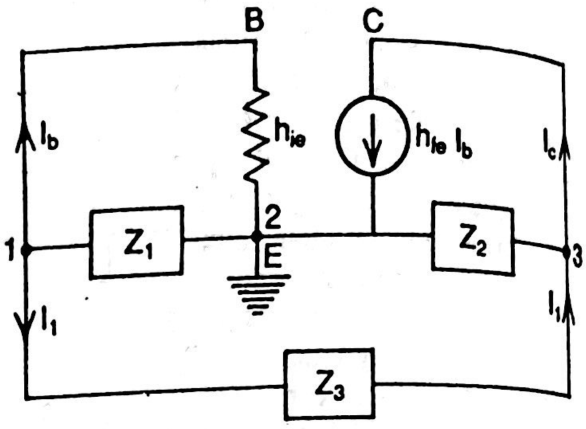

Fig. 7. General equivalent circuit of Hartley oscillator

Now the load impedance Z_L between output terminals 2 and 3 is equivalent to the equivalent impedance of Z_2 in parallel with the series combination of Z'_1 and Z_3. Hence

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{(Z_3 + Z'_1)} = \frac{Z_3 + Z'_1 + Z_2}{Z_2(Z_3 + Z'_1)}$$

or

$$Z_L = \frac{Z_2(Z_3 + Z'_1)}{Z_2 + (Z_3 + Z'_1)} \quad \text{...(2)}$$

The voltage gain without feedback is given by

$$A_{ve} = -(h_{fe} Z_L / h_{ie}) \quad \text{... (3)}$$

In order to obtain the feedback fraction β, we consider the output voltage between the terminals 2 and 3. The output voltage is given by

$$V_o = I_1 (Z'_1 + Z_3)$$

The voltage fedback to the input terminals 1 and 2 is given by

$$V_{fb} = I_1 Z'_1 \qquad \therefore \qquad \beta = V_{fb} / V_o = Z'_1 / (Z'_1 + Z_3) \quad \text{... (4)}$$

Applying the condition $A_{ve} \beta = 1$ for oscillation, we get

$$-\frac{h_{fe} Z_L}{h_{ie}} \times \frac{Z'_1}{(Z'_1 + Z_3)} = 1$$

Substituting the value of Z_L, we get

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2(Z'_1 + Z_3)}{Z_2 + (Z_3 + Z'_1)} \times \frac{Z'_1}{(Z'_1 + Z_3)} = -1$$

or

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2 Z'_1}{Z_2 + (Z_3 + Z'_1)} = -1$$

or

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2 \{Z_1 h_{ie} / (Z_1 + h_{ie})\}}{Z_2 + Z_3 + \{Z_1 h_{ie} / (Z_1 + h_{ie})\}} = -1$$

or

$$\frac{h_{fe}}{h_{ie}} \left[\frac{Z_1 Z_2 h_{ie}}{Z_2(Z_1 + h_{ie}) + Z_3(Z_1 + h_{ie}) + Z_1 h_{ie}} \right] = -1$$

$$h_{fe}\, Z_1 Z_2 = -Z_2 (Z_1 + h_{ie}) - Z_3 (Z_1 + h_{ie}) - Z_1 h_{ie}$$

$$Z_1 Z_2 (h_{fe}) + Z_1 Z_2 + Z_2 h_{ie} + Z_1 Z_3 h_{ie} + Z_1 h_{ie} = 0$$

$X_C = -j/\omega C$

$X_L = j\omega L$

$$\boxed{h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 (1 + h_{fe}) + Z_1 Z_3 = 0} \quad \ldots (5)$$

This is the general equation for the oscillator.

alysis of Hartley Oscillator.

Suppose in Hartley oscillator, the resistances of inductors are negligibly small and M be the ual inductance between the inductors. Now we have

$$Z_1 = j\omega L_1 + j\omega M, \quad Z_2 = j\omega L_2 + j\omega M \quad \text{and} \quad Z_3 = 1/j\omega C = -j/\omega C$$

Substituting these values in eq. (5), we get

$$h_{ie}[(j\omega L_1 + j\omega M) + (j\omega L_2 + j\omega M) - j/\omega C] + (j\omega L_1 + j\omega M)(j\omega L_2 + j\omega M)$$
$$\times (1 + h_{fe}) + (j\omega L_1 + j\omega M)(-j/\omega C) = 0$$

$$h_{ie}\, j\omega[L_1 + L_2 + 2M - (1/\omega^2 C)] - \omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) + \{(L_1 + M)/C\} = 0 \quad \ldots (6)$$

The frequency of oscillation can be obtained by equating the imaginary part of eq. (6) to zero, *i.e.*,

$$L_1 + L_2 + 2M - (1/\omega^2 C) = 0 \quad \text{or} \quad \omega^2 C = 1/(L_1 + L_2 + 2M)$$

$$\omega = \frac{1}{\sqrt{\{(L_1 + L_2 + 2M)C\}}} \quad \therefore \quad f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\{(L_1 + L_2 + 2M)C\}}} \quad \ldots (7)$$

The condition for the maintenance of the oscillations can be obtained by equating the real part of (6) to zero. Thus

$$-\omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) + \{(L_1 + M)/C\} = 0$$

$$1 + h_{fe} = \frac{1}{(L_2 + M)\,\omega^2 C} = \frac{(L_1 + L_2 + 2M)C}{(L_2 + M)C}$$

$$h_{fe} = \frac{(L_1 + L_2 + 2M)C}{(L_2 + M)} - 1 = \frac{(L_1 + M)}{(L_2 + M)} \quad \ldots (8)$$

Equation (8) gives the condition for the maintenance of the oscillations.

