Ques-5: con a simple graph exist with 15 vertices each of degree five?

Amounts. In graph theory, Handrohaking theorem rotates in any given graph, sum of degree of all the vertices is twice the number of edges contained in it.

Let br: (V,E) be an undirected graph with e edges. Then

2e = Zuev deg (u)

The sum of the degree of the vertices 5.15 = 75 is odd

Therefore by handshaking theorem a simple graph with 15 vertices each of degree the connot exist.

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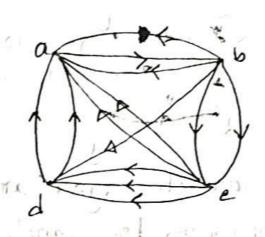
Asolar of the property of

In exercises 7-9 determine the number of vertices and edges and bind the in-degree and out-degree of each retriex for the given directed multigraph. (7) Answers vertices are 4. edges arre7. The in-degree ten lig-7 are degta) = 3 deg(b)=1, deg(c)=2, deg(d)=1 The out-degree in tig-7 are degt(a) = 1, degt(b) = 2, degt(c) = 1, degt(d) = 3 the authorities is bit & one B ADTOLOGICE. work. ventices are 4. Edger are 8. The in-degree in tig-8 are deg (a)=2, deg (b)=3, deg (e) = 2, deg(d) = 1

The out-degree in fig-8 and deg+(a)=2, deg(b)=44

deg+(e) = 1, deg+(d) = 01

(g) Amwen? ax



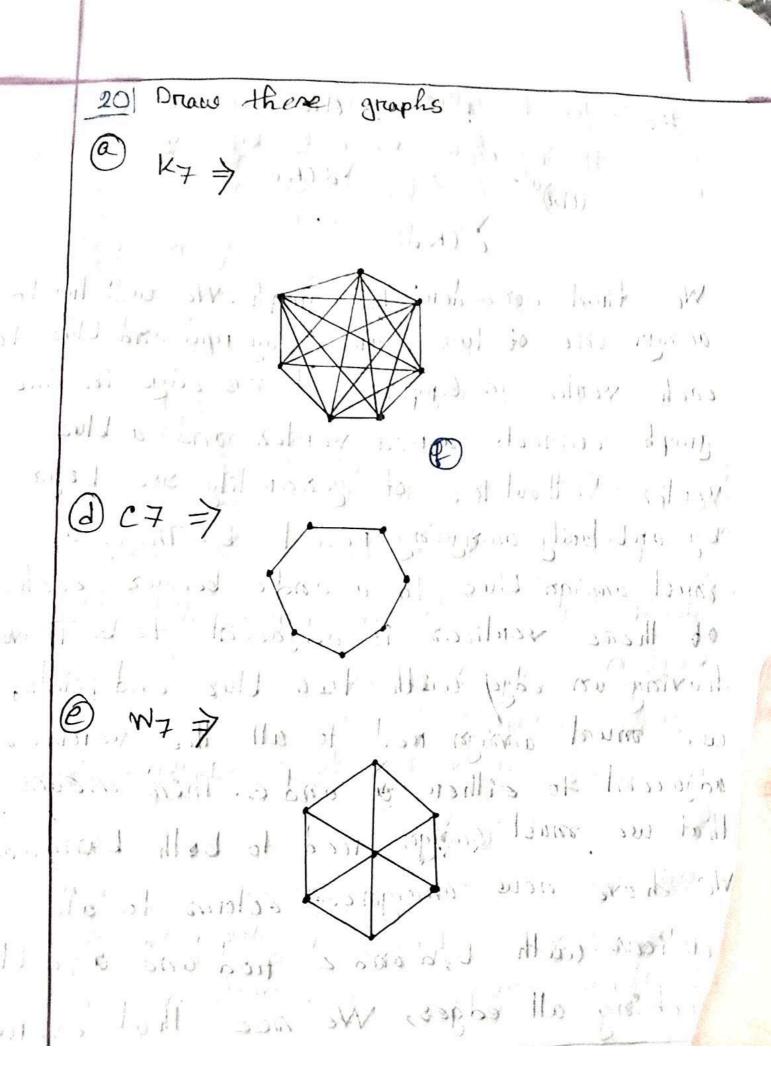
Ventices one 5. Edges one 13.

