Universiti Teknologi MARA Cawangan Kedah Kampus Sungai Petani

2021 ICMS INTERNATIONAL CONFERENCE ON COMPUTING, MATHEMATICS AND STATISTICS

RADIATIVE CASSON FLUID OVER A SLIPPERY VERTICAL RIGA PLATE WITH VISCOUS DISSIPATION AND BUOYANCY EFFECTS

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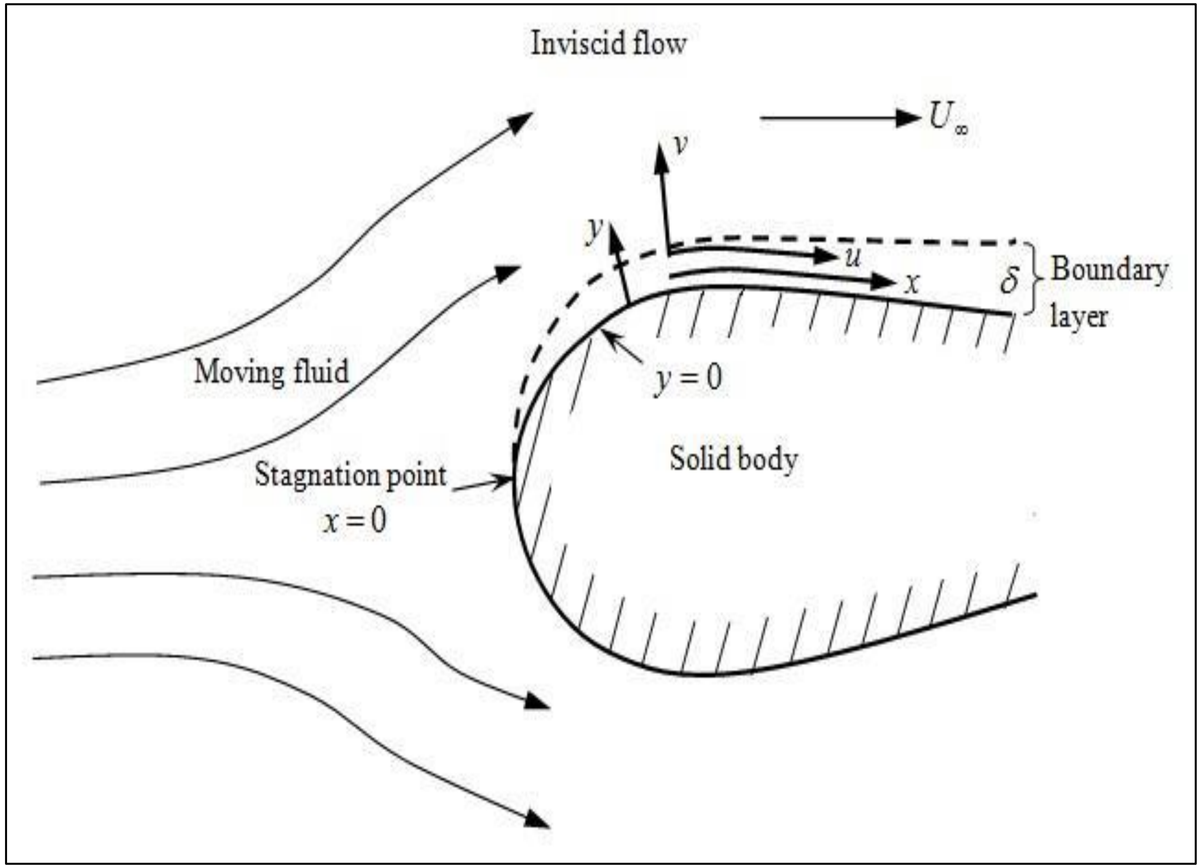
Introduction

Boundary layer



Boundary layer formation and separation on a circular surface.