Example 1. *Find the operating frequency of a transistor Hartley oscillator if L_1 = 100 μH, L_2 = mH, mutual inductance between the coils M = 20 μH and C = 20 pF.*

In case of Hartley oscillator

$$f = \frac{1}{2\pi} \times \frac{1}{\sqrt{(L_1 + L_2 + 2M)C}} = \frac{1}{2 \times 3{\cdot}14} \times \frac{1}{\sqrt{[(100 + 1000 + 2 \times 20)\,10^{-6}\,(20 \times 10^{-12})]}}$$

$$= \frac{1}{2 \times 3{\cdot}14} \times \frac{1}{\sqrt{[1140 \times 10^{-6} \times (20 \times 10^{-12})]}} = 1052 \text{ k Hz}$$

Example 2. *A Hartley oscillator is to span a frequency range from 50 kHz to 150 kHz. The riable capacitance has the values in the range 50 pF to 450 pF. The transistor to be used has h_{fe} = 50 d Δ_{he} = 0·5 [$\Delta_{he} = h_{fe}\, h_{oe} - h_{ie}\, h_{re}$]. Determine the values of the inductances. Neglect mutual ductance between the coils and use CE circuit configuration.*

We know that

$$f = \frac{1}{2\pi\sqrt{\{(L_1 + L_2)C\}}}$$

The oscillator frequency has to be varied in the ratio 1 : 3. The variable capacitances have the lues in the ratio 1 : 9 so the frequency of oscillations can be varied in the ratio 1 : 3.

$$\therefore \quad 50 \times 10^3 = \frac{1}{2 \times 3{\cdot}14\sqrt{\{(L_1 + L_2) \times 450 \times 10^{-12}\}}}$$

Solving we get $\quad L_1 + L_2 = 0{\cdot}0225 \text{ H} = 22{\cdot}5 \text{ mH} \quad \ldots (1)$

Again $\qquad \dfrac{L_2}{L_1} = \dfrac{h_{fe}}{\Delta_{he}} = \dfrac{50}{0{\cdot}5} = 100$

$\therefore \qquad L_2 = 100\,L_1$

Solving eqs. (1) and (2), we get

$L_1 = 0{\cdot}225$ mH $\quad$ and $\quad L_2 = 22{\cdot}275$ mH

14. COLPITT'S OSCILLATOR :

Fig. (8) shows the circuit of a Colpitt's oscillator. The circuit of Colpitt oscillator is the same that of Hartley oscillator except that the emitter tap is connected between the capacitances C_1 and C_2. In this circuit the parallel combination of R_e and C_e in conjuction with R_1 and R_2 combination provides stabilized self-bias. The frequency determining network is made up of inductance L and capacitors C_1 and C_2. The function of C_c is to block d.c. and provides an a.c. path from collector to the tank circuit. RFC (Radio frequency choke) provides the necessary d.c. load resistance for collector and also prevents a.c. signal from entering the d.c. supply V_{cc}. The condenser C_b conveys feedback from collector to base circuit.

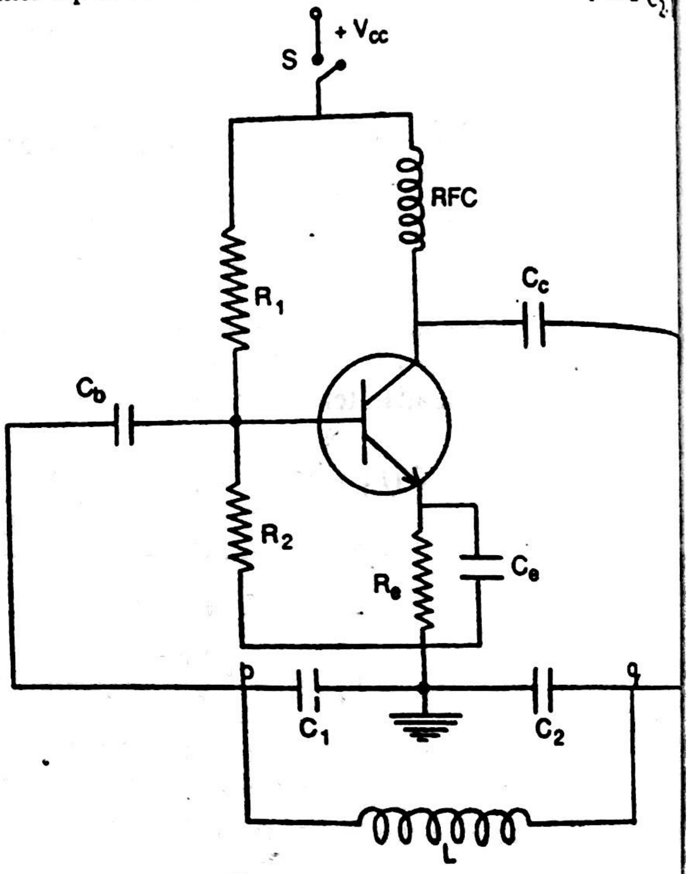

Fig. 8. Colpitt's Oscillator.

Circuit operation : When the switch S is closed, the capacitors C_1 and C_2 are charged. These capacitors are discharged through the inductance L and thereby set up damped harmonic oscillations in the tank circuit. The oscillations across C_1 are applied to the base-emitter junction and appears in the amplified form in the collector circuit and supply losses to the tank circuit. The amount of feedback depends upon relative capacitance values of C_1 and C_2. Higher is the value of C_1, smaller is the feedback.

Now we shall show that the energy supplied to the tank circuit is in phase with the generated oscillations. Hence the capacitors C_1 and C_2 act as an alternating voltage divider. So points p and q are 180° out of phase. Further, we know that a CE transistor produces a phase change of 180° between input voltage and output voltage. Thus a total phase change of 360° occurs. In this way, continuous undamped oscillations are produced.

Analysis : Fig. (9) shows the equivalent circuit of Colpitt's oscillator.