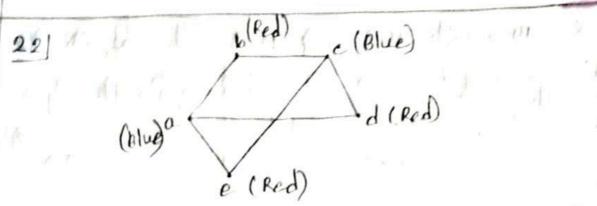
The in-degree degin tig-8 ane deg-(a)=6 deg-(b)=1 deg-(c)=2 deg-(d)=4 deg-(e)=0

the out-degree in lig-8 are $deg^{\dagger}(0)=1$ $deg^{\dagger}(0)=5$, $deg^{\dagger}(0)=2$, $deg^{\dagger}e=0$

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(a) The and a git in single - or will

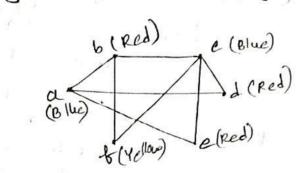




We find consider. the grouph. We will try to arordign one of two colons, may mad and blue, to each vertex in Graphso that no edge in the graph connects a new ventex and a blue ventex. Without loss of generality we begin by aribitarily arrigating red to b. Then we must arrign blue to a and'c because each of there ventices is adjacent to b. To avoid having an edge with two blue end points, we must assign red to all the vertices adjacent to either a and c. This means that we must arrigh , red to both board ande We have now arrigored colors to all vertices with b, d and e ned and a, c blue checking all edges, We see that every

edge connects a red ventex and a blue ventex. Hence, By theorem 4 the graph is bipartite.

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We brust consider the graph. We will try to assign Red andorblue to each veritex in graph so that no edge in graph connects a ned veritex and blue ventex. Without loss of generality use antitarily ansign red to b. Then we must assign blue to a,e, t and c- because each is adjacent to b. But this is not possible because fand a are adjacent, so both connot be aprign blue. This argument shows that we cannot assign one of two colors to each of the ventices of the graph so that no adjacent ventices are

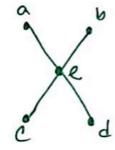
apprign the same colon tence, by theory 4 the graph is not siparatite. 21 April 15 mi Ash y alm of shired not in and the state of the district of the and the state of t a rest for some some some some some I love is not on allower the low the super of commiss transfer and and and marie as pr the contrate of the contrate o and important the

Amswern 19 on it south the it is south all and it -Number of vertices = 5 Number of edges=13 In degree of ventex, deg (a) = 6 In destree of vertex des (b=1 In degree of venter degt (c)=2 In degree of ventur, degt (d) = y In degree of vontex, deg (e) =0

Out degree of venter ident (6) = 1 Out degree of ventex degt (b)=5 Out degree of venter, dept ()=5 Out degree of venter, deat (d) = 2 out degree of venter, deat (e) = 0

In Exercise 21-25 determine whether the grouph is biportite. You may find it useful to apply Theorem 4 and answers the question by determining whether it is possible to assign either fread or blue to each nentless so that two adjacent vertex are assigned the same colors.

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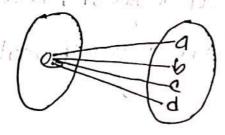


Anso To show that this greaph is bipare tite, we can exhibit the parts and note that indeed every edges and joint veretices in different points. Take jes to be one pant and jaibile

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to be the other (in fact there is no choice in the mother Fach edges joins a ventex in one pand to a vertex in the other. This graph is the (b) Complète biparctite graph K1,4.



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colue) de (ned)

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At hirest we consider the graph. We will try to arrogan either ned on blue to each vertex in growth so that no edge in the graph connects a ned vertex and a blue vertex Without loss of generality we aristarily arrigor ned to a then we must arroign tand a because each is adjacent to a Then we must aronigon ned to all vertices adjacent to either f and c. This means that we must assign ned to a, b, d ande. We have now assigned colors to all beretices with a, b, de red and t, c blue. We nee that every vertex

connects a ned veritex and a blue veritex. Hence, by theorem 4 the graph is bipartite.

a(blue) b (ned)
c (tred)
d (blue)

We consider the graph. We will try to arrigh either ned on blue to each venter in graph so that no edge in graph connects a ned vertex and a blue wentex. Wethout loss of generality we aribitarily assign red to b. Then we must assign blue to a, t, e,d. because each of their adjacent to b. But it is not possible be cause Le andf are adjacent, so both cannot be arrigor blue we arrigor e and f as prinky ellow and pink. This argument shows that we connot appign one of two colors

to each of the vertices of the graph so that no odjacent vertices are arounged the same color. Hence, by theorem 4 that the graph is not bipartitle.

bipardite.

a) kn=>
Amwer: Ka is bipartite if we allow one of the nets to be empty.

K2 is bipartite because we can let one vetitex be in V2 and the other vertex to be in V2. be in V3 and the other vertex to be in V2.

Kn for n>,3 is not bipartite. Choose any three vertices. They all are pair wise connected, therefore there is no way to partition them into two disjoint sets v2 or V2 such that there are no edges within V2.

