Exercises on

Quantum Field Theory

(Based on the lectures by *Dr. M Arshad Momen*)

Contents

1 Exercise for Class-2 [Lecture (-1)]

Derive the Euler–Lagrange equations corresponding to this action:

$$S[x] = m \int \sqrt{\eta_{ab}\, dx^a dx^b} = m \int d\tau = m \int \sqrt{\eta_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda}}\, d\lambda$$

2 Exercise for Class-4 [Lecture (1)]

2.1 Assignment-1

How does $\hat{J}^{rs}$ transform under Poincaré transformation? (It's a semi direct product)

2.2 Assignment-2

Recover $3D$ rotation algebra from angular momentum.

3 Exercise for Class-5 [Lecture (2)]

3.1 Exercise-1

Show that we can recover the Lorentz force from the action:

$$S = \frac{1}{2} \int m\, \dot{\xi}^2 d\tau + q \int A_b(\xi) \dot{\xi}^b\, d\tau$$

[This latter interaction term is geometrical as well]

3.2 Exercise-2

Check the commutations:

$$\begin{aligned} [J, p] &\sim p \\ [p, p] &\sim 0 \\ [J, J] &\sim J \\ [J, p^2] &\sim 0 \\ [J, p_a p^a] = [J, p_a]p^a + p_a[J, p^a] &\sim 0 \end{aligned}$$

4 Exercise for Class-6 [Lecture (3)]

4.1 Exercise-1

Show that the lie derivatives of vector field satisfies:

$$\mathfrak{L}_{\hat{V}} \mathfrak{L}_{\hat{W}} - \mathfrak{L}_{\hat{W}} \mathfrak{L}_{\hat{V}} = \mathfrak{L}_{[\hat{V}, \hat{W}]}$$

4.2 Exercise-2

Derive the lie derivative for a tensor $\mathfrak{L}_{\hat{V}}\, \tilde{T}^a{}_b$.