2.2. Show that the set of three elements $\{0,1,2\}$ and the two binary operators + and · as defined by the above table is not a Boolean algebra. State which of the Huntington postulates is not satisfied.

Answer: To show that the set of three elements $\{0,1,2\}$ with the binary operators + and · is not a Boolean algebra we need to demonstrate that it fails to satisfy one of the Huntington postulates. The Huntington postulates are:

1. Closure Axioms: Every operation in the alegbra must be closed on the set.

2. Identity Axioms: There exist two distinct elements, 0 and 1, such that for all x in the set, $x+0=x$ and $x \cdot 1=x$

3. Complement Axioms: For each element x in the set, there exists another element y such that $x+y=1$ and $x \cdot y=0$

4. Idempotent Axioms: For each element x in the set, $x+x=x$ and $x \cdot x=x$

5. Commutative Axioms: For all x and y in the set $x+y=y+x$ and $x \cdot y=y \cdot x$

6. Associative Axioms: For all x, y and z in the set, $(x+y)+z=x+(y+z)$ and $(x \cdot y) \cdot z=x \cdot (y \cdot z)$

7. Distributive Axioms: For all x, y and z in the set,

$x+(y \cdot z) = (x+y) \cdot (x+z)$ and $x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

Let's examine whether the given set and operation satisfy these axioms:

Set: $\{0,1,2\}$

Operators: + (addition) and · (multiplication)

$0+1 = 1 \in \{0,1,2\}$

$0 \cdot 1 = 0 \in \{0,1,2\}$

$1+2 = 3 \notin \{0,1,2\}$ (since 3 is not in the set)

$1 \cdot 2 = 2 \in \{0,1,2\}$

$2+2 = 4 \notin \{0,1,2\}$ (since 4 is not in the set)

$2 \cdot 2 = 4 \notin \{0,1,2\}$ (since 4 is not in the set)

The closure axioms fails for addition and multiplication because some of the results fall outside the set.

Since the closure axioms is not satisfied, this set with the given operators is not a Boolean algebra.

2.3. Demonstrate by means of truth tables the validity of the following theorems of Boolean algebra.

a. The associative laws.

b. De Morgan's theorems for three variables.

c. The distributive law of + over.

Answer:

(a)

The associative laws:

The associative law state that for any three variables A, B and C the following expression is true:

$$(A+B)+C = A+(B+C)$$

To demonstrate this, let's create a truth table for both side of the equations:

A	B	C	(A+B)+C	A+(B+C)
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

(b)

De Morgans Theorem for three variables:

De Morgans theorem states that for three variables A, B and C, the following expressions are true:

1. ~(A+B+C) = ~A*~B*~C
2. ~(A*B*C) = ~A+~B+~C

Lets create a truth table to demonstrate the validity of these expression:

A	B	C	~(A+B+C)	~A*~B*~C	~(A*B*C)	(~A+~B+~C
0	0	0	1	1	1	1
0	0	1	0	0	1	1
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	0	1	1
1	0	1	0	0	1	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

(C)

The distributive law of "+" over the Boolean operator "·" states that for any Boolean variables, A, B and C.

$A+(B\cdot C)=(A+B)\cdot(A+C)$

Let's create a truth table with columns for A, B, C; $A+(B*C)$, $(A+B)\cdot(A+C)$ and compare their values.

A	B	C	A+(B*C)	(A+B)*(A+C)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

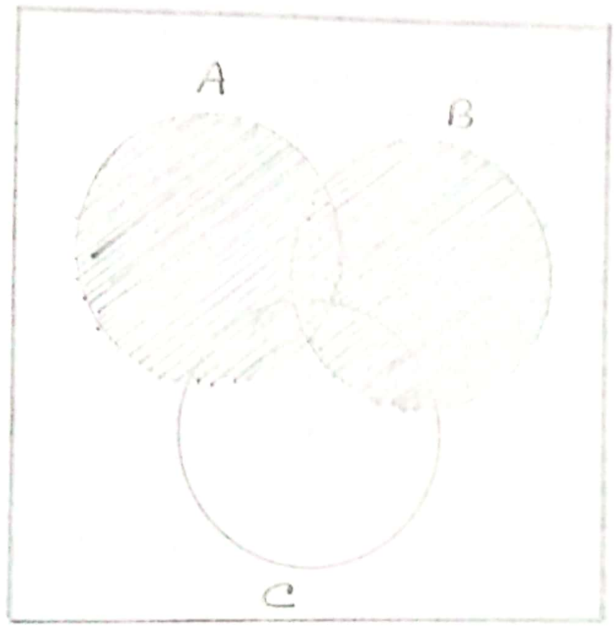
2-4. Repeat problem 2-3 using venn diagrams.