Let $C_s = \dfrac{C_1 C_2}{C_1 + C_2}$, then the circuit equations may be written as

$$j\left(\omega L - \frac{1}{\omega C_s}\right) I_1 - \frac{j}{\omega C_1} I_b + \frac{j}{\omega C_1} I_c = 0, \qquad \ldots (1)$$

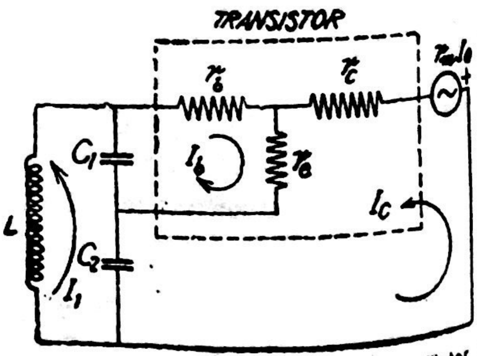

Fig. 9. Equivalent circuit of Colpitt's oscillator.

$$\frac{-j}{\omega C_1} I_1 + \left(r_b + r_e - \frac{j}{\omega C_1}\right) I_b + r_e I_c = 0, \qquad \ldots (2)$$

$$\frac{j}{\omega C_2} I_1 + \left(r_e - r_m\right) I_b + \left(r_e + r_c - r_m - \frac{j}{\omega C_2}\right) I_c = 0. \qquad \ldots (3)$$

For oscillations to occur, the determinant of the above equations must be zero. Equating it to zero, real and imaginary terms of the determinant separately, we obtain the feedback condition and erating frequency. Hence,

$$\begin{vmatrix} j\left(\omega L - \frac{1}{\omega C_s}\right) & \frac{-j}{\omega C_1} & \frac{j}{\omega C_2} \\ -\frac{j}{\omega C_1} & \left(r_b + r_e - \frac{j}{\omega C_1}\right) & r_e \\ \frac{j}{\omega C_2} & \left(r_e - r_m\right) & \left(r_e + r_c - r_m - \frac{j}{\omega C_2}\right) \end{vmatrix} = 0 \qquad \ldots (4)$$

$$j\left(\omega L - \frac{1}{\omega C_s}\right)\left\{\left(r_b + r_e - \frac{j}{\omega C_1}\right)\left(r_e + r_c - r_m - \frac{j}{\omega C_2}\right) - r_e\left(r_e - r_m\right)\right\}$$

$$-\frac{j}{\omega C_1}\left\{\left(r_e\right)\left(\frac{j}{\omega C_2}\right) - \left(\frac{j}{\omega C_1}\right)\left(r_e + r_c - r_m - \frac{j}{\omega C_1}\right)\right\}$$

$$+\frac{j}{\omega C_2}\left\{\left(\frac{-j}{\omega C_1}\right)\left(r_e - r_m\right) - \frac{j}{\omega C_2}\left(r_b + r_e - \frac{j}{\omega C_1}\right)\right\} = 0. \qquad \ldots (5)$$

Equating real part to zero, we get

$$\left(\omega L - \frac{1}{\omega C_s}\right)\left(\frac{r_b + r_e}{\omega C_2} + \frac{r_e + r_c - r_m}{\omega C_1}\right)\Bigg\} + \left\{\frac{r_e}{\omega^2 C_1 C_2} + \frac{r_e + r_c - r_m}{\omega^2 C_1^2}\right\} + \left\{\frac{r_e - r_m}{\omega^2 C_1 C_2} + \frac{r_b + r_e}{\omega^2 C_2^2}\right\} = 0$$

or

$$\left(\omega L - \frac{1}{\omega C_s}\right)\left(\frac{r_b + r_e}{\omega C_2} + \frac{r_e + r_c - r_m}{\omega C_1}\right) + \frac{r_e + r_c - r_m}{\omega^2 C_1^2} + \frac{r_b + r_e}{\omega^2 C_2^2} + \frac{2r_e - r_m}{\omega^2 C_1 C_2} = 0. \qquad \ldots (6)$$

The frequency at which oscillations start is very near to the resonant frequency of resonant circuit, i.e. $\omega L - \frac{1}{\omega C_s} = 0$. It may also be assumed that $r_e << r_m$.

The equation (6), then becomes $$\frac{r_e + r_c - r_m}{\omega^2 C_1^2} + \frac{r_b + r_e}{\omega^2 C_2^2} + \frac{2r_e - r_m}{\omega^2 C_1 C_2} = 0$$

or

$$\left(r_b + r_e\right)\left(\frac{C_1}{C_2}\right)^2 - r_m\left(\frac{C_1}{C_2}\right) + \left(r_e - r_m\right) = 0 \qquad \ldots (7)$$

since $r_e << r_m$.

Equation (7) is quadratic, hence $\dfrac{C_1}{C_2} = \dfrac{r_m + \sqrt{\left\{r_m^2 - 4(r_b + r_e)(r_e - r_m)\right\}}}{2(r_b + r_e)}$ ...

The second term in square root is small in comparison to r_m.

$$\therefore \qquad \frac{C_1}{C_2} \geq \frac{r_m + r_m}{2(r_b + r_e)} \geq \frac{r_m}{(r_b + r_e)}$$ ...

Equating the imaginary part to zero, we get

$$\left(\omega L - \frac{1}{\omega C_s}\right)\left\{(r_b + r_e)(r_e + r_c - r_m) - \frac{1}{\omega^2 C_1 C_2}\right\} - \frac{1}{\omega^3 C_1^2 C_2} - \frac{1}{\omega^3 C_1 C_2^2} = 0$$

or $$\left(\omega L - \frac{1}{\omega C_s}\right)\left(A - \frac{1}{\omega^2 C_1 C_2}\right) - \frac{1}{\omega^3 C_1 C_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 0,$$ ...(

where $A = (r_b + r_e)(r_e + r_c - r_m)$.

