* Optimal Control * Bonus Assignment – 1 *

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Problem-1 :- Hyper Sensitivity optimal Control problem.

Determine $x(t)$ & $u(t)$ $t \in [0, t_f)$

minimize the Objective functional

$$J = \frac{1}{2} \int_0^{t_f} x^2(t) + u^2(t)\, dt.$$

STC.

$$\dot{x} = -x + u,$$

Boundary Conditions $x(0) = x_0 = 1.$

$y(t_f) = x_f = 1 \quad t_f = \text{fixed}$

Sol'n Given: $J = \int_0^{t_f} x^2(t) + u^2(t)\, dt$.

STC $\dot{x} = -x+u$.

Boundary Conditions $x(0) = x_0 = 1$

$y(t_f) = x_f = 1$ $t_f =$ fixed $\Rightarrow \delta t_f = 0$.

Hamiltonian equation:

$$H = L + \lambda^T \underline{f} \quad \text{where} \quad L = \frac{1}{2}(x^2+u^2) \quad f = (-x+u)$$

$$= \frac{1}{2}(x^2+u^2) + \lambda^T(-x+u)$$

$\lambda =$ euler multiplier.

State Equation:

$$\boxed{\dot{x} = -x+u}$$

$$\frac{\partial H}{\partial \lambda} = \dot{x}^*(t)$$

Co-state equation:

$$\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x}(x^*(t), u^*(t), \lambda^*(t), t)$$

$$\Rightarrow \dot{\lambda}^*(t) = -\frac{\partial}{\partial x}\left(\frac{1}{2}(x^2+u^2)\right) + \lambda(-x+u)$$

$$\boxed{\dot{\lambda}^*(t) = -x^*(t) + \lambda^*(t)}$$

\# $\frac{\partial H}{\partial u} = 0$; $\Rightarrow \frac{\partial}{\partial u}\left(\frac{1}{2}(x^2+u^2) + \lambda^T(-x+u)\right)$

$$= u + \lambda \Rightarrow u + \lambda = 0.$$

Optimal Control $\boxed{u^*(t) = -\lambda^*(t).}$

For solving this equations we have boundary conditions as

$x(0) = x_0 = 1$; $y(t_f) = x_f = 1$; t_f is fixed.

As all the values of x_0, x_f, t_0, t_f are fixed so transversality conditions are not applied.

Solving for $x(t)$ & $\lambda(t)$ analytically.

$$\dot{x} = -x + u = -x - \lambda$$
$$\dot{\lambda} = -x + \lambda$$

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

After computing the code for solving problem we got λ_0 value as 0.4142.

Using $x_0 = 0$; $x_f = 1$; $\lambda_0 = 0.4142$;

$$\dot{x} = -x + u \quad ; \quad u^* = -\lambda^*$$

$$u = \dot{x} + x.$$

Code is attached to the file.

Evaluated results for problem-1:

$\lambda(t_f) = 0.4142.$

$x(t_f) = 1;$

Problem-2 :

Minimize t_f.

Subjected to Constraints

$$\dot{x} = v \sin\theta$$
$$\dot{y} = v \cos\theta$$
$$\dot{v} = g \cos\theta$$

$x(t_0) = x_0 = 0$; $\quad x(t_f) = x_f = 2$
$y(t_0) = y_0 = 0$; $\quad y(t_f) = y_f = 2$
$v(t_0) = v_0 = 0$; $\quad v(t_f) = t_f = \text{free}$.

Hamiltonian :

$$H = \lambda_x v \sin\theta + \lambda_y v \cos\theta + \lambda_v g \cos\theta$$

$$\Rightarrow H = \lambda_x v \sin\theta + (\lambda_y v + \lambda_v g) \cos\theta$$

for optimal Control : $\frac{\partial H}{\partial \theta} = 0$.

$$\frac{\partial H}{\partial \theta} = \lambda_x v \cos\theta - (\lambda_y v + \lambda_v g) \sin\theta = 0$$

$$\lambda_x v \cos\theta - (\lambda_y v + \lambda_v g) \sin\theta = 0$$

$$\dot{\lambda}_x = 0 = -\frac{\partial H}{\partial x}$$

$$\dot{\lambda}_y = 0 = \frac{-\partial H}{\partial y}$$

$$\dot{\lambda}_v = \frac{-\partial H}{\partial v} = -\lambda_x \sin\theta - \lambda_y \cos\theta$$

Matlab Codes for solving $x(t), y(t), v(t)$ for final time are calculated and the results are as follows.

$$\boxed{\begin{array}{l} \text{① Minimized value of } t_f = 0.8165 \text{ seconds.} \\ \text{② } \lambda_x^* = -0.1477; \\ \text{③ } \lambda_y^* = -0.0564; \\ \text{④ } \lambda_v^* = -0.1000; \end{array}}$$