Finite Thrust Planar transfer of Spacecraft between Heliocentric Orbits

Abstract:

An optimal orbital transfer is developed for a finite thrust planar transfer from a heliocentric circular orbit of radius $r_o = 1$ to a heliocentric circular orbit of radius $r_f = 1.5$. The optimal orbital transfer problem is modelled using a two-point boundary value problem (TPBVP). To minimize the fuel consumption, or what is the same as the final time, the TPBVP is solved using the shooting methods. The optimal control problem is formulated by deriving the dynamics of the given system using the Newton's second law for a particle and Lagrange's equations. The second order differential equations obtained from the above methods is rewritten into a system of four first order equations. System is solved for Optimal control using Direct and Indirect, Single and Multiple shooting techniques and the results are compared.

Given Data:

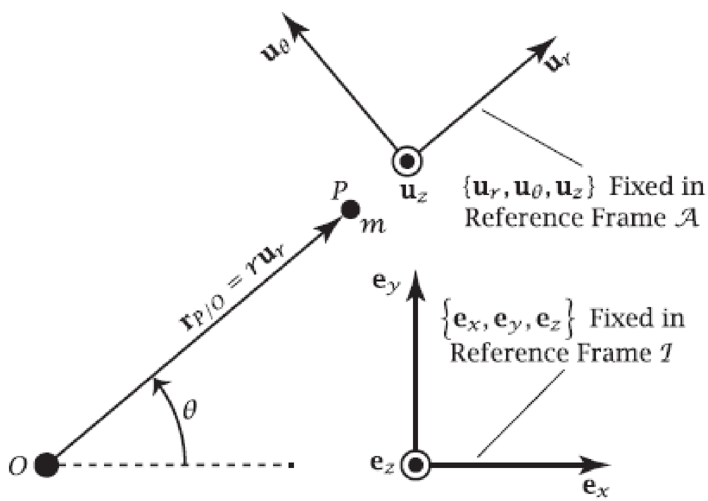


Figure 1: Schematic of particle moving in an inertially fixed plane.

Mass of spacecraft = m.

Position of spacecraft relative to sun $\boldsymbol{r}_{P/O} = r\boldsymbol{u}_r$.

$\{\boldsymbol{u}_r, \boldsymbol{u}_\theta, \boldsymbol{u}_z\}$ is obtained by rotating fixed reference frame by an angle θ about axis $\boldsymbol{e}_z$.

Forces acting on the spacecraft

1. Gravitational Force $$G = -m\mu \frac{\mathbf{r}_{P/O}}{\|\mathbf{r}_{P/O}\|^3}. \quad (1)$$
2. Thrust Force $$\mathbf{T} = T\mathbf{w}. \quad (2)$$

Where **w** is the unit vector that lies at an angle β from the direction of $\boldsymbol{u}_\theta$.

Derivation of Second Order Differential equations using:

A) Newton's Second law for a particle.

Kinematics of Particle

We know that $$r_{P/O} = r\boldsymbol{u}_r \tag{3}$$

Velocity of particle in the Inertial Reference frame is given by

$$^{I}V_P = \frac{^{I}\mathrm{d}r_{P/O}}{\mathrm{d}t} = \frac{^{A}\mathrm{d}r_{P/O}}{\mathrm{d}t} + {}^{I}\omega^{A} \times r_{P/O}$$

$$= \dot{r}.\boldsymbol{u}_r + \dot{\theta}.\boldsymbol{u}_z \times r.\boldsymbol{u}_r$$

$$^{I}V_P = \dot{r}.\boldsymbol{u}_r + r.\dot{\theta}.\boldsymbol{u}_\theta \tag{4}$$

Acceleration of particle in Inertial Frame:

$$^{I}a_P = \frac{^{I}d\left({}^{I}V_P\right)}{dt} = \frac{^{A}d\left({}^{I}V_P\right)}{dt} + {}^{I}\omega^{A} \times {}^{I}V_P$$

$$= \left(\ddot{r} - r.\dot{\theta}^2\right)\boldsymbol{u}_r + \left(2.\dot{r}.\dot{\theta} + r.\ddot{\theta}\right)\boldsymbol{u}_\theta \tag{5}$$

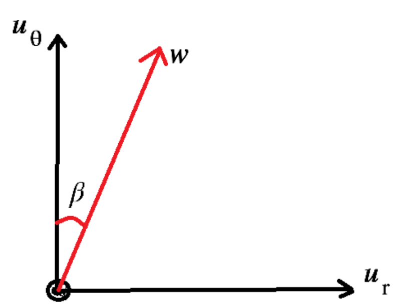


Fig 2 showing the unit vector **w** in reference frame **A**

From Fig 2

$$w = \boldsymbol{u}_r.\sin\beta + \boldsymbol{u}_\theta.\cos\beta$$
$$T = T(\,\mathbf{u}_r.\sin\beta + \mathbf{u}_\theta.\cos\beta) \tag{6}$$

From Newton's second law of motion

We know that $\mathbf{F} = \mathbf{m*a}$

Total Forces acting on the spacecraft are $\mathbf{G} + \mathbf{T}$

$$\mathbf{G} + \mathbf{T} = -m\mu \frac{\boldsymbol{r}_{P/O}}{\|\mathbf{r}_{P/O}\|^3} + T\left(\boldsymbol{u}_r.\sin\beta + \boldsymbol{u}_\theta.\cos\beta\right) \tag{7}$$

$$m*\mathbf{a} = m*\left(\left(\ddot{r} - r.\dot{\theta}^2\right)\mathbf{u}_r + \left(2.\dot{r}.\dot{\theta} + r.\ddot{\theta}\right)\mathbf{u}_\theta\right) \tag{8}$$

By Equating equation (7) and (8)

$$-m\mu \cdot \frac{\boldsymbol{u}_r}{r^2} + T\cdot\sin\beta\ .\ \boldsymbol{u}_r + T\cos\beta\ .\ \boldsymbol{u}_\theta = m\cdot\left(\ddot{r} - r.\dot{\theta}^2\right)\boldsymbol{u}_r + m.\left(2.\dot{r}.\ \dot{\theta} + r.\ddot{\theta}\right)\boldsymbol{u}_\theta \tag{9}$$

By projecting equation (9) onto $\boldsymbol{u}_r$ *and* $\boldsymbol{u}_\theta$ and simplifying we get below equation respectively.

$$\ddot{r} = -\frac{\mu}{r^2} + \frac{T}{m}\cdot\sin\beta + r.\dot{\theta}^2 \tag{10}$$

$$2.\dot{r}.\dot{\theta} + r.\ddot{\theta} = \frac{T}{m}\cdot\cos\beta \tag{11}$$

Equation 10 and 11 are the two second order differential equations obtained from the Newton's second law.

B) Lagrange's Equation

From Given Data:

Points to be considered While applying Lagrange's equation are

Generalized co-ordinates are r and Θ

Lagrange's Equation is given by $L = T - U$

$$T = \frac{1}{2}m\cdot {}^{I}\mathbf{V}_P\ .\ {}^{I}\mathbf{V}_P \tag{12}$$

$$U = -\frac{m\mu}{r} \tag{13}$$

Substituting equation (4) in (12) we get

$$T = \frac{1}{2}\cdot m\cdot\left(\dot{r}.\mathbf{u}_r + r\cdot\dot{\theta}\cdot\mathbf{u}_\theta\right).\left(\dot{r}.\mathbf{u}_r + r\cdot\dot{\theta}\cdot\mathbf{u}_\theta\right)$$

$$T = \frac{m}{2}\left(\dot{r}^2 + \left(r.\dot{\theta}\right)^2\right)$$

$$L = T - U = \frac{m}{2}\left(\dot{r}^2 + \left(r\dot{\theta}\right)^2\right) - \frac{m\mu}{r};$$ **(14)**

By LaGrange Mechanics

$$\frac{\mathrm{d}\dfrac{\partial L}{\partial \dot{r}}}{\mathrm{d}t} - \frac{\partial L}{\partial r} = Q_o{}'$$ **(15A)**

$$\frac{\mathrm{d}\dfrac{\partial L}{\partial \dot{\theta}}}{\mathrm{d}t} - \frac{\partial L}{\partial \theta} = Q_o{}'$$ **(15B)**

Now solving for Equation **15A**

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 + \frac{m\mu}{r^2};$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r};$$

$$\frac{\mathrm{d}\dfrac{\partial L}{\partial \dot{r}}}{\mathrm{d}t} = m\ddot{r};$$

substituting in (15A) we get

$$Q_0{}' = \boldsymbol{T} \cdot \frac{\mathbf{dr}}{\mathrm{dr}} = \boldsymbol{T}.\boldsymbol{u}_{\mathrm{r}} = (\mathrm{T}\sin\beta.\mathbf{u}_{\mathrm{r}} + \mathrm{T}\cos\beta.\,\mathbf{u}_{\theta}).\mathbf{u}_{\mathrm{r}} = \mathrm{T}\sin\beta$$

$$m\ddot{r} - mr.\dot{\theta}^2 - \frac{m\mu}{r^2} = T\sin\beta;$$ **(16A)**

$$\ddot{r} = \frac{T\cdot\sin\beta}{m} + r.\dot{\theta}^2 - \frac{\mu}{r^2};$$

Now solving for equation **15B**

$$\frac{\partial L}{\partial \theta} = 0;$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{m\,2.r.r.\dot{\theta}}{2} = m.r^2.\dot{\theta};$$

$$\frac{\mathrm{d}\dfrac{\partial L}{\partial \dot{\theta}}}{\mathrm{d}t} = 2.m.r.\dot{r}.\dot{\theta} + m.\,r^2.\,\ddot{\theta}\,;$$

$$Q_0{}' = \boldsymbol{T}.\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\theta} = \boldsymbol{T}.r.\frac{d\boldsymbol{u}_{\mathrm{r}}}{d\theta} = \boldsymbol{T}.r.\boldsymbol{u}_{\theta} = T.r.\cos\beta;$$

By substituting these in 15B We get

$$2.\dot{r}.\dot{\theta} + \left(r.\ddot{\theta}\right) = \frac{T}{m}\cdot\cos\beta; \qquad \textbf{(16B)}$$

It is evident that Equation **(10) = (16A) and (11) = (16B)**

Therefore, the second order differential equations obtained using Both newton's second law and Lagrange's equation are same and are the final equations of motion.

Converting the 2nd order differential equations into 1st order equations
By substitute

$$\dot{r} = v_r; \qquad \textbf{(17)}$$
$$\dot{\theta} = \frac{v_0}{r};$$

Differentiating above equation (17) gives

$$\dot{v}_r = \ddot{r};$$
$$\dot{v}_\theta = \dot{r}.\dot{\theta} + r\,\ddot{\theta}; \qquad (18)$$

By substituting them into second order differential equations we get

From equation (10)

$$\dot{v}_r = -\frac{\mu}{r^2} + \frac{T}{m}\sin\beta + \frac{v_\theta^{\,2}}{r};$$
$$\dot{v}_\theta = \frac{T}{m}\cos\beta - \frac{v_r.v_\theta}{r}; \qquad (19)$$

Mass flow rate of the system is given as

$$\dot{m} = -\frac{T}{v_e}; \qquad (20)$$

Where v_e = Exhaust speed of Engine

Now we have 5 first order differential equations defining the unstable system.

Formulation of Optimal control Problem:

The spacecraft shown as point mass P in the figure 1 is in the initially rotating in heliocentric circular orbit $r_0 = 1$ is to be terminated into the heliocentric circular orbit of radius $r_f = 1.5$. Assume that the motion is planar, the aerodynamic forces are negligible, and the thrust magnitude is constant. The control variable is the thrust direction $\beta(t)$, which is measured from the local vertical $\boldsymbol{u}_\theta$. The mass of the spacecraft is m. The gravitational force exerted on the vehicle is $\boldsymbol{G} = -\frac{m\mu\,\mathbf{r}_{P/O}}{||\mathbf{r}_{P/O}||^3}$;

Optimize the time required to transfer the space craft from heliocentric circular orbit of radius 1 to radius 1.5. Which is same as the maximize the mass of the spacecraft as the thrust is constant.

The objective function is given as $J = \int_{t_0}^{t_f} 1 \; dt$ or $J = -\int_{m_0}^{m_f} 1 \, dm$ subjected to constrains

$$\dot{r} = v_r;$$

$$\dot{\theta} = \frac{v_\theta}{r};$$

$$\dot{v}_r = -\frac{\mu}{r^2} + \frac{T}{m}\sin\beta + \frac{{v_\theta}^2}{r} \;;$$

$$\dot{v}_\theta = \frac{T}{m}\cos\beta - \frac{v_r . v_\theta}{r} \;;$$

$$\dot{m} = -\frac{T}{v_e};$$

state equations ………… **(21)**

Subjected to boundary conditions

$$\begin{aligned}
&r_0 = 1; && r_f = 1.5; \\
&\theta_0 = 0; && \theta_f = free; \\
&m_0 = 1; && m_f = free; \\
&v_{r_0} = 0; && v_{r_f} = 0; \;\; \textit{to acheive desired energy orbit radial velocity needs to be zero.} \\
&\textit{initial time } t_0 = 0; && \textit{final time is free}
\end{aligned}$$

(22)

Also, for a spacecraft to be in orbit the gravitational force should be equal to the centrifugal force acting on the spacecraft.

i.e. centrifugal force acting on the space craft is given by equation

$$F_c = m.r.\dot{\theta}^2 = \frac{m.{v_\theta}^2}{r}; \qquad (23)$$

Gravitational force acting on the spacecraft is given by

$$\boldsymbol{G} = -\frac{m\mu \, \mathbf{r}_{P/O}}{\|\mathbf{r}_{P/O}\|^3} = m \cdot \frac{\mu}{r^2} \;; \qquad (24)$$

By equating and simplifying the equation (23) and (24)

$$v_\theta = \sqrt{\frac{\mu}{r}};$$

$$v_{\theta_0} = \sqrt{\frac{\mu}{r_0}} \;; \qquad (25)$$

$$v_{\theta_f} = \sqrt{\frac{\mu}{r_f}} \;;$$

Also given the values of constants are as follows

$$T = 0.1405$$

$$\mu = 1$$

$$v_e = 1.8758344$$

Let $\lambda_r(t)$, $\lambda_\theta(t)$, $\lambda_{v_r}(t)$, $\lambda_{v_\theta}(t)$, $\lambda_m(t)$ be the LaGrange multipliers for the states $r,\ \theta,\ v_r,\ v_\theta,\ m$ respectively.

Then the Hamiltonian can is given as

$$H = 1 + \lambda_r^T(v_r) + \lambda_\theta^T\left(\frac{v_\theta}{r}\right) + \lambda_{v_r}\left(-\frac{\mu}{r^2} + \frac{T}{m}\sin\beta + \frac{v_\theta^2}{2}\right)^T + \lambda_{v_\theta}^T\left(\frac{T}{m}\cos\beta - \frac{v_r.v_\theta}{r}\right) + \lambda_m^T\left(-\frac{T}{v_e}\right); \quad (26)$$

Deriving first order optimality conditions:

Necessary conditions to satisfy the for a solution to be optimal is (ignoring to write * for simplicity)

$$\frac{\partial H}{\partial \boldsymbol{u}} = 0;$$

$$\frac{\partial H}{\partial \beta} = \frac{T}{m}\left(\lambda_{v_r}\cos\beta - \lambda_{v_\theta}\sin\beta\right) = 0; \quad (27)$$

$$\tan\beta = \frac{\lambda_{v_r}}{\lambda_{v_\theta}};$$

Deriving the co-state equation:

$$\frac{\partial H}{\partial \boldsymbol{x}} = -\dot{\lambda}_x^T;$$

Applying for various states in our problem:

$$\frac{\partial H}{\partial r} = -\dot{\lambda}_r^T = -\frac{\lambda_\theta \cdot v_\theta}{r^2} + \lambda_{v_r}\left(\frac{2.\mu}{r^3} - \frac{v_\theta^2}{r^2}\right) + \lambda_{v_\theta} \cdot \frac{v_\theta \cdot v_r}{r^2}$$

$$\dot{\lambda}_r^T = \frac{\lambda_\theta \cdot v_\theta}{r^2} - \lambda_{v_r}\left(\frac{2.\mu}{r^3} - \frac{v_\theta^2}{r^2}\right) - \lambda_{v_\theta} \cdot \frac{v_\theta \cdot v_r}{r^2}; \quad (28)$$

Similarly, the other state equations are computed and are simplified to get as follows:

$$\dot{\lambda}_{\theta} = 0;$$

$$\dot{\lambda}_{v_r} = -\lambda_r + \lambda_{v_\theta} \cdot \frac{v_\theta}{r};$$

$$\dot{\lambda}_{v_\theta} = -\frac{\lambda_\theta}{r} - 2 \cdot \lambda_{v_r} \cdot \frac{v_\theta}{r} + \lambda_{v_\theta} \cdot \frac{v_r}{r}; \tag{29}$$

$$\dot{\lambda}_m = \frac{T}{m^2}\left(\lambda_{v_r} \sin\beta + \lambda_{v_\theta} \cos\beta\right);$$

As the values of final time, final mass and terminal longitude are free transversality conditions are applied at those variables.

Transversality condition at the δt_f

$$H_{t_f} + \frac{\partial M}{\partial t_f} - \boldsymbol{v}^{\mathrm{T}} . \frac{\partial b}{\partial t_f} = \mathbf{0};$$

$$M = \left(m_0 - m_f\right);$$

$$\frac{\partial b}{\partial t_f} = 0; \tag{30}$$

$$H_{t_f} = 0;$$

Transversality condition on Final mass

$$\lambda_{m_f}{}^T + \frac{\partial M}{\partial m_f} - \boldsymbol{V}^{\mathrm{T}} \frac{\partial b}{\partial t_f} = \mathbf{0};$$

$$M = \left(m_0 - m_f\right);$$

$$\frac{\partial b}{\partial t_f} = 0; \tag{31}$$

$$\lambda_{m_f}{}^T - 1 = 0;$$

Transversality condition on Terminal Longitude

$$\lambda_{m_\theta}{}^T + \frac{\partial M}{\partial m_f} - \boldsymbol{V}^{\mathrm{T}} \frac{\partial b}{\partial t_f} = \mathbf{0};$$

$$M = \left(m_0 - m_f\right); \ \frac{\partial b}{\partial t_f} = 0; \tag{32}$$

$$\lambda_{m_\theta}{}^T = 0;$$

Now we have 5 state differential equations **(17), (19), (20)** and 5 co-state differential equations **(28), (29)** to be solved. To solve 10 differential equations, we need 10 boundary conditions, but we have only 5 initial boundary conditions states in equation **(22), (25).** The other five boundary conditions are known at the final time according to the desired final characteristics of the space craft. To achieve the desired energy orbit, the radial velocity of the vehicle needs to be zero.

Terminal state constraint provides only 3 of the 5 necessary conditions and remaining are obtained using the transversality conditions.

The terminal state matrix is defined as follows

$$\begin{pmatrix} r(t_f) - r_f; \\ v_r(t_f)\ ; \\ v_\theta(t_f) - \sqrt{\dfrac{\mu}{r_f}}; \\ H(t_f)\ ; \\ \lambda_m(t_f) - 1\ ; \\ \lambda_\theta(t_f); \end{pmatrix} = 0; \qquad \textbf{(33)}$$

The TPBVP is stated from the state equation (17), (19), (20) and co-state equations (28), (29) as equations to be solved and Initial boundary value as equations (22), (25) and final boundary values as equation (33).

