44. Use quantifiers and logical connectives to express the fact that a quadratic polynomial with real number coefficients has at most two real roots.
45. Determine the truth value of the statement $\forall x \exists y(xy = 1)$ if the domain for the variables consists of
   a) the nonzero real numbers.
   b) the nonzero integers.
   c) the positive real numbers.
46. Determine the truth value of the statement $\exists x \forall y(x \leq y^2)$ if the domain for the variables consists of
   a) the positive real numbers.
   b) the integers.
   c) the nonzero real numbers.
47. Show that the two statements $\neg\exists x \forall y P(x, y)$ and $\forall x \exists y \neg P(x, y)$, where both quantifiers over the first variable in $P(x, y)$ have the same domain, and both quantifiers over the second variable in $P(x, y)$ have the same domain, are logically equivalent.
48. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x \forall y(P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain, are logically equivalent. (The new variable y is used to combine the quantifications correctly.)

*49 a) Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x \exists y (P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.
   b) Show that $\forall x P(x) \vee \exists x Q(x)$ is equivalent to $\forall x \exists y (P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain.

A statement is in prenex normal form (**PNF**) if and only if it is of the form

$$Q_1 x_1 Q_2 x_2 \ldots Q_k x_k P(x_1, x_2, \ldots, x_k),$$

where each Q_i, $i = 1, 2, \ldots, k$, is either the existential quantifier or the universal quantifier, and $P(x_1, \ldots, x_k)$ is a predicate involving no quantifiers. For example, $\exists x \forall y(P(x, y) \wedge Q(y))$ is in prenex normal form, whereas $\exists x P(x) \vee \forall x Q(x)$ is not (because the quantifiers do not all occur first).

Every statement formed from propositional variables, predicates, T, a nd F using logical connectives and quantifiers is equivalent to a statement in prenex normal form. Exercise 51 asks for a proof of this fact.

*50. Put these statements in prenex normal form. [Hint: Use logical equivalence from Tables 6 and 7 in Section 1.2, Table 2 in Section 1.3, Example 19 in Section 1.3, Exercises 45 and 46 in Section 1.3, and Exercises 48 and 49 in this section.]
   a) $\exists x P(x) \vee \exists x Q(x) \vee A$, where A is a proposition not involving any quantifiers.
   b) $\neg(\forall x P(x) \vee \forall x Q(x))$ c) $\exists x P(x) \rightarrow \exists x Q(x)$

**51. Show how to transform an arbitrary statement to a statement in prenex normal form that is equivalent to the given statement.

*52. Express the quantification $\exists! x P(x)$, introduced on page 37, using universal quantifications, existential quantifications, and logical operators.

1.5 RULES OF INFERENCE

Introduction Later in this chapter we will study proofs. Proofs in mathematics are valid arguments that establish the truth of mathematical statements. By an **argument,** we mean a sequence of statements that end with a conclusion. By **valid,** we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or **premises,** of the argument. That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. To deduce new statements from statements we already have, we use rules of inference which are templates for constructing valid arguments. Rules of inference are our basic tools for establishing the truth of statements.

Before we study mathematical proofs, we will look at arguments that involve only compound propositions. We will define what it means for an argument involving compound propositions to be valid. Then we will introduce a collection of rules of inference in propositional logic. These rules of inference are among the most important ingredients in producing valid arguments. After we illustrate how rules of inference are used to produce valid arguments, we will describe some common forms of incorrect reasoning, called **fallacies,** which lead to invalid arguments.

After studying rules of inference in propositional logic, we will introduce rules of inference for quantified statements. We will describe how these rules of inference can be used to produce valid arguments. These rules of inference for statements involving existential and universal quantifiers play an important role in proofs in computer science and mathematics, although they are often used without being explicitly mentioned.

Finally, we will show how rules of inference for propositions and for quantified statements can be combined. These combinations of rule of inference are often used together in complicated arguments.

Argument → sequence of statement
Conclusion → final statement of the Argument.
valid → Last result.

Valid Arguments in Propositional Logic Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

"If you have a current password, then you can log onto the network."
"You have a current password."
Therefore,
"You can log onto the network."

We would like to determine whether this is a valid argument. That is, we would like to determine whether the conclusion "You can log onto the network" must be true when the premises "If you have a current password, then you can log onto the network" and "You have a current password" are both true.

Before we discuss the validity of this particular argument, we will look at its form. Use p to represent "You have a current password" and q to represent "You can log onto the network." Then, the argument has the form

$$\begin{array}{l} p \rightarrow q \\ \underline{p \quad\quad} \\ \therefore q \end{array}$$

where $\therefore$ is the symbol that denotes "therefore."