From equation (10), we get

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{L C_s} + \frac{1}{A C_1 C_2}\right)}$$ ...(

Equation (11) gives the frequency of oscillations.

Alternative treatment.:

We have derived the general equation of oscillator as

$$h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 (1 + h_{fe}) + Z_1 Z_3 = 0$$ ...

See eq. (5) of previous treatment.

In case of Colpitt's oscillator

$$Z_1 = 1/j\omega C, \quad Z_2 = 1/j\omega C_2 \quad \text{and} \quad Z_3 = j\omega L$$ ...

Substituting these values in eq. (1), we get

$$h_{ie}\left(-\frac{j}{\omega C_1} - \frac{j}{\omega C_2} + j\omega L\right) + \left(-\frac{j}{\omega C_1}\right)\left(-\frac{j}{\omega C_2}\right)(1 + h_{fe}) + \left(-\frac{j}{\omega C_1}\right) j\omega L = 0$$

$$-j h_{ie}\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L\right) - \frac{(1 + h_{fe})}{\omega^2 C_1 C_2} + \frac{L}{C_1} = 0$$ ...

The frequency of oscillation can be obtained by equating the imaginary part to zero

$$\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L = 0 \quad \text{or} \quad \frac{(C_1 + C_2)}{L C_1 C_2} = \omega^2$$

or $$\omega = \sqrt{\frac{(C_1 + C_2)}{L C_1 C_2}} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\left\{\frac{(C_1 + C_2)}{L C_1 C_2}\right\}}$$ ...

Equation (4) gives the frequency of oscillations.

To obtain the condition of maintenance of oscillations, we compare the real parts on both sid eq. (3). Hence

$$-\frac{(1 + h_{fe})}{\omega^2 C_1 C_2} + \frac{L}{C_1} = 0 \quad \text{or} \quad (1 + h_{fe}) = \omega^2 C_2 L$$

$$h_{fe} = \omega^2 C_2 L - 1 = \frac{(C_1 + C_2)}{L C_1 C_2} \times C_2 L - 1$$

$$h_{fe} = 1 + \left(\frac{C_2}{C_1}\right) - 1 = \frac{C_2}{C_1} \quad \ldots (5)$$

Equation (5) gives the condition of maintenance of oscillations.

Example 1. *Find the operating frequency of a transistor Collpitt's oscillator if* $C_1 = 0{\cdot}001\ \mu F$. $_2 = 0{\cdot}01\ \mu F$ *and* $L = 15\ \mu H$.

For Collpitt's oscillator $f = \frac{1}{2\pi}\sqrt{\left[\frac{(C_1 + C_2)}{L C_1 C_2}\right]}$

$$\therefore \quad f = \frac{1}{2 \times 3{\cdot}14}\left[\frac{(0{\cdot}001 + 0{\cdot}01)\,10^{-6}}{(15 \times 10^{-6})(0{\cdot}001 \times 10^{-6})(0{\cdot}01 \times 10^{-6})}\right]^{1/2}$$

$$= 1361 \times 10^3 \text{ Hz} = \mathbf{1361\ kHz.}$$

4·6. PHASE SHIFT OSCILLATORS :

4·6·1. RC OSCILLATOR :

The phase shift transistor oscillator is similar to vacuum tube phase shift oscillator. Figure 10 shows the circuit diagram of phase shift oscillator. To obtain a positive feedback essential for oscillations, the frequency determining circuit must introduce a phase change of 180°. This phase shift of 180° is obtained with three cascade sections *CR, CR, CR* (each section consists of a series coupling capacitor and a shunt resistor) each shifting the signal by 60°. The phase shift comes about because *R* and *C* provides a current which leads the applied voltage by certain angle. The smaller is the capacitance more will the current lead the voltage for a given resistance. With a proper choice of *R* and *C*, a phase shift of 60° per section is achieved.

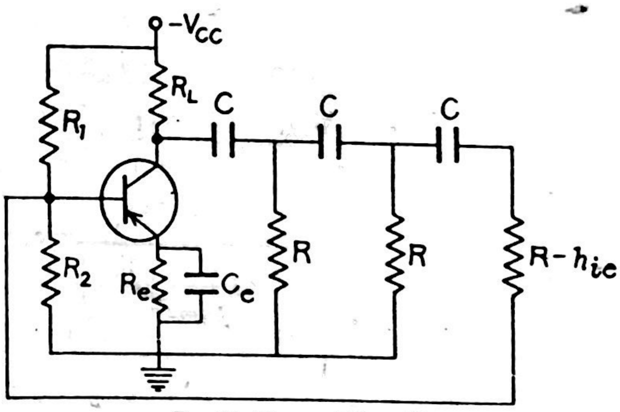

Fig. 10. Phase shift oscillator.

Analysis.

For convenience, the following assumptions are made :

(i) The three *RC* sections are made identical, of course, the third resistor is taken as $(R - h_{ie})$ because the input resistance of the transistor h_{ie} is added to give a total resistance of R.

(ii) The biasing resistances R_1 and R_2 have no effect on the a.c. operation of the circuit since they are larger.

(iii) Since $1 / h_{oe}$ is much larger than R_L, its effect can be neglected.

(iv) Since h_{re} of the transistor is usually very small, $h_{re} V_2$ can be neglected.

With all these considerations, the equivalent circuit is shown in fig. 11 (a & b).

