Chapter – 7.3

Ques– 7

Determine whether the relation represented by the matrix in Exercise 3 are reflexive, irreflexive, symmetric, antisymmetric and /or transitive.

Exercise – 3 List the ordered pair in the relation on $\{1,2,3\}$ corresponding to these matrics (where the rows and columns correspond to the integers listed in increasing order).

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Answer

$\Rightarrow$ The given matrics is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

The rows and ~~etc~~ columns of the matric's correspond to the integer 1, 2, 3 listed in increasing order.

Now, Let's list the ordered pair based on the non-zero entires in each row:

For the first row: (1,1), (1,3)

for the second row: (2,2)

For the third row: (3,1), (3,3)

So, the ordered pairs in the relation are:

(1,1), (1,3), (2,2), (3,1), (3,3)

Now lets determine the properties of the relation.

1. Reflexive: A relation is reflexive if every element is related to itself. In the given relation, all element (1,2,3) are related to themselves., So, the relation is reflexive

2. Irreflexive: A relation is irreflexive if no element is related to itself. In the given relation, all element are related to themselves. So, the relation is not irreflexive

3. Symmetric: A relation is symmetric if whenever (a,b) is in the relation, then (b,a) is also in the relation. In the given relation, we have (1,3) and (3,1) is also in the relation. Similarly, we have (3,1) and (1,3)

is also in the relation. So, the relation is symmetric.

4. Anti-symmetric: A relation is anti-symmetric if whenever (a, b) and (b, a) are both in the relation, then $a = b$. In the given relation, we have (1, 3) and (3, 1), but $1 \neq 3$. Therefore the relation is not antisymmetric.

5. Transitive: A relation is transitive if whenever (a, b) and (b, c) are both in the relation, then (a, c) is also in the relation. The relation is Transitive.

Ques-8 Determine whether the relation represented by the matrics in Exercise 4 are reflexive, irreflexive, symmetric, anti-symmetric and/or transitive.

Exercise: 4: List the ordered pairs in the relations on {1, 2, 3, 4} corresponding to these matrics (where the rows and coloumn correspond to the integers listed in increasing order). (a)

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Answer

The given matrix is: $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

Therefore, the ordered pairs corresponding to the matrics: ~~{(1,1), (1,4), 2~~

{(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)}

1. Reflexive: A relation is reflexive if every element is related is itself. In the given relation, all element ~~are not~~ (1, 2, 3, 4) are not related to themselve. So, the relation is not reflexive.

2. Irreflexive: A relation is irreflexive if every element is not related to itself. Here all element (1, 2, 3, 4) are not related to themselves. So, the relation is not irreflexive.

3. Symmetric: A relation is symmetric if whenever (a,b) is in the relation then (b,a) is also the relation. So, the relation is symmetric.

4. Antisymmetric: A relation is anti-symmetic if whenever (a,b) and (b,a) are both in the relation, then $a=b$. So, the relation is not anti-symmetric.

5. Transitive: A relation is transitive if whenever (a,b) and (b,c) are present, (a,c) must be also present. By evalauting the ordered pair, we find that there are no instances where this contition is violated. Thus, the relation is transitive.

13 Let R be the relation represent by the matrics

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Answer

⇒ Here the given matrix ;

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The ordered pair in the relation

$R = \{(1,2), (1,3), (2,1), (2,2), (3,1), (3,3)\}$

∴ R^2 is the composite of R with itself and it consists of ordered of pair (a,c) such that (a,b) ∈ R and (b,c) ∈ R .

∴ $R^2 = R \circ R$

$R = \{(1,2), (1,3), (2,1)\ (2,2), (3,1), (3,3)\}$

$R = \{(1,2), (1,3), (2,1)\ (2,2), (3,1)\ (3,3)\}$

∴ $R \circ R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2)\ (3,3)\}$

RoR written in matrix:

$$R \circ R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

19 Let R_1 and R_2 be the relations on a set A represented by the matrics

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_2 \circ R_1$ (d) $R_1 \circ R_2$

Answer

The orders pair in the relation

$R_1 = \{(1,2), (2,1), (2,2), (2,3)\ (3,1)\}$

$R_2 = \{(1,2), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

a

$\therefore R_1 \cup R_2 = \{(1,2), (2,1), (2,2), (2,3), (3,1)\} \cup \{(1,2), (2,2), (2,3), (3,1), (3,2)\ (3,3)\}$

$= \{(1,2), (2,1)\ (2,2)\ (2,3), (3,1), (3,2), (3,3)\}$

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(b) $R_1 \cap R_2 = \{(1,2),(2,1),(2,2),(2,3),(3,1)\} \cap \{(1,2),$
$(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$= \{(1,2),(2,2),(2,3),(3,1)\}$

(d) $R_1 \circ R_2$ $\cancel{R_2 \circ R_1} = \{(1,2),(1,3),(3,2),\cancel{(2,2)},(2,3),(2,1),$
$\cancel{(2,3)},(3,2)\}$

represented by matrix,

$$\cancel{R_2 \circ R_1}\ R_1 \circ R_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) $R_2 \circ R_1 = \{(1,2),(2,2),(2,3),(3,1),(3,2),(3,3)\}$
$\circ \{(1,2),(2,1),(2,2),(2,3),(3,1)\}$

