Department of Information and Communication Engineering
Pabna University of Science and Technology, Pabna
Faculty of Engineering and Technology
B.Sc. (Engineering) 2nd Year 1st Semester Examination-2017
Session: **2015-2016**
Course Code: **Math-2101** Course Title: **Vector, Matrix and Linear Algebra**

NB:	1. Answer any **<u>SIX</u>**(THREE from each PART) questions. 2. Figures in the right margin indicate marks. 3. Parts of the same question should be answered together and in the same sequence. 4. Separate answer script must be used for answering the questions of PART-A and PART-B

Time: 3 Hours **Total Marks: 70**

PART-A

1. a) Define Vector with examples. Find the projection of the vector $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on the vector $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. $3\frac{2}{3}$

   b) Determine an unit vector perpendicular to the plane of $\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$. 4

   c) Prove that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$. 4

2. a) What do you mean by box product? If $\vec{V} = \vec{w} \times \vec{r}$, prove that $\vec{w} = \frac{1}{2} curl\, \vec{V}$ where $\vec{w}$ is a constant vector. $3\frac{2}{3}$

   b) What do you mean by linearly independence and dependence of a vector? Show that the divergence of the curl of a vector is zero. 4

   c) Define vector differentiation and vector integration. Suppose $\vec{A} = sin u\, \hat{\imath} + cos u\, \hat{\jmath} + u\hat{k}$ and $\vec{B} = cos u\, \hat{\imath} - sin u\, \hat{\jmath} - 3\,\hat{k}$, and $\vec{C} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$. Find $\frac{d}{du}\left(\vec{A} \times (\vec{B} \times \vec{C})\right)$ at $u = 0$. 4

3. a) Suppose $\vec{A} = x^2z^2\hat{\imath} - 2y^2z^2\hat{\jmath} + xy^2z\hat{k}$. Find curl $\vec{A}$ at the point (1, -1, 1). 4

   b) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$. $5\frac{2}{3}$

   c) Let $\emptyset = x^2yz - 4xyz^2$. Find the directional derivative of $\emptyset$ at P(1,3,1) in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. 2

4. a) State and prove the Green's theorem in plane. 6

   b) Verify the Green's theorem in the plane for $\oint_C \{(2x - y^3)dx - xy\, dy\}$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. $5\frac{2}{3}$

PART-B

5. a) Define rank of a matrix. Find the rank of the matrix 6

$$A = \begin{bmatrix} 6 & 2\,0 & 4 \\ -2 & -13 & 4 \\ -1 & -16 & 10 \end{bmatrix}$$

   b) Prove that the following system of linear equations is inconsistent: $5\frac{2}{3}$

$$3x_1 + 4x_2 - x_3 + 2x_4 = 1$$
$$x_1 - 2x_2 + 3x_3 + x_4 = 2$$
$$3x_1 + 14x_2 - 11x_3 + x_4 = 3$$

6. a) Define sum and direct sum of two subspaces. Let $V(\mathbb{R})$ be a vector space of $n \times n$ matrices and let S and T are two subspaces of $V(\mathbb{R})$ such that $S = \{A: A' = A\}$ and $T = \{A: A' = -A\}$. Then prove that $V(\mathbb{R}) = S \oplus T$. 4

   b) Define linear combination of vectors. Consider the vectors $v_1 = (2, 1, 3)$, $v_2 = (1, -1, 3)$, and $v_3 = (3, 2, 5)$ in $\mathbb{R}^3$, then show that $v = (5, 9, 5)$ is a linear combination of v_1, v_2, v_3. $3\frac{2}{3}$

   c) Define basis and dimension of a vector space. Let $W = L(S)$, where $S = \{(1, -2, 5, -3), (2, 3, 1, -4), (3, 8, -3, -5)\}$, is a subset of $\mathbb{R}^4$. Find the basis and dimension of W. 4

7. a) Define Laplace's equation in parabolic cylindrical co-ordinates. $4\frac{2}{3}$

   b) Show that set of vectors $\{(3, 0, 1, -1), (2, -1, 0, 1), (1, 1, 1, -2)\}$ is linearly independent. 7