Applying Kirchoff's voltage law for three loops in fig. (11 b), we have

$$\{R + R_L + (1 / j\,\omega C)\}\, I_1 - R I_2 + h_{fe} R_L I_b = 0 \quad \ldots (1)$$

$$-R I_1 + (2R + 1 / j\,\omega C)\, I_2 - R I_b = 0 \quad \ldots (2)$$

$$-R I_2 + (2R + 1 / j\,\omega C)\, I_b = 0 \quad \ldots (3)$$

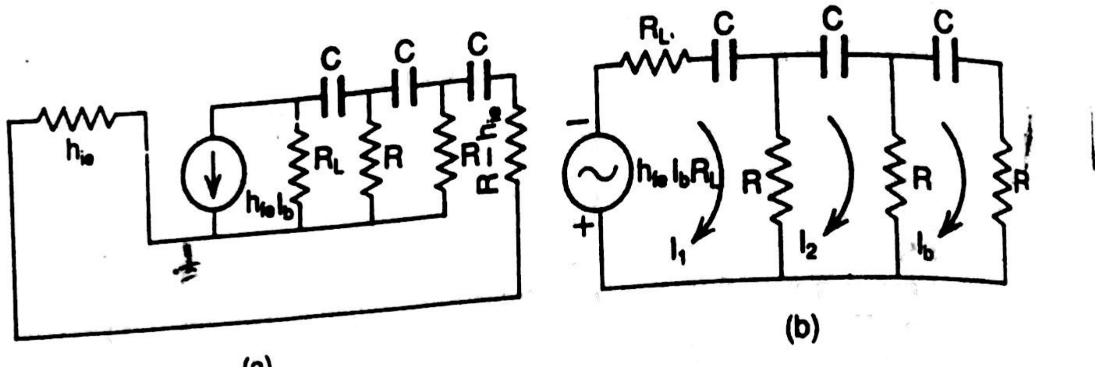

Fig. 11. Equivalent circuit of Phase shift oscillator.

The determinant form of above equations is given by

$$\begin{vmatrix} R+R_L+1/j\omega C & -R & h_{fe}R_L \\ -R & (2R+1/j\omega C) & -R \\ 0 & -R & (2R+1/j\omega C) \end{vmatrix} = 0$$

or $(R+R_L+1/j\omega C)\{3R^2+4R/j\omega C-1/\omega^2C^2\}-R^2(2R+1/j\omega C)+h_{fe}R_LR^2=0$

or $(R+R_L)(3R^2-1/\omega^2C^2)-(R+R_L)(j\,4R/\omega C)-j(3R^2/\omega C-1/\omega^3C^3)$
$\qquad -4R/\omega^2C^2-2R^3+(jR^2/\omega C)+h_{fe}R_LR^2=0$

Equating the imaginary part to zero, we get

or $\qquad -(R+R_L)(4R/\omega C)-\{(3R^2/\omega C)-(1/\omega^3C^3)\}+R^2/\omega C=0$

or $\qquad -(R+R_L)4R-3R^2+(1/\omega^2C^2)+R^2=0$

or $\qquad -4R^2-4RR_L-3R^2+(1/\omega^2C^2)+R^2=0$

$\qquad -6R^2-4RR_L+1/\omega^2C^2=0 \quad$ or $\quad 1/\omega^2C^2=6R^2+4RR_L$

$$\therefore \quad \omega^2=\frac{1}{C^2(6R^2+4RR_L)} \quad \text{or} \quad \omega=\frac{1}{C\sqrt{(6R^2+4RR_L)}}$$

or
$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi C\sqrt{(6R^2+4RR_L)}}$$

This equation gives the frequency of oscillations.

To obtain the condition of maintenance of oscillations, we compare the real part of eq. (4) t

Hence

$$(R+R_L)(3R^2-1/\omega^2C^2)-4R/\omega^2C^2-2R^3+h_{fe}R_LR^2=0$$

or $3R^3-R/\omega^2C^2+3R^2R_L-R_L/\omega^2C^2-4R/\omega^2C^2-2R^3+h_{fe}R_LR^2=0$

or $3R^3-R\{(6R^2+4RR_L)\}+3R^2R_L-R_L(6R^2+4RR_L)-4R(6R^2+4RR_L)$
$\qquad -2R^3+h_{fe}R_L$

or $3R^3-6R^3-4R^2R_L+3R^2R_L-6R^2R_L-4RR_L^2-24R^3-16R^2R_L-2R^3+h_{fe}R_LR^2=0$

or $\qquad h_{fe}R_LR^2 = 29R^3+23R^2R_L+4RR_L^2$

or
$$h_{fe}=29\frac{R}{R_L}+23+4\frac{R_L}{R}$$

$$\therefore \quad h_{fe}=23+29\frac{R}{R_L}+4\frac{R_L}{R}$$

Eq. (6) gives the condition for sustained oscillations. In practice R_L is taken equal to R. Hence from eqs. (5) and (6), we have

$$f = \frac{1}{2\sqrt{10}\,.(\pi R C)} \qquad \ldots (7)$$

and

$$h_{fe} = 56$$

So for the phase shift oscillators with $R_L = R$, h_{fe} should be 56 for sustained oscillations.

14.6.3. WIEN BRIDGE OSCILLATOR :

This is also audio frequency oscillator. The advantage of this oscillator is that the frequency may be varied over a frequency range 1 : 10, whereas in RC oscillators the frequency cannot be varied, *i.e.* RC oscillator is a fixed frequency oscillator. In RC oscillator both frequency determining network and amplifier introduce a phase change in the signal and positive feedback is obtained. On the other hand, the oscillations may be obtained by arranging both network and amplifier to introduce zero phase shift and actually this is the principle of Wien-bridge oscillator.

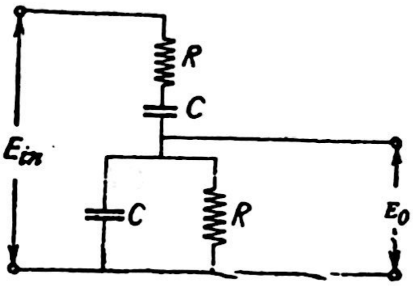

Fig. 12. Wien bridge network.