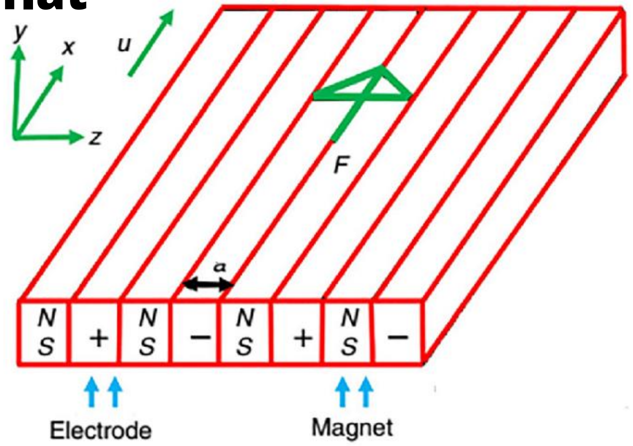
- **Was introduced by Ludwig Prandtl in 1904.**
- **The first region is a very thin layer close to the surface where viscosity and friction effects cannot be neglected.**

Introduction

Casson Fluid:

- One of non-Newtonian fluid
- The viscosity decreases or increases when shear is applied
- Example: tomato sauce, concentrated fruit juice

Riga Plate: An electromagnetic actuator which includes span-aligned series of alternating electrodes and permanent magnets placed on a flat surface



Viscous dissipation:
Responsible for the instability during the process of heat transfer which can lead to an unpredictable distribution of the temperature

Objectives

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1

To investigate the radiative Casson fluid in the presence of the magnetic field, velocity slip, thermal slip, viscous dissipation and buoyancy effects over permeable slippery vertical Riga stretching and shrinking plate.

2

To transform the governing partial differential equations (PDEs) to ordinary differential equations (ODEs) using similarity transformations and solve numerically using a boundary value problem solver (BVP4C) in MATLAB software.

3

To examine the effects of governing parameters such as Casson parameter, modified Hartmann number and suction/injection and slip boundary conditions on velocity and temperature profiles as well as skin friction coefficient and local Nusselt number

Literature Review/Justifications

Boundary layer flow (Prandtl, 1904)

Non-Newtonian fluid and Casson Fluid

- Alwawi et al. (2019)
- Ullah et al. (2017)
- Haldar et. al (2018)
- Awais et al. (2021)
- Anwar et al. (2021)

Stretching/ Shrinking Sheet

- Crane (1970)
- Wang (1990)
- Vajravelu & Mukhopadhyay (2018)
- Lok et al. (2011)
- Lund et al. (2021)
- Mousavi et al. (2021)

Viscous Dissipation

- Requile (2020)
- Hussanan et al. (2016)
- Iqbal et. al (2018)
- Nayak et al. (2021)
- Yusof et al. (2020)
- Eldabe et al. (2021)

Study the viscous dissipation on Casson fluid over a vertical Riga plate with radiation and buoyancy effects

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

Methodology

GOVERNING EQUATION IN PDEs

Continuity Equation :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Momentum Equation :

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\upsilon\left(1+\frac{1}{B}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\pi j_o M}{8\rho}e^{\left(\frac{\pi}{\alpha_1}y\right)}+g\beta\left(T-T_\infty\right)$$

Energy Equation :

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3k^*\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(1+\frac{1}{B}\right)\left(\frac{\partial u}{\partial y}\right)^2$$

Boundary Conditions :

$$u=\lambda u_w\left(x\right)+N\frac{\partial u}{\partial y},\quad v=-V_w\left(x\right),\quad T=T_w+D\frac{\partial T}{\partial y}\text{ at } y=0$$

$$u\rightarrow u_e\left(x\right),\ T\rightarrow T_\infty,\quad \text{as}\quad y\rightarrow\infty$$

Methodology

TRANSFORMED MOMENTUM EQUATION:

$$\left(1+\frac{1}{B}\right)f'''(\eta)+2-2f'^{2}(\eta)+f(\eta)f''(\eta)+2Qe^{(-A\eta)}+2\sigma\theta=0$$

