Date: 22/05/23
Class NO: 01

Matrix: A matrix is a rectangular array of numbers (real or complex) enclosed by a pair of Brackets and the numbers in the array are called the entities or the elements of the matrix.

i.e. a rectangular array of (real or complex) numbers of the form
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$
is called a Matrix

Rectangular Matrix:

If the numbers of row and columns are not the same the matrix is called a rectangular matrix.
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Square Matrix:

If the row and columns are same.

Diagonal Matrix:-

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

A square matrix whose elements $a_{ij}=0$ if $i \neq j$ is called diagonal matrix.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Identity matrix: A square matrix whose elements $a_{ij}=0$ if $i \neq j$ and $a_{ij}=1$ if $i=j$ is called the identity matrix or unit matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Zero Matrix: In matrix where every elements is zero called a null matrix or zero matrix.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Symetric Matrix: A Matrix 'A' is called symetric if $A^T = A$.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 2 & 6 \\ 3 & 6 & 3 \end{bmatrix}$$

Skew - Symetric Matrix: A matrix 'A' is called skew - symetric if $A^T = -A$

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$= -A$$

Hermition Matrix: If a matrix "A" is $A = A^* = (\bar{A})^T$ then it is called hermition matrix

$$A = \begin{pmatrix} 2 & 2-3i & 3 \\ 2+3i & 5 & 1+i \\ 3 & 1-i & 0 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 2 & 2+3i & 3 \\ 2-3i & 5 & 1-i \\ 3 & 1+i & 0 \end{pmatrix}$$

$$(\bar{A})^T = \begin{pmatrix} 2 & 2-3i & 3 \\ 2+3i & 5 & 1+i \\ 3 & 1-i & 0 \end{pmatrix}$$

$$= A$$

Skew Hermition Matrix: $A = -A^* = -(\bar{A})^T$

$$A = \begin{pmatrix} 2i & 2-3i & 3 \\ -2-3i & 5i & 1+i \\ -3 & -1+i & 0 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} -2i & 2+3i & 3 \\ -2+3i & -5i & 1-i \\ -3 & -1-i & 0 \end{pmatrix}$$

$$(\bar{A})^T = \begin{pmatrix} -2i & -2+3i & -3 \\ 2+3i & -5i & -1-i \\ 3 & 1-i & 0 \end{pmatrix}$$

$$-(\bar{A})^T = \begin{pmatrix} 2i & 2+3i & 3 \\ -2-3i & 5i & 1+i \\ 3 & -1+i & 0 \end{pmatrix}$$

$$= A$$

Orthogonal Matrix:- A real square matrix ~~with b~~ A is said to be orthogonal if

$$AA^T = A^TA = I$$

Idempotent matrix: A square matrix A is called an idempotent matrix if $A^2 = A$.

Nilpotent matrix: A square A is called a nilpotent matrix of order n if $A^n = 0$ and $A^{n-1} \neq 0$ where n is positive integer and 0 is the null matrix.

Unitary matrix: $AA^* = A^*A = I$ where $A^* = (\bar{A})^T$.

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

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(i) Prove that (i) $(A^T)^T = A$; (ii) $(A+B)^T = A^T + B^T$
(iii) $(AB)^T = B^T A^T$ (iv) $(\alpha A)^T = \alpha A^T$ where α is a scalar.

Sol:
(i) let, $A = [a_{ij}]$; $i = 1, 2, \dots m$
$j = 1, 2, \dots n$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

By definition $A^T = [a_{ij}]^T = [a_{ji}]$

again, $(A^T)^T = [a_{ji}]^T = [a_{ij}] = A$

(ii) let, $A = [a_{ij}]$
$B = [b_{ji}]$

where, $i = 1, 2, \dots m$
$j = 1, 2, \dots n$

let, $C = A + B$

where,
$[c_{ij}] = [a_{ij}] + [b_{ij}]$

(iii)

Now, $(A+B)^T = C^T = [c_{ij}]^T$

$= c_{ji}$

$= [a_{ji}] + [b_{ji}]$

$= A^T + B^T$

(iv)

$(AB)^T = B^T A^T$

$A = [a_{ij}]$; $i = 1, 2, \ldots m$
$j = 1, 2, \ldots n$

$B = [b_{ij}]$; $i = 1, 2 \ldots n$
$j = 1, 2 \ldots p$

$A^T = [a_{ji}]$ is an $n \times m$ matrix

$B^T = [b_{ji}]$ is a $p \times n$ Matrix

Let, $C = AB$

$\Rightarrow [C_{ij}] = AB$

Note:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$[C_{ik}] = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$$

$(m \times n) \cdot n \times p$

$$c_{11} = a_{11} b_{11} + a_{12} b_{21} = \sum_{j=1}^{2} a_{ij} b_{j1} = a_{11} b_{11} + a_{12} b_{21}$$

$$c_{12} = \sum_{j=1}^{2} a_{ij} b_{j2} = a_{11} b_{12} + a_{12} b_{22}$$

Again:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} \quad ; \quad i = 1, 2 \ldots, m$$
$$\quad \& \quad k = 1, 2 \ldots, p$$

Therefore, the $(k,i)^{th}$ element of $(AB)^T$ = The $(i,k)^{th}$ element of AB

$$= \sum_{j=1}^{n} a_{ij} b_{jk}$$

$$= \sum_{j=1}^{n} a_{ji}^T b_{kj}^T$$

$$= \sum_{j=1}^{n} b_{kj}^T a_{ji}^T$$

$\Rightarrow (AB)^T = (k,i)^{th}$ element of $B^T A^T$

$\therefore$ Hence, $(AB)^T = B^T A^T$ (Proved)