The Wien-bridge network is shown in figure 12. The upper arm consists of resistance and capacitance in series, while the lower arm has some resistance and capacitance in parallel. The network is supplied from a constant voltage source and is terminated in an infinite impedance. The phase shift and attenuation introduced by the network can be calculated as follows :

The impedance of parallel RC network is $\dfrac{-(R\,.\,j/\omega C)}{R-(j/\omega C)}$ and of series RC network is $\left(R - \dfrac{j}{\omega C}\right)$.

$$\therefore \quad \frac{E_0}{E_{in}} = \frac{\text{impedance of parallel combination}}{\text{Total impedance}} \qquad \ldots (1)$$

$$= \frac{\dfrac{-(R\,j/\omega C)}{R-(j/\omega C)}}{R - \dfrac{j}{\omega C} - \dfrac{(R\,j/\omega C)}{R-(j/\omega C)}} = \frac{-\dfrac{jR}{\omega C}}{\left(R - \dfrac{j}{\omega C}\right)^2 - \dfrac{jR}{\omega C}}$$

$$= \frac{-\dfrac{jR}{\omega C}}{\left(R^2 - \dfrac{1}{\omega^2 C^2} - \dfrac{3jR}{\omega C}\right)} \qquad \ldots (2)$$

frequency of oscillation
wien bridge network

There is zero phase shift if the term in j vanishes, *i.e.*

$$R^2 - \frac{1}{\omega^2 C^2} = 0 \quad \text{or} \quad \omega = \frac{1}{RC}. \quad \therefore \quad f = \frac{1}{2\pi RC}. \qquad \ldots (3)$$

Again from equation (2), we get $\dfrac{E_0}{E_{in}} = \dfrac{-jR/\omega C}{-3jR/\omega C} = \dfrac{1}{3}$. $\qquad \ldots (4)$

The maintaining amplifier thus requires a gain just exceeding 3 to sustain oscillations.

As pointed out earlier, the output voltage V_1 from the bridge must not be zero for oscillation to occur. This can be achieved by taking the ratio $R_4/(R_3 + R_4)$ smaller than (1/3). Thus

$$\frac{R_4}{R_3+R_4}=\frac{1}{3}-\frac{1}{K}$$

where $K > 3$. The bridge now will be slightly unbalanced and gives a feedback voltage V_i.

Fig. (13) gives the circuit diagram of the Wien-bridge oscillator.

At balance

$$\frac{R_3}{R_4}=\frac{(R+1/j\,\omega C)}{R\times(1/j\,\omega C)/(R+1/j\,\omega C)}$$

$$=\frac{(R+1/j\,\omega C)(R+1/j\,\omega C)\,j\,\omega C}{R}$$

$$=\frac{j\,\omega R^2 C+2-j/\omega C}{R}$$

$$=\frac{2}{R}+j\ (\omega C R-1/\omega R\,C)$$

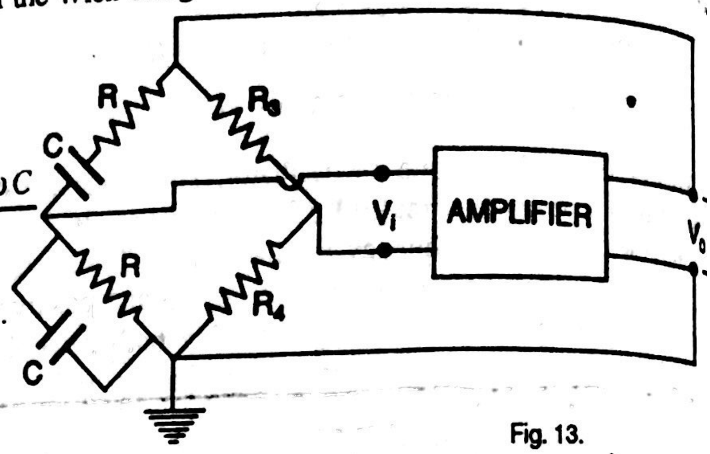

Fig. 13.

Equating the imaginary term to zero, we get

$$\omega C R = 1/\omega CR \quad \text{or} \quad \omega^2 = 1/C^2R^2 \quad \text{or} \quad \omega = 1/CR.$$

$$\therefore \quad f=\frac{\omega}{2\pi}=\frac{1}{2\pi\,C\,R}$$

where f is the frequency of oscillations.

Example 1. *A Wien bridge oscillator is to cover a frequency range from* 20 Hz *to* 20 k Hz. *variable capacitance has a value from* 30 pF *to* 300 pF. *Calculate the resistance values required to co the frequency range. If the gain of the amplifier be* 5, *find the ratio of the resistances in the other a of the bridge.*

In case of Wien bridge oscillator, the frequency of oscillations in given by

$$f=\frac{1}{2\pi\,R\,C}$$

We know that the capacitance is maximum at lower frequency, hence f = 20 Hz and C = 300 pF

$$\therefore \quad 20=\frac{1}{2\times 3{\cdot}14\times R\times 300\times 10^{-12}}$$

or

$$R=\frac{10^{12}}{(2\times 3{\cdot}14)\times 300\times 20}$$

$$= 26{\cdot}5\times 10^6\ \Omega = \mathbf{26{\cdot}5\ M\Omega}$$

It is obvious from question that capacitance changes in the ratio 1 : 10. Hence with the resista $26{\cdot}5 \times 10^6\ \Omega$, a frequency range from 20 Hz to 200 Hz can be covered. For the next frequency ra 200 Hz to 2 kHz, the resistance should be decreased by a factor of 10. Thus R = 2·65 MΩ. Simila for the frequency range from 2 kHz to 20 kHz another resistance of R = 0·265 MΩ or 265 kΩ required. Therefore, the three values of resistances are 265 kΩ, 2·65 MΩ and 26·5 MΩ.

Given that the gain of the amplifier is 5. So, K = 5.

