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Random variable : A random variable is a function that assigns a real number to ~~a sa~~ each sample point in a sample space.

Descrete R.V : obtained from descrete sample space

Continious R.V : " " continious " "

Probability function → Descrete : The set of ordered pairs $(x, P(x))$ is called P.F.

শর্ত :

(i) $P(x) \geq 0$

(ii) $\sum P(x) = 1$

(iii) $P[X=x] = P(x)$

$$\boxed{X \sim P[x, P(x)]}$$

Probability function → Continious : A function $f(x)$ of a continious random variable X is called probability density function if,

শর্ত :

(i) $f(x) \geq 0$

(ii) $\int_{-\alpha}^{\alpha} f(x)\, dx = 1$

Mean, $E[X]$: Let X be a descrete random variable which can takes values $x_1, x_2, \dots x_n$ with associates probabilities $P(x_1), P(x_2), P(x_n)$ then mathematical expectation or mean of X is defined by

$$E[X] = \sum_{i=1}^{n} x_i\, P(x_i)$$

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<u>Variance</u>: ... then the variance of X denoted by σ^2 is defined as

$$\sigma^2 = E[X]^2 - [E[X]]^2$$

$$E[X]^2 = \sum_{i=1}^{n} x_i^2 \, P(x_i)$$

<u>Mean and Variance of Continious R.V</u>: if X is a continious R.V with probability density function $f(x)$ then,

$$\mu = E[X] = \int_{-\alpha}^{+\alpha} x \, f(x) \, dx \qquad \boxed{f(x), \text{ limit}}$$

variance, $\sigma^2 = E[X]^2 - \mu^2$

where, $E[X]^2 = \int_{-\alpha}^{\alpha} x^2 \, f(x) \, dx$

<u>Binomial distribution</u>: A descrete R.V X is said to have a binomial distribution if its p.f is defined by

$$P(x) = {}^nC_x (p)^x (q)^{n-x} \qquad x = 0 \dots n$$

Mean = np

variance = npq

$$\boxed{X \sim B(n, p)}$$

<u>Parameter</u>: The unknown constants required to define a distribution are called the parameter

10.3.2. Some important properties of the distribution

1. It is a discrete probability distribution with parameters n and p.
2. The mean of the distribution is np and its variance is npq. The mean of the distribution is greater than variance since $q < 1$.
3. The distribution is positively skewed if $p < 1/2$ and negatively skewed if $p > 1/2$
4. The distribution is symmetric if $p = q = 1/2$.
5. The distribution tends to Poisson distribution if the number of trials, n tends to infinity.

6. The distribution tends to normal distribution if n tends to infinity and p or q is not so small.
7. $P(X = n) = p^n$ and $P(X = 0) = q^n$

Definition of Normal distribution: A continuous random variable X is said to have a normal distribution if its probability density function is defined by

$$f(x;\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad ; -\infty < x < \infty$$

where $\pi = 3.1416$; $e = 2.7183$. Here π and e are mathematical constants. μ and σ^2 are the two parameters of the distribution. Actually μ is the mean of the distribution and σ^2 is the variance of the distribution. It is symbolically expressed as $X \sim N(\mu, \sigma^2)$.

There are many ways we can get normal distributions.

10.5.2. Some important properties of normal distribution.

1) The distribution is symmetrical about μ.
2) Mean, median and mode of the distribution are equal.
3) The mean of the distribution is μ and the variance is σ^2.
4) The curve has a single peak, i.e. it is unimodal.
5) $\mu \pm \sigma$, $\mu \pm 2\sigma$, $\mu \pm 3\sigma$, covers 68.27%, 95.45% and 99.73% area respectively.
6) All odd central moments of the distribution are zer 0.
7) For large sample most of the distributions tend to normal distribution.
8) Skewness of the distribution is zero. That is $\beta_1 = 0$.
9) The distribution is mesokurtic and the value of $\beta_2 = 3$.
10) $\mu \pm \sigma$ are the points of inflection of the curve.

10.5.4. Standard normal distribution. A continuous random variable Z is said to have a standard normal distribution if it probability density function is defined by

$$f(z; 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}; -\infty < z < \infty$$

The variable Z is called standard normal variate. The mean of the distribution is zero and the variance is one. In symbols, it can be expressed as

$$Z \sim N(0, 1)$$

10.5.5. Finding probabilities for a normal distribution. Normal curve depends on mean and variance. Once mean and variance are specified, the normal curve is completely determined. The area under the normal curve between two ordinates depends upon the values of μ and σ^2. It is difficult task to make normal integral tables for different values of μ and σ^2. Fortunately, we are able to transfer any normal random variable to standard normal variate. This can be done by means of following transformation

$$Z = \frac{X - \mu}{\sigma}$$

It can be easily shown that $E[Z] = 0$ and $Var[Z] = 1$.

That is Z is normally distributed with mean zero and variance one. So normal variates with different means and variances can be converted into a standard normal variate. Hence a single table for a standard normal integral can serve to find the probability of normal distributions with different means and standard deviations. Table 1 gives the area under the standard normal curve corresponding to $P[Z \leq z]$ for values of Z from -3.4 to $+3.4$.

However, area for right tail such as $P[Z \geq z_1]$ is computed using the relation $P[Z \geq z_1] = 1 - P[Z \leq z_1]$, area between two values of z such as $P[z_1 \leq Z \leq z_2]$ is computed using the relation $P[z_1 \leq Z \leq z_2] = P[Z \leq z_2] - P[Z \leq z_1]$. Again, in some Tables area only for positive values of z are given, in that case, area corresponding to negative values can be computed using the relation $P[Z \leq -z_1] = 1 - P[Z \leq z_1]$.