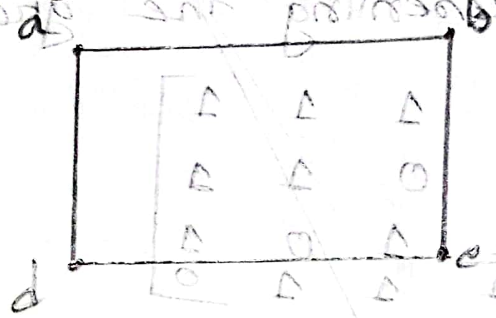
8.3

9) Represent each of these graphs with an adjacency matrix.

d) C_4

Answer: The graph of C_4 is shown below:

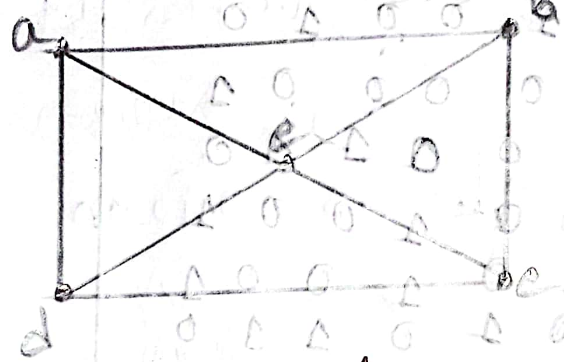


We order the vertices as a, b, c, d. The matrix representing the graph is -

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(d) W_4

Ans: The graph of W_4 is shown below:



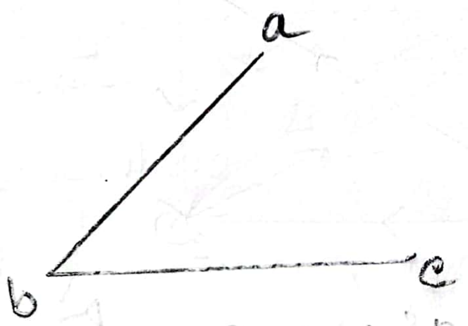
We order the vertices as a, b, c, d, e. The matrix representing the graph is:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

In exercises 10-12 draw a graph with the given adjacency matrix.

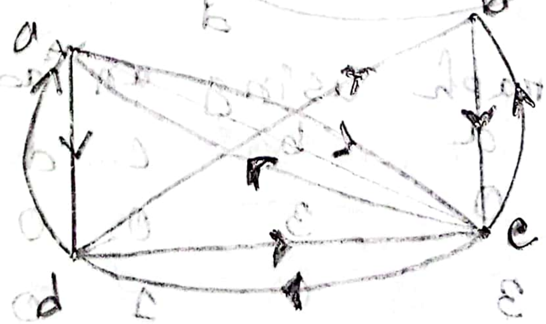
(10) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Ans:



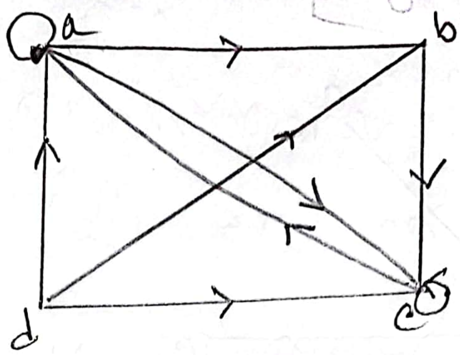
(11) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Ans: The graph is shown below:



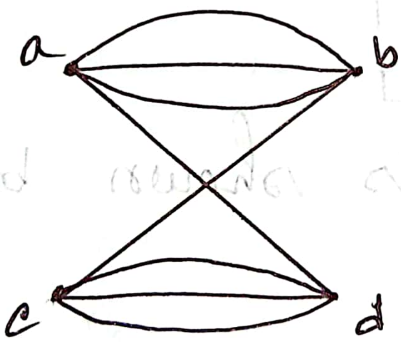
(12)
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Ans: The graph is shown below:



(13) Represent the given graph with adjacency matrix.

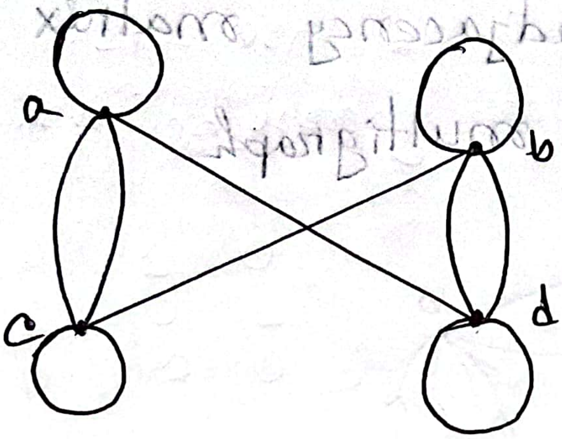
(14)



