Ques-5: Can a simple graph exist with 15 vertices each of degree five?

Answer: In graph theory, Handshaking theorem states in any given graph, sum of degree of all the vertices is twice the number of edges contained in it.

Let $G = (V, E)$ be an undirected graph with e edges. Then

$$2e = \sum_{u \in V} \deg(u)$$

The sum of the degree of the vertices

$5 . 15 = 75$ is odd

Therefore by handshaking theorem a simple graph with 15 vertices each of degree five cannot exist.

In exercises 7-9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

⑦ Answer:

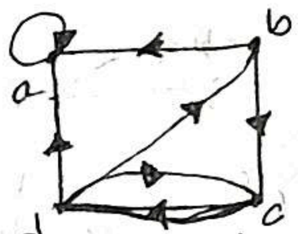


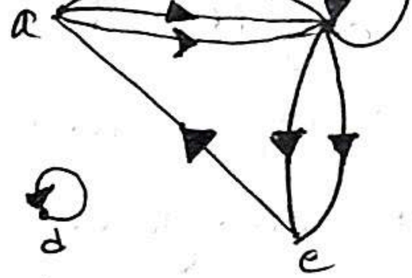
fig-7

vertices are 4. edges are 7.

The in-degree in fig-7 are $deg^-(a) = 3$, $deg^-(b) = 1$, $deg^-(c) = 2$, $deg^-(d) = 1$.

The out-degree in fig-7 are $deg^+(a) = 1$, $deg^+(b) = 2$, $deg^+(c) = 1$, $deg^+(d) = 3$

⑧ Answer:



vertices are 4. Edges are 8.

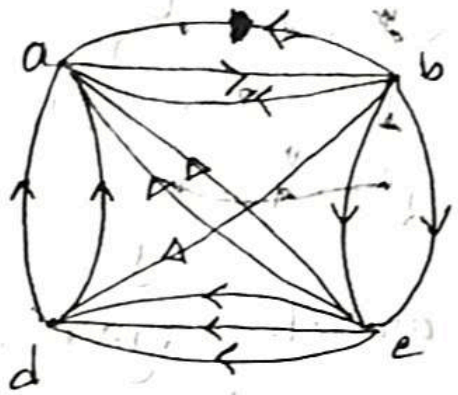
The in-degree in fig-8 are $deg^-(a) = 2$, $deg^-(b) = 3$,

$deg^-(c) = 2$, $deg^-(d) = 1$

The out-degree in fig-8 are $deg^+(a) = 2$, $deg^+(b) = 4$

$deg^+(c) = 1$, $deg^+(d) = 1$

(9) Answer:



• e

Vertices are 5. Edges are 13.

The in-degree ~~deg~~ in fig-8 are $deg^-(a) = 6$ $deg^-(b) = 1$

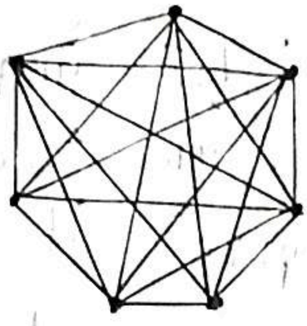
$deg^-(c) = 2$ $deg^-(d) = 4$ $deg^-(e) = 0$

The out-degree in fig-8 are $deg^+(a) = 1$

$deg^+(b) = 5$, $deg^+(c) = 5$, $deg^+(d) = 2$, $deg^+ e = 0$

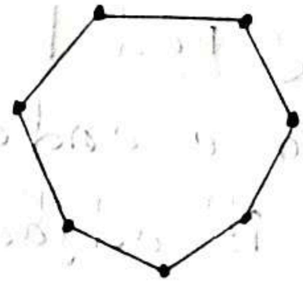
20| Draw these graphs

(a) $K_7 \Rightarrow$

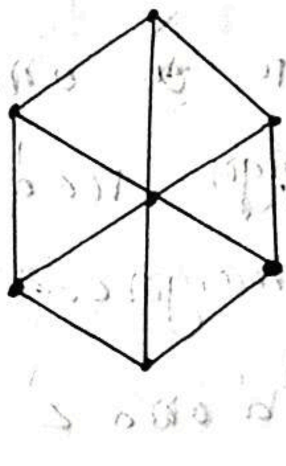


(f)

