15.07.23

Chapter-9: Eigen values and Eigenvectors

Definition: If A is an $n \times n$ matrix, then a non-zero vector v in R^n is called an eigenvector of A if Av is a scalar multiple of v, that is $Av = \lambda v$ —(1) for some scalar λ. The scalar λ is called an eignvalue of A and v is said to be an eigenvector of A corresponding to λ.

Example-2: The charateristics matrix of A is

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{bmatrix}$$

Now the determinant of $\lambda I - A$ is

$$|\lambda I - A| = \begin{vmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2$$

Therefore, the characteristics equation of A is $\lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$\Rightarrow \lambda(\lambda - 2) - 1(\lambda - 2) = 0$

$\Rightarrow (\lambda-2)(\lambda-1)=0$

$\therefore \lambda = 1, 2$ which are the eigen values of the matrix.

Example-4:

The characteristics matrix of A is

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-2 & -3 \\ -1 & \lambda-4 \end{bmatrix}$$

Now the determinant of $\lambda I - A$ is

$$|\lambda I - A| = (\lambda-2)(\lambda-4)-3$$

The characteristic equation of A is

$(\lambda-2)(\lambda-4)-3=0$

$\Rightarrow \lambda^2-4\lambda-2\lambda+8-3=0$

$\Rightarrow \lambda^2-6\lambda+5=0$

$\Rightarrow \lambda^2-5\lambda-\lambda+5=0$

$\Rightarrow \lambda(\lambda-5)-1(\lambda-5)=0$

$\Rightarrow (\lambda-5)(\lambda-1)=0$

$\therefore \lambda = 5, 1$ which are the eigen values of the matrix A.

Now, by definition $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of A corresponding to λ if and only if X is non-trival solution of $(\lambda I - A) = 0$, that is of

$$\begin{bmatrix} \lambda-2 & -3 \\ -1 & \lambda-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ ——— } (1)$$

If $\lambda = 5$, equation (1) becomes

$$\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 3x_1 - 3x_2 = 0$$
$$-x_1 + x_2 = 0$$

$$\Rightarrow x_1 - x_2 = 0$$

This system is in echelon form and consistent. There are more unknowns than equation in echelon form, the system has an infinite number of solutions. Again the equation begins x_1 only, unknown x_2 is free variable.
Let, $x_2 = a$, the eigenvectors of A corresponding to the eigenvalue $\lambda = 5$ are non-zero vector of ~~matri~~ the form $X = \begin{bmatrix} a \\ a \end{bmatrix}$. Let $a = 1$ then $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigen vector corresponding to the eigenvalue 5.

If $\lambda=1$, equation ① becomes,

$$\begin{bmatrix} -1 & -3 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -x_1 - 3x_2 = 0$

$-x_1 - 3x_2 = 0$

$\Rightarrow x_1 + 3x_2 = 0$

This system is also in echelon form and has more unknown values than equation. The equation begins with x_1 and x_2 is free variable.

Let $x_2 = b$

$\therefore x_1 = -3b$

The eigen vector of A corresponding to the eigen value $\lambda=1$ are the non-zero vectors of the form.

$$X = \begin{bmatrix} -3b \\ b \end{bmatrix}$$

Let, $b=1$, then $X = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ is an eigenvector corresponding to the eigenvalue $\lambda=1$.

11.01.22

Cayley – Hamilton theorem: Every square matrix satisfies its own characteristics equation i.e. if the characteristic equation of the nth order matrix, A is $f(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n = 0$
then cayley-Hamiltom states that
$f(A) = A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots + a_{n-1}A + a_n I = 0$
I is the nth order unit matrix and O is the zero matrix.

Verify Caley–Hamilton theorem:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$|\lambda I - A|$

satisfies, $f(\lambda) = \lambda^2 - 2\lambda - 3$
Hence the characteristic equation is
$\lambda^2 - 2\lambda - 3 = 0$
Hence, by caley–Hamilton theorem we get:
$A^2 - 2A - 3I = 0$
Here, $A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

LHS $= A^2 - 2A - 3I$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$=$ RHS

Hence, the Caley-Hamilton thm is verified.