Ans: The graph using an adjacency matrix is shown below:

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 3 & 0 & 0 \\ b & 3 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 3 \\ d & 1 & 0 & 3 & 0 \end{array}$$

(15)



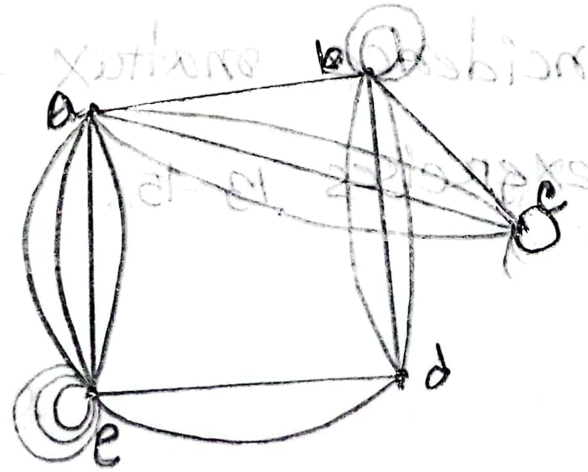
Ans:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$

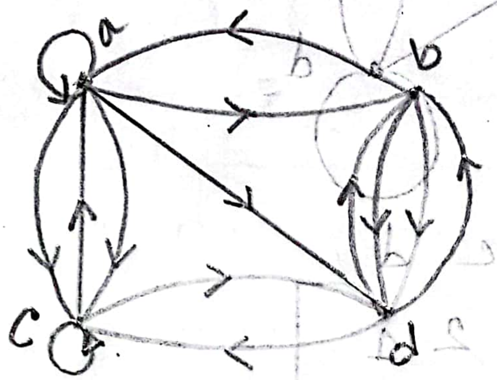
(18) Draw an undirected graph represent by the given adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

Ans:



(21) Find the adjacency matrix of the given directed multigraph.



Ans: The adjacency matrix of the given directed multigraph is shown below:

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 2 & 1 \\ b & 1 & 0 & 0 & 2 \\ c & 1 & 0 & 1 & 1 \\ d & 0 & 2 & 1 & 0 \end{array}$$

(27) Use an incidence matrix to represent graph in exercises 13-15.

34 ~~#~~ In Exercises determine whether the given pair of graph is Isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

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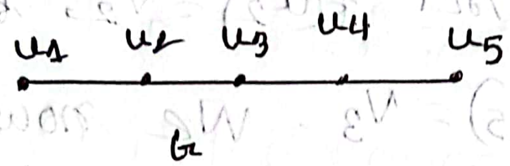

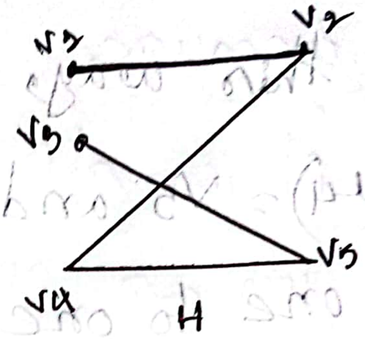

Ans:

~~Both~~ Determine the graph 1 as G and graph 2 as H. Both G and H have four vertices of degree two and two vertices of degree of degree three. ~~It is~~ also We now will define a function f and then determine whether it is an isomorphism. Because $deg(u_1) = 1$ because it is not adjacent to any other vertex of degree one, the image of ~~v_1 or v_3~~ u_1, must be either v_1 or v_3, the only vertices of degree on in H not adjacent to a vertex of degree one. We arbitarily set $f(u_1) = v_1$

[If we found that this choice did not

lead to isomorphism; we would then try $f(u_1) = v_3$]. Because u_2 is adjacent to u_1, the possible images of u_2 are v_2. Continuing in this way, we set $f(u_2) = v_2$, $f(u_3) = v_4$, $f(u_4) = v_5$ and $f(u_5) = v_3$. We now have a one to one correspondence between the vertex set of G and the vertex set of H, namely, $f(u_1) = v_1$, $f(u_2) = v_2$, $f(u_3) = v_4$, $f(u_4) = v_5$ and $f(u_5) = v_3$.

Now we examine the adjacency matrix of G,

$$A_G = \begin{array}{c|ccccc} & u_1 & u_2 & u_3 & u_4 & u_5 \\ \hline u_1 & 0 & 1 & 0 & 0 & 0 \\ u_2 & 1 & 0 & 1 & 0 & 0 \\ u_3 & 0 & 1 & 0 & 1 & 0 \\ u_4 & 0 & 0 & 1 & 0 & 1 \\ u_5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

The adjacency matrix of H with the rows and columns labeled by the images of the corresponding vertices in G.

$$A_H = \begin{array}{c|ccccc} & 1 & 2 & 4 & 5 & 3 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Beacause $A_G = A_H$, it follows that f preserves edges. We conclude that f is an isomorphism, so, G and H are isomorphic.