Singular and Non-Singular Matrix

$$A^{-1} = \frac{1}{|A|} \operatorname{Adj} |A|$$

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~~Roll:~~ class: 03

Example 12(b)

Find the inverse of $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{bmatrix}$

step:

$|A| = 53 \neq 0$

co-factor → $C(1,1) = \underbrace{(-1)^{1+1}}_{\text{sign}} \underbrace{\begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix}}_{\text{value}}$ → Adj(A)

→ Minor

$= 0-(-3)$

$= 3$

$C(1,2) = (-1)^{1+2} \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix}$

$= -(8+3)$

$= -11$

$C(1,3) = (-1)^{1+3} \begin{vmatrix} 4 & 0 \\ 3 & 3 \end{vmatrix} = 12$

$C(2,1) = -5$

$C(2,2) = -5$

$C(2,3) = -9$

$C(3,1) = 1$

$C(3,2) = 14$ | $C(3,3) = 4$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$= \frac{1}{53} \begin{bmatrix} 3 & -11 & 12 \\ -5 & -5 & -9 \\ 1 & 14 & 4 \end{bmatrix}^T$$

ge/.133 Topic: find the inverse Matrix using row cononical form.

$$\text{Matrix} = A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Now,

$$[A I_2] = \begin{bmatrix} 2 & 5 & \vdots & 1 & 0 \\ 1 & 3 & \vdots & 0 & 1 \end{bmatrix} \rightarrow \text{Interchanging row 1 and row 2 we ha}$$

$$= \begin{bmatrix} 1 & 3 & \vdots & 0 & 1 \\ 2 & 5 & \vdots & 1 & 0 \end{bmatrix} \rightarrow R_2' = R_2 - 2R_1$$

$$= \begin{bmatrix} 1 & 3 & \vdots & 0 & 1 \\ 0 & -1 & \vdots & 1 & -2 \end{bmatrix} \rightarrow R_1' = R_1 + 3R_2$$

$$[A\ I_2] = \left[\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & -1 & 1 & -2 \end{array}\right] \rightarrow P_2' = P_2 \times -1$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

Hence A is invertible and $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Proving:

$$A A^{-1} = I$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(Proved)

CamScanner

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Example 17 → Solve the linear equation

$$2x + y = 1 \qquad x \text{ \& } y = ?$$
$$x - 2y = 3$$

Solution:

From, $2x + y = 1$
$x - 2y = 3$

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Method →

$$\Rightarrow A \cdot X = L$$

$$\Rightarrow A^{-1} A \cdot X = A^{-1} L$$ → সবসময় বামে অথবা ডানে গুণ করতে হবে, [যেকোনো একপাশে]

$$\Rightarrow I X = A^{-1} L$$

$$\Rightarrow \boxed{X = A^{-1} L} \longrightarrow (i)$$

First we have to find the A^{-1}

here, $A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$

Hence. From eq (i)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{-2}{5} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{6}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Hence, $x = 1$

$y = -1$

Ans

Rank of a Matrix → Page - 155

Echelon Matrix → first Non Zero element of their row [illegible] Echelon Matrix [illegible]

$$\begin{pmatrix} 0 & 1 & 3 & -2 \\ 0 & 0 & -13 & 11 \\ 0 & 0 & 0 & 4 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 2 & 5 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 3 \end{pmatrix}$$

[Echelon Matrix] [Non Echelon Matrix]

Page: 171. Example 4

$$A = \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$ → find the row echelon from.

$R_2' = R_2 - 2R_1 \; ; \; R_3' = R_3 - 3R_1$

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 3 & -6 & 1 \\ 0 & 0 & 5 & -12 & 2 \end{pmatrix} \text{ ——— } A$$

$R_3' = 3R_3 - 5R_2$

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 3 & -6 & 1 \\ 0 & 0 & 0 & -6 & 1 \end{pmatrix}$$

This Matrix is in row Echolon form.

Now Reduce echelon form:

এইখানে সব প্রথম Non zero form কে 1 হওয়া লাগবে।

From eq (A)

$$R_2' = R_2/3 \; ; \; R_3' = R_3/5$$

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{-12}{5} & \frac{2}{5} \end{pmatrix}$$

$$R_3' = R_3 - R_2$$

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{-2}{5} & \frac{1}{15} \end{pmatrix}$$

$R_3' = R_3 \times \frac{-3}{2}$

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{6} \end{pmatrix}$$

Chapter: 4

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Rank of a Matrix

→ কোনো matrix এর সর্বোচ্চ independent row সংখ্যাই হচ্ছে তার Rank.

The rank of a matrix A is the maximum number of linearly independent rows or columns in the matrix.

The row and column ranks of a matrix are equal.

[Example 5] [Page: 173] # Reduce the matrix A to the normal (or canonical) form and hence obtain it's rank.

$$A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{pmatrix}$$

Soln :

The canonical form of the given matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Hence the rank of the given matrix is 3.

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Example : 12 → এইটা আসবেনা i think.

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 1$$

$$2x_1 + 3x_2 + 4x_3 + 5x_4 + 6x_5 = -1$$

$$3x_1 + 5x_2 + 6x_3 + 7x_4 + 4x_5 = 2$$

$$4x_1 + 7x_2 + 10x_3 + 13x_4 + 16x_5 = 1$$

$$5x_1 + 8x_2 + 9x_3 + 10x_4 + 3x_5 = 3$$