(d) $C_7 \Rightarrow$



(e) $W_7 \Rightarrow$



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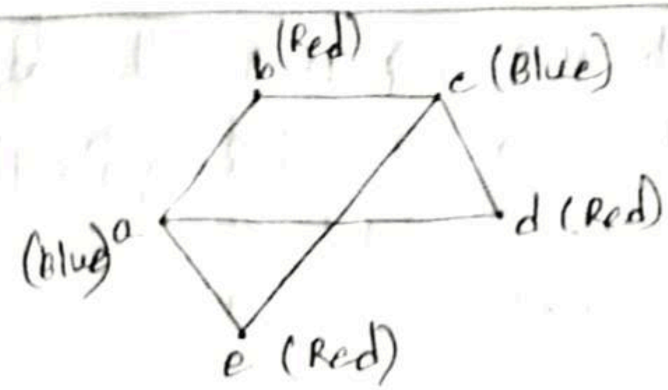
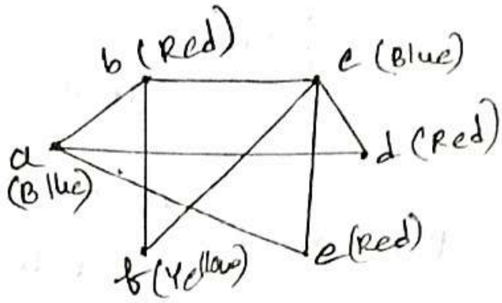


We first consider the graph. We will try to assign one of two colors, say red and blue, to each vertex in Graph so that no edge in the graph connects a red vertex and a blue vertex. Without loss of generality we begin by arbitarily assigning red to b. Then we must assign blue to a and c because each of these vertices is adjacent to b. To avoid having an edge with two blue end points, we must assign red to all the vertices adjacent to either a and c. This means that we must assign red to both b ~~and~~, d and e. We have now assigned colors to all vertices with b, d and e red and a, c blue. Checking all edges, We see that every edge connects a red vertex and a blue vertex. Hence, By theorem 4 the graph is bipartite.

23|



We first consider the graph. We will try to assign Red ~~and~~ or blue to each vertex in graph so that no edge in graph connects a red vertex and blue vertex. Without loss of generality we arbitarily assign red to b. Then we must assign blue to a, e, f and c because each is adjacent to b. But this is not possible because f and c are adjacent, so both cannot be assign blue. This argument shows that we cannot assign one of two colors to each of the vertices of the graph so that no adjacent vertices are assign the same color. Hence, by theorem 4 the graph is not bipartite.

21|

Answer : 19

Number of vertices = 5
Number of edges = 13

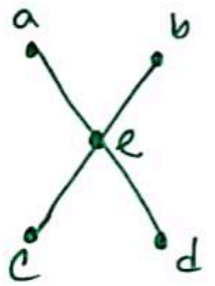
In degree of vertex, $deg^-(a) = 6$
In degree of vertex, $deg^-(b) = 1$
In degree of vertex, $deg^+(c) = 2$
In degree of vertex, $deg^+(d) = 4$
In degree of vertex, $deg^-(e) = 0$

Out degree of vertex, $deg^+(a) = 1$
Out degree of vertex, $deg^+(b) = 5$
Out degree of vertex, $deg^+(c) = 5$
Out degree of vertex, $deg^+(d) = 2$
Out degree of vertex, $deg^+(e) = 0$

Ref

In Exercise 21-25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either ~~Read~~ Red or blue to each vertex so that two adjacent vertex are assigned the same color.

21.



Ans: To show that this graph is bipartite, we can exhibit the parts and note that indeed every edges and joint vertices in different parts.

Take $\{e\}$ to be one part and $\{a, b, c,$

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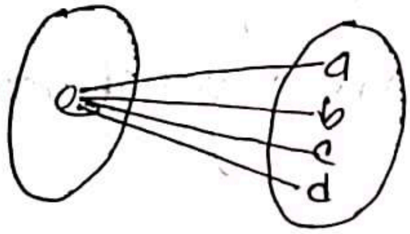
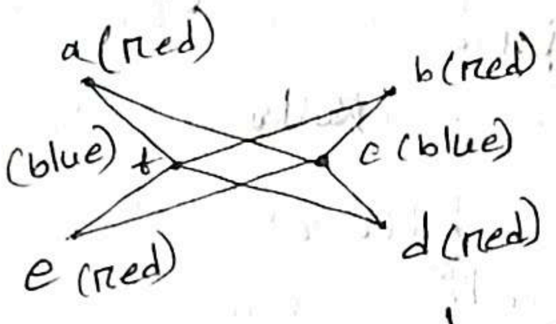
to be the other (in fact there is no choice in the matter. Each edges joins a vertex in one part to a vertex in the other. This graph is the complete bipartite graph $K_{1,4}$.