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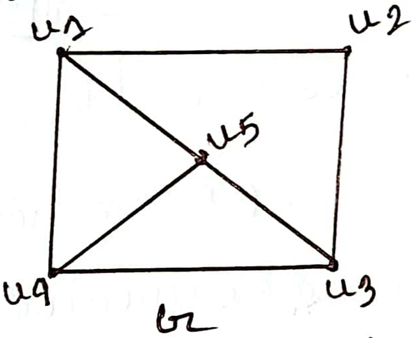

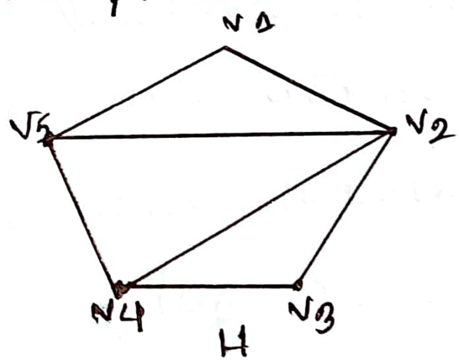

3 v1 2
2 v2 3
3 v3 2
3 v4 3
3 v5 3

Ans: Determine the graph 1 as G and graph 2 as H. The graphs G and H both have five vertices and seven edges.

But G and H are not isomorphic.

The graph G have 4 vertices of degree 3 and 1 vertices of degree 2.

The graph H have 3 vertices of degree 3 and 2 vertices of degree 2. Because these invariants all disagree, it is still conceivable that these graphs are ~~isomor~~ not isomorphic.

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degree ⟹ $u_1 = 3$ $u_2 = 3$ $u_3 = 3$ $u_4 = 3$ $u_5 = 3$ $u_6 = 3$

$v_1 = 3$ $v_2 = 3$ $v_3 = 3$ $v_4 = 3$ $v_5 = 3$ $v_6 = 3$

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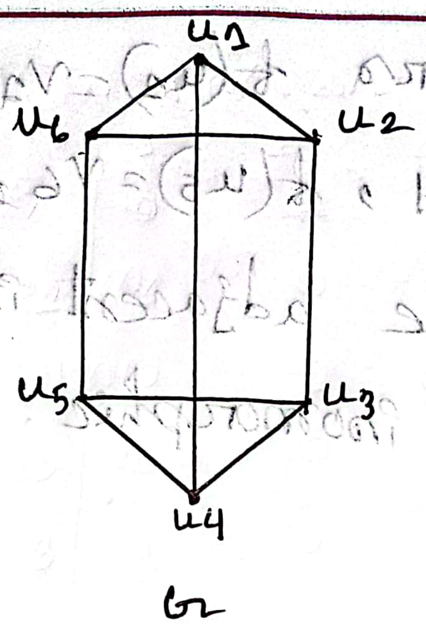
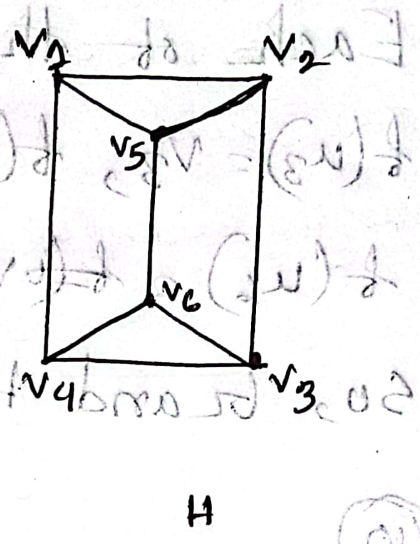
39)





Ans: Determine the graph 1 as G and graph 2 as H. The graph G and H both have 6 vertices and 9 edges. They also both have six vertices of degree three. The function f with $f(u_1) = v_1$, $f(u_2) = v_2$, $f(u_3) = v_3$, $f(u_4) = v_4$, $f(u_5) = v_6$, $f(u_6) = v_5$ is a one-to-one correspondence between G and H. To see that the correspondence preserves adjacency, note that adjacent vertices in G are u_1 and u_2, u_1 and u_6, u_2 and u_3, u_1 and u_3, u_4 and u_5, and u_5 and u_6.

Each of the pairs $f(u_1)=v_1$, $f(u_2)=v_2$, $f(u_3)=v_3$, $f(u_4)=v_4$, $f(u_5)=v_6$ and $f(u_6)=\cancel{f(}v_5$. are adjacent in H.

So, G and H are isomorphic.