Example-11: The characteristics of matrix A is

$$\lambda I - A = \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-1 & -2 & -2 \\ -3 & \lambda-1 & 0 \\ -1 & -1 & \lambda-1 \end{bmatrix}$$

Now the determinant of the matrix $\lambda I - A$ is :

$$|\lambda I - A| = \begin{vmatrix} \lambda-1 & -2 & -2 \\ -3 & \lambda-1 & 0 \\ -1 & -1 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1)^3 + 2\{-3(\lambda-1)\} - 2(3+\lambda-1)$$

$$= (\lambda-1)^3 - 6\lambda + 6 - 4 - 2\lambda$$

$$= \lambda^3 - 3\lambda^2 + 3\lambda - 1 - 8\lambda + 2$$

$$= \lambda^3 - 3\lambda^2 - 5\lambda + 1$$

The characteristic equation of A is

$\lambda^3 - 3\lambda^2 - 5\lambda + 1 = 0$

Using C-H thm we get

$A^3 - 3A^2 - 5A + I = 0$

Multiplying both sides by A^{-1} we get

$A^2 - 3A - 5I + A^{-1} = 0$

$\Rightarrow A^{-1} = 3A + 5I - A^2$

Now,

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 & 4 \\ 6 & 7 & 6 \\ 5 & 4 & 3 \end{bmatrix}$$

$$A^{-1} = 3A + 5I - A^2$$

$$= 3\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 9 & 6 & 4 \\ 6 & 7 & 6 \\ 5 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 6 \\ 9 & 3 & 0 \\ 3 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 9 & 6 & 4 \\ 6 & 7 & 6 \\ 5 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+5-9 & 6+0-6 & 6+0-4 \\ 9+0-6 & 3+5-7 & 0+0-6 \\ 3+0-5 & 3+0-4 & 3+5-3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{bmatrix}$$

Chapter-1: System of linear equation

A straight line in the cartesian xy-plane can be represented algebrically by an equation of the form $ax+by=c$, where, a,b,c are real constants and x,y are variables. An equation of this kind is called a linear equation of two variables.

$$a_1x_1+a_2x_2+ \cdots + a_nx_n=b \quad \text{--- (1)}$$

if $b=0$ then (1) is homogenous

$b\neq0$ then (1) is non-homogeneous linear equation.

system of linear equation

- Inconsistent
  - No solution
- Consistent
  - Unique solution
  - More than one solution

Example – 4:

$$\left.\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ 2x_1 + x_2 - x_3 &= 1 \\ x_1 - x_2 - x_3 &= -6 \end{aligned}\right\} \quad (1)$$

Let us represent the three linear equation of the system (1) by L_1, L_2 and L_3 respectively. Reduce the system to echelon form by the elementary operations. Eliminate x_1 from second and third equation:

$L_2' \rightarrow -2L_1 + L_2$

$$\begin{aligned} -2L_1 &: -2x_1 + 4x_2 - 6x_3 = -14 \\ L_2 &: 2x_1 + x_2 - x_3 = 1 \\ \hline -2L_1 + L_2 &: 5x_2 - 7x_3 = -13 \end{aligned}$$

$L_3' \rightarrow -1L_1 + L_3$

$$\begin{aligned} -1L_1 &: -x_1 + 2x_2 - 3x_3 = -7 \\ L_3 &: x_1 - x_2 - x_3 = -6 \\ \hline -L_1 + L_3 &: x_2 - 4x_3 = -13 \end{aligned}$$

Thus we obtain the equivalent system

$$\left.\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ 5x_2 - 7x_3 &= -13 \\ x_2 - 4x_3 &= -13 \end{aligned}\right\} \quad (2)$$

Interchanging second and third equations we have the equivalent system:

$$\left.\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ x_2 - 4x_3 &= -13 \\ 5x_2 - 7x_3 &= -13 \end{aligned}\right\} \quad (3)$$

