****Simpson’s rule****

****Simpson’s rule****is one of the numerical methods which is used to evaluate the definite integral. Usually, to find the definite integral, we use the fundamental theorem of calculus, where we have to apply the antiderivative techniques of integration. However, sometimes, it isn’t easy to find the antiderivative of an integral, like in Scientific Experiments, where the function has to be determined from the observed readings. Therefore, numerical methods are used to approximate the integral in such conditions. Other numerical methods used are[trapezoidal rule](https://byjus.com/maths/trapezoidal-rule/), midpoint rule, left or right approximation using Riemann sums. Here, we will discuss Simpson’s rule formula, 1/3 rule, 3/8 rule and examples.

****Table of Contents:****

* [Formula](https://byjus.com/" \l "formula)
* [Simpson’s 1/3 Rule](https://byjus.com/" \l "simpsons-1/3-rule)
  + [1/3 Rule for Integration](https://byjus.com/" \l "1/3-rule-for-integration)
* [Simpson’s 3/8 Rule](https://byjus.com/" \l "simpsons-3/8-rule)
* [Error](https://byjus.com/" \l "error)
* [Example](https://byjus.com/" \l "example)

# Simpson’s Rule Formula

Simpson’s rule methods are more accurate than the other numerical approximations and its formula for n+1 equally spaced subdivision is given by;



Where n is the even number, △x = (b – a)/n and xi = a + i△x

If we have f(x) = y, which is equally spaced between [a, b] and if a = x0, x1 = x0 + h, x2 = x0 + 2h …., xn = x0 + nh, where h is the difference between the terms. Or we can say that y0 = f(x0), y1 = f(x1), y2 = f(x2),……,yn = f(xn) are the analogous values of y with each value of x.

## ****Simpson’s 1/3 Rule****

Simpson’s 1/3rd rule is an extension of the trapezoidal rule in which the integrand is approximated by a second-order polynomial. Simpson rule can be derived from the various way using Newton’s divided difference polynomial,  Lagrange polynomial and the method of coefficients. Simpson’s 1/3 rule is defined by:

|  |
| --- |
| ∫ab f(x) dx = h/3 [(y0 + yn) + 4(y1 + y3 + y5 + …. + yn-1) + 2(y2 + y4 + y6 + ….. + yn-2)] |

This rule is known as Simpson’s****One-third rule****.

### ****Simpson’s ⅓ Rule for Integration****

We can get a quick approximation for definite integrals when we divide a small interval [a, b] into two parts. Therefore, after dividing the interval, we get;

x0= a, x1= a + b, x2 = b

Hence, we can write the approximation as;

∫ab f(x) dx ≈ S2 = h/3[f(x0) + 4f(x1) + f(x2)]

S2 = h/3 [f(a) + 4 f((a+b)/2) + f(b)]

Where h = (b – a)/2

This is the Simpson’s ⅓ rule for integration.

## **Simpson’s 3/8 Rule**

Another method of numerical integration is called “Simpson’s 3/8 rule”. It is completely based on the cubic interpolation rather than the quadratic interpolation. Simpson’s 3/8 or three-eight rule is given by:

|  |
| --- |
| ∫ab f(x) dx = 3h/8 [(y0 + yn) + 3(y1 + y2 + y4 + y5 + …. + yn-1) + 2(y3 + y6 + y9 + ….. + yn-3)] |

This rule is more accurate than the standard method, as it uses one more functional value. For 3/8 rule, the composite Simpson’s 3/8 rule also exists which is similar to the generalized form. The 3/8 rule is known as Simpson’s second rule of integration.

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## **Simpson’s Rule Error**

Although in Simpson’s rule method we get a more accurate approximation for definite integral, still the error occurs which is defined when n = 2;

-(1/90)[(b-a)/2]5 f(4) (ξ)

Where ξ is some number between a and b.

## **Simpson’s Rule Example**

****Example: Evaluate ∫01exdx, by Simpson’s ⅓ rule.****

****Solution:****

Let us divide the range [0, 1] into six equal parts by taking h = 1/6.

If x0 = 0 then y0 = e0 = 1.

If x1 = x0 + h = ⅙, then y1 = e1/6 = 1.1813

If x2 = x0 + 2h = 2/6 = 1/3 then, y2 = e1/3 = 1.3956

If x3 = x0 + 3h = 3/6 = ½ then y3 = e1/2= 1.6487

If x4 = x0 + 4h = 4/6 ⅔ then y4 = e2/3 = 1.9477

If x5 = x0 + 5h = ⅚ then y5 = e5/6 = 2.3009

If x6 = x0 + 6h = 6/6 = 1 then y6 = e1 = 2.7182

We know by Simpson’s ⅓ rule;

∫ab f(x) dx = h/3 [(y0 + yn) + 4(y1 + y3 + y5 + …. + yn-1) + 2(y2 + y4 + y6 + ….. + yn-2)]

Therefore,

∫01exdx = (1/18) [(1 + 2.7182) + 4(1.1813 + 1.6487 + 2.3009) + 2(1.39561 + 1.9477)]

=  (1/18)[3.7182 + 20.5236 + 6.68662]

= 1.7182 (approx.)

**Reference:**

https://byjus.com/maths/simpsons-rule/