Answer:

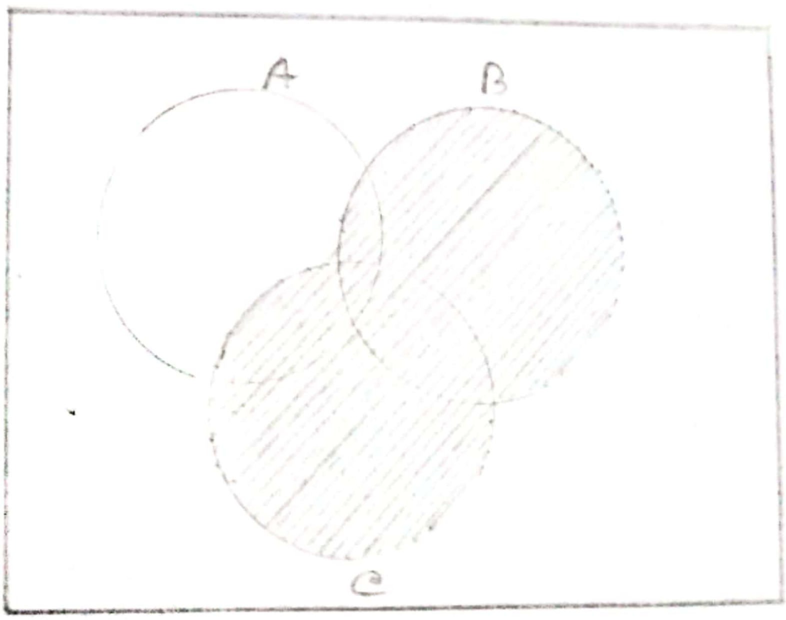
(a)

The associative law; $(A+B)+C = A+(B+C)$

Venn diagram for associative law;



$(A+B)+C$



$A+(B+C)$

2.5. Simplify the following Boolean functions to a minimum number of literals.

Answer:

(a)

$xy + xy'$

Ans: $xy + xy'$

$= x(y+y')$

$= x \cdot 1$

$= x$

(b)

$(x+y)(x+y')$

Ans: $(x+y)(x+y')$

$= x \cdot x + xy' + xy + yy'$

$= x + x(y'+y) + 0$

$= x + x \cdot 1$

$= x + x$

$= x$

(c)

$xyz + x'y + xyz'$

Ans: $xyz + x'y + xyz'$

$= xy(z+z') + x'y$

$= xy \cdot 1 + x'y$

$= xy + x'y$

$= (x+x')y$

$= 1 \cdot y$

$= y$

(d)

$zx + zx'y$

Ans: $zx + zx'y$

$= zx(1+y) + zx'y$

$= zx + zxy + zx'y$

$= zx + zy(x+x')$

$= zx + zy \cdot 1$

$= zx + zy$

$= z(x+y)$

(e)

$(A+B)'(A'+B')'$

Ans: $(A+B)'(A'+B')'$

$= (A+B)'(A')'(B')'$

$= A' \cdot B' \cdot A \cdot B$

$= A'A \cdot B'B$

$= 0 \cdot 0$

$= 0$

(F)

$y(wz' + wz) + xy$

Ans: $y(wz' + wz) + xy$

$= wyz' + wyz + xy$

$= wy(z' + z) + xy$

$= wy \cdot 1 + xy$

$= wy + xy$

$= y(w+x)$

$= y(x+w)$

2.6. Reduce the following Boolean expression to the required number of literals.

Answer:

(a)

$$ABC + A'B'C + A'BC + ABC' + A'B'C'$$

$$= A'B'C' + A'B'C + A'BC + ABC + ABC + ABC'$$

$$= A'B'(C'+C) + BC(A+A') + AB(C+C')$$

$$= A'B' + BC + AB$$

$$= A'B' + B(C+A)$$

A \ BC	B'C'	B'C	BC	BC'
A'	1	1	1	
A			1	1

$$F = A'B' + BC + AB$$

$$= A'B' + B(C+A)$$

(b)

$BC + AC' + AB + BCD$ to four literals.

$= BC(1+D) + AC' + AB$

$= BC + AC' + AB(C+C')$

$= BC + AC' + ABC + ABC'$

$= BC(1+A) + AC'(1+B)$

$= BC + AC'$

(C)

$[(CD)' + A]' + A + CD + AB$ to three literals.

$= [C' + D' + A]' + A + CD + AB$

$= A'CD + A + CD + AB$

$= CD(1+A') + A(1+B)$

$= CD + A$

(D)

$(A+C+D)(A+C+D')(A+C'+D)(A+B')$ to four literals.

$= (A + AC + AD' + AC + C + CD' + AD + CD + DD')(A+C'+D)(A+$

$= \{A(1+C+D'+C+D) + C(1+D'+D)\}(A+C'+D)(A+B')$

$= (A+C)(A+C'+D)(A+B')$

$= (A + AC' + AD + AC + AC' + CD)(A+B')$

$= \{A(1+C'+D+C+C) + CD\}(A+B')$

$= (A+CD)(A+B')$

$= A + AB' + ACD + B'CD$

$A(1+B'+CD) + B'CD = A + B'CD$