

Exercises on

Quantum Field Theory

(Based on the lectures by *Dr. M Arshad Momen*)

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1 Exercise for Class-2 [Lecture-1]

Derive the Euler–Lagrange equations corresponding to this action:

$$S[x] = m \int \sqrt{\eta_{ab} dx^a dx^b} = m \int d\tau = m \int \sqrt{\eta_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda}} d\lambda$$

2 Exercise for Class-4 [Lecture-1]

2.1 Assignment-1

How does \hat{J}^{rs} transform under Poincaré transformation? (It's a semi direct product)

2.2 Assignment-2

Recover 3D rotation algebra from angular momentum.

3 Exercise for Class-5 [Lecture-2]

3.1 Exercise-1

Show that we can recover the Lorentz force from the action:

$$S = \frac{1}{2} \int m \dot{\xi}^2 d\tau + q \int A_b(\xi) \dot{\xi}^b d\tau$$

[This latter interaction term is geometrical as well]

3.2 Exercise-2

Check the commutations:

$$\begin{aligned} [J, p] &\sim p \\ [p, p] &\sim 0 \\ [J, J] &\sim J \\ [J, p^2] &\sim 0 \\ [J, p_a p^a] &= [J, p_a] p^a + p_a [J, p^a] \sim 0 \end{aligned}$$

4 Exercise for Class-6 [Lecture-3]

4.1 Exercise-1

Show that the lie derivatives of vector field satisfies:

$$\mathfrak{L}_{\hat{V}} \mathfrak{L}_{\hat{W}} - \mathfrak{L}_{\hat{W}} \mathfrak{L}_{\hat{V}} = \mathfrak{L}_{[\hat{V}, \hat{W}]}$$

4.2 Exercise-2

Derive the lie derivative for a tensor $\mathfrak{L}_{\hat{V}} \tilde{T}^a{}_b$.

5 Exercise for Class-7 [Lecture-4]

- Calculate $\mathfrak{L} g_{ab}$

Hint : Let choose,

$$g_{ab} = P_a Q_b$$

$$\mathfrak{L}_v(P_a Q_b) = (\mathfrak{L}_v P_a) Q_b + P_a (\mathfrak{L}_v Q_b)$$

$$\mathfrak{L}_v g_{ab} = V^c \partial_c g_{ab} + g_{ac} \partial_b V^c + g_{cb} \partial_a V^c$$

- Verify if the partial derivative is switched with the covariant derivative ($\partial \rightarrow \nabla$) the Lie derivative won't change.

6 Exercise for Class-8 [Lecture-5]

Derive the equation of motion for this action and compare with the KG equation to determine the constants -

$$S[\phi] = \alpha \phi_i \phi_i + \beta \partial \phi_i \partial \phi_i + \gamma_i \phi_i + K_i \partial \phi_i$$

Note: The last term is important for π -meson.