TRANSFORMED ENERGY EQUATION:

$$\left(1+\frac{4}{3}Rd\right)\theta''(\eta)+\Pr\theta'(\eta)f(\eta)-\Pr f'(\eta)\theta(\eta)+Ec\left(1+\frac{1}{B}\right)f''^{2}(\eta)=0$$

TRANSFORMED BOUNDARY CONDITIONS:

$$f'(0)=\lambda+\omega f''(0),\quad f(0)=S,\quad \theta(0)=1+\varepsilon\theta'(0)$$

$$f'(\infty)\to 1,\quad \theta(\infty)\to 0 \quad \text{as} \quad y\to\infty.$$

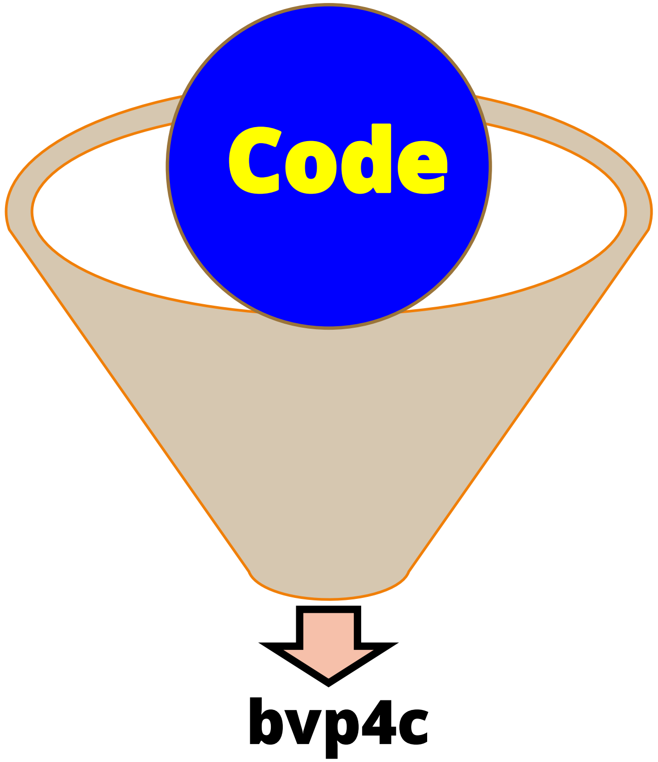
Methodology

BVP4C IN MATLAB

$$\sqrt{\frac{2L}{x}}\,\frac{\sqrt{\mathrm{Re}}}{\left(1+\frac{1}{B}\right)}C_f = f''(0)$$

$$\frac{Nu}{\sqrt{\frac{x}{2L}}\sqrt{\mathrm{Re}}\left(1+\frac{4}{3}Rd\right)} = -\theta'(0)$$

Mathematical Formulation of Governing Parameters	Name of Governing Parameters
$C_f = \frac{\tau_w}{\rho u_w^2(x)}$	**Skin Friction**
$Nu = \frac{x\, q_w}{k(T_w - T_\infty)}$	**Local Nusselt number**
$\tau_w = \left(\mu + \frac{p_y}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}$	**Wall shear stress**
$q_w = -k\left(1+(4/3)Rd\right)(\partial T/\partial y)_{y=0}$	**Rate of heat transfer**
$\mathrm{Re} = x u_w(x)/\upsilon$	**Local Reynolds number**



Results and Discussions

Table 1: Comparison of heat transfer coefficient $-\theta'(0)$ for the various value of B as $Q = Rd = Ec = A = \omega = 0$, $S = 5.0$, $\varepsilon = 3.33$, $\Pr = 0.7$, $\lambda = -1.0$ and $\sigma = 0.0$.

ε	B	Haldar at el. (2018)	Yusof et al. (2020)	Present study
3.33	1.0	0.2755	0.2755	0.2763
	1.4	0.2756	0.2756	0.2764
	2.0	0.2756	0.2756	0.2766
	2.4	0.2756	0.2756	0.2766

Results and Discussions

Table 2: Skin friction coefficient $f''(0)$ and local Nusselt Number $-\theta'(0)$ for different values of B when $Q = Rd = Ec = A = \omega = S = \Pr = \varepsilon = 1, \sigma = 0.2$ for $\lambda = -1$ and 1.