Let us represent the three linear equations of the new system (3) by L_1', L_2', L_3' respectively. Eliminate x_2 from the third equation by the operation:

$L_3' \rightarrow L_3' - 5L_2'$

L_3' : $5x_2 - 7x_3 = -13$

$-5L_2'$: $-5x_2 + 20x_3 = 65$

$L_3' - 5L_2'$: $13x_3 = 52$

Thus we obtain another equivalent system:

$$\left.\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ x_2 - 4x_3 &= -13 \\ 13x_3 &= 52 \end{aligned}\right\} \quad (4)$$

Mutiplying third equation by $1/3$ we get:

$$\left.\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ x_2 - 4x_3 &= -13 \\ x_3 &= 4 \end{aligned}\right\} \quad (5)$$ which is in echelon form.

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Substituting $x_3=4$ in second equation of ⑤ we get

$x_2 = -13 + 4\times4$

$\therefore x_2 = 3$

Substituting $x_2=3, x_3=4$ in first equation of ⑤ we get.

$x_1 = 7 + 2\times3 - 3\times4$

$= 13 - 12 = 1$

$\therefore x_1 = 1$

Thus $x_1=1, x_2=3, x_3=4$.

Chapter 6: Vector Space

The two operations are required to satisfy the following axioms:

(A1) Addition is commutative:
For all vectors $u, v \in V$, $u+v = v+u$

(A2) Addition is associative:
for all vectors $u, v, w \in V$ $(u+v)+w = u+(v+w)$

(A3) Existence of zero (zero vector)
There exists a vector $0 \in V$ such that for all $v \in V$
$v+0 = 0+v = v$

(A4) Existence of negative.
For each $v \in V$ there is a vector $-v \in V$ for which
$v+(-v) = (-v)+v = 0$

M(1) For any scalars $\alpha, \beta \in F$ and any vector $u, v \in V$,
$\alpha(u+v) = \alpha u + \alpha v$

(M2) For any scalars $\alpha, \beta \in F$ and any vector $v \in V$,
$(\alpha+\beta)v = \alpha v + \beta v$

(M3) For any scalars $\alpha, \beta \in F$ and any vector $v \in V$,
$(\alpha\beta)v = \alpha(\beta v)$

(M4) For each $v \in V$, $1v = v$; where 1 is the unit scalar and $1 \in F$.

Subspace of a vector space: W is a subspace of V whenever $w_1, w_2 \in W$, $\alpha, \beta \in F$ implies that $\alpha w_1 + \beta w_2 \in W$

225 **Example-3:** Consider the vector space.

$$\mathbb{R}^3 = \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$$

Then $W_1 = \{(a,b,0) \mid a,b \in \mathbb{R}\}$, $W_2 = \{(0,b,c) \mid b,c \in \mathbb{R}\}$ and $W_3 = \{(a,0,c) \mid a,c \in \mathbb{R}\}$ are subspace of $\mathbb{R}^3$. W_1, W_2, W_3 represents the sets of all points in the X-Y, Y-Z and Z-X planes respectively.

231 **Example-3:**

For $0 \in \mathbb{R}^3$, $0 = (0,0,0) \notin W$

Since $0 - 2.0 + 3.0 = 0 \neq 5$

$\therefore a - 2b + 3c = 5$ is not a subspace of $\mathbb{R}^3$.

Example-6:

① Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Then $A, B \in W$, since $\det(A) = 0$ and $\det(B) = 0$

But $A + B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \notin W$

Since $\det(A+B) = -1 \neq 0$

Hence W is not a subspace of V.

(ii) The unit matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ belongs W, since,

$$I^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

But $4I = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ does not belongs to W.

Since, $(4I)^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} \neq 4I$

Hence, W is not a subspace of V.