Now

$$\frac{R_4}{R_3+R_4}=\frac{1}{3}-\frac{1}{K}=\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$$

or

$$1+\frac{R_3}{R_4}=\frac{15}{2} \quad \text{or} \quad \frac{R_3}{R_4}=\frac{13}{2}$$

$$\therefore \quad R_3 : R_4 = 13 : 2$$

14·7. CRYSTAL OSCILLATOR :

We have seen that the frequency of *LC* oscillators depends upon the values of tank circuit parameters. These values change with time, temperature changes, etc. Hence the frequency of oscillations does not remain constant at desired value. For excellent stability of oscillation, piezo-electric quartz crystal is used in place of the tuned circuit in oscillator. Such an oscillator is called a crystal oscillator.

The natural shape of a quartz crystal is hexagonal. There are three axes : the *z*-axis is called the optic axis, the *x*-axis is called the electric axis and the *y*-axis is called the mechanical axis. The quartz crystal exhibits the property that when mechanical stress is applied across the faces of the crystal, a potential difference is developed across the opposite faces of the crystal. Conversely, if a voltage is applied across one faces, a mechanical stress is produced along the other faces. This effect is called piezo-electric effect. Thus when a piezo-electric crystal is subjected to a proper alternating potential, it vibrates mechanically. The amplitude of mechanical oscillations become maximum when frequency of applied alternating voltage is equal to the natural frequency of the crystal.

The equivalent electrical circuit of a vibrating crystal can be represented by a series *LCR* circuit shunted by C_M as shown in fig. (14). The inductor *L* and capacitor *C* represent electrical equivalents of crystal mass and mechanical compliance respectively. The resistance *R* is the electrical equivalent of

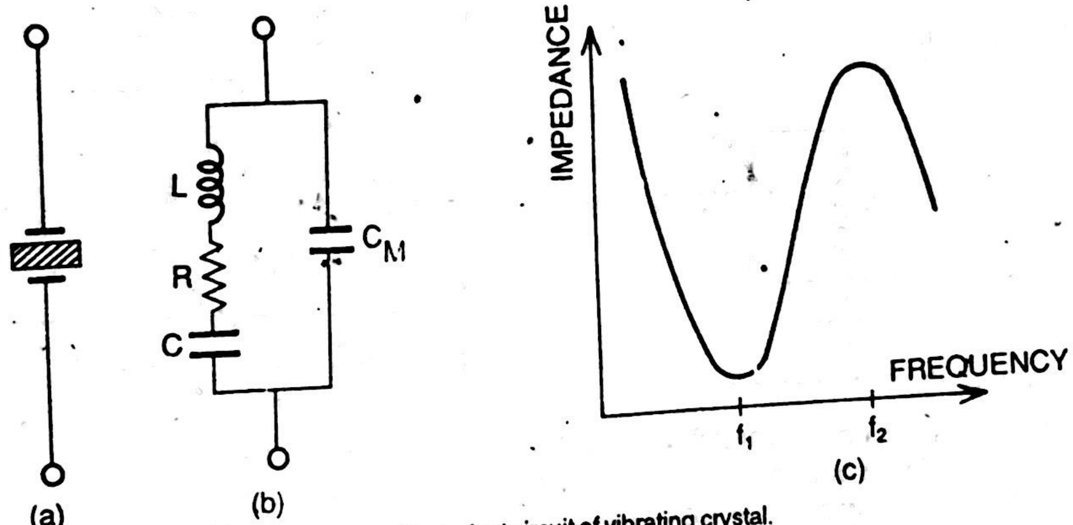

Fig. 14. Equivalent circuit of vibrating crystal.

the crystal structure's internal friction. The shunt capacitance C_M represents the capacitance due to mechanical mounting (between electrodes which are usually electroplated in position) when the crystal is not vibrating. For a crystal with dimensions 30 × 4 × 1·5 mm and at a frequency of 90 KHz, L = 137H, C = 0·0235 PF, R = 15 KΩ and C_M = 3·5 PF. The impedance v/s frequency curve is shown in fig. (14 c). It is obvious from the figure that there exists one resonant condition when the reactances of the series *RLC* leg are equal and opposite. At this condition, the series-resonant impedance is very low (equal to *R*). Thus at series-resonance

$$\omega_s L - \frac{1}{\omega_s C} = 0 \quad \text{or} \quad \omega_s^2 = \frac{1}{LC} \quad \text{or} \quad f_s = \frac{\omega_s}{2\pi} = \frac{1}{2\pi\sqrt{(L\,C)}} \qquad \ldots (1)$$

The other resonant condition occurs at a higher frequency when the reactance of the series resonant leg equals the reactance of capacitor C_M. This is a parallel resonance or antiresonance condition of the crystal. At this frequency the crystal offers a very high impedance to the external circuit. In this case.

$$\omega_p L - \frac{1}{\omega_p C} = \frac{1}{\omega_p C_M} \quad \text{or} \quad \omega_p L = \frac{1}{\omega_p}\left[\frac{1}{C_M} + \frac{1}{C}\right] = \frac{(C + C_M)}{\omega_p C C_M}$$

$$\text{or} \quad \omega_p^2 = \frac{(C + C_M)}{L C C_M} \quad \text{or} \quad \omega_p = \sqrt{\left\{\frac{(C + C_M)}{L C C_M}\right\}} \quad \therefore \; f_p = \frac{1}{2\pi}\sqrt{\left\{\frac{(C + C_M)}{L C C_M}\right\}} \quad \dots (2)$$

Fig. (15) shows the crystal controlled oscillator using crystal in series feedback path. The excite the crystal for operation in series-resonant mode it must be connected as a series element in feedback path. The reason is that at series resonant frequency of the crystal its impedance is smallest and the amount of positive feedback is largest. In the cricuit, R_1, R_2 and R_e provide a voltage divider stabilized d.c. bias circuit. Capacitor C_e provides a.c. bypass of emitter resistor. The capacitor C blocks any d.c. between collector and base.