We know that when p and q are propositional variables, the statement $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology (see Exercise 10(c) in Section 1.2). In particular, when both $p \rightarrow q$ and p are true, we know that q must also be true. We say this form of argument is **valid** because whenever all its premises (all statements in the argument other than the final one, the conclusion) are true, the conclusion must also be true. Now suppose that both "If you have a current password, then you can log onto the network" and "You have a current password" are true statements. When we replace p by "You have a current password" and q by "You can log onto the network," it necessarily follows that the conclusion "You can log onto the network" is true. This argument is **valid** because its form is valid. Note that whenever we replace p and q by propositions where $p \rightarrow q$ and p are both true, then q must also be true.

What happens when we replace p and q in this argument form by propositions where not both p and $p \rightarrow q$ are true? For example, suppose that p represents "You have access to the network" and q represents "You can change your grade" and that p is true, but $p \rightarrow q$ is false. The argument we obtain by substituting these values of p and q into the argument form is

"If you have access to the network, then you can change your grade."
"You have access to the network."
$\therefore$ "You can change your grade."

The argument we obtained is a valid argument, but because one of the premises, namely the first premise, is false, we cannot conclude that the conclusion is true. (Most likely, this conclusion is false.)

In our discussion, to analyze an argument, we replaced propositions by propositional variables. This changed an argument to an **argument form.** We saw that the validity of an argument follows from the validity of the form of the argument. We summarize the terminology used to discuss the validity of arguments with our definition of the key notions.

Definition 1 An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is valid if the truth of all its premises implies that the conclusion is true.

An *argument form* in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is *valid* if no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

From the definition of a valid argument form we see that the argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion q is valid, when $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q$ is a tautology.

The key to showing that an argument in propositional logic is valid is to show that its argument form is valid. Consequently, we would like techniques to show that argument forms are valid. We will now develop methods for accomplishing this task.

Rules of Inference for Propositional Logic We can always use a truth table to show that an argument form is valid. We do this by showing that whenever the premises are true, the conclusion must also be true. However, this can be a tedious approach. For example, when an argument form involves 10 different propositional variables, to use a truth table to show this argument form is valid requires $2^{10} = 1024$ different rows. Fortunately, we do not have to resort to truth tables. Instead, we can first establish the validity of some relatively simple argument forms, called **rules of inference.** These rules of inference can be used as building blocks to construct more complicated valid argument forms. We will now introduce the most important rules of inference in propositional logic.

The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is the basis of the rule of inference called **modus ponens,** or the **law of detachment.** (Modus ponens is Latin for *mode that affirms.*) T his tautology leads to the following valid argument form, which we have already seen in our initial discussion about arguments (where, as before, the symbol $\therefore$ denotes "therefore."):

$$\begin{array}{rl} & p \\ & \underline{p \rightarrow q} \\ \therefore & q \end{array}$$

Using this notation, the hypotheses are written in a column, followed by a horizontal bar, followed by a line that begins with the therefore symbol and ends with the conclusion. In particular, modus ponens tells us that if a conditional statement and the hypothesis of this conditional statement are both true, then the conclusion must also be true. Example 1 illustrates the use of modus ponens.

Example 1 Suppose that the conditional statement "If it snows today, then we will go skiing" and its hypothesis, "It is snowing today," are true. Then, by modus ponens, it follows that the conclusion of the conditional statement, "We will go skiing," is true. ◀

As we mentioned earlier, a valid argument can lead to an incorrect conclusion if one or more of its premises is false. We illustrate this again in Example 2.

Example 2 Determine whether the argument given here is valid and determine whether its conclusion must be true because of the validity of the argument.

"If $\sqrt{2} > \frac{3}{2}$, then $(\sqrt{2})^2 > (\frac{3}{2})^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $(\sqrt{2})^2 = 2 > (\frac{3}{2})^2 = \frac{9}{4}$."

Solution Let p be the proposition "$\sqrt{2} > \frac{3}{2}$" and q the proposition "$2 > (\frac{3}{2})^2$." The premises of the argument are $p \rightarrow q$ and p, and q is its conclusion. This argument is valid because it is constructed by using modus ponens, a valid argument form. However, one of its premises, $\sqrt{2} > \frac{3}{2}$, is false. Consequently, we cannot conclude that the conclusion is true. Furthermore, note that the conclusion of this argument is false, because $2 < \frac{9}{4}$. ◀

Table 1 lists the most important rules of inference for propositional logic. Exercises 9, 10, 15, and 30 in Section 1.2 ask for the verifications that these rules of inference are valid argument forms. We now give examples of arguments that use these rules of inference. In each argument, we first use propositional variables to express the propositions in the argument. We then show that the resulting argument form is a rule of inference from Table 1.