λ	B	$f''(0)$	$-\theta'(0)$
-1	1	1.44642945	0.31453586
	3	1.58940614	0.39861559
	5	1.62494292	0.41526553
	1000	1.68425844	0.43984523
1	1	0.16805332	0.53200768
	3	0.19405006	0.53362593
	5	0.20077725	0.53402820
	1000	0.21225288	0.53469730

Results and Discussions

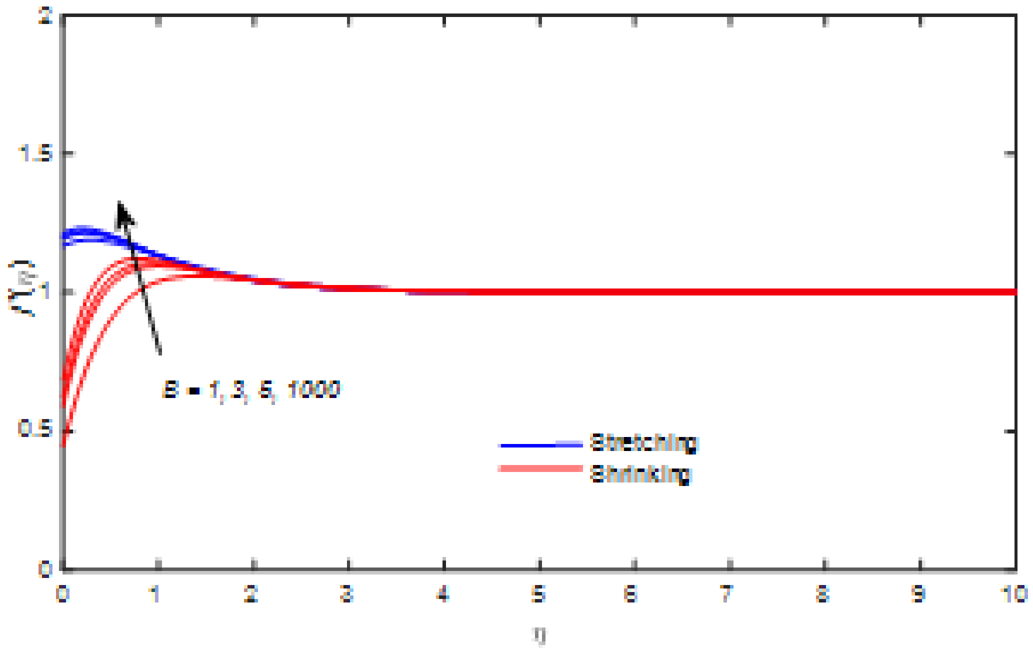


Figure 1: Velocity profile, $f'(\eta)$ for various values of B when $Q = Rd = Ec = A = \omega = S = \Pr = \varepsilon = 1, \sigma = 0.2$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