8. a) Define kernel and range of a linear transformation. 3

   b) Define linear transformation with example. $1\frac{2}{3}$

   c) Show that the following transformation defines a linear operation on $\mathbb{R}^3$: $T(x, y, z) = (x + y, -x - y, z)$. 7

...tment of Information and Communication Engineering
Pabna University of Science and Technology
Faculty of Engineering and Technology
B.Sc. (Engineering) 2nd Year 1st Semester Examination-2018
Session: **2016-2017**

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Course Code: **Math-2101**

Course Title: **Vector, Matrix and Linear Algebra**

NB:
1. Answer any <u>**SIX**</u> (THREE from each PART) questions.
2. Figures in the right margin indicate marks.
3. Parts of the same question should be answered together and in the same sequence.
4. Separate answer script must be used for answering the question of **PART-A** and **PART-B.**

Time: 3 Hours

Total Marks: 70

PART-A

1. a) Without making use of the cross product, determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{\imath} - \hat{6}\jmath - 3\hat{k}$, and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$. 4
   b) What do you mean by dot product of two vectors? Find the projection of the vectors $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on the vector $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. $3\frac{2}{3}$
   c) Suppose $\vec{c}_1$ and $\vec{c}_2$ are constant vectors and λ is a constant scalar. Show that $\vec{H} = e^{-\lambda x}(\vec{c}_1 \sin \lambda y + \vec{c}_2 \sin \lambda y)$ satisfies the partial differential equation 4
$$\frac{\partial^2 \vec{H}}{\partial x^2} + \frac{\partial^2 \vec{H}}{\partial y^2} = 0.$$

2. a) Find the unit tangent vector to any point on the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$. $3\frac{2}{3}$
   b) Find the value of the constants a, b, c so that the vector $\vec{V} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrational. 4
   c) Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. 4

3. a) Show that $\vec{A} = (2x^2 + 8xy^2z)\hat{\imath} + (3x^3y - 3xy)\hat{\jmath} - (4y^2z^2 + 2x^3z)\hat{k}$ is not solenoidal but $\vec{B} = xyz^2\vec{A}$ is solenoid. $4\frac{2}{3}$
   b) Let $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$. Evaluate $\int_C \vec{A}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the straight lines from (0, 0, 0) to (1, 0, 0), then to (1, 1, 0), and then to (1,1,1). 5
   c) Suppose $\vec{A}$ and $\vec{B}$ are irrotational. Prove that $\vec{A} \times \vec{B}$ is solenoidal. 2

4. a) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$, where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. $6\frac{2}{3}$
   b) Use Divergence theorem to evaluate $\iint_s \vec{F}.\hat{n}\,ds$, where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 5

PART-B

5. a) Define Square matrix, Identity matrix, Diagonal matrix and Scalar matrix with example. 4
   b) Prove that $(AB)^{-1} = b^{-1}A^{-1}$. $2\frac{2}{3}$
   c) What is echelon form of a matrix? Find the echelon form of the matrix 5
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

6. a) Define inverse of a matrix. Find the inverse of the matrix $6\frac{2}{3}$

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{pmatrix}$$

b) What do you mean by minor and cofactor of a matrix? Find the rank of the matrix 5

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$$

7. a) Determine the eigen values and the eigen vectors of the matrix 5

$$B = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$$

b) State Cayley-Hamilton theorem. Verify it for $6\frac{2}{3}$

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

8. a) Solve the following system of equations $5\frac{2}{3}$

$$\begin{aligned} 2x + 4y - z &= 9 \\ 3x - y + 5z &= 5 \\ 8x + 2y + 9z &= 19 \end{aligned}$$

b) Determine whether or not the vectors $(1, -2, 1)$, $(2, 1, -1)$ and $(7, -4, 1)$ are linearly independent. 3

c) Show that the vectors $(1, 1, 1)$, $(1, 2, 3)$ and $(2, -1, 1)$ are dependent and form a basis of $\mathbb{R}^3$. 3

Department of Information and Communication Engineering
Pabna University of Science and Technology
Faculty of Engineering and Technology
B.Sc. (Engineering) 2nd Year 1st Semester Examination-2019
Course Code: **Math-2101** Course Title: **Vector, Matrix and Linear Algebra**

NB:
1. Answer any **SIX** (THREE from each PART) questions.
2. Figures in the right margin indicate marks.
3. Parts of the same question should be answered together and in the same sequence.
4. Separate answer script must be used for answering the question of **PART-A** and **PART-B**.