Table 1 Rules of inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Example 3 *State which rule of inference is the basis of the following argument: "It is below freezing now. Therefore, it is either below freezing or raining now."*

Solution Let p be the proposition "It is below freezing now" and q the proposition "It is raining now." Then this argument is of the form

P $P\wedge$

pvq

$$\frac{p}{\therefore p \vee q}$$

This is an argument that uses the addition rule. ◀

Example 4 *State which rule of inference is the basis of the following argument: "It is below freezing and raining now. Therefore, it is below freezing now."*

Solution Let p be the proposition "It is below freezing now," and let q be the proposition "It is raining now." This argument is of the form

$$\frac{p \wedge q}{\therefore p}$$

This argument uses the simplification rule. ◀

Example 5 *State which rule of inference is used in the argument:*

If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

Solution Let p be the proposition "It is raining today," let q be the proposition "We will not have a barbecue today," and let r be the proposition "We will have a barbecue tomorrow." Then this argument is of the form

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

◀

Hence, this argument is a hypothetical syllogism.

Using Rules of Inference to Build Arguments When there are many premises, several rules of inference are often needed to show that an argument is valid. This is illustrated by Examples 6 and 7, where the steps of arguments are displayed on separate lines, with the reason for each step explicitly stated. These examples also show how arguments in English can be analyzed using rules of inference.

Example 6 *Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."*

Solution Let p be the proposition "It is sunny this afternoon," q the proposition "It is colder than yesterday," r the proposition "We will go swimming," s the proposition "We will take a canoe trip," and t the proposition "We will be home by sunset." Then the hypotheses become $\neg p \wedge q$, $r \to p$, $\neg r \to s$, and $s \to t$. The conclusion is simply t. We need to give a valid argument with hypotheses $\neg p \wedge q$, $r \to p$, $\neg r \to s$, and $s \to t$ and conclusion t.

Extra Examples

We construct an argument to show that our hypotheses lead to the desired conclusion as follows.

Step	Reason
1. $\neg p \wedge q$	Hypothesis
2. $\neg p$	Simplification using (1)
3. $r \to p$	Hypothesis
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \to s$	Hypothesis
6. s	Modus ponens using (4) and (5)
7. $s \to t$	Hypothesis
8. t	Modus ponens using (6) and (7)

Note that we could have used a truth table to show that whenever each of the four hypotheses is true, the conclusion is also true. However, because we are working with five propositional variables, p, q, r, s, and t, such a truth table would have 32 rows. ◀

Example 7 *Show that the hypotheses "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."*

Solution Let p be the proposition "You send me an e-mail message," q the proposition "I will finish writing the program," r the proposition "I will go to sleep early," and s the proposition "I will wake up feeling refreshed." Then the hypotheses are $p \to q$, $\neg p \to r$, and $r \to s$. The desired conclusion is $\neg q \to s$. We need to give a valid argument with hypotheses $p \to q$, $\neg p \to r$, and $r \to s$ and conclusion $\neg q \to s$.

This argument form shows that the hypotheses p lead to the desired conclusion.

Step	Reason
1. $p \to q$	Hypothesis
2. $\neg q \to \neg p$	Contra positive of (1)
3. $\neg p \to r$	Hypothesis
4. $\neg q \to r$	Hypothetical syllogism using (2) and (3)
5. $r \to s$	Hypothesis
6. $\neg q \to s$	Hypothetical syllogism using (4) and (5)

Example 8 Show that the following argument is valid. If today is Tuesday, I have a test in Mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics.

Converting to logical notation.

Let T denote 'Today is Tuesday'

M denote 'I have a test in Mathematics'

E denote 'I have a test in Economics'

S denote 'My Economics Professor is sick'

1. $T \to (M \vee E)$
2. $S \to \neg E$
3. $T \wedge S$

$\therefore M$

From 3 we get $\left.\begin{array}{l}4.\ T\\ 5.\ S\end{array}\right\}$ (simplification)

From 4 and 1 we get 6. $M \vee E$ (modus ponens)

From 5 and 2 we get 7. $\neg E$ (modus ponens)

From 6 and 7 we get 8. M (disjunctive syllogism)

Example 9 *Show that the following argument is valid. If Mohan is a lawyer, then he is ambitious. If Mohan is an early riser, then he does not like idlies. If Mohan is ambitious, then he is an early riser. Then if Mohan is a lawyer, then he does not like idlies.*

Solution

Mohan is a lawyer be denoted by L

Mohan is ambitious be denoted by A

Mohan is an early riser be denoted by E

Mohan likes idlies be denoted by I

The premises and conclusion can be given by

1. $L \to A$
2. $E \to \neg I$
3. $A \to E$

$\therefore L \to \neg I$

From 1 and 3 by hypothetical syllogism we get 4. $L \to E$

From 4 and 2 by hypothetical syllogism we get $L \to \neg I$

Hence the argument is correct.