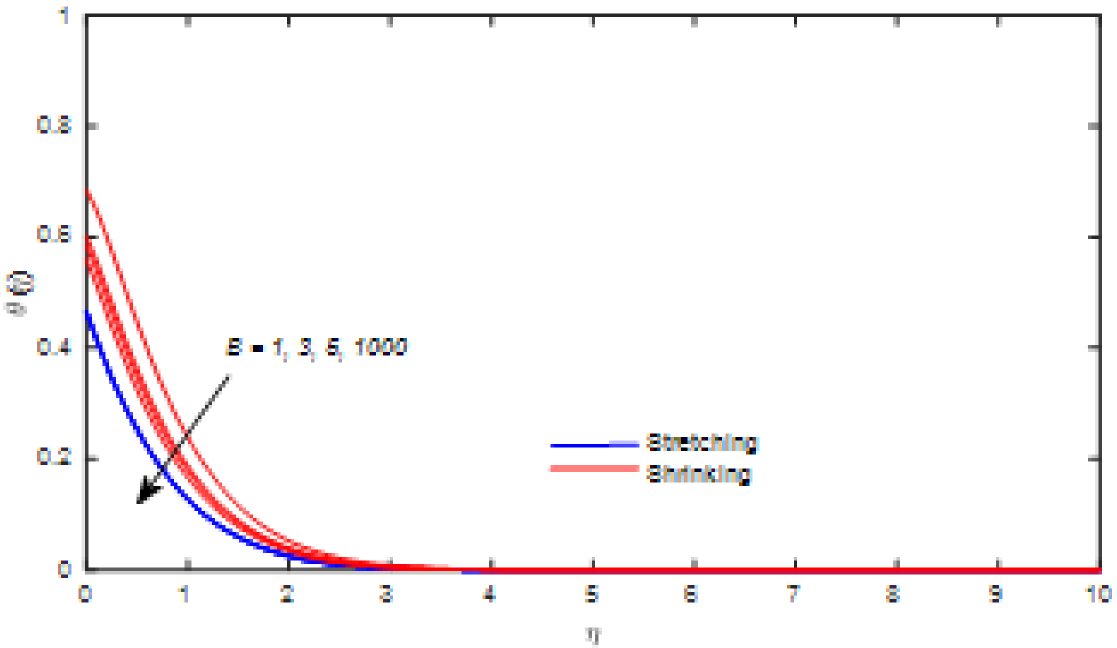


Figure 2: Temperature profile, $\theta(\eta)$ for various values of B when $Q = Rd = Ec = A = \omega = S = \Pr = \varepsilon = 1, 1, \sigma = 0.2$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

$B \uparrow \quad f'(\eta) \uparrow \quad \theta(\eta) \downarrow$

Results and Discussions

Table 3: Skin friction coefficient, $f''(0)$ and local Nusselt number $-\theta'(0)$, for different values Q and λ when $Rd = \Pr = Ec = A = \omega = S = B = 1$ and $\varepsilon = 1$.

λ	Q	$f''(0)$	$-\theta'(0)$
-1.0	0.5	1.35277636	0.30396290
	2.5	1.69979144	0.32938878
	4.5	1.99302104	0.32663310
1.0	0.5	0.09264741	0.52814975
	2.5	0.37861293	0.53509000
	4.5	0.63159076	0.52633580