Time: 3 Hours **Total Marks: 70**

PART-A

1. a) Define matrix, null matrix, nilpotent matrix and periodic matrix with examples. $3\frac{2}{3}$
   b) Prove the associative law for matrix multiplication. 4
   c) Prove that the matrix which is commutative for matrix multiplication with a diagonal matrix with distinct diagonal elements is diagonal matrix. 4

2. a) Define an idempotent matrix. Show that the matrix $A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$ is idempotent. $3\frac{2}{3}$
   b) Define an orthogonal matrix, If A and B are orthogonal matrices of order n, then prove that the matrices AB and BA are also orthogonal. 3
   c) Define Hermitian and skew-Hermitian matrices. Show that the diagonal elements of a Hermitian matrix are necessary real but for skew-Hermitian matrix they are either purely imaginary or zero. 5

3. a) Define adjoint and inverse of a matrix. Find the adjoint and inverse of the matrix 6
$$A = \begin{pmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}.$$
   b) Define rank of a matrix. Find the rank of the following matrix using the normal form $5\frac{2}{3}$
$$A = \begin{pmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{pmatrix}.$$

4. a) State Cayley-Hamilton theorem. Find the characteristic polynomial $\Delta(t)$ of the matrix $6\frac{2}{3}$
$$A = \begin{pmatrix} 1 & 6 & -2 \\ -3 & 2 & 0 \\ 0 & 3 & 4 \end{pmatrix}.$$
   b) State Stoke's theorem. Verify Stoke's theorem for $\vec{A} = (x - z + 2)\hat{\imath} + (yz + 4)\hat{\jmath} - xz\hat{k}$, where S is the surface for the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy plane. 5

PART-B

5. a) Define subspace of a vector space. Prove that W is a subspace of a vector space V if and only if (i) W is nonempty, (ii) W is closed under vector addition and (iii) W is closed under scalar multiplication. 6
   b) Define linear combinations, linear spans, sum and direct sum. Write the vector $v = (1, -2, 5)$ as a linear combination of the vectors $e_1 = (1, 1, 1), e_2 = (1, 2, 3)$ and $e_3 = (2, -1, 1)$ $5\frac{2}{3}$

6. a) Define vector and scalar with examples. Determine a unit vector perpendicular to the plane of $\vec{A} = 2i - 6j - 3k$ and $\vec{B} = 4i + 3j - k$. 4
   b) Prove that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$ 3
   c) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = -t - 5$, where t is the time. Find the components of its velocity and acceleration at time t =1 in the direction $i - 2j + 2k$. $4\frac{2}{3}$

7. a) Determine whether (1, 1, 1), (1, 2, 3) and (2, -1, 1) are linearly independent. 3

b) Define basis and dimension. Determine whether (1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5) form a basis of $\mathbb{R}^4$. 4

c) Let $\{(1,1,1,1),(1,2,1,2)\}$ be linearly independent subset of the vector space $\mathbb{R}^4$. Extend it to a basis for $\mathbb{R}^4$. $4\frac{2}{3}$

8. a) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where C is the boundary of the region defined by $y = \sqrt{x},\ y = x^2$. $5\frac{2}{3}$

b) State Divergence theorem. Verify this theorem for $\vec{A} = 4xi - 2y^2j + z^2k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. 6