Linear combination of vectors: Let V be a vector space over the field F and let $v_1 - - - - v_n \in V$ then any vecto $v \in V$ is called linear combination of $v_1, v_2, - - - - , v_n$ if and only if there exists scalars $\alpha_1, \alpha_2, - - - \alpha_n$ in F such that $v = \alpha_1 v_1 + \alpha_2 v_2 + - - - + \alpha_n v_n$

$$= \sum_{i=1} \alpha_i v_i$$

Example-7: In order to show that v is a ~~vector~~ linear combination of v_1, v_2 and v_3 there must be scalars α_1, α_2 and α_3 in F such that $v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

$$(5,9,5) = \alpha_1(2,1,4) + \alpha_2(1,-1,3) + \alpha_3(3,2,5)$$
$$= (2\alpha_1, \alpha_1, 4\alpha_1) + (\alpha_2, -\alpha_2, 3\alpha_2) + (3\alpha_3, 2\alpha_3, 5\alpha_3)$$
$$= (2\alpha_1 + \alpha_2 + 3\alpha_3,\ \alpha_1 - \alpha_2 + 2\alpha_3,\ 4\alpha_1 + 3\alpha_2 + 5\alpha_3)$$

$$\left.\begin{array}{l} 2\alpha_1 + \alpha_2 + 3\alpha_3 = 5 \\ \alpha_1 - \alpha_2 + 2\alpha_3 = 9 \\ 4\alpha_1 + 3\alpha_2 + 5\alpha_3 = 5 \end{array}\right\} (1)$$

Reduce the system (1) to echelon form by elementary operation. Interchanging first and second equation, we get:

$$\left.\begin{array}{l} \alpha_1 - \alpha_2 + 2\alpha_3 = 9 \\ 2\alpha_1 + \alpha_2 + 3\alpha_3 = 5 \\ 4\alpha_1 + 3\alpha_2 + 5\alpha_3 = 5 \end{array}\right\} (2)$$

Multiplying first equation by 2 and 4 then subtract from the second and third equations we get:

$$\left.\begin{array}{l} \alpha_1 - \alpha_2 + 2\alpha_3 = 9 \\ \quad 3\alpha_2 - \alpha_3 = -13 \\ \quad 7\alpha_2 - 3\alpha_3 = -31 \end{array}\right\} (3)$$

multiply second equation by 7/3 and substract from third equation we get:

$$\left.\begin{aligned} \alpha_1 - \alpha_2 + 2\alpha_3 &= 9 \\ 3\alpha_2 - \alpha_3 &= -13 \\ -\tfrac{2}{3}\alpha_3 &= -\tfrac{2}{3} \end{aligned}\right\} \quad (4)$$

From the third equation we get, $\alpha_3 = 1$, keeping the value of α_3 in second equation we get,

$$3\alpha_2 = -13 + 1$$
$$\alpha_2 = -12/3$$
$$\therefore \alpha_2 = -4$$

Again keeping the value of $\alpha_2 = -4$, $\alpha_3 = 1$ in first equation we get.

$$\alpha_1 - (-4) + 2 \cdot 1 = 9$$
$$\Rightarrow \alpha_1 = 9 - 6$$
$$\therefore \alpha_1 = 3$$

Hence, the linear combination of v_1, v_2 and v_3 is
$v = 3v_1 - 4v_2 + v_3$

Example-9: Let, $v = \alpha_1 v_1 + \alpha_2 v_2$ where α_1, α_2 are unknown scalars. $(1, \lambda, 5) = \alpha_1(1,-3,2) + \alpha_2(2,-1,1)$

$$= (\alpha_1, -3\alpha_1, 2\alpha_1) + (2\alpha_2, -\alpha_2, \alpha_2)$$

$$= (\alpha_1 + 2\alpha_2, -3\alpha_1 - \alpha_2, 2\alpha_1 + \alpha_2)$$

$$\left.\begin{aligned} \alpha_1 + 2\alpha_2 &= 1 \\ -3\alpha_1 - \alpha_2 &= \lambda \\ 2\alpha_1 + \alpha_2 &= 5 \end{aligned}\right\} \text{①}$$

Multiply 1st equation by -3 and 2nd equation by 2 and substr from second and third equations respectively. We get

$$\left.\begin{aligned} \alpha_1 + 2\alpha_2 &= 1 \\ 5\alpha_2 &= \lambda + 3 \\ -3\alpha_2 &= 3 \end{aligned}\right\} \text{②}$$