Results and Discussions

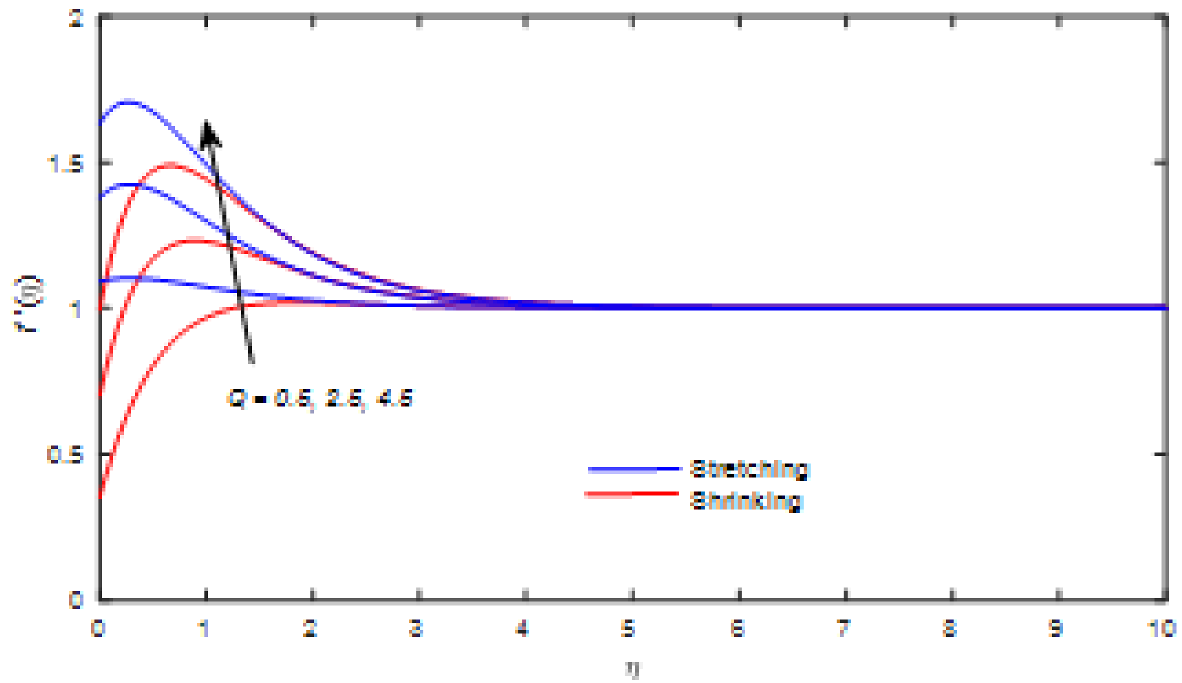


Figure 3: Velocity profile, $f'(\eta)$ for various values of Q when $Rd = Pr = Ec = A = \omega = S = B = \varepsilon = 1$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

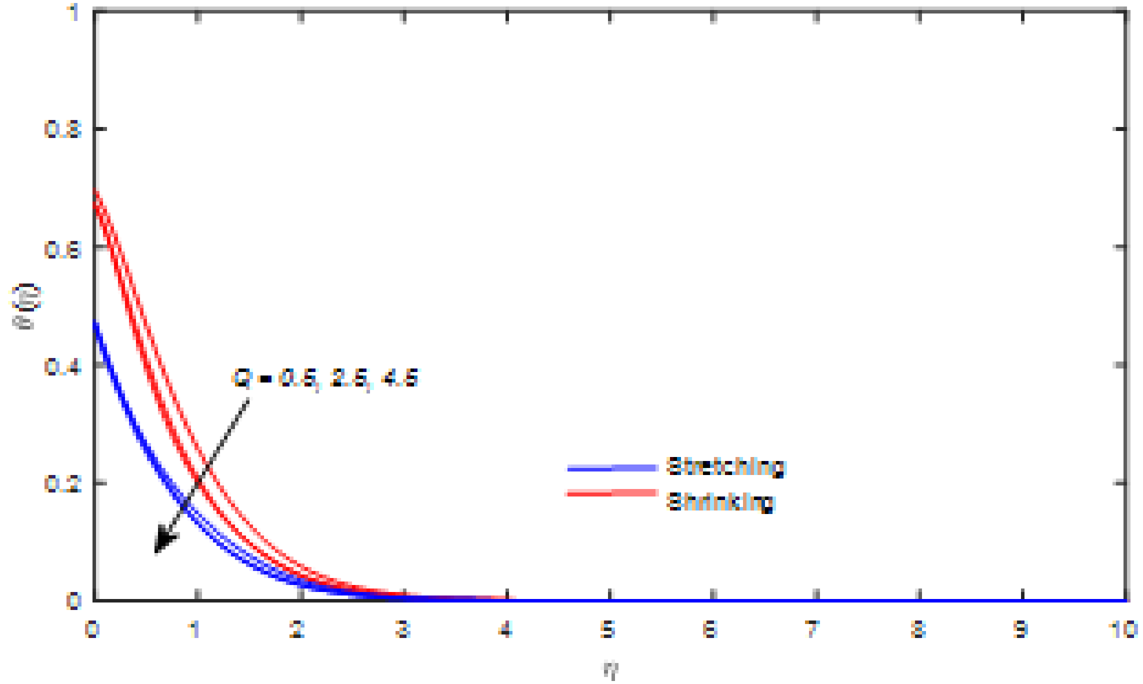


Figure 4: Temperature profile, $\theta(\eta)$ for various values of Q when $Rd = Pr = Ec = A = \omega = S = B = \varepsilon = 1$