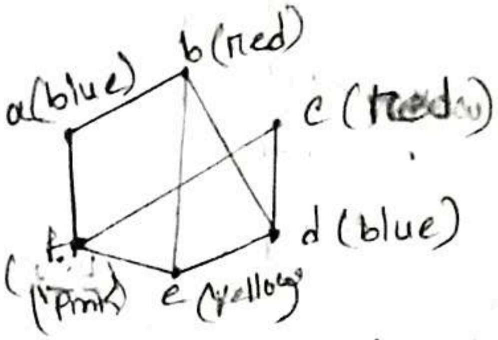


2(4)



At first we consider the graph. We will try to assign either red or blue to each vertex in graph so that no edge in the graph connects a red vertex and a blue vertex. Without loss of generality we arbitrarily assign red to a. Then we must assign f and c because each is adjacent to a. Then we must assign red to all vertices adjacent to either f and c. This means that we must assign red to a, b, d and e. We have now assigned colors to all vertices with a, b, d, e red and f, c blue. We see that every vertex connects a red vertex and a blue vertex. Hence, by theorem 4 the graph is bipartite.

(25)



We consider the graph. We will try to assign either red or blue to each vertex in graph so that no edge in graph connects a red vertex and a blue vertex. Without loss of generality we arbitarily assign red to b. Then we must assign blue to a, f, e, d. because each of this adjacent to b. But it is not possible because d, e and f are adjacent, so both cannot be assign blue. We assign e and f as ~~Pink~~ yellow and pink. This argument shows that we cannot assign one of two colors to each of the vertices of the graph so that no adjacent vertices are assigned the same color. Hence, by theorem 4 that the graph is not bipartite.

26| For which values of n are these graphs bipartite.

(a) $K_n \Rightarrow$

Answer: K_1 is bipartite if we allow one of the sets to be empty.

K_2 is bipartite because we can let one vertex be in V_1 and the other vertex to be in V_2.

~~(b) C_n~~ K_n for $n \geq 3$ is not bipartite. Choose any three vertices. They all are pair wise connected, therefore there is no way to partition them into two disjoint sets V_1 or V_2 such that there are no edges within V_1 and no edges within V_2.

That is, K_n is bipartite when $n < 3$.

(b) C_n

Answer: C is bipartite if and only if n is even. Label the vertices by $1, 2 \cdots$ consecutively along the cycle. If vertex 1 is in V_1 then vertex 2 must be in V_2, vertex 3 must be in V_1, vertex 4 must be in V_2 and so on. All vertices with odd number are in V_1 and all vertices with even number in V_2. The last vertex is in V_1 if n is odd and it is in V_2 if n is even. But it is connected to vertex 1. We see that if n is odd, the graph is not bipartite and if n is even, the graph is bipartite.

In summary, C_n is bipartite if and only if n is even.

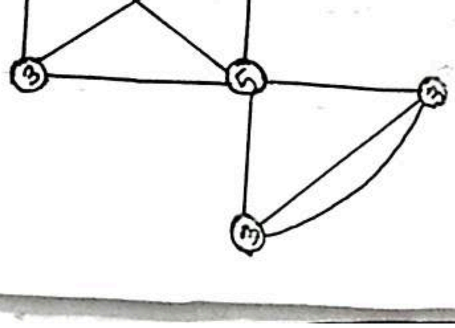
(d) Q_n

Answer: Q_n is bipartite for any n. Let V_1 consist of all vertices whose sum of coordinates is odd and let V_2 consist of all vertices whose sum of coordinates is even. Two vertices in Q_n are connected if and only if their coordinates differ in only one position. Therefore, the sum of their coordinates have different parity, so they are in different sets.

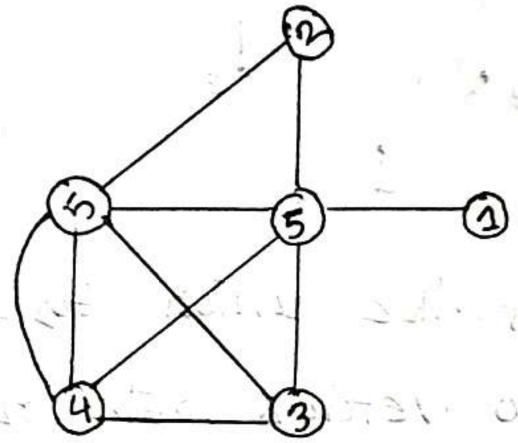
2.

36| Determine whether each of these sequence is graphic. For those that are, draw a graph having the given degree sequence.

(a) 5, 3, 3, 3, 3, 3

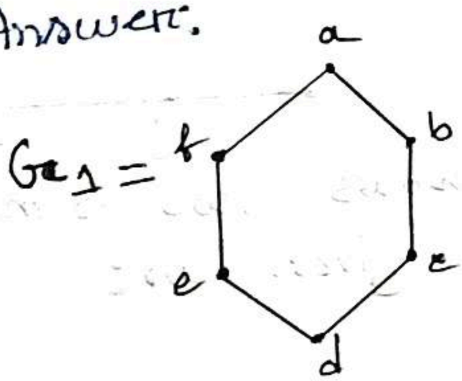


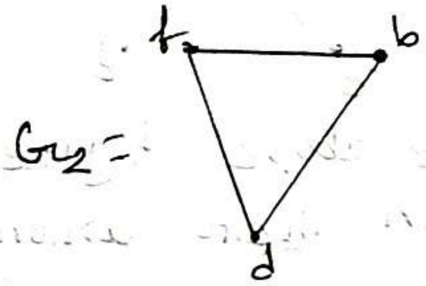
(h) 5, 5, 4, 3, 2, 1



In Exercise 50-52 find the union of the given pair of simple graph (Assume edges with the same endpoints are the same.