Numerical solution of optimal control problem:

The optimal control problem is first solved using the Indirect shooting methods.

Steps Involved in indirect single shooting method:

1. Form the Meta state vector: $Z = \begin{bmatrix} X \\ \lambda \end{bmatrix}$; and from the meta state vector obtain the Error Deviation Vector $dZ = \begin{bmatrix} dX \\ d\lambda \end{bmatrix}$.
2. The initial co-state values are to be guesses and the initial boundary value vector consisting of both state and co-state values is formed as follows: $Z(t_0) = \begin{bmatrix} X(t_0) \\ \lambda(t_0) \end{bmatrix}$.
3. Also produce the guess for final time.
4. Now numerically integrate the state and co-state differential equations using the MATLAB’s ODE113 solver.
5. The integrated state and co-state vectors are obtained after solving. If the final state meta state matrix is satisfied those are acceptable path of the state and co-state vectors.

6. Then once the initial values of state and co-state are obtained those values are introduced into the fsolve (a non-linear equation solver) to solve the error function and the final values of the co-states are generated.
7. Using those values optimal control is obtained from the equation $\beta = a\tan2\left(\lambda_{v_r}, \lambda_{v_\theta} \right)$;

**** Code for the Indirect single shooting is attached to this document in the Appendix-1.**

Results for Indirect single shooting:

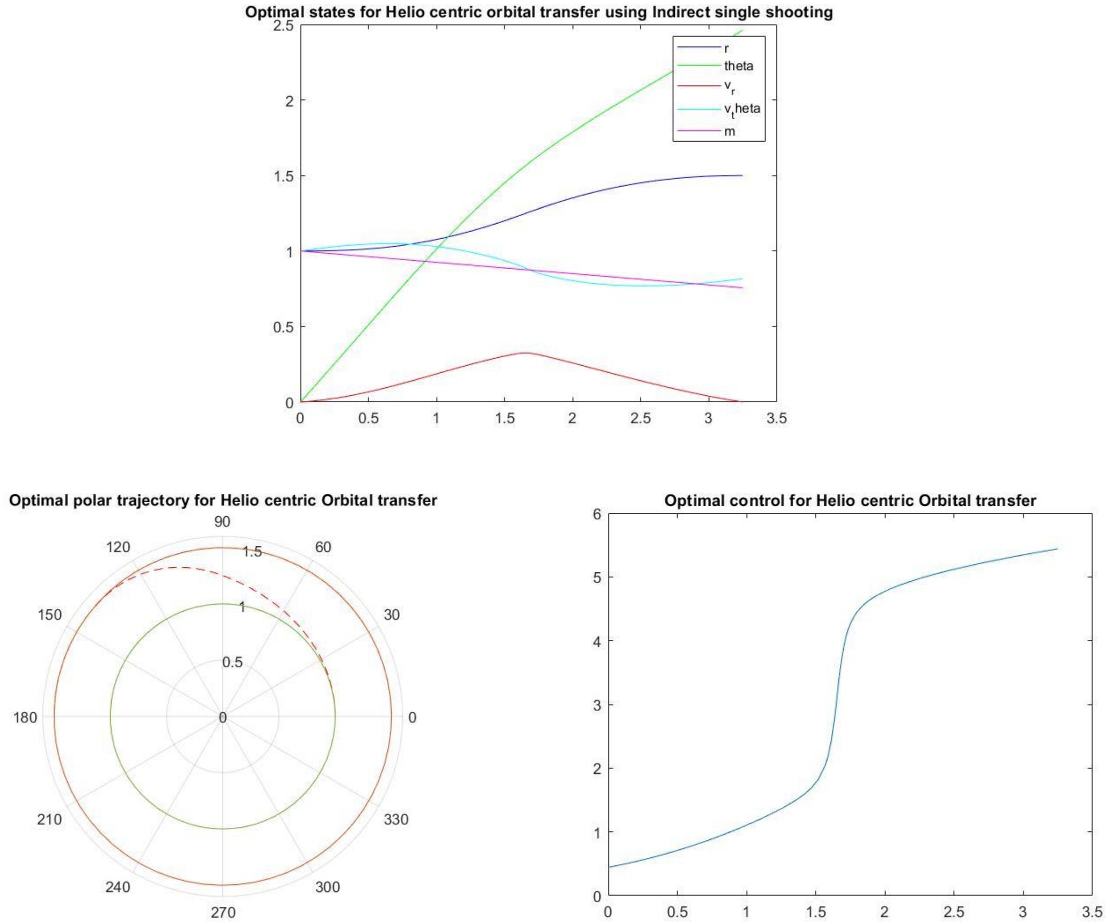


Fig 3 Optimal states, Optimal polar trajectory, Input Control for Indirect Single shooting

Final Iteration values for Indirect single shooting					
Iteration	**Function-count**	**f(x)**	**Norm of step**	**1st order optimality**	**Trust Region radius**
16	107	1.092430E-23	1.441900E-06	3.670000E-12	2.440
Objective Function		Optimized Final Mass			7.567196692090550E-01
Performance of code		Time Elapsed for Computation (Seconds)			9.21459700e-01

Analysis and conclusions:

The TPBVP of Helio-centric circular orbital transfer was solved using the Indirect single shooting technique. It was found that the optimal final time for orbital transfer was 3.248068422356106 seconds, and the space craft enter the terminal trajectory at a polar angle of 1.410380330062125e+02°. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 3. It was observed that the computation time for plotting the results was more. The Computation time excluding the plots was 3.74400300e-01 seconds.

Indirect Multiple shooting:

Steps Involved in indirect Multiple shooting method:

1. Form the Meta state vector: $Z = \begin{bmatrix} X \\ \lambda \end{bmatrix}$; and from the meta state vector obtain the Error Deviation Vector $dZ = \begin{bmatrix} dX \\ d\lambda \end{bmatrix}$.
2. As this is multiple shooting initial values for states and the co-states are to be guessed for all the intervals.
3. Say k is the no. of intervals and n is no. of states then the initial guesses for the states and co-states in intervals are computed as follows: p0 guess = ones (2*n, k-1); let z0guess be the initial guess vector of the co-states and final time guess. z0guess will be a 6 X 1 column matrix.
4. For the transcription of the optimal control problem into nonlinear control problem the indirect shooting with NLP partitions the overall time interval into k – sub arcs: $\left[t_{i-1}, t_i\right]_{i = 1,2,3....k}$ with $t_k = t_f$. The length of each sub arc will be $\nabla T = t_i - t_{i-1} = \left(t_f - t_0\right) / 2$.
5. The initial boundary value vector consisting of both state and co-state values is formed as follows: $Z\left(t_0\right)_{guess} = \begin{bmatrix} \lambda_{initial\ guess} \\ t_{f\ guess} \\ P0_{guess\ (2 \cdot n\ X\ k-1)} \end{bmatrix}$;
6. Now numerically integrate the state and co-state differential equations using the MATLAB's ODE113 solver.
7. For the first interval Meta state guess vector includes initial state values and initial co-state guess values.
8. For the remaining intervals the values of guess are extracted from the $P0_{guess}(:, i-1)$ matrix. i stands for i^{th} interval.
9. The integrated state and co-state vectors are obtained after solving for each interval. Up to the kth iteration the final state meta matrix will be $P(tau(i-1)) - P0_{guess}(i)$;
10. For the final interval i.e., i = k; the final meta state vector will be same terminal state matrix defined in the **equation (33).** If the final state meta state matrix is satisfied those are acceptable path of the state and co-state vectors.
11. Then once the initial values of state and co-state are obtained those values are introduced into the fsolve (a non-linear equation solver) to solve the error function and the final values of the co-states are generated over the k sub-intervals.
12. Using those values optimal control is obtained from the equation $\beta = a\tan2\left(\lambda_{v_r}, \lambda_{v_0}\right)$;

**** Code for the Indirect multiple shooting is attached for this document in the Appendix-2.**

Results for Indirect multiple shooting with 2 intervals:

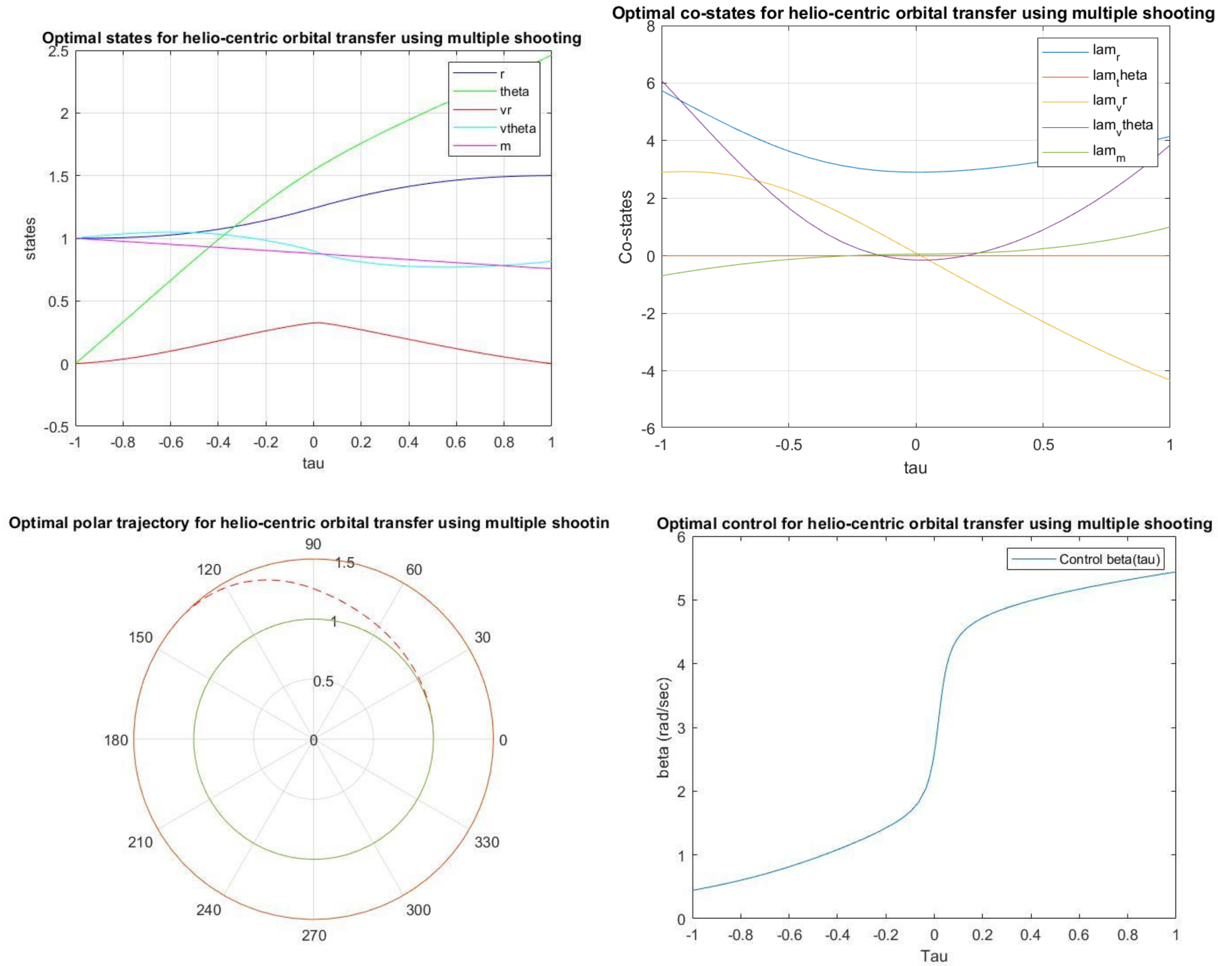


Fig 4 Optimal states, co-states, Optimal polar trajectory, Input Control for Indirect Multiple shooting

Final Iteration values for Indirect multiple shooting for 2 intervals					
Iteration	**Function-count**	**f(x)**	**Norm of step**	**1st order optimality**	**Trust Region radius**
15	256	5.784280E-25	5.941480E-07	2.020000E-12	3.910
Objective Function		**Optimized Final Mass**			7.567196623621370E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.41242390E+00

Analysis and Conclusions:

An optimal control problem is transcribed into a Nonlinear control problem partitioning into 2 sub arcs and is solved using a Fsolve NLP solver using indirect multiple shooting technique and was found that the final mass was 7.567196623621370E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 4. It was observed that the computation time for plotting the results was 1.41242390E+00 seconds. The spacecraft enters the terminal orbital at 1.410380330062125e+02°.

Results for Indirect multiple shooting with 4 intervals:

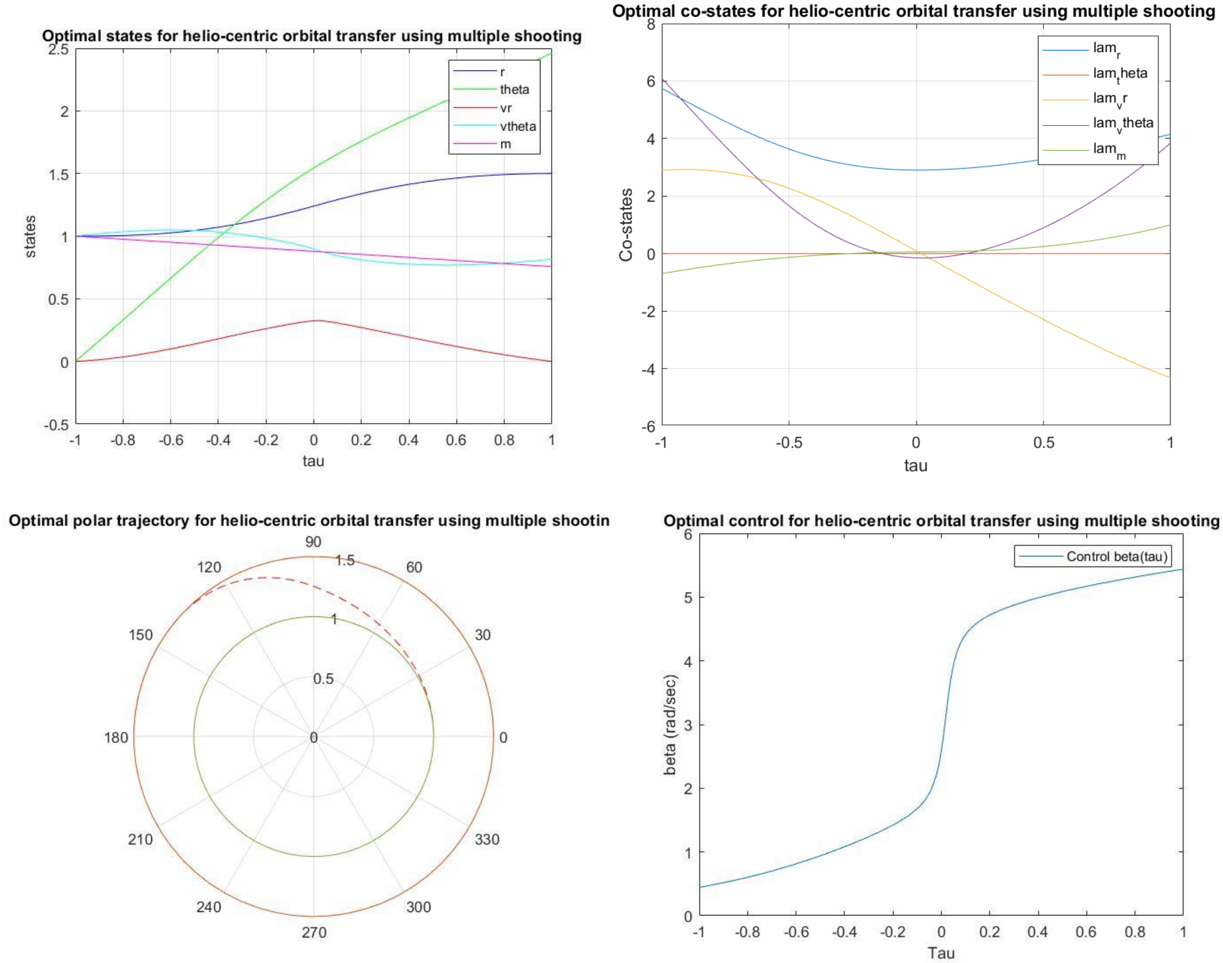


Fig 5 Optimal states, Co-states, Optimal polar trajectory, Input Control for Indirect Multiple shooting.

Final Iteration values for Indirect multiple shooting for 4 intervals					
Iteration	**Function-count**	**f(x)**	**Norm of step**	**1st order optimality**	**Trust Region radius**
8	333	8.870060E-26	3.243560E-07	6.390000E-13	15.400
Objective Function		**Optimized Final Mass**			7.567196648894210E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.90640830E+00

Analysis and Conclusions:

An optimal control problem is transcribed into a Nonlinear control problem partitioning into 4 sub arcs and is solved using a Fsolve NLP solver using indirect multiple shooting technique and was found that the final mass was 7.567196648894210E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 5. The spacecraft enters the terminal orbital at 1.410380690259635e+02°.