Department of Information and Communication Engineering

Pabna University of Science and Technology

Faculty of Engineering and Technology

B.Sc. (Engineering) 2nd Year 1st Semester Examination-2020

Session: **2018-2019, 2017-2018, & 2016-2017**

Course Code: **Math-2101**

Course Title: **Vector, Matrix and Linear Algebra**

NB:	1. Answer any **SIX** (THREE from each PART) questions. 2. Figures in the right margin indicate marks. 3. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours **Total Marks: 70**

$\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

PART-A

1. a) Define unit vector. Determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and, $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$. 4

   b) What do you mean by dot product of two vectors? Find the projection of the vectors $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on the vector $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. 4

   c) Find the angles which the vector $\vec{A} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ makes with the coordinate axes. $3\frac{2}{3}$

2. a) Suppose $\vec{A} = x^2yz\hat{\imath} - 2xz^3\hat{\jmath} + xz^2\hat{k}$ and $\vec{B} = 2z\hat{\imath} + y\hat{\jmath} - x^2\hat{k}$. Find $\frac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$. 4

   b) Define irrotational vector. If $curl\ \vec{v} = 0$, find the constants a, b and c so that $\vec{v} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\jmath} + (4x + cy + 3z)\hat{k}$ is irrotational. 4

   c) Find an equation of the tangent plane to the surface $x^2yz - 4xyz^2 = 6$ at the point $P(1,2,1)$. $3\frac{2}{3}$

3. a) Show that $\vec{A} = (2x^2 + 8xy^2z)\hat{\imath} + (3x^3y - 3xy)\hat{\jmath} - (4y^2z^2 + 2x^3z)\hat{k}$ is not solenoidal but $\vec{B} = xyz^2\vec{A}$ is solenoid. $4\frac{2}{3}$

   b) Suppose, $\vec{F} = -3x^2\hat{\imath} + 5xy\hat{\jmath}$. Evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve in the xy-plane, $y = 2x^2$ from (0,0) to (1,2). 5

   7

   c) Suppose, $\vec{v} = \vec{\omega} \times \vec{r}$. Prove that $\vec{\omega} = \frac{1}{2} curl \vec{v}$, where $\vec{\omega}$ is a constant vector. 2

4. a) Let $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$. Evaluate $\int_C \vec{A}.d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following path C: 6
   (i) $x = t, y = t^2, z = t^3$.
   (ii) The straight line from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to (1,1,1).
   (iii) The straight line from $(0,0,0)$ to $(1,1,1)$.
   (iv) The straight line joining $(0,0,0)$ to $(1,1,1)$.

   $a = 9, b = -3, c = 5$

   $6\hat{i} + 3\hat{j} + 14\hat{k} = -6$

   b) State Stoke's theorem, Divergence theorem, and Green's theorem. $5\frac{2}{3}$

PART-B

5. a) Define matrix and order of a matrix. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{bmatrix}$. 6
   Find: (i) AB and (ii) BA.

   b) Define square matrix. If $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$, prove that $A^3 - 4A^2 - A + 4I = 0$. $5\frac{2}{3}$

6. a) Show that the matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is nilpotent. 2

   b) If A and B are conformable matrices, then prove that $(A + B)' = A' + B'$. 3

   c) If $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{pmatrix}$, then verify that $(AB)' = B'A'$. $6\frac{2}{3}$

7\. a) Define inverse of a matrix. Find the adjoint and inverse of the matrix $6\frac{2}{3}$

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}.$$

b) What do you mean by normal form of matrix? Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{pmatrix}$ into normal form and hence find its rank. 5

8\. a) Solve the following system of equations $5\frac{2}{3}$

$$2x + y - 2z + 3w = 1$$
$$3x + 2y - z + 2w = 4$$
$$3x + 3y + 3z - 3w = 5.$$

b) Determine whether or not the vectors $(1, -2, 1)$, (2, 1, -1) and $(7, -4, 1)$ are linearly independent. 3

c) Show that the vectors $(1, 1, 1)$, $(1, 2, 3)$ and $(2, -1, 1)$ are dependent and form a basis of $\mathbb{R}^3$. 3

$$\begin{bmatrix} -2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix}$$