Mutiplying third equation by $-1/3$ we get,

$$\left.\begin{aligned} \alpha_1 + 2\alpha_2 &= 1 \\ 5\alpha_2 &= \lambda + 3 \\ \alpha_2 &= -1 \end{aligned}\right\}$$

$\therefore 5 \times (-1) = \lambda + 3$

$\Rightarrow \therefore \lambda = -5 - 3$

$= -8$

$\therefore \alpha_1 = 1 - 2(-1)$

$= 1 + 2 = 3$

Hence, v is a linear combination v_1 and v_2 ~~the solution is~~ if $\lambda = -8$.

Example-11:

Set A as a linear combination of A_1, A_2, A_3 using unknown scalars α_1, α_2 and α_3

$\therefore A = \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3$

$$\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 & \alpha_1 \\ \alpha_1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \alpha_2 & \alpha_2 \end{bmatrix} + \begin{bmatrix} 0 & 2\alpha_3 \\ 0 & -\alpha_3 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 & \alpha_1 + 2\alpha_3 \\ \alpha_1 + \alpha_2 & \alpha_2 - \alpha_3 \end{bmatrix}$$

$\therefore \alpha_1 = 3$

$\alpha_1 + 2\alpha_3 = 1$

$\alpha_1 + \alpha_2 = 1$

$\alpha_2 - \alpha_3 = -1$

From the first equation, we get $\alpha_1 = 3$

$$\alpha_2 = 1 - \alpha_1$$
$$= 1 - 3$$
$$\therefore \alpha_2 = -2$$

Again, $\alpha_2 - \alpha_3 = -1$

$$-\alpha_3 = -1 - \alpha_2$$
$$= -1 + 2$$
$$= 1$$
$$\therefore \alpha_3 = -1$$

Therefore, $A = 3A_1 - 2A_2 - A_3$.

Linear dependent and linear independent (page: 253)

Exampl-16 (page-259)

Chapter: 7 : Linear transformation

Definition: Let U and V be two vector spaces over the same field F. A linear transformation ~~of~~ T of U into V written as $T: U \to V$, is a transformation of U into V such that

(i) $T(u_1+u_2) = T(u_1) + T(u_2)$ for all $u_1, u_2 \in U$

(ii) $T(\alpha u) = \alpha T(u)$ for all $u \in U$ and all $\alpha \in F$.

Example-1:

If $u_1 = (x_1, y_1)$ and $u_2 = (x_2, y_2)$ then

$$u_1 + u_2 = \left((x_1+x_2), (x_1+x_2) + (y_1+y_2)\cdot(x_1+x_2) - (y_1 - y_2)\right)$$

$$= (x_1 \cdot x_1 + y_1 x_1 - y_1) + (x_2 \cdot x_2 + y_2 \cdot x_2 - y_2)$$

$$= T(u_1) + T(u_2)$$

Also if u is a ~~set~~ scalar, $\alpha u_1 = \alpha(x_1, y_1) = (\alpha x_1, \alpha y_1)$

$$\therefore T(\alpha u_1) = T(\alpha x_1, \alpha y_1)$$

$$= (\alpha x_1, \alpha x_1 + \alpha y_1 \cdot \alpha x_1 - \alpha y_1)$$

$$= \alpha(x_1 \cdot x_1 + y_1 \cdot x_1 - y_1)$$

$$= \alpha T(u_1)$$

Kernel: Let T be a linear transformation of U into V, then the kernel (or nullspace) of T is the subset of U consisting of all $x \in U$ for which $T(x) = 0$ where $0 \in V$. The kernel of T is generally denoted by $Ker(T)$.

Range: The range of T is the subset of V consisting of all $y \in V$ such that $T(x) = y$ for all $x \in U$. It is generally denoted by $R(T)$.

Rank and nulity: If $T: U \to V$ is a linear transformation then demention of the range of T is called the rank of T and the dimension of the ~~image o~~ kernel of T is called the nullity of T.

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