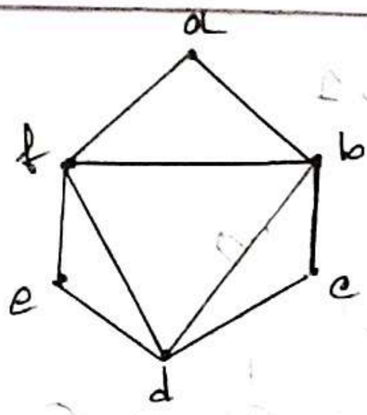
50| Answer.





The above figure shows two simple graphs. Then their union is given as:

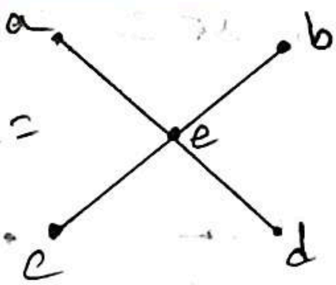
$G_1 \cup G_2 =$



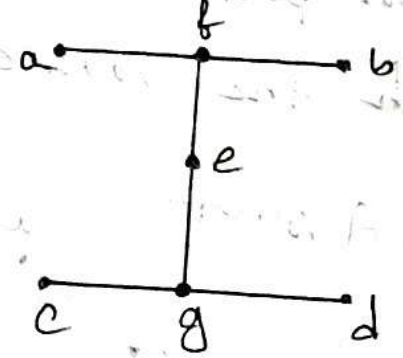
The vertex set of the union $G_1 \cup G_2$ is the union of the two vertex sets, namely $\{a, b, c, d, e, f\}$.

51 Answer:

$G_1 =$

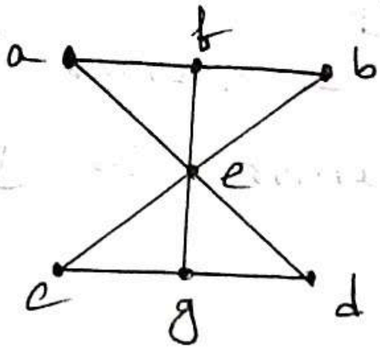


$G_2 =$



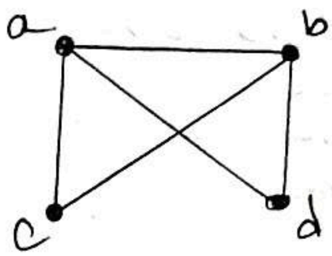
The above figure shows two simple graphs. Then their union is given as:

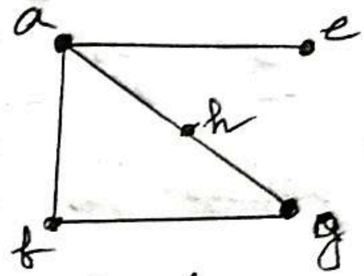
$G_1 \cup G_2 =$



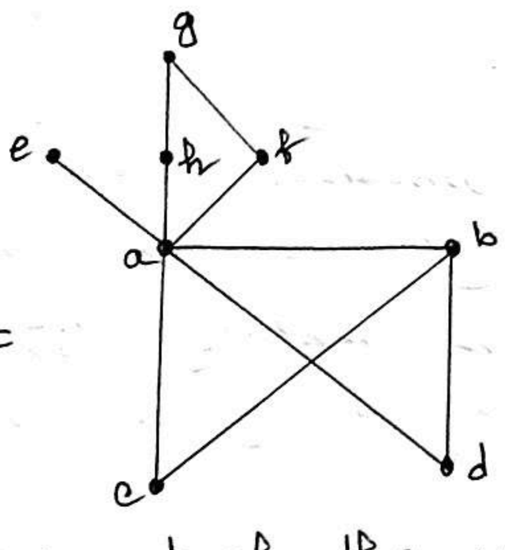
The vertex set of the union $G_1 \cup G_2$ is the union of the two vertex sets, namely $\{a, b, c, d, e, f, g\}$

(52) Answer:

$G_1 =$ 

$G_2 =$ 

The above graphs show two simple graphs. Then their union is shown as:

$G_1 \cup G_2 =$ 

The vertex set of the union $G_1 \cup G_2$ is the union of the two vertex sets, namely $\{a, b, c, d, e, f, g, h\}$