$= \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),$
$(3,2),(3,1),(3,3)\}$

$R_2 \circ R_1$

represented by matrices :

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_1 \cup R_2$ represeted by the matrix :

$$R_1 \cup R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_1 \cap R_2$ represented by the matrix :

$$R \cap R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

15 Let R be the relation represented by matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

find the matrix that represent

(a) R^2 (b) R^3 (c) R^4

Answer

(a) given that matrix: $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

The order pair in the relation

$R = \{(1,2), (2,3), (3,1), (3,2)\}$

R^2 is the composite of R with itself and it consists of order pair (a,c) such that $(a,b) \in R$ and $(b,c) \in R$.

$\therefore R^2 = R \circ R$

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$\therefore R \circ R = \{(1,2), (2,3), (3,1), (3,2)\} \circ \{(1,2), (2,3), (3,1), (3,2)\}$

$= \{(1,3), (2,1), (2,2), (3,2), (3,3)\}$

represented by matrix:

$$R \circ R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(b) $R^3 = R \circ R \circ R$

$\therefore R \circ R \circ R = \{(1,3), (2,1), (2,2), (3,2), (3,3)\} \circ \{(1,2), (2,3), (3,1), (3,2)\}$

$= \{(1,1), (1,2), (2,2), (2,3), (3,3), (3,1), (3,2)\}$

represented by matrix:

$$R \circ R \circ R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

c $R^4 = R \circ R \circ R \circ R$

$\therefore R \circ R \circ R \circ R = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,2), (3,3)\} \circ \{(1,2), (2,3), (3,1), (3,2)\}$

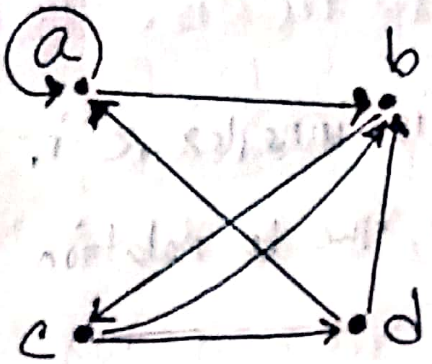
$= \{(1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

represented by matrix:

$$R^4 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

22 Draw the direct graph that represent the relation
{(a,a), (a,b), (b,c), (c,b), (c,d), (d,a), (d,b)}

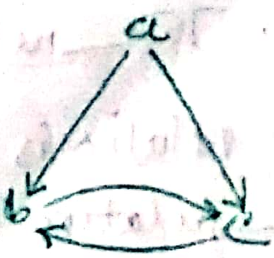
Answer



(31) Determine whether the relation represented by directed graph shown in Exercise (23–25) are reflexive, irreflexive, symmetric, anti-symmetric, asymetric and/or transitive.

Answer

Exercise – 23 :



Answer

Here the relation $R = \{(a,b), (a,c), (b,c), (c,b)\}$

Reflexive : A relation is reflexive if every element in in the set is related itself like (a,a), (b,b) etc. So, the relation is not reflexive.

Irreflexive : A relation is irreflexive if no element is related to itself. So, the ~~to~~ relation is irreflexive.

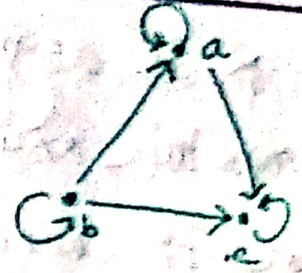
Symmetric : A relation is symmetric if for every pair (a,b) in the relation, the pair (b,a) also in the relation. So, the relation is not symmetric.

Asymmetric : A relation is asymmetric if for every pair (a,b) in the relation, the pair (b,a) is not in the relation. The relation is not asymmetric.

Transitive : A relation is transitive if for every (a,b) an (b,c) in the relation and also (a,c) in the relation. So, the relation is transitive.

Exercise:

24



Answer:

Here $R = \{(a,a), (a,c), (b,b), (b,a), (b,c), (c,c), (c,a)\}$

Reflexive: A relation R is reflexive if every element in the set related with itself. we can see that (a,a), (b,b), (c,c) is present. So, the relation is reflexive.

Irreflexive: A relation is irreflexive if no element in the set related with itself. So, the relation is not irreflexive.

Symmetric: A relation R is symmetric if for every pair (a,b) in the relation, the pair (b,a) also in the relation. So, the relation is not symmetric.

Antisymmetric: A relation is antisymmetric if whenever (a,b) and (b,a) are in the relation R. then a=b. So, the relation is anti-symmetric.

Asymmetric: A relation R is asymmetric if it is both anti-symmetric and not symmetric.

Since, R is not symmetric, it cannot be asymmetric.

So, the relation is not asymmetric.

Transitive: A relation is transitive if whenever (a,b) and (b,c) are in the relation, then (a,c) must be in the relation. Here, (a,c) and (c,a) in the relation and (a,a) are also in the relation. So, the relation is transitive.