That is, kon is bipartite when n<3.

(6) Cn

Amousen: Cis bipartite it and only if n is even. Label the ventices by 1,2 -... consecutively along the cycle. It variex 1 is in va then vertex 2 must be in V2, veritex ×3 must be in V1, veritex4 must be in v2 and no on. All ventices with odd number are in Va and all vertices with even number in Ve. The last ventex is in va it on is odd and it is vin ve it n is even. But it is connected to vertex 1. We nee that if n is odd. the graph is not bipartite and if on is even, the graph is bipartite.

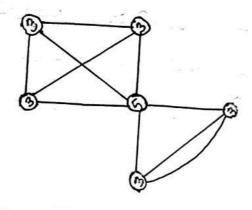
Inna nummary, Conis biparitite of and only if on is even.

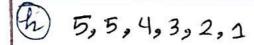
(d) Qn

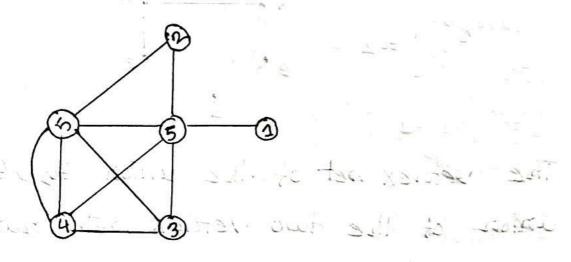
Armwers On is any n. Let vs consist of all vertices whose sum of coordinates is odd and let vs consist of all vertices whose sum of coordinates is even. Two vertices in On are connected if and only if their coordinates differ in only one possition therefore, the sum of their coordinate have different paraty, so they are in different sets.

36 Deterorine whether each of these sequence in graphic. For those that are draw a graph having the given degree sequence.

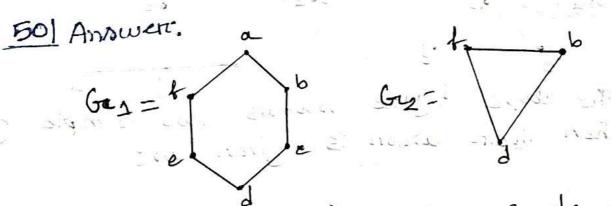
9 5,3,3,3,3,3



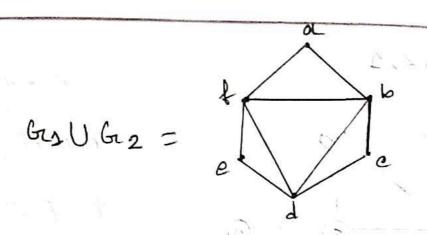




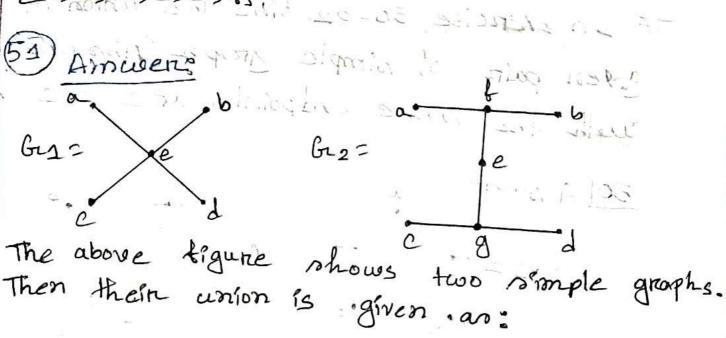
In Exercise 50-52 tind the union of the given pair of simple grouph (Assume edges with the same endpoints are the same.



The above tigure shows two simple graphs. Then their union is given aso



The vertex set of the union Gulber is the union of the two vertex sets, namely 2a, b, c, d, e, ff.

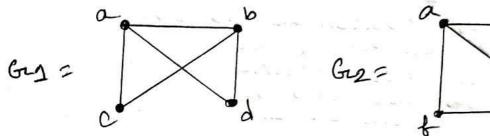


Groupen is given an:

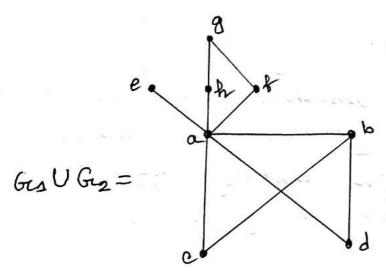
Gist Giz 2

The vertex set of the union GravGra is the union of the two verdex sets, namely La, b, c, d, e, f, g}

52 Amwer:



The above graphs show two simple graphs, Then their union is shown as:



The vertex set of the usion GraUG2 is the union of the two vettex sets, namely la, b, C, d, e, t, g, h?