Combining Rules of Inference for Propositions and Quantified Statements We have developed rules of inference both for propositions and for quantified statements. Note that in our arguments in Examples 15 and 16 we used both universal instantiation, a rule of inference for quantified statements, and modus ponens, a rule of inference for propositional logic. We will often need to use this combination of rules of inference. Because universal instantiation and modus ponens are used so often together, this combination of rules is sometimes called **universal modus ponens.** This rule tells us that if $\forall x(P(x) \to Q(x))$ is true, and if $P(a)$ is true for a particular element a in the domain of the universal quantifier, then $Q(a)$ must also be true. To see this, note that by universal instantiation, $P(a) \to Q(a)$ is true. Then, by modus ponens, $Q(a)$ must also be true. We can describe universal modus ponens as follows:

$$\begin{array}{l} \forall x(P(x) \to Q(x)) \\ \underline{P(a), \text{ where } a \text{ is a particular element in the domain}} \\ \therefore\ Q(a) \end{array}$$

Universal modus ponens is commonly used in mathematical arguments. This is illustrated in Example 17.

Example 17 *Assume that "For all positive integers n, if n is greater than 4, then n^2 is less than 2^n" is* true. Use universal modus ponens to show that $100^2 < 2^{100}$.

Solution Let $P(n)$ denote "$n > 4$" and $Q(n)$ denote "$n^2 < 2^n$." The statement "For all positive integers n, if n is greater than 4, then n^2 is less than 2^n" can be represented by $\forall n(p(n) \to Q(n))$, where the domain consists of all positive integers. We are assuming that $\forall n(p(n) \to Q(n))$ is true. Note that $P(100)$ is true because $100 > 4$. It follows by universal modus ponens that $Q(n)$ is true, namely that $100^2 < 2^{100}$ ◀

Another useful combination of a rule of inference from propositional logic and a rule of inference for quantified statements is **universal modus tollens.** Universal modus tollens combines universal instantiation and modus tollens and can be expressed in the following way:

$$\begin{array}{l} \forall x(P(x) \to Q(x)) \\ \underline{\neg Q(a), \text{where } a \text{ is a particular element in the domain}} \\ \therefore \neg P(a) \end{array}$$

We leave the verification of universal modus tollens to the reader (see Exercise 25). Exercise 26 develops additional combinations of rules of inference in propositional logic and quantified statements.

Exercises

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

   If Socrates is human, then Socrates is mortal.
   Socrates is human.
   ∴ Socrates is mortal.

2. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

   If George does not have eight legs, then he is not an insect.
   George is an insect.
   ∴ George has eight legs.

3. What rule of inference is used in each of these arguments?

   a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
   b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
   c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
   d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
   e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

4. What rule of inference is used in each of these arguments?

   a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

5. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

6. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

7. What rules of inference are used in this famous argument? "All men are mortal. So crates is a man. Therefore, So crates is mortal."

8. What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

9. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

b) "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

c) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

d) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

e) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

f) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

10. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."

b) "If I work, it is either sunny or partly sunny." "I worked last Monday or I worked last Friday." "It was not sunny on Tuesday." "It was not partly sunny on Friday."

c) "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies."

d) "Every student has an Internet account." "Homer does not have an Internet account." "Maggie has an Internet account."

e) "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat."

f) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road."

11. Show that the argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion $q \to r$ is valid if the argument form with premises $p_1, p_2, \ldots, p_n, q$, and conclusion r is valid.

12. Show that the argument form with premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, and $\neg s$ and conclusion $q \to r$ is valid by first using Exercise 11 and then using rules of inference from Table 1.

13. For each of these arguments, explain which rules of inference are used for each step.

a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

b) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

c) "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

d) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."

14. For each of these arguments, explain which rules of inference are used for each step.

a) "Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket."

b) "Each of five room mates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five room mates can take a course in algorithms next year."