Results for Indirect multiple shooting with 8 intervals:

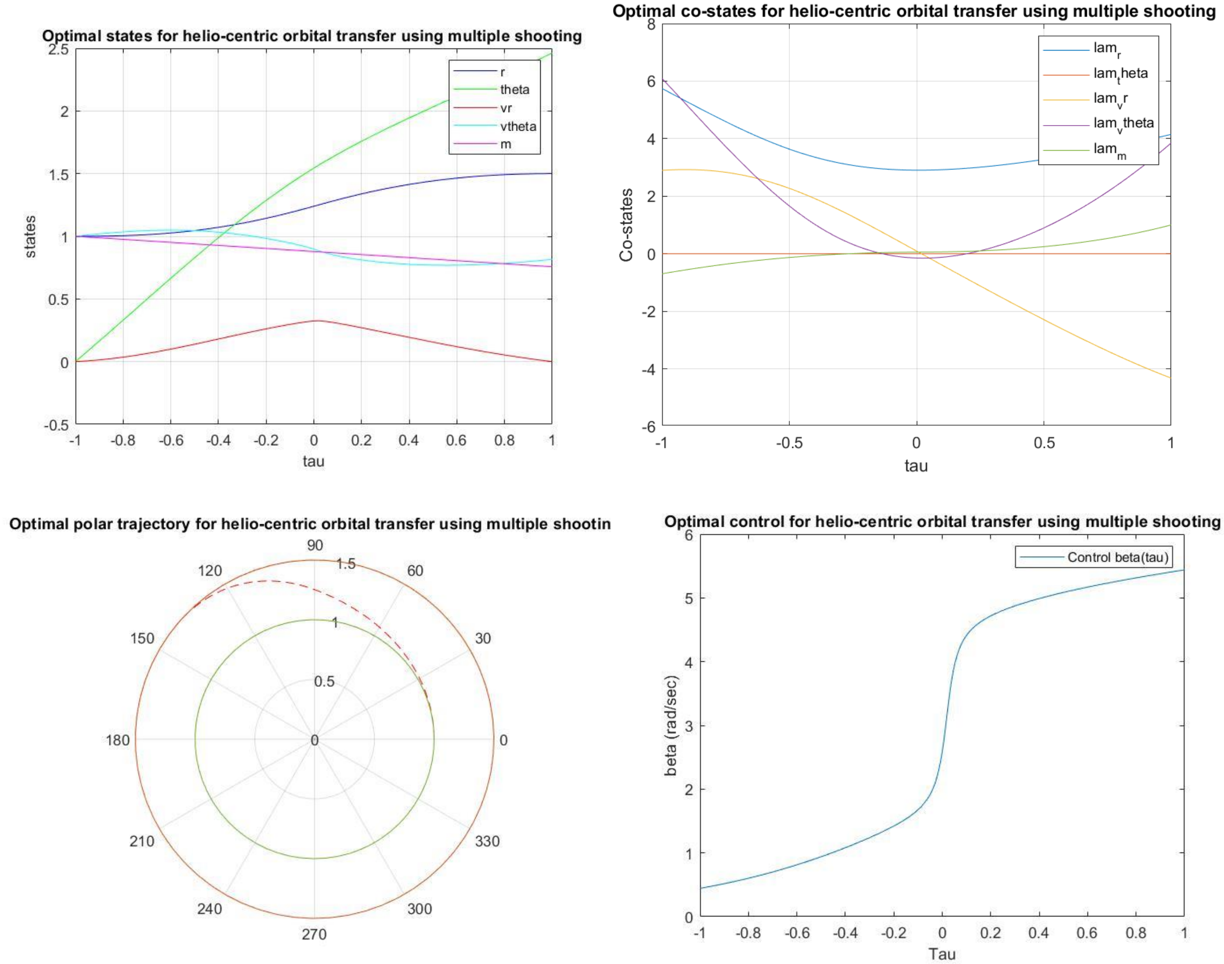


Fig 6 Optimal states, Co-states, Optimal polar trajectory, Input Control for Indirect Multiple shooting.

Final Iteration values for Indirect multiple shooting for 8 intervals					
Iteration	**Function-count**	**f(x)**	**Norm of step**	**1st order optimality**	**Trust Region radius**
13	1002	6.108600E-25	1.109370E-06	2.030000E-12	7.070
Objective Function		**Optimized Final Mass**			7.567196033049570E-01
Performance of code		Time Elapsed for Computation (Seconds)			4.72733210e+00

Analysis and Conclusions:

An optimal control problem is transcribed into a Nonlinear control problem partitioning into 8 sub arcs and is solved using a Fsolve NLP solver using indirect multiple shooting technique and was found that the final mass was 7.567196033049570E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 6. The spacecraft enters the terminal orbital at 1.410380474511374e+02 °.

Results for Indirect multiple shooting with 16 intervals:

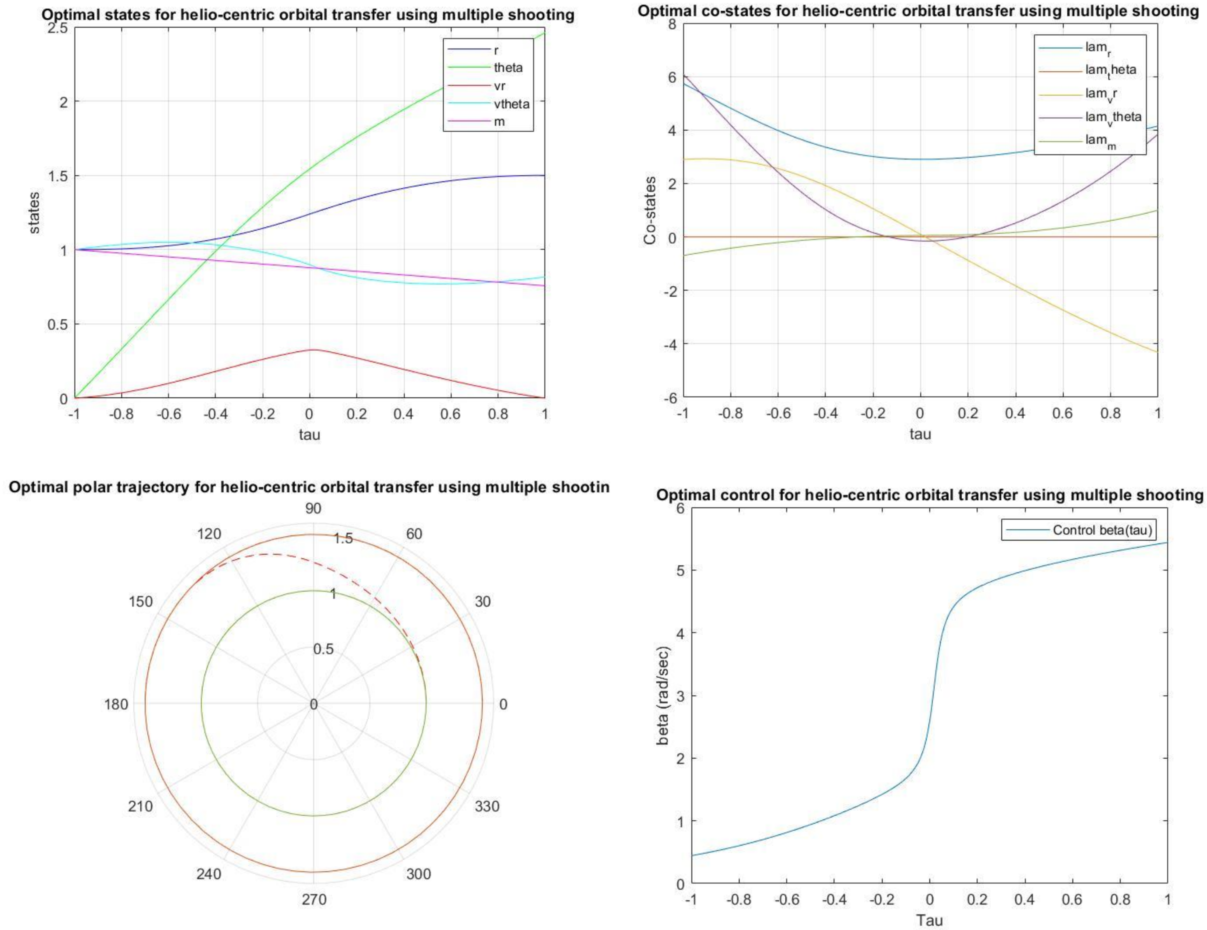


Fig 7 Optimal states, co-states, Optimal polar trajectory, Input Control for Indirect Multiple shooting.

Final Iteration values for Indirect multiple shooting for 16 intervals					
Iteration	**Function-count**	**f(x)**	**Norm of step**	**1st order optimality**	**Trust Region radius**
12	1885	2.316610E-20	8.757100E-05	1.060000E-10	9.770
Objective Function		**Optimized Final Mass**			7.567195838948950E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.272448210000000e+01

Analysis and Conclusions:

An optimal control problem is transcribed into a Nonlinear control problem partitioning into 16 sub arcs and is solved using a Fsolve NLP solver using indirect multiple shooting technique and was found that the final mass was 7.567195838948950E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 7. The spacecraft enters the terminal orbital at 1.410380291667426e+02°.

Observations in Indirect Multiple shooting:

An optimal control problem is solved using the Indirect multiple shooting technique by partitioning into 2, 4, 8, 16 sub-arcs and the solutions were documented. It was observed that the final mass and the polar position of the space craft while entering the terminal helio-centric circular orbit remains almost same. While the computational times for solving the NLP increased. Up on increasing the number of intervals it was observed that the time taken to execute single iteration increased which is due to increase in the number of the sub-arcs the states and co-states were partitioned. For all the intervals the equation was solved. The solver stopped as the function value of the computed terminal state matrix in the equation 33 reached near equal to zero which is evident from the final Iteration Values tabulated. Also, the states and the co-state trajectories remained same for every multiple shooting.

Results for Direct single shooting with control parameterized to 2^{nd} degree:

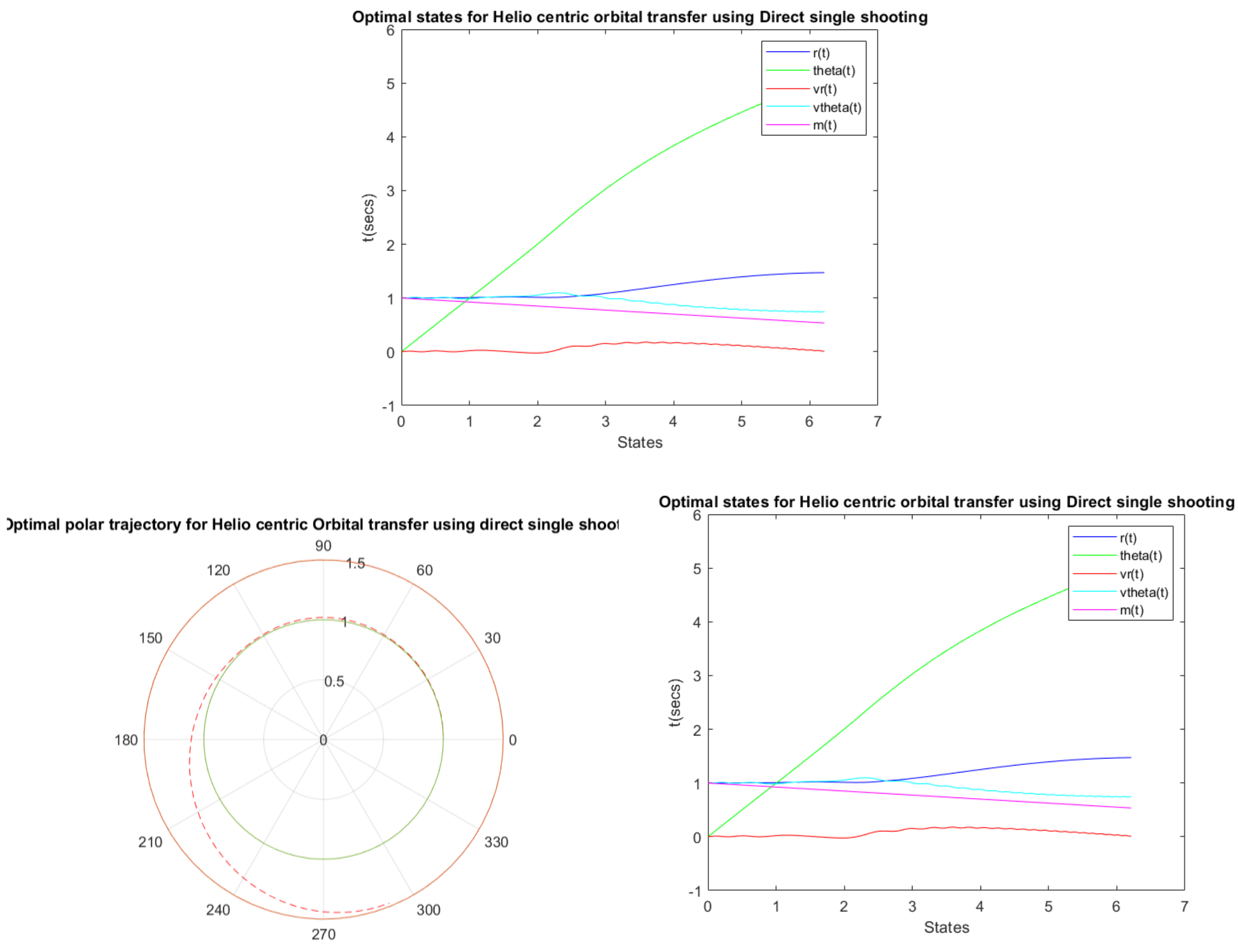


Fig 8 Optimal states, Optimal polar trajectory, Input Control for Direct Single shooting.

Final Iteration values for Direct single shooting with polynomial degree 2					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of Step**
50	457	6.211213E+00	6.885000E-02	7.544000E-01	9.873000E-08
Objective Function		**Optimized Final Mass**			5.347801609756590E-01
Performance of code		Time Elapsed for Computation (Seconds)			3.95057E+00
**** Converged to infeasible point**					

Analysis and Conclusions:

An optimal control problem is solved using direct single shooting (Numerical technique) and was found that the final mass was 5.347801609756590E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 8. The spacecraft was unable to enters the terminal orbital due to infeasibility with this input control. This is not optimal solution for spacecraft orbital transfer.

Results for Direct single shooting with control parameterized to 3rd degree:

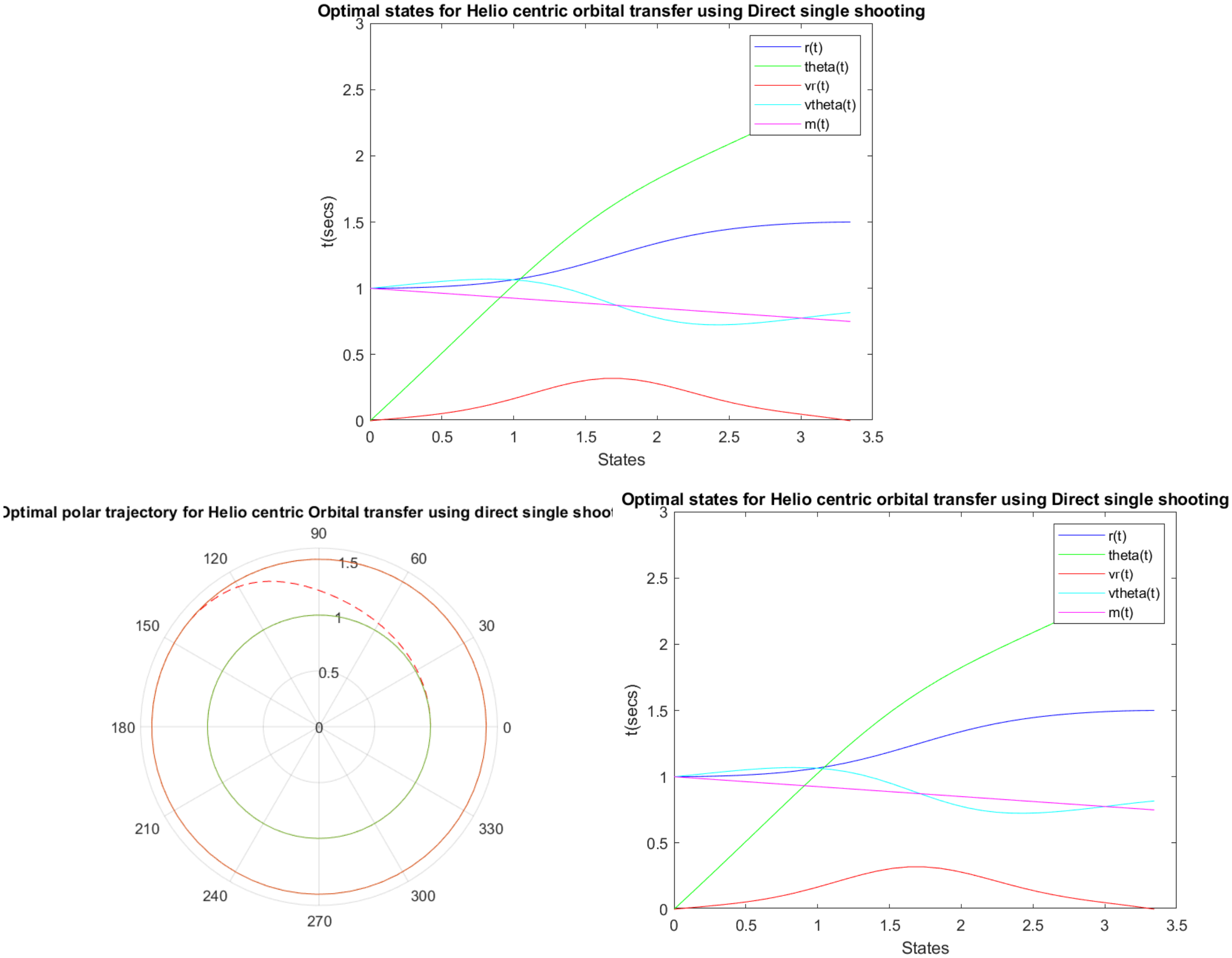


Fig 9 Optimal states, Optimal polar trajectory, Input Control for Direct Single shooting.

Final Iteration values for Direct single shooting with polynomial degree 3					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
40	289	3.344200E+00	1.884000E-14	5.235000E-06	1.662000E-04
Objective Function		**Optimized Final Mass**			7.495194370799330E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.619080E+00
**** local minima possible and constraints satisfied**					

Analysis and Conclusions:

An optimal control problem is solved using direct single shooting (Numerical technique) and was found that the final mass was 7.495194370799330E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 9. The spacecraft enters the terminal orbital at 1.446724938313424e+02°. The optimized time was found to be 3.344200E+00 seconds.

Results for Direct single shooting with control parameterized to 4th degree:

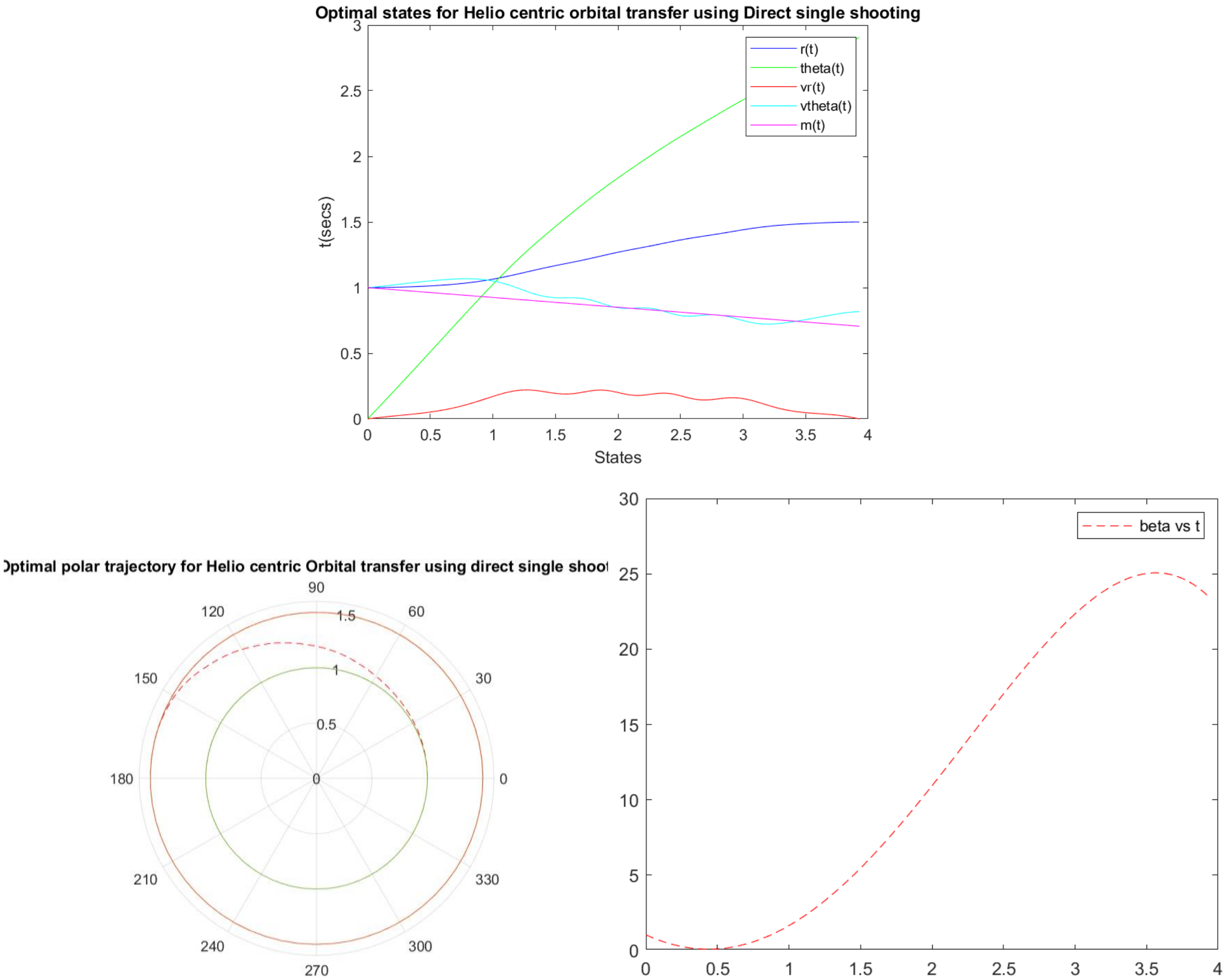


Fig 10 Optimal states, Optimal polar trajectory, Input Control for Direct Single shooting.