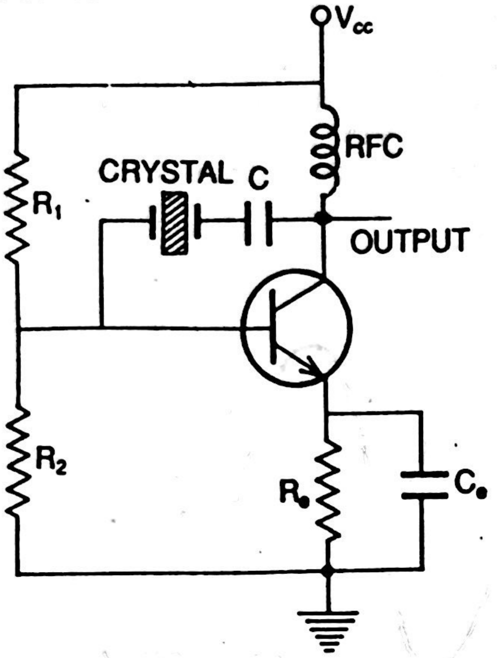

Fig. 15. Crystal controlled oscillator (series feedback).

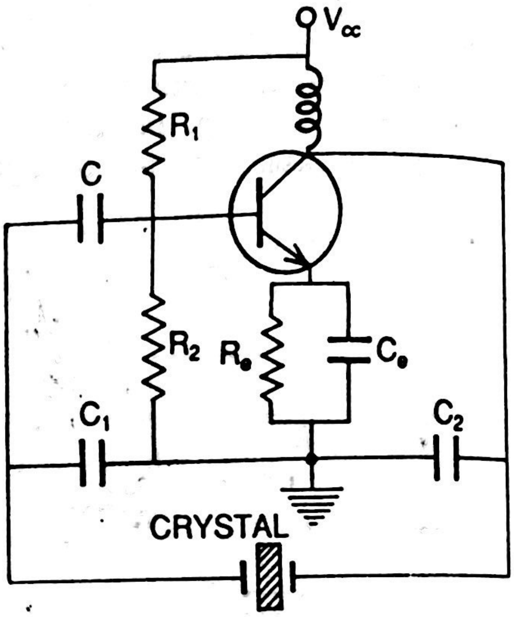

Fig. 16. Crystal controlled oscillator operating in parallel resonant mode.

Fig. (16) shows the crystal controlled oscillator operating in parallel resonant mode. We know that the parallel impedance of a crystal is maximum and hence it is connected in shunt. At the parallel resonant operating frequency a crystal appears as an inductive reactance of largest value. If we compare this circuit with the Colpitt's oscillator circuit, the two circuits are identical except that the crystal takes the place of the inductor. It is important to note that the frequency of the oscillator is determined by the crystal parameters. Since the resistance R of the crystal is very small, its Q is very high. Hence the frequency stability of the oscillator is very high.

Example 1. *The a.c. equivalent circuit of a crystal has the values :* $L = 1$ H, $C = 0{\cdot}01$ pF, $R = 1000\ \Omega$ *and* $C_M = 20$ pF. *Calculate* f_s *and* f_p *of the crystal.*

Given that $L = 1$ *H*, $C = 0{\cdot}01$ pF $= 0{\cdot}01 \times 10^{-12}$ F and $C_M = 20$ pF $= 20 \times 10^{-12}$ F.

We know that

$$f_s = \frac{1}{2\pi\sqrt{(L\,C)}}$$

Substituting the given values, we get

$$f_s = \frac{1}{2 \times 3{\cdot}14 \times \sqrt{(1 \times 10{\cdot}01 \times 10^{-12})}} = 1589 \times 10^3 \text{ Hz}$$

$= \mathbf{1589\ kHz}$

Further

$$f_p = \frac{1}{2\pi}\sqrt{\left\{\frac{(C + C_M)}{L\,C\,C_M}\right\}}$$

$$= \frac{1}{2 \times 3{\cdot}14}\sqrt{\left\{\frac{\left(0{\cdot}01 \times 10^{-12} + 20 \times 10^{-12}\right)}{1 \times 0{\cdot}01 \times 10^{-12} \times 20 \times 10^{-12}}\right\}}$$

$$= 1590 \times 10^3 \text{ Hz} = \mathbf{1590\ kHz}$$

EXERCISES AND PROBLEMS

1. Answer the following :
   (i) Is it correct that oscillator is an amplifier with infinite gain ?
   (ii) Why do you prefer *CE* configuration in oscillator circuit ?
   (iii) How are oscillations normally produced in most transistor oscillators ?
2. Draw the circuit diagrams of Hartley and Colpitt oscillators and obtain the expressions for the frequency of oscillations.
3. Explain with the aid of circuit diagram the working of a transistor *R-C* oscillator.
4. Derive the condition for sustained oscillations in a feedback oscillator.
5. Draw the circuit diagram of phase shift oscillator and explain its operation by deriving expression for frequency of oscillation.
6. Discuss the operation of a phase shift oscillator.
   A phase shift oscillator using a *PNP* transistor has the following circuit constants ?
   $C_1 = C_2 = C_3 = 0{\cdot}02\ \mu$ f $V_{cc} = -10$ volts, $R_1 = R_2 = R_3 = 3{\cdot}2$ KΩ, $R_L = 10$ KΩ,
   Calculate the current and frequency of oscillations.
7. Describe the crystal oscillator.
8. A Colpitt's oscillator is to generate a frequency of 16 MHz. The coil to be used has an inductance of 10 μH and the transistor has $h_{fe} = 50$ and $\Delta_{he} = 0{\cdot}5$. Find the values of the capacitances.
   [Ans. $C_1 \approx 1000$ pF, $C_2 \approx 10$ pF]