Final Iteration values for Direct single shooting with polynomial degree 4					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
62	584	3.929003E+00	3.153000E-13	1.794000E-06	4.301000E-08
Objective Function		**Optimized Final Mass**			7.057176889249940E-01
Performance of code		Time Elapsed for Computation (Seconds)			3.244720E+00
**** local minima possible and constraints satisfied**					

Analysis and Conclusions:

An optimal control problem is solved using direct single shooting (Numerical technique) and was found that the final mass was 7.057176889249940E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 9. The spacecraft enters the terminal orbital at 1.665435037649477e+02°. The optimized time was found to be 3.929003E+00 seconds. This cannot be optimal solution as it consumes more fuel.

Results for Direct single shooting with control parameterized to 5th degree:

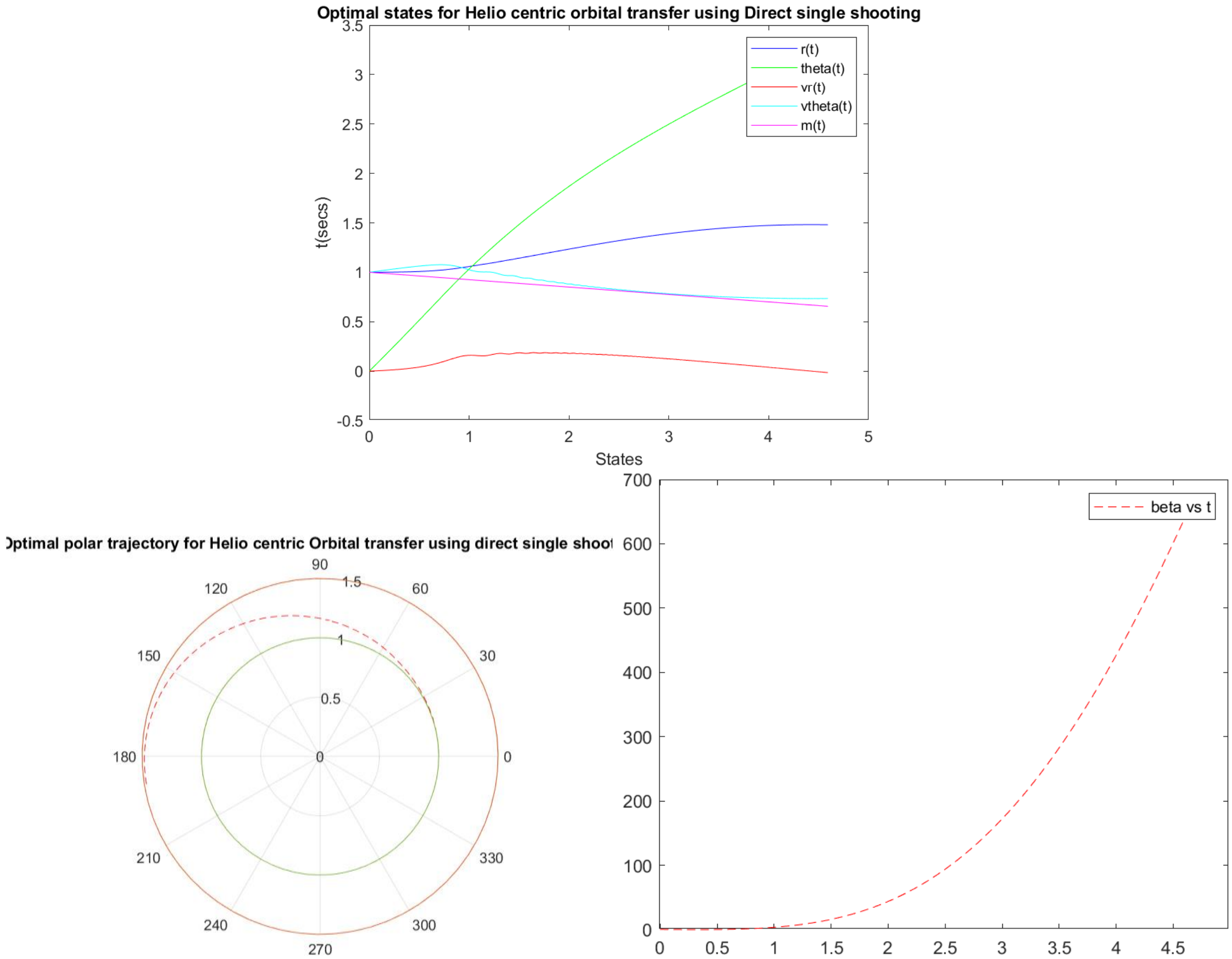


Fig 11 Optimal states, Optimal polar trajectory, Input Control for Direct Single shooting.

Final Iteration values for Direct single shooting with polynomial degree 5					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
449	5007	4.591737E+00	8.106000E-02	8.264000E-01	6.517000E-04
Objective Function		**Optimized Final Mass**			6.560788566071440E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.034756E+02
**** Solver stopped prematurely, upon increasing the Max Func Evals it led to infeasible point**					

Analysis and Conclusions:

An optimal control problem is solved using direct single shooting (Numerical technique) and was found that the final mass was 6.560788566071440E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 11. The spacecraft was unable to enters the terminal orbital due to infeasibility with this input control. This is not optimal solution for spacecraft orbital transfer. The states are also infeasible.

Results for Direct single shooting with control parameterized to 6th degree:

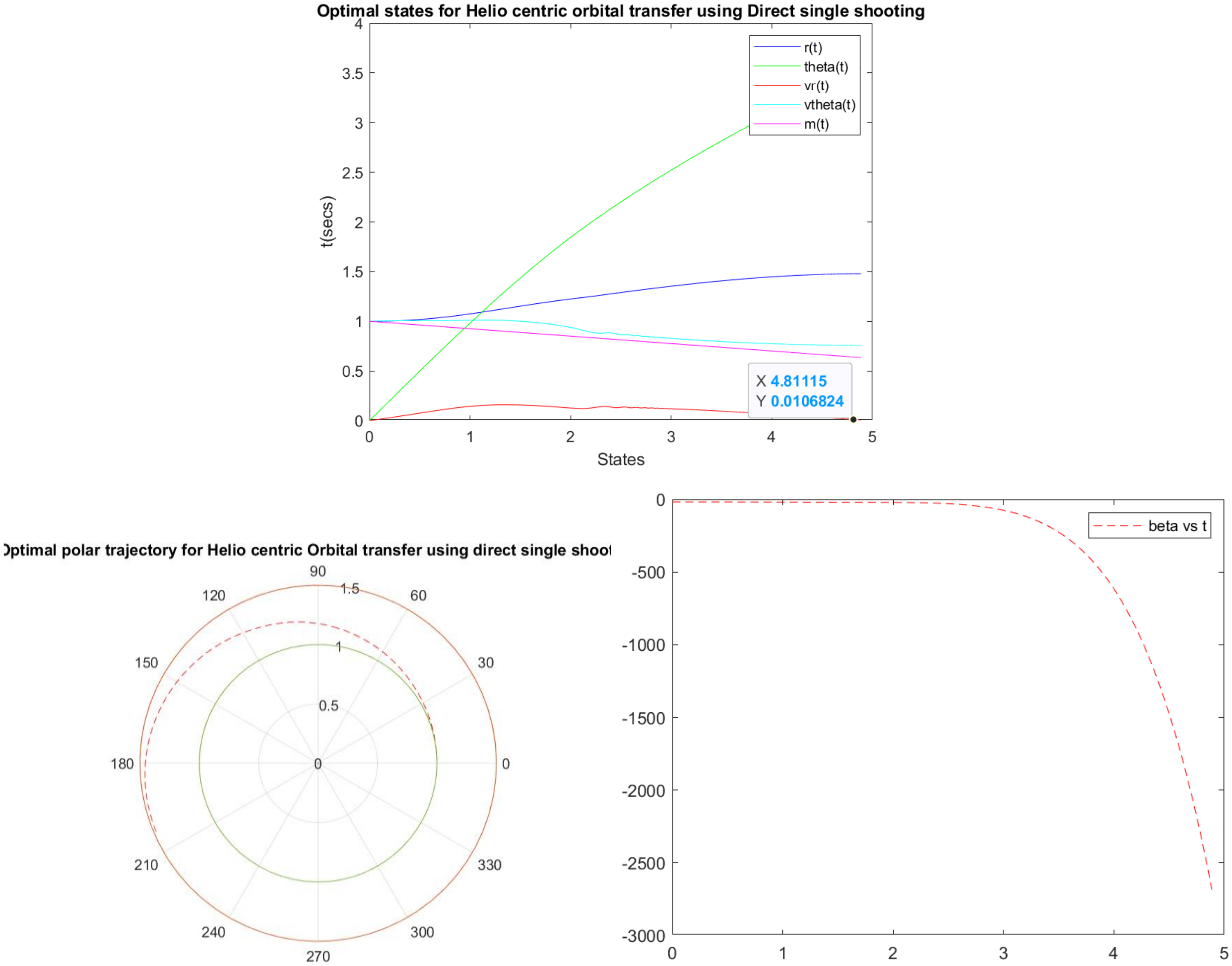


Fig 12 Optimal states, Optimal polar trajectory, Input Control for Direct Single shooting.

Final Iteration values for Direct single shooting with polynomial degree 6					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
27	368	4.887693E+00	5.921000E-02	2.570000E-01	2.022000E-08
Objective Function		**Optimized Final Mass**			6.339118166641830E-01
Performance of code		Time Elapsed for Computation (Seconds)			1.792437E+02
**** Converged to Infeasible point**					

Analysis and Conclusions:

An optimal control problem is solved using direct single shooting (Numerical technique) and was found that the final mass was 6.560788566071440E-01. The optimal solutions for states, polar trajectory and Thrust direction angle are shown in the figure 12. The spacecraft was unable to enters the terminal orbital due to infeasibility with this input control. This is not optimal solution for spacecraft orbital transfer. The states are infeasible.

Conclusion for Direct Single Shooting Technique:

Direct single shooting was used to minimize the time taken for a space craft to enter the terminal orbit. It is evident that the control with 3rd degree gave an optimum solution using direct single shooting technique. The optimal final time was found to be 3.3442 seconds. The maximum mass of space craft was found to be 7.495194370799330E-01. As the solution of the n = 2 code says local minima possible but it was not found it can be further minimized using the correct guesses and it can be further optimized.

Results for Direct multiple shooting with control parameterized to 2nd degree and 2 partitions:

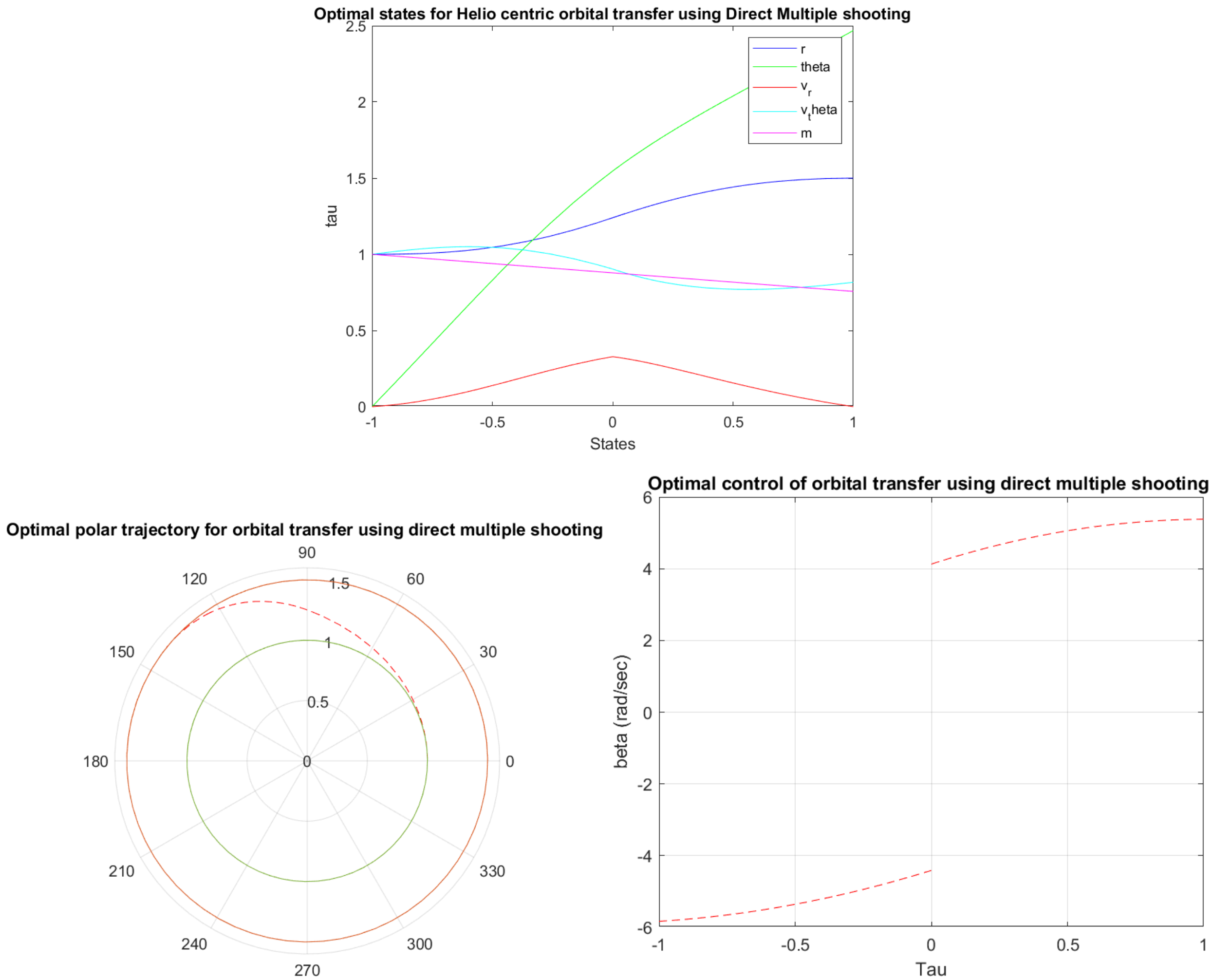


Fig 13 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting.

Final Iteration values for Direct single shooting with polynomial degree 2, partitions - 2					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
58	848	3.249986E+00	1.152000E-07	6.775000E-05	1.603000E-02
Objective		**Optimized Final Mass**			7.565760177131490E-01
Final time (secs)		3.249986E+00	**Angle of entry to terminal orbit (Deg)**		1.414610E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.536573E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 2^{nd} degree and 4 partitions:

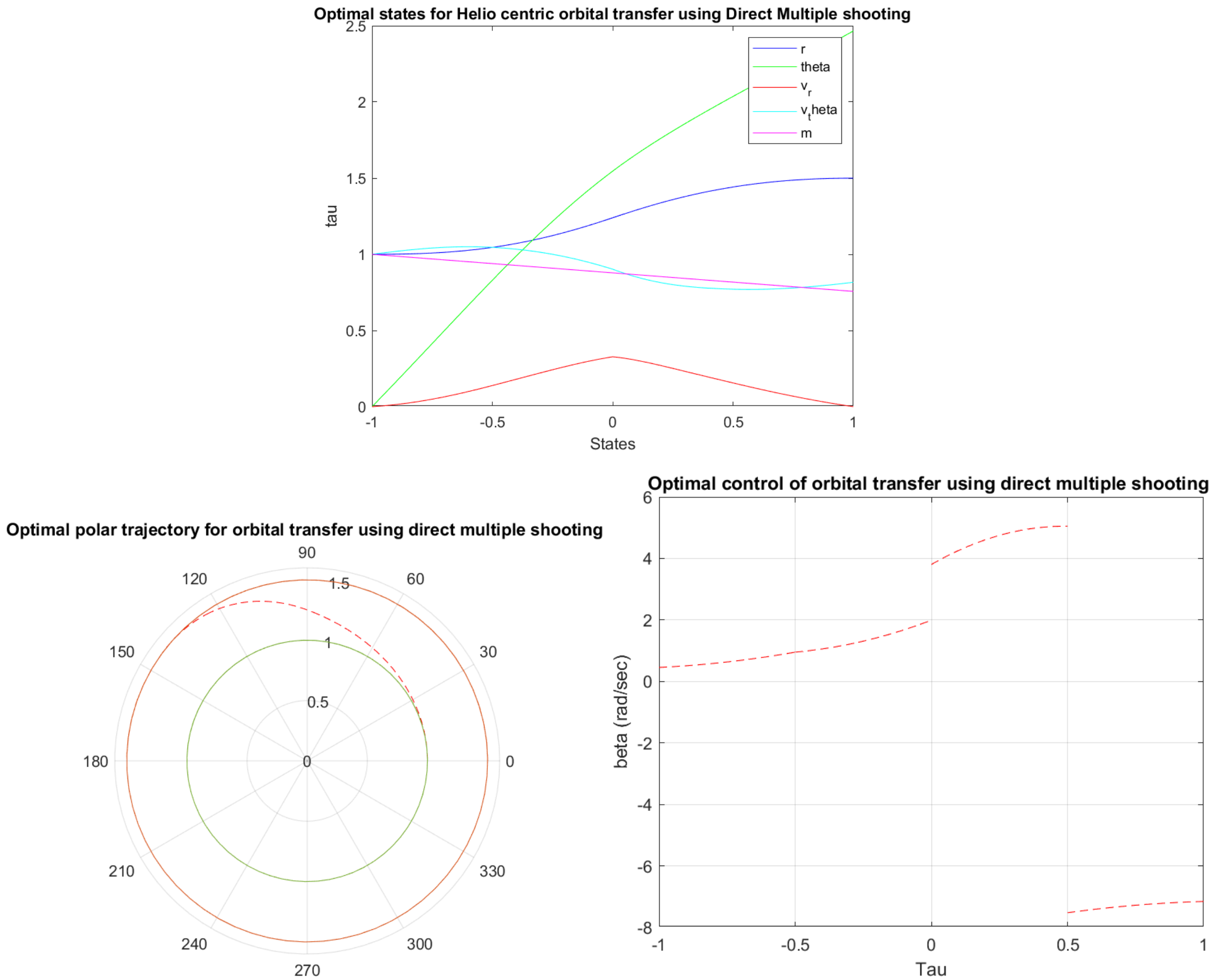


Fig 14 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting.

Final Iteration values for Direct single shooting with polynomial degree 2, partitions - 4					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
50	1494	3.249022E+00	1.486000E-07	8.015000E-05	2.578000E-02
Objective		**Optimized Final Mass**			7.566482578078050E-01
Final time (secs)		3.249022E+00	**Angle of entry to terminal orbit (Deg)**		1.412875E+02
Performance of code		Time Elapsed for Computation (Seconds)			4.339505E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 2nd degree and 8 partitions:

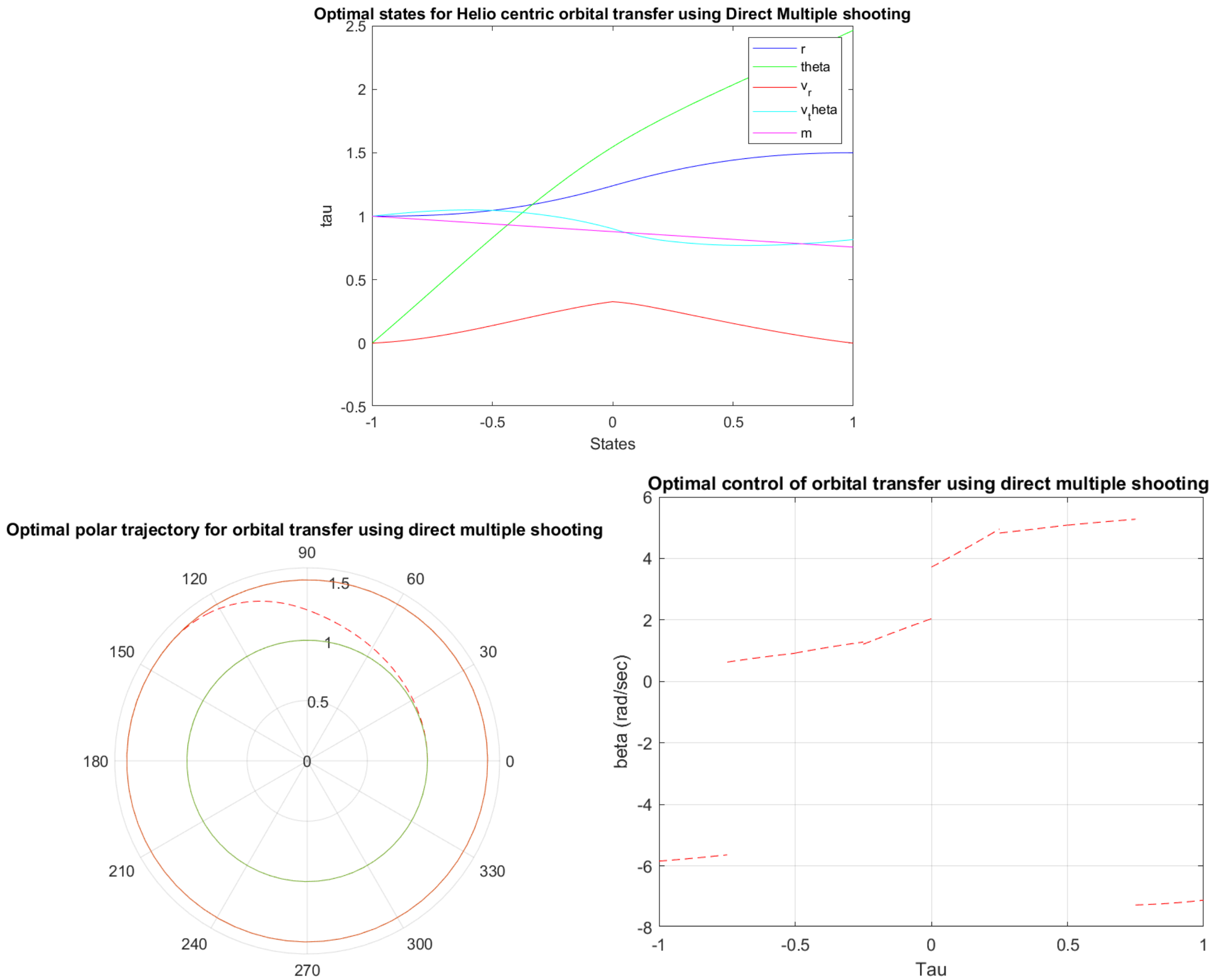


Fig 15 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 2, partitions - 8					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
84	5223	3.248899E+00	8.920000E-13	7.965000E-05	1.029000E-02
Objective		**Optimized Final Mass**			7.566574580680050E-01
Final time (secs)		3.248899E+00	**Angle of entry to terminal orbit (Deg)**		1.412313E+02
Performance of code		Time Elapsed for Computation (Seconds)			1.841098E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 2nd degree and 16 partitions:

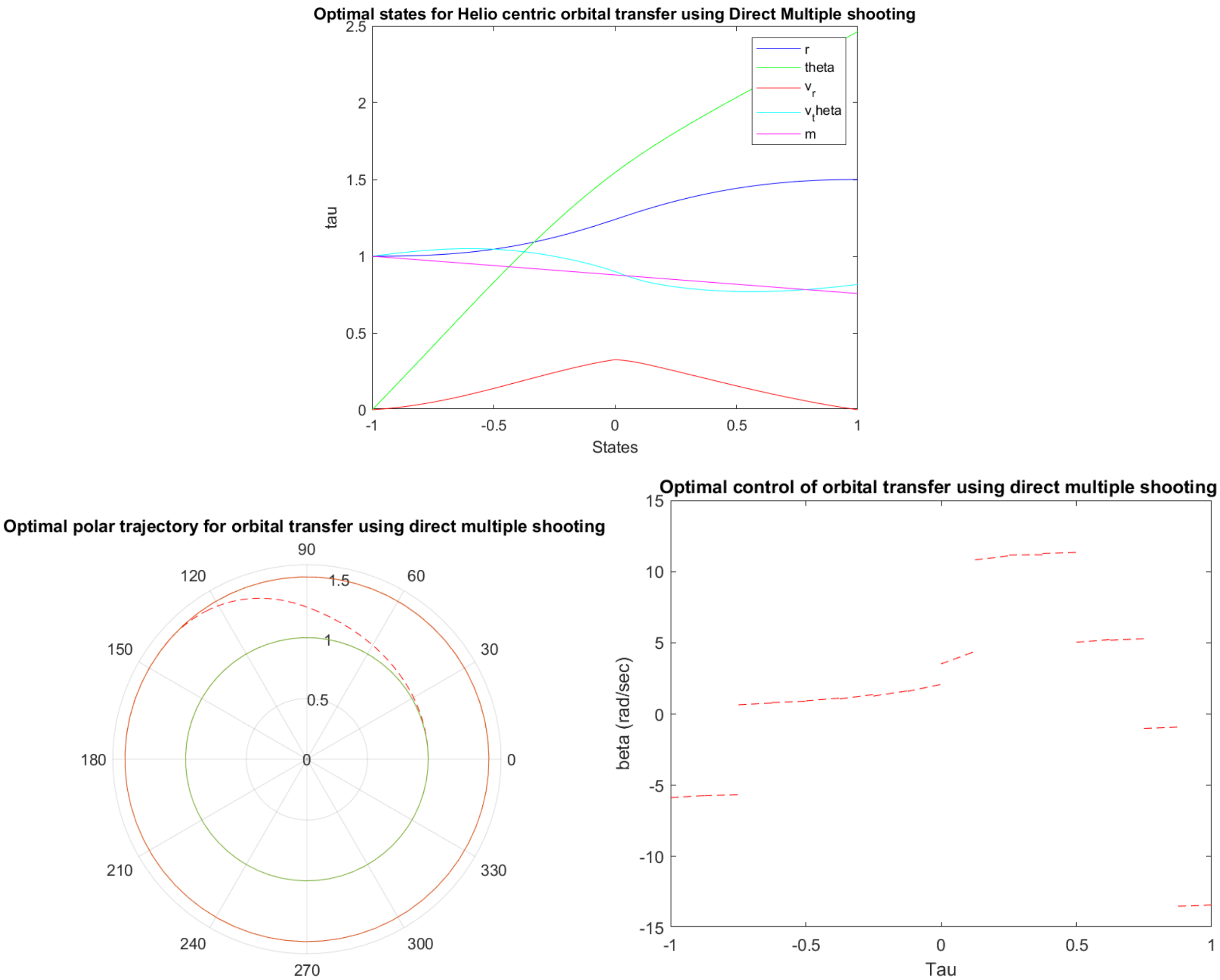


Fig 16 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 2, partitions - 16					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
128	16147	3.248644E+00	7.832000E-08	9.783000E-05	8.121000E-02
Objective		**Optimized Final Mass**			7.566765432209500E-01
Final time (secs)		3.248644E+00	**Angle of entry to terminal orbit (Deg)**		1.411638E+02
Performance of code		Time Elapsed for Computation (Seconds)			9.543735E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 3rd degree and 2 partitions:

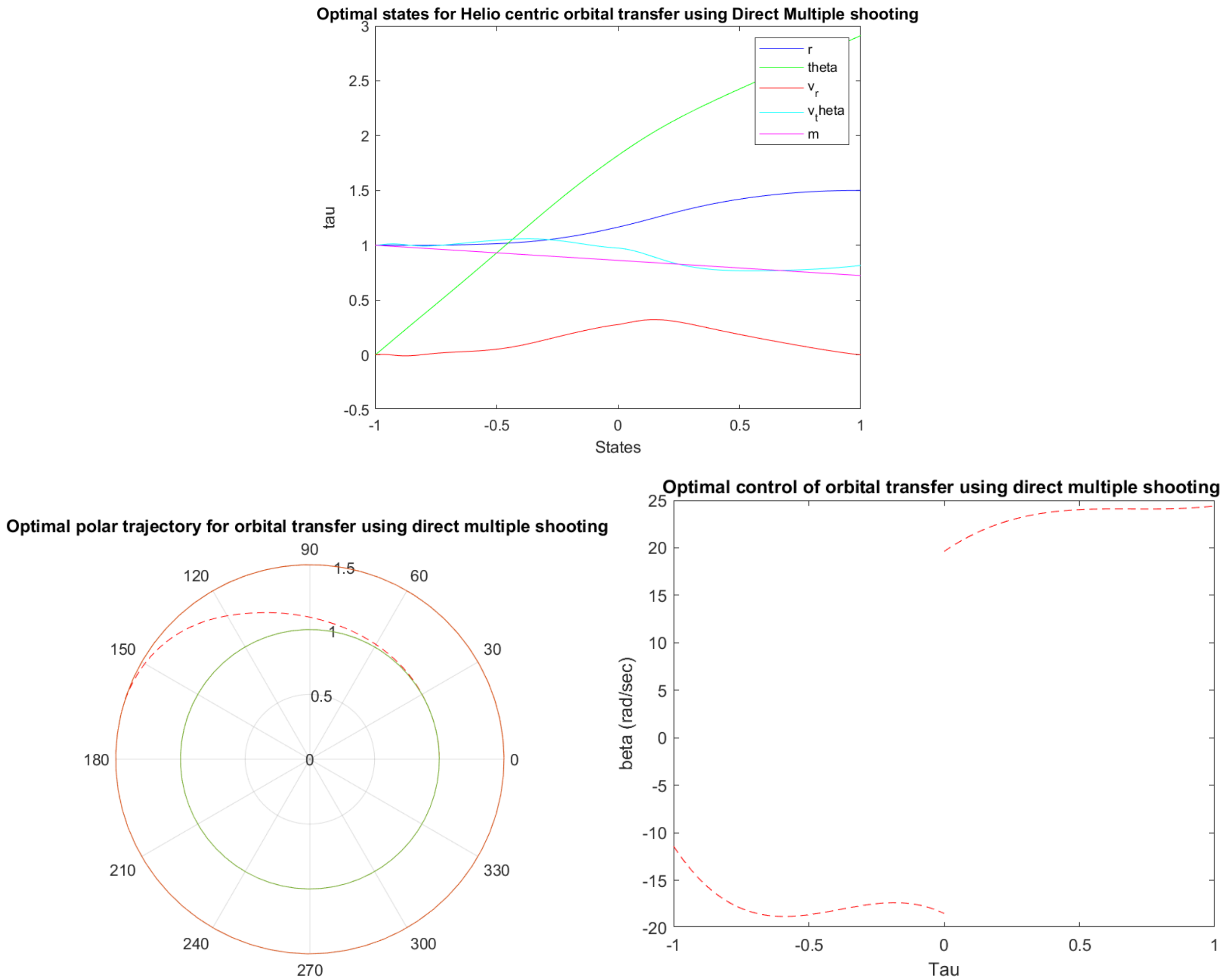


Fig 17 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 3, partitions - 2					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
126	2301	3.688175E+00	9.630000E-11	3.200000E-05	2.242000E-03
Objective		**Optimized Final Mass**			7.237557162095160E-01
Final time (secs)		3.688175E+00	**Angle of entry to terminal orbit (Deg)**		1.668823E+02
Performance of code		Time Elapsed for Computation (Seconds)			7.558162E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 3rd degree and 4 partitions:

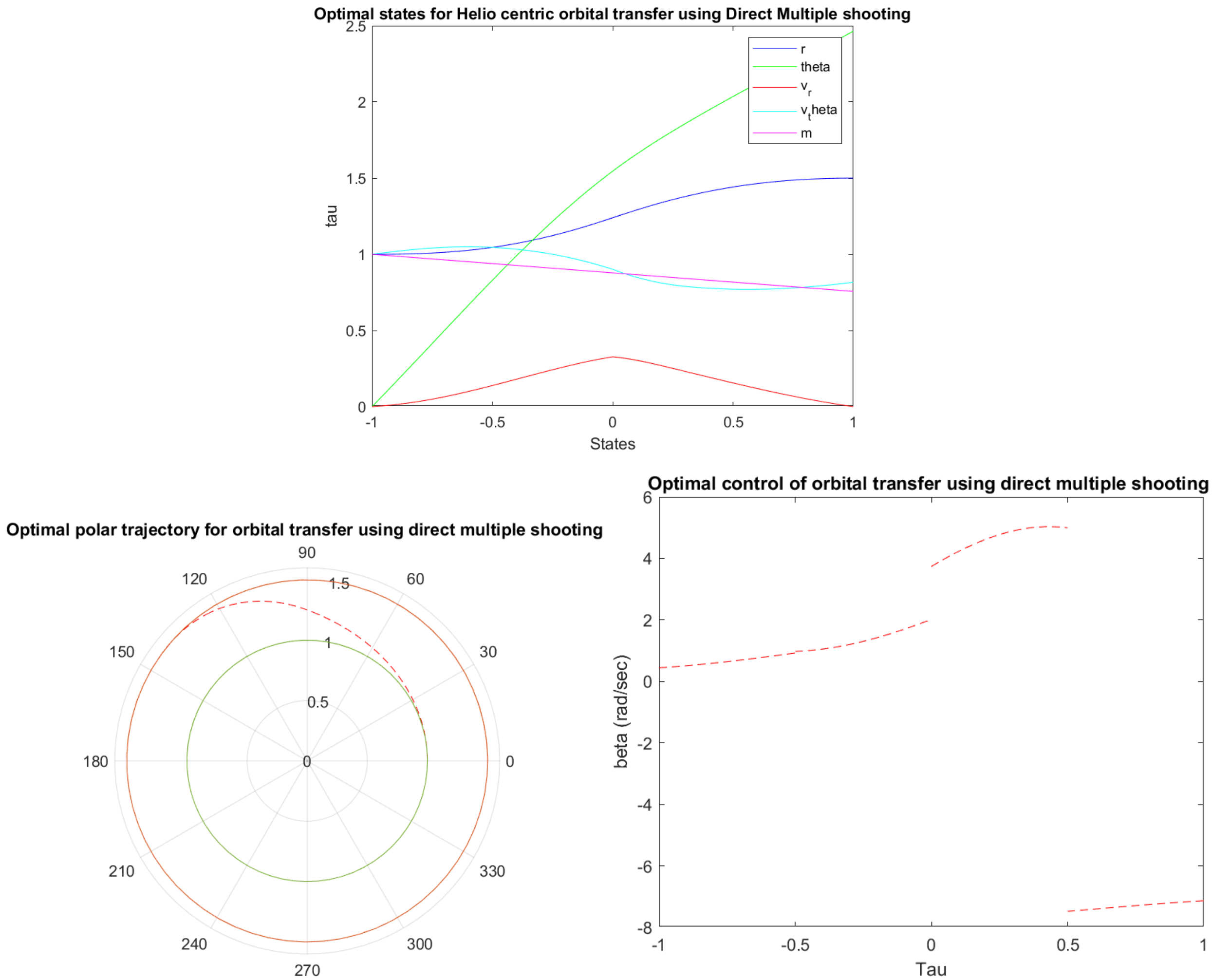


Fig 18 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 3, partitions - 4					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
68	2294	3.248977E+00	4.783000E-07	2.774000E-05	1.566000E-01
Objective		**Optimized Final Mass**			7.566516230342230E-01
Final time (secs)		3.248977E+00	**Angle of entry to terminal orbit (Deg)**		1.412447E+02
Performance of code		Time Elapsed for Computation (Seconds)			6.375949E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 3rd degree and 8 partitions:

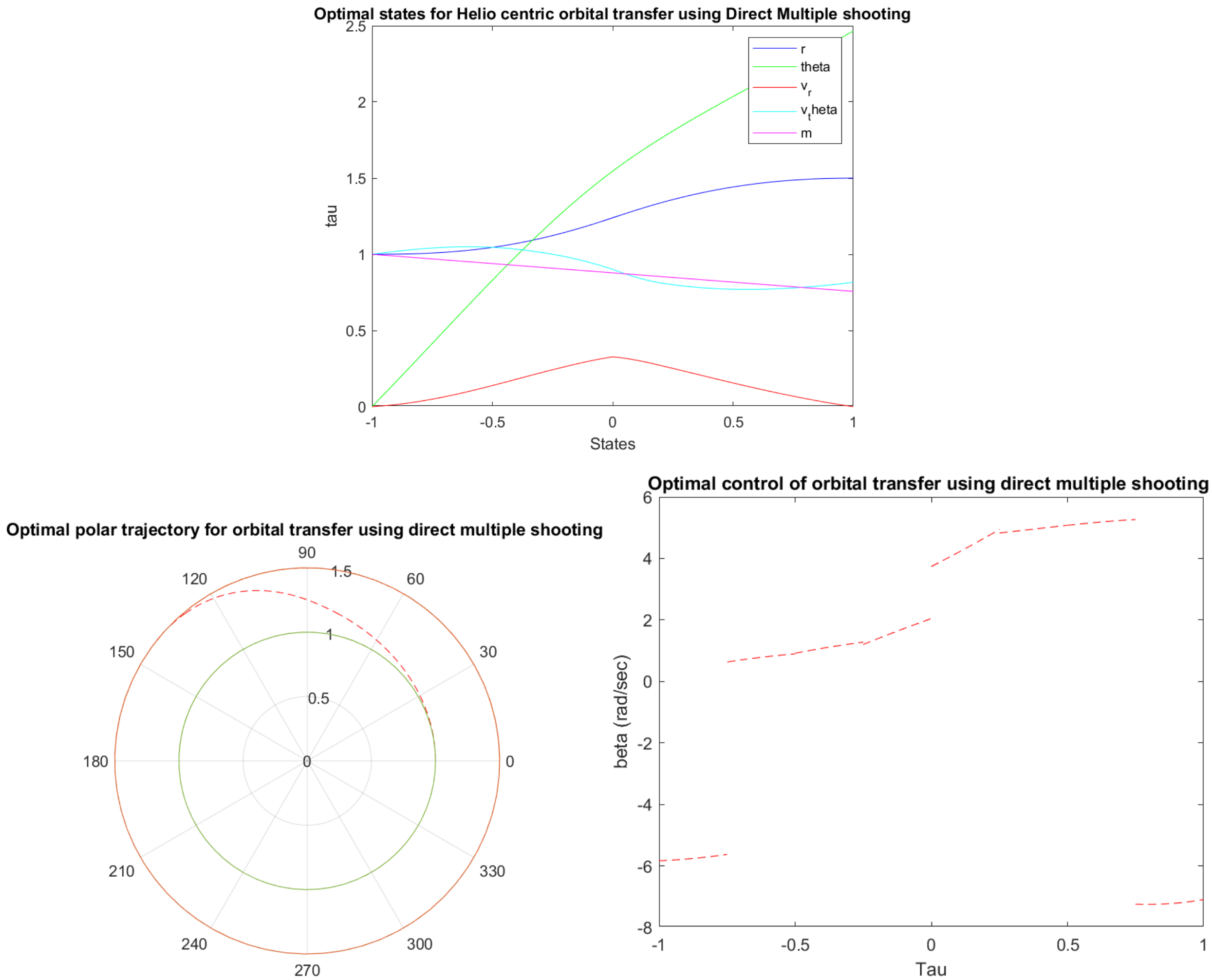


Fig 19 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 3, partitions - 8					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
85	5969	3.248975E+00	1.848000E-10	8.179000E-05	7.787000E-02
Objective		**Optimized Final Mass**			7.566517619431560E-01
Final time (secs)		3.248975E+00	**Angle of entry to terminal orbit (Deg)**		1.412507E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.303063E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 3rd degree and 16 partitions:

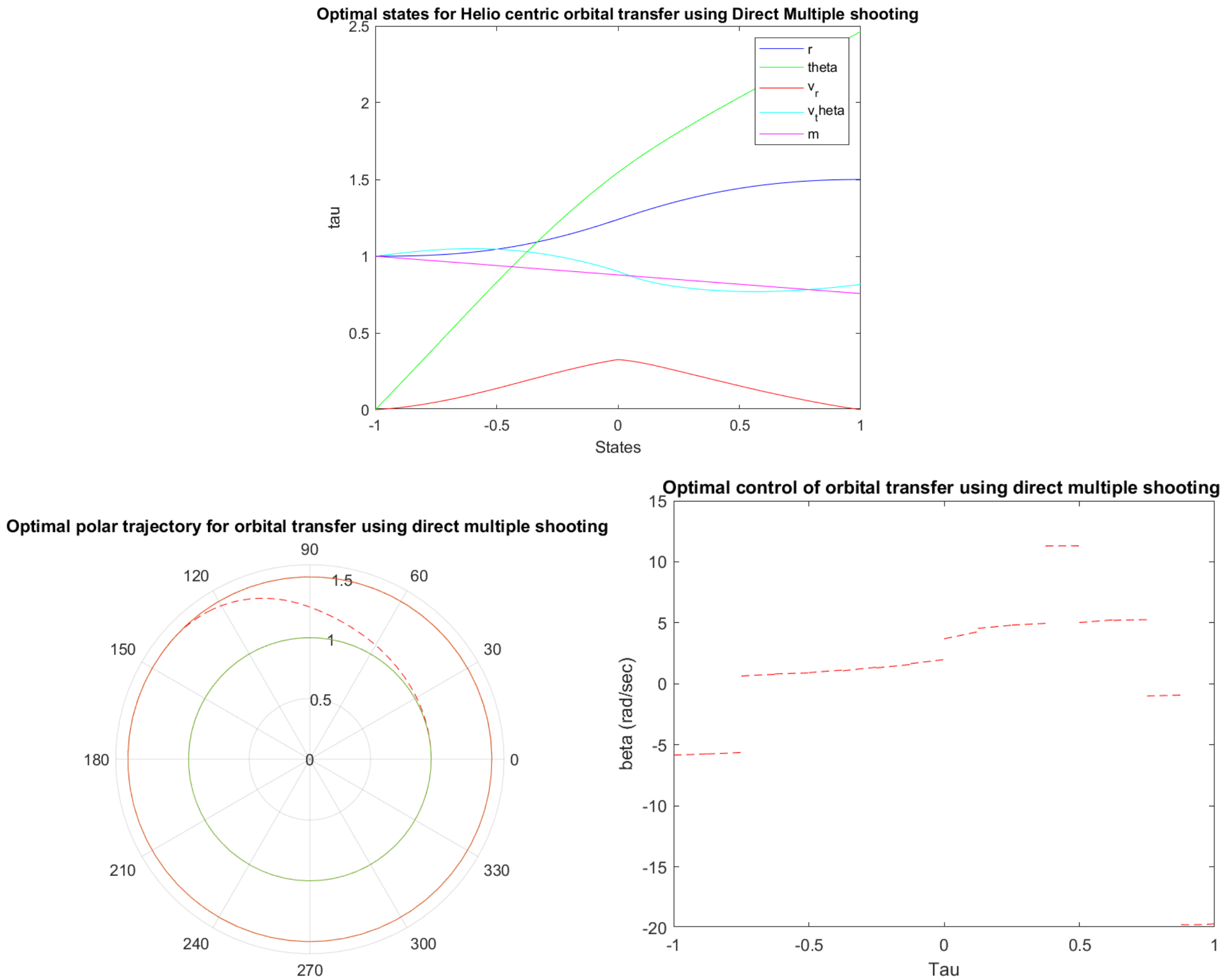


Fig 20 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 3, partitions - 16					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
134	19070	3.248884E+00	1.220000E-08	9.762000E-05	3.633000E-02
Objective		**Optimized Final Mass**			7.566586166311090E-01
Final time (secs)		3.248884E+00	**Angle of entry to terminal orbit (Deg)**		1.411958E+02
Performance of code		Time Elapsed for Computation (Seconds)			1.302095E+02
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 4th degree and 2 partitions:

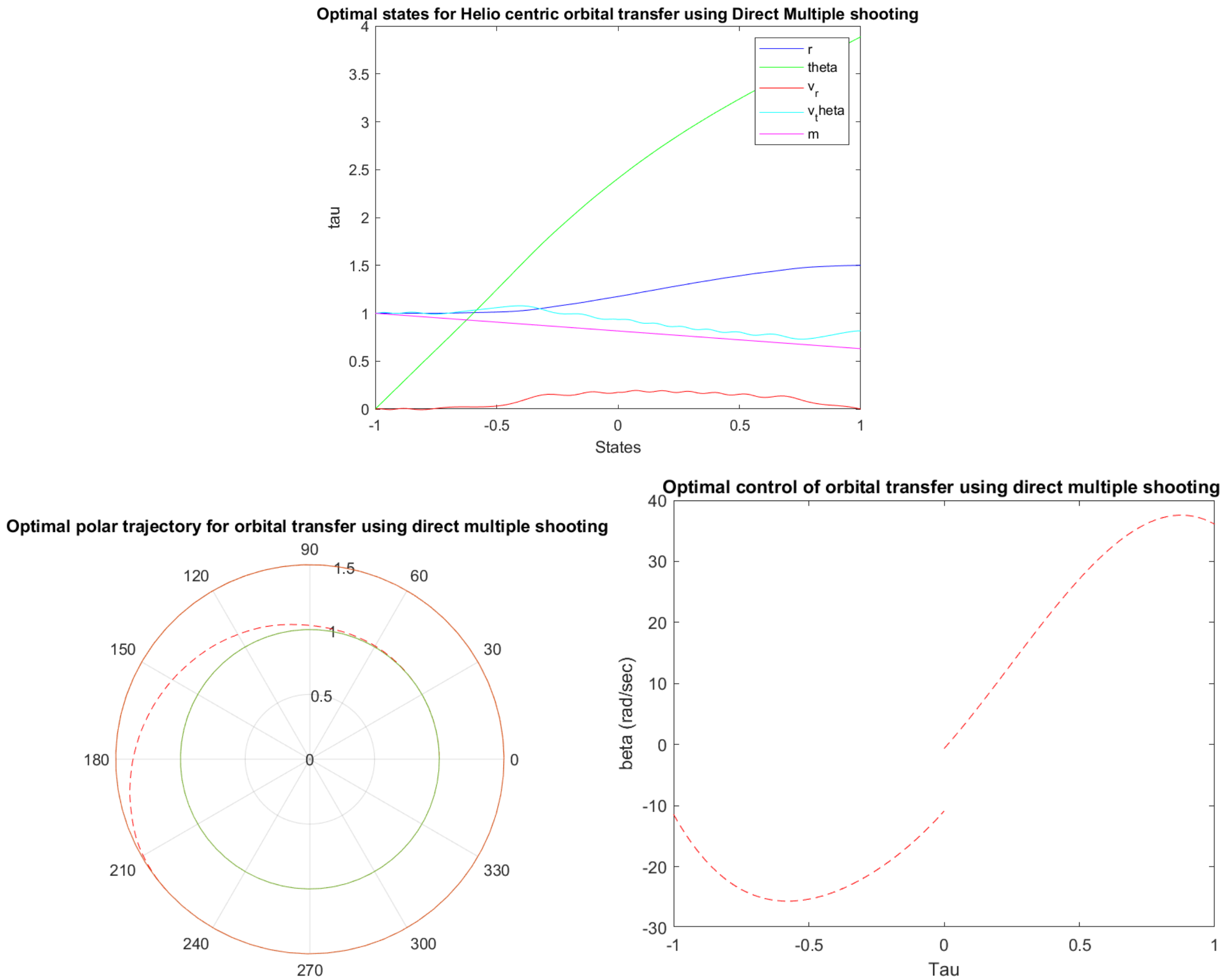


Fig 21 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 4, partitions - 2					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
129	2356	4.933047E+00	1.411000E-10	3.378000E-05	1.218000E-04
Objective		**Optimized Final Mass**			6.305147356058990E-01
Final time (secs)		4.933047E+00	**Angle of entry to terminal orbit (Deg)**		2.227497E+02
Performance of code		Time Elapsed for Computation (Seconds)			1.121940E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 4th degree and 4 partitions:

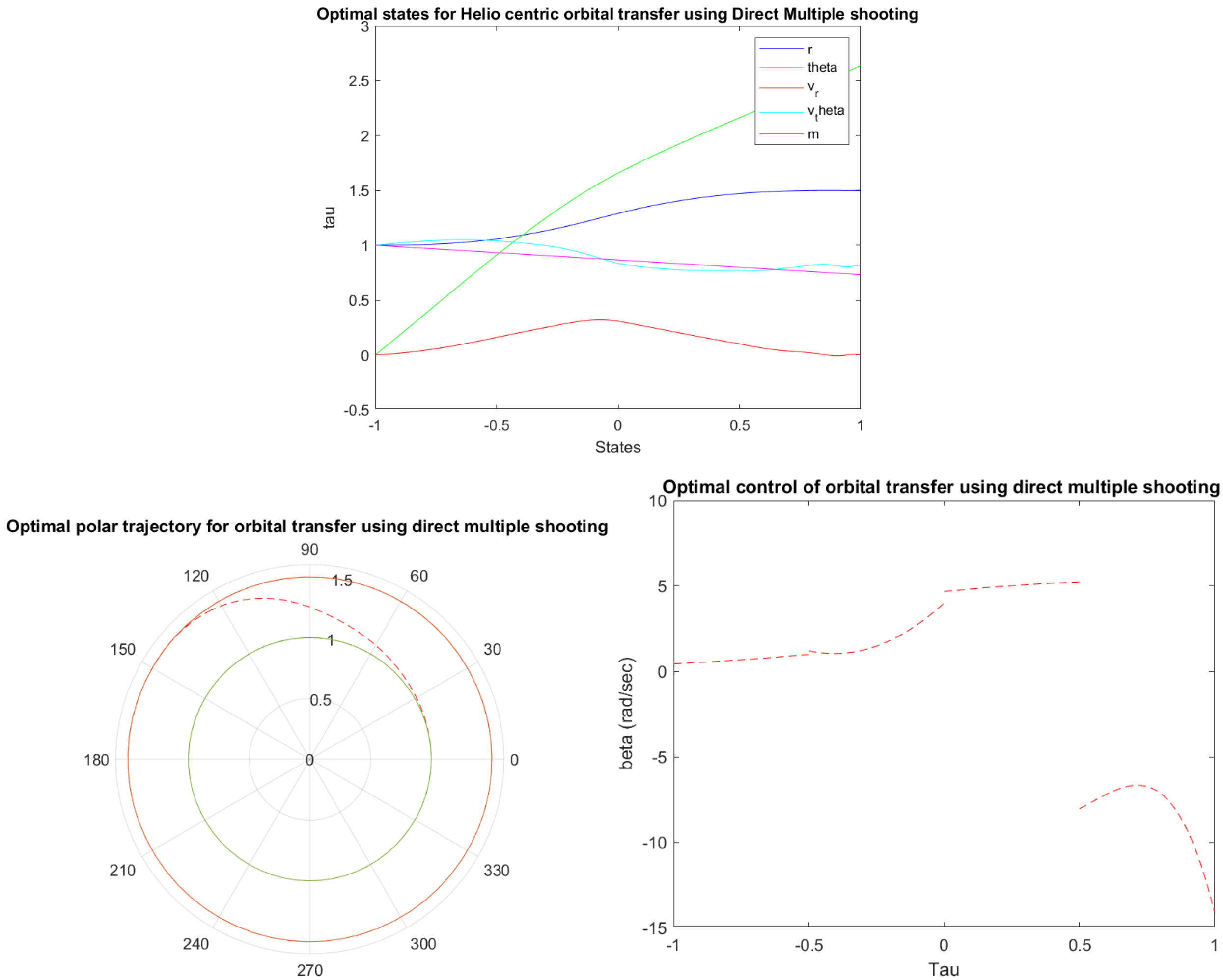


Fig 22 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 4, partitions - 4					
Iteration	**Function-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
127	4767	3.575033E+00	2.572000E-08	8.293000E-05	1.821000E-02
Objective		**Optimized Final Mass**			7.322300222819660E-01
Final time (secs)		3.575033E+00	**Angle of entry to terminal orbit (Deg)**		1.511883E+02
Performance of code		Time Elapsed for Computation (Seconds)			1.372696E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 4th degree and 8 partitions:

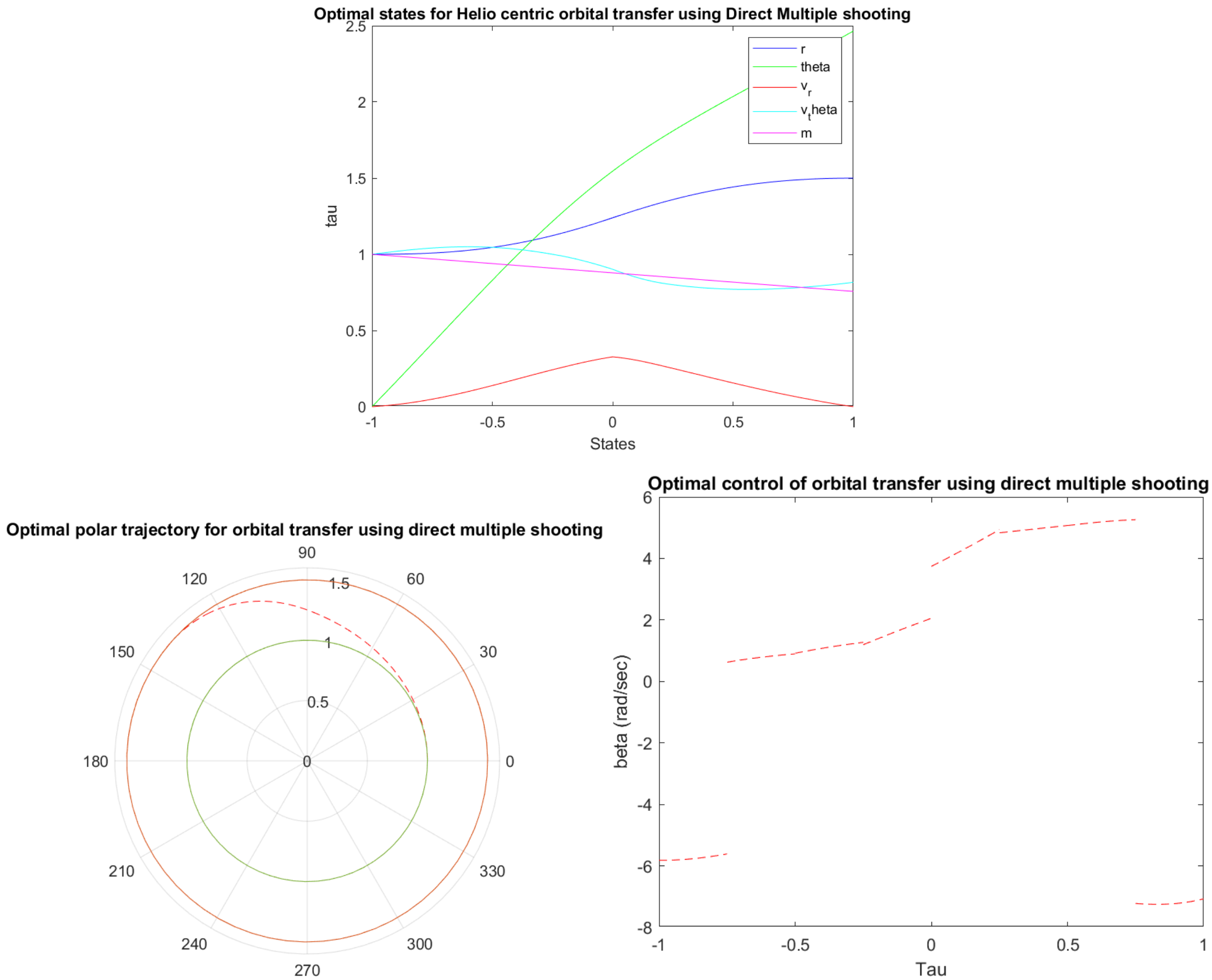
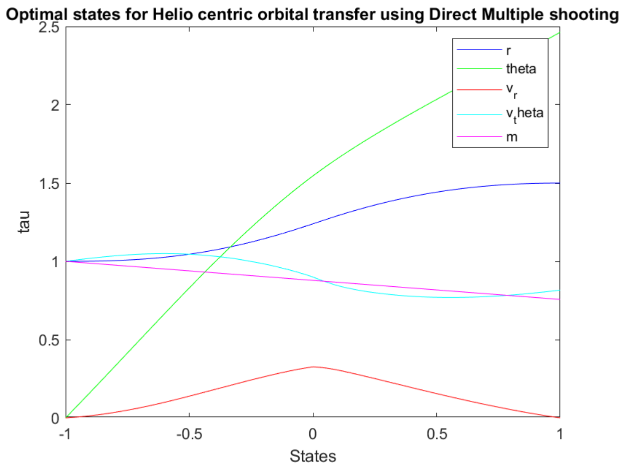
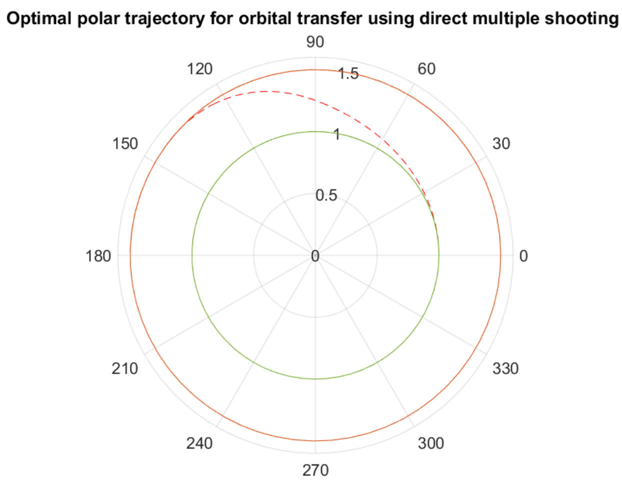


Fig 23 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 4, partitions - 8					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
78	6096	3.249097E+00	1.463000E-07	9.991000E-05	4.066000E-02
Objective		**Optimized Final Mass**			7.566426533018470E-01
Final time (secs)		3.249097E+00	**Angle of entry to terminal orbit (Deg)**		1.412613E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.533239E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 4th degree and 16 partitions:





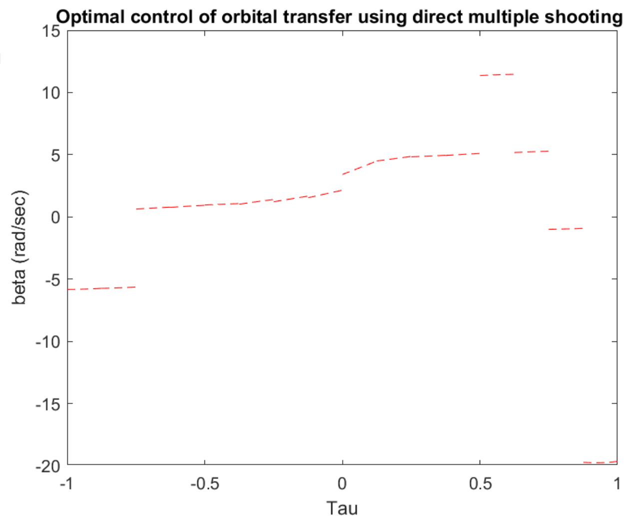
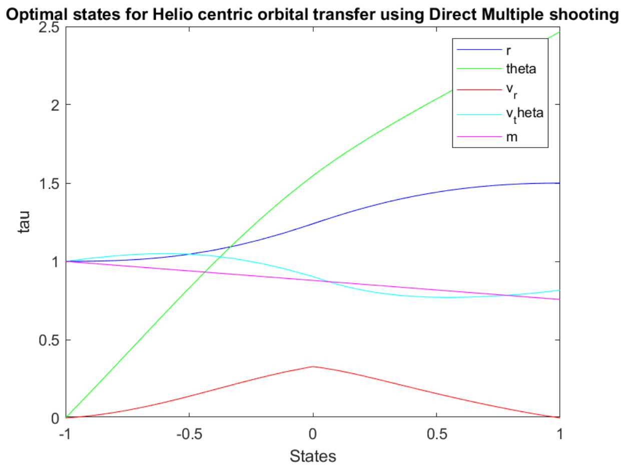
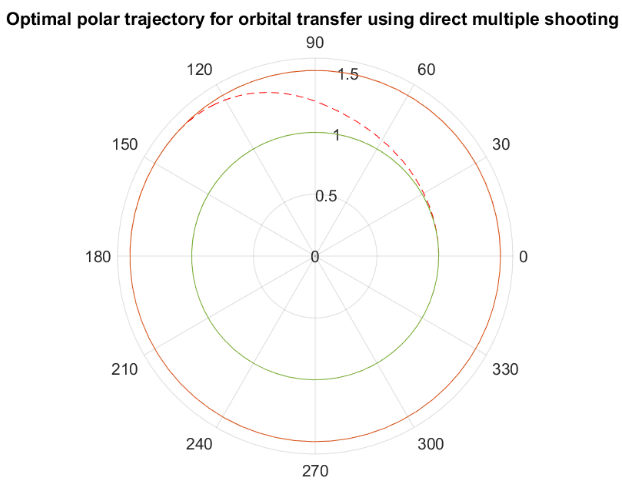


Fig 24 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 4, partitions - 16					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
154	24364	3.248560E+00	1.579000E-07	7.845000E-05	1.294000E-01
Objective		**Optimized Final Mass**			7.566828678897650E-01
Final time (secs)		3.248560E+00	**Angle of entry to terminal orbit (Deg)**		1.411420E+02
Performance of code		Time Elapsed for Computation (Seconds)			1.639299E+02
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 5th degree and 2 partitions:





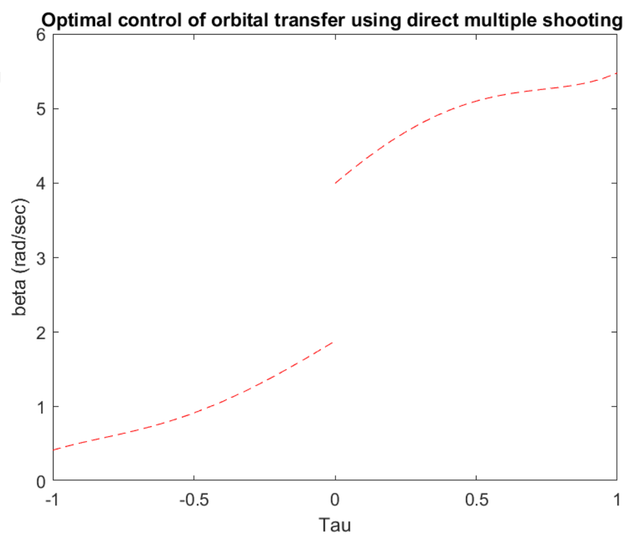


Fig 25 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 5, partitions - 2					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
53	1067	3.249576E+00	3.551000E-09	5.090000E-05	3.498000E-03
Objective		**Optimized Final Mass**			7.566067451885590E-01
Final time (secs)		3.249576E+00	**Angle of entry to terminal orbit (Deg)**		1.413692E+02
Performance of code		Time Elapsed for Computation (Seconds)			3.203460E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 5th degree and 4 partitions:

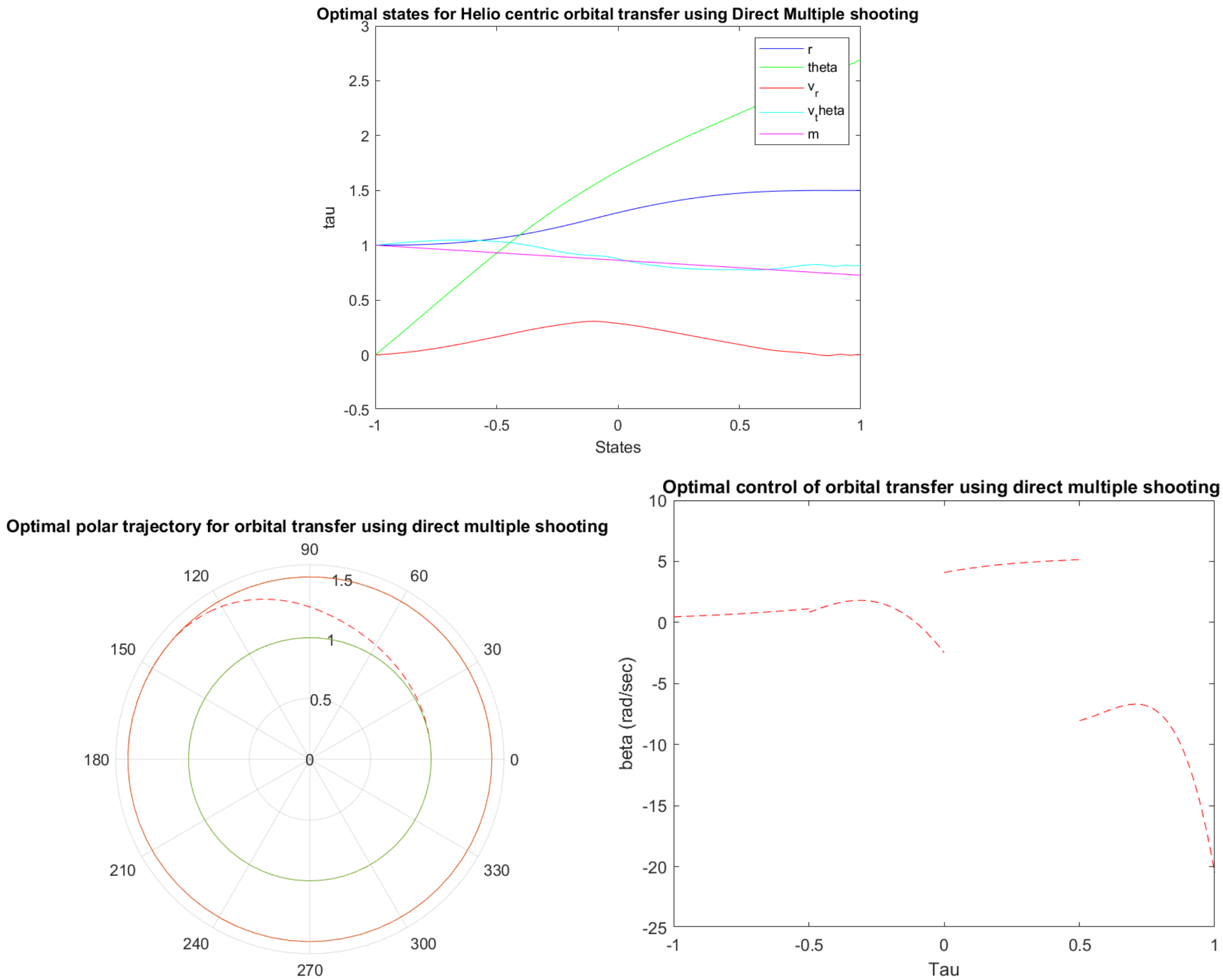


Fig 26 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 5, partitions - 4					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
172	7112	3.652131E+00	1.271000E-07	4.797000E-05	6.547000E-02
Objective		**Optimized Final Mass**			7.264554061558000E-01
Final time (secs)		3.652131E+00	**Angle of entry to terminal orbit (Deg)**		1.542556E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.072417E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 5th degree and 8 partitions:

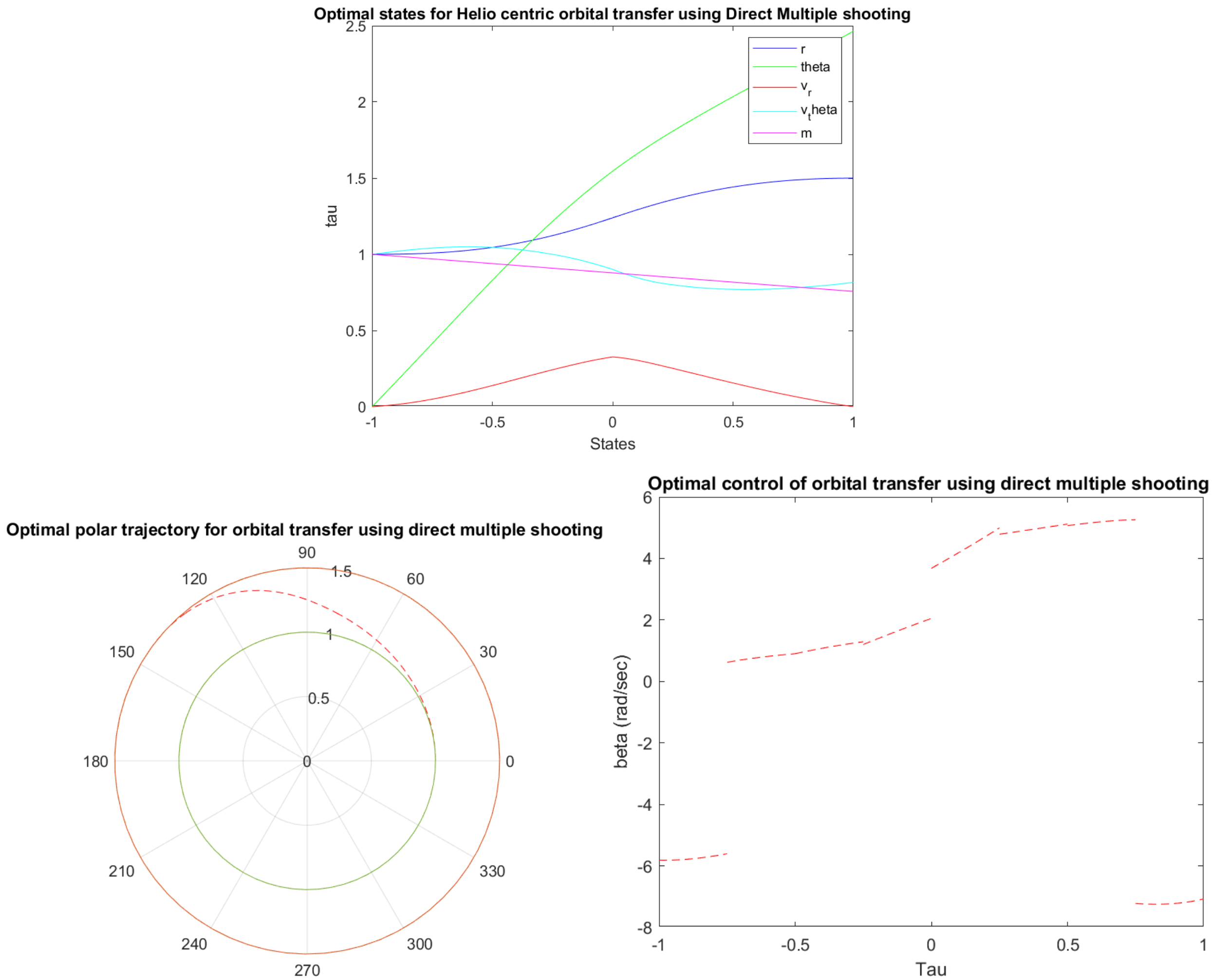


Fig 27 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 5, partitions - 8					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
92	7917	3.249074E+00	4.239000E-08	9.887000E-05	2.120000E-02
Objective		**Optimized Final Mass**			7.566443647593720E-01
Final time (secs)		3.249074E+00	**Angle of entry to terminal orbit (Deg)**		1.412120E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.748499E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 5th degree and 16 partitions:

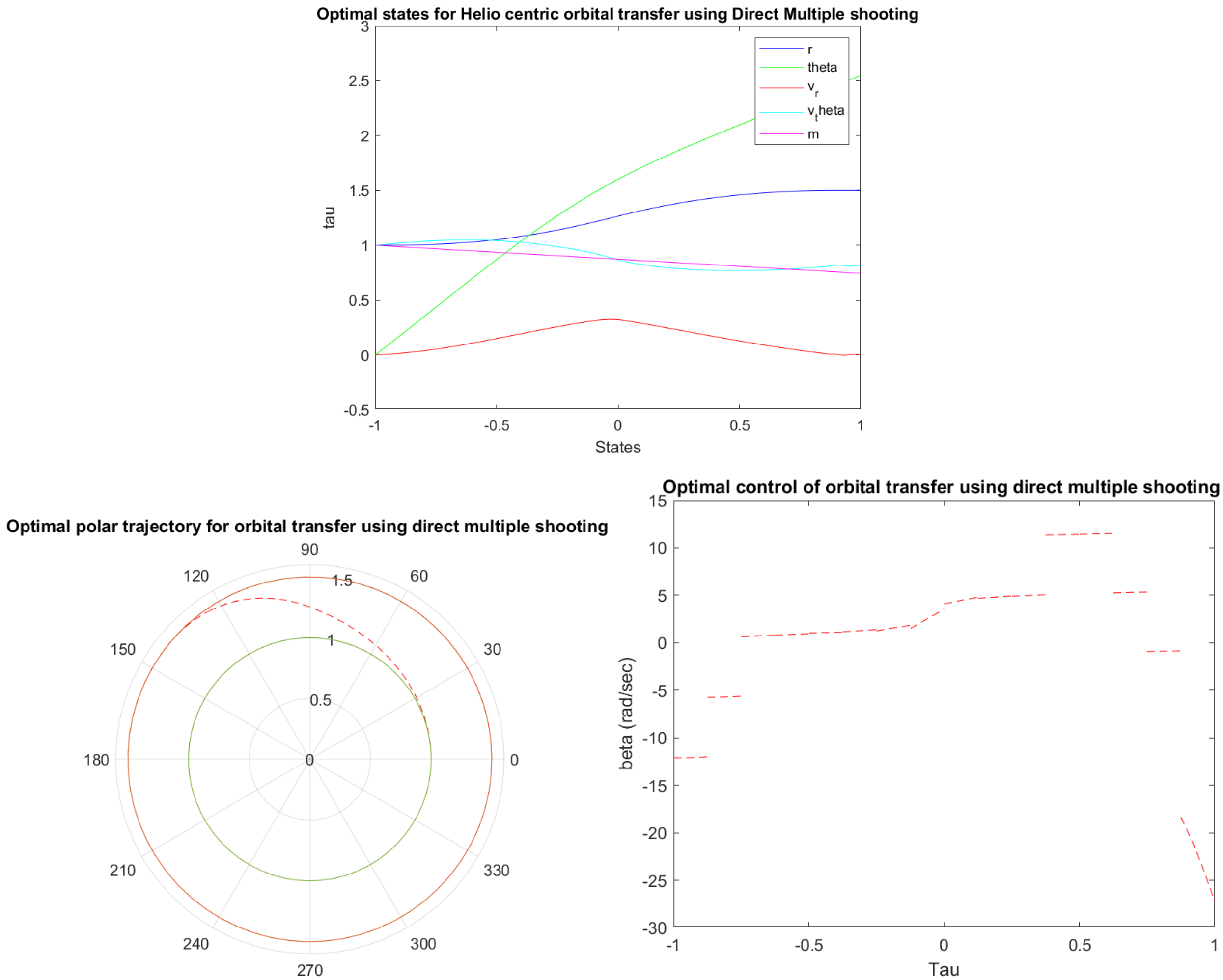


Fig 28 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 5, partitions - 16					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
179	31168	3.411013E+00	1.528000E-07	9.133000E-05	7.605000E-02
Objective		**Optimized Final Mass**			7.445151464199550E-01
Final time (secs)		3.411013E+00	**Angle of entry to terminal orbit (Deg)**		1.461155E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.214874E+02
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 6th degree and 2 partitions:

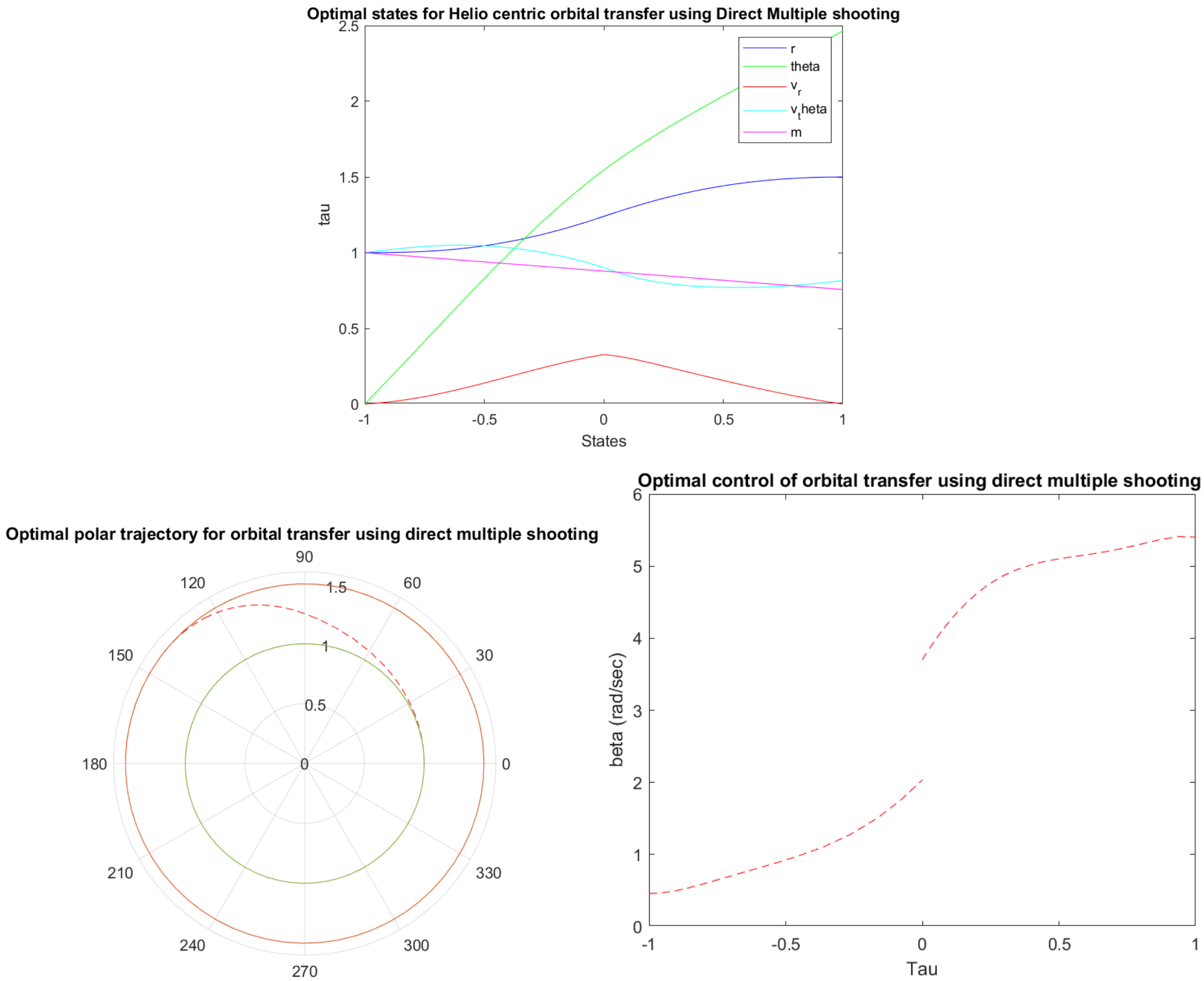


Fig 29 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 6, partitions - 2					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
93	2079	3.248943E+00	3.002000E-07	6.626000E-05	1.470000E-01
Objective		**Optimized Final Mass**			7.566541385009600E-01
Final time (secs)		3.248943E+00	**Angle of entry to terminal orbit (Deg)**		1.412396E+02
Performance of code		Time Elapsed for Computation (Seconds)			5.304387E+00
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 6th degree and 4 partitions:

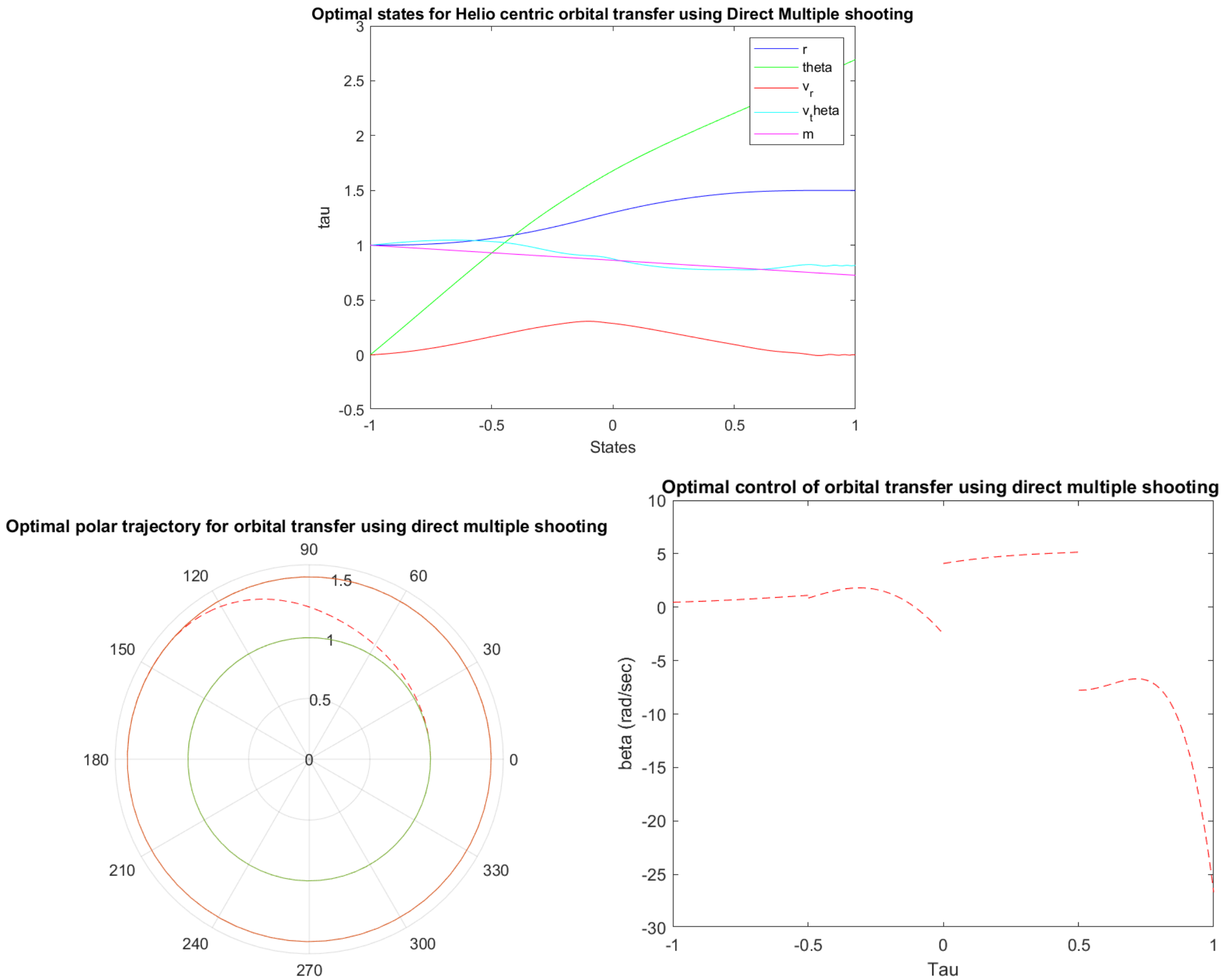


Fig 30 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 6, partitions - 4					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
194	8881	3.655418E+00	2.217000E-08	8.817000E-05	1.230000E-02
Objective		**Optimized Final Mass**			7.262091848377180E-01
Final time (secs)		3.655418E+00	**Angle of entry to terminal orbit (Deg)**		1.543417E+02
Performance of code		Time Elapsed for Computation (Seconds)			3.446440E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 6th degree and 8 partitions:

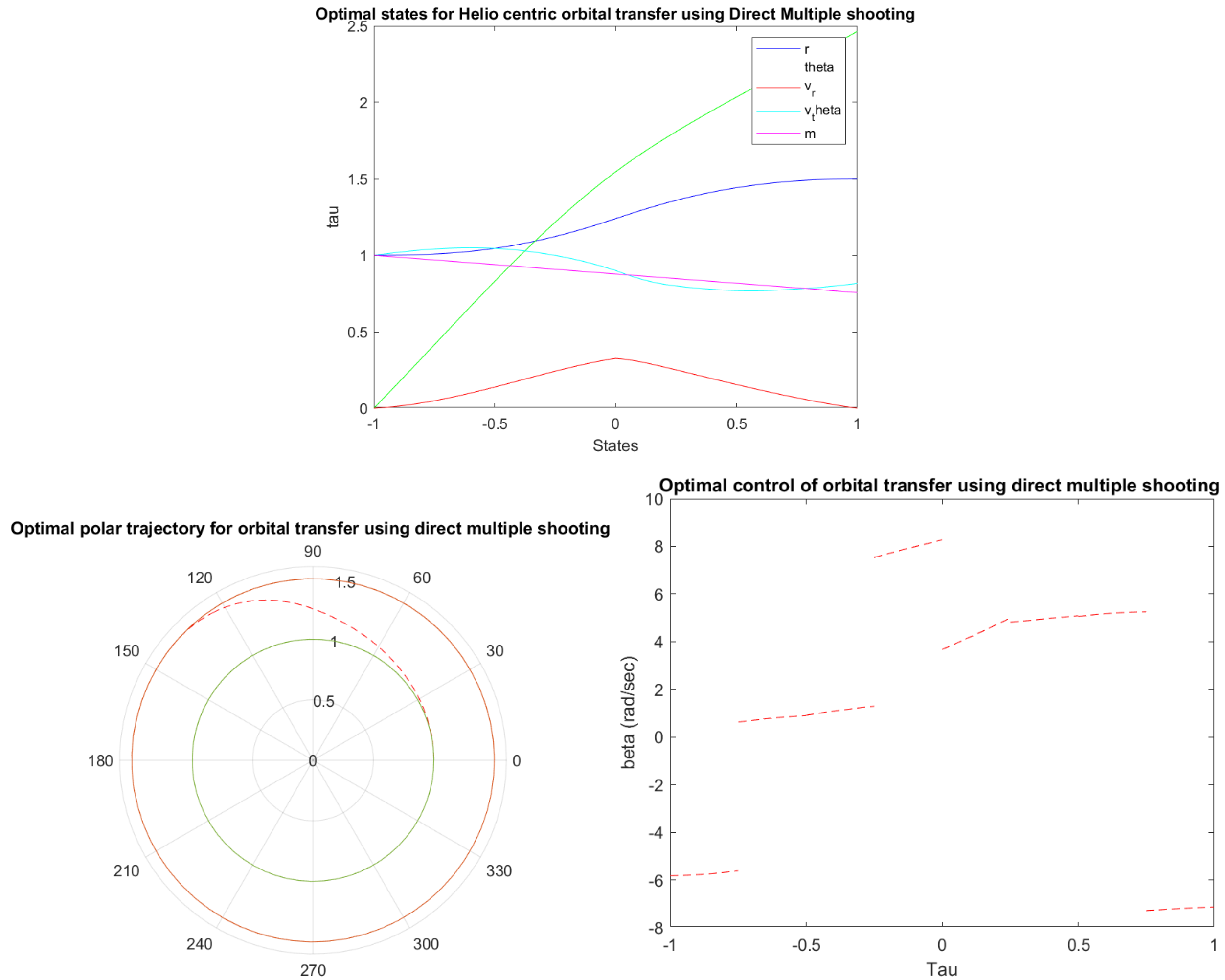


Fig 31 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 6, partitions - 8					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
124	11662	3.248903E+00	7.318000E-09	3.847000E-05	6.103000E-03
Objective		**Optimized Final Mass**			7.566571500847960E-01
Final time (secs)		3.248903E+00	**Angle of entry to terminal orbit (Deg)**		1.411774E+02
Performance of code		Time Elapsed for Computation (Seconds)			4.845673E+01
**** Local minima found that satisfies the constraints**					

Results for Direct multiple shooting with control parameterized to 6th degree and 16 partitions:

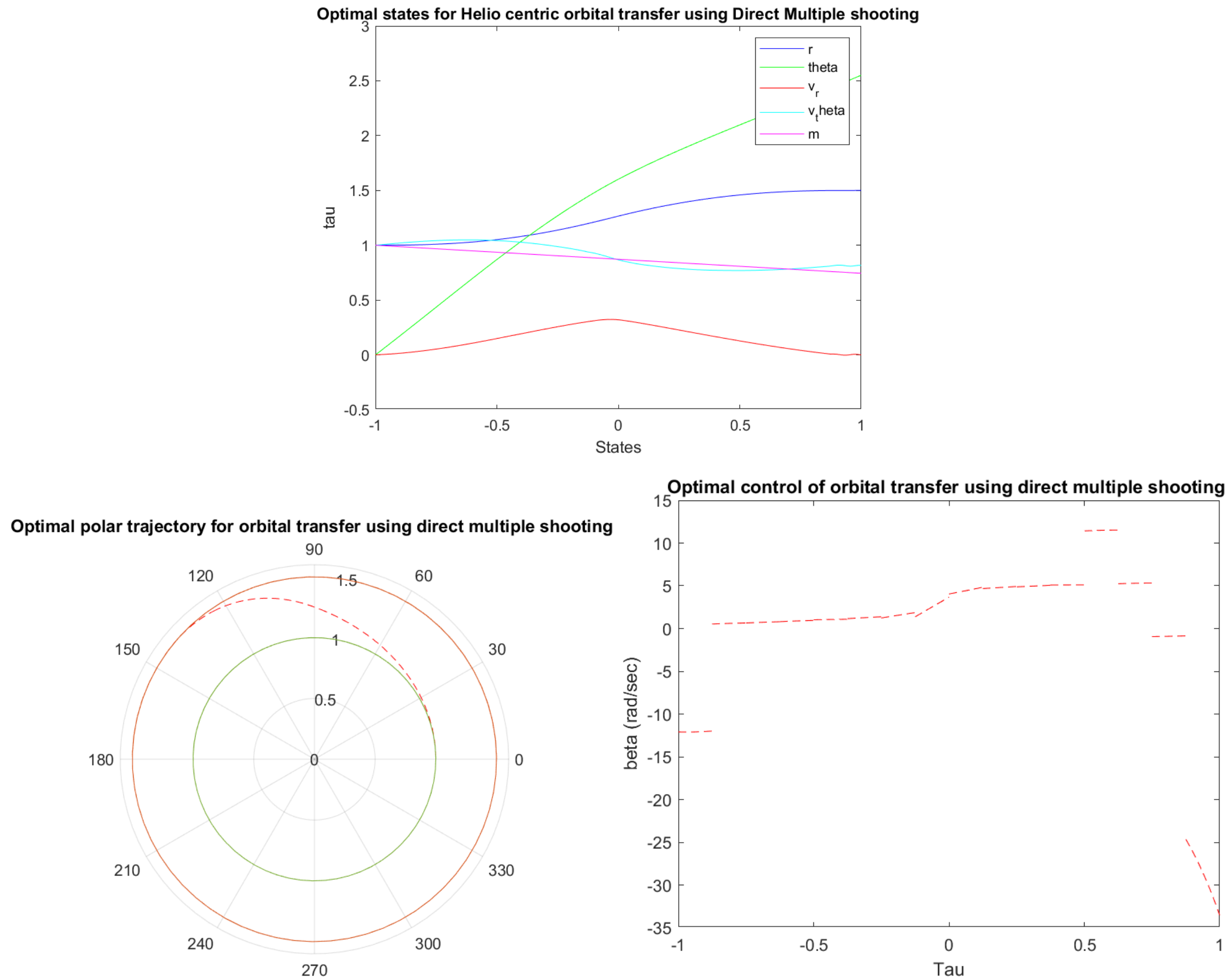


Fig 32 Optimal states, Optimal polar trajectory, Input Control for Direct Multiple shooting

Final Iteration values for Direct single shooting with polynomial degree 6, partitions - 16					
Iteration	**Func-count**	**f(x)**	**Feasibility**	**1st order optimality**	**Norm of step**
182	34634	3.410641E+00	3.445000E-07	8.007000E-05	1.347000E-01
Objective		**Optimized Final Mass**			7.445429461070460E-01
Final time (secs)		3.410641E+00	**Angle of entry to terminal orbit (Deg)**		1.461383E+02
Performance of code		Time Elapsed for Computation (Seconds)			2.459830E+02
**** Local minima found that satisfies the constraints**					

Conclusions for Direct Multiple Shooting:

A Non-linear problem was optimized using the Direct Multiple shooting for minimizing the time taken by a spacecraft to enter an helio-centric circular orbit of radius 1.5 from an helio-centric circular orbit of radius 1.5. It was observed that the optimal solution for transfer time using the Direct multiple shooting was found to be 3.248560E+00 seconds. Maximized mass was 7.566828678897650E-01. The angle of spacecraft position while entering the terminal orbit with respect to the local horizontal is 1.411420E+02° (appr.) to 141. 142°.These values were found for a combination of degree of polynomial to be 4 and No. of partitions to be 16.

Observations from code performance:

In-order to reduce the computation time for the Direct multiple shooting the "*Tolerance on Function was reduced to 1e-4*". It was observed that the overall time for processing increases due to increase in both the Number of partitions, Degree of input polynomial and combined as well. By keeping polynomial as constant and increasing no. of the partitions leads to increase in the time taken to compute individual iteration. By keeping no. of partitions constant and increasing the degree of leads to increase in overall time for computation along with the number of Iterations.

Observations in Direct Single Shooting vs Direct Multiple shooting:

It was observed that the using the Direct single shooting only the input polynomial with degrees 3 and 4 were solved. Whereas local minima were found for all the input polynomial degrees ranging from 2 to 6 and for all the intervals of 2,4,8,16. Also the final time was better optimized using the multiple shooting technique.

Observations in In-Direct Single Shooting vs In-Direct Multiple shooting:

It was observed that the terminal state equation was solved using both the techniques and results were almost the same for final time and final mass. As the no. of intervals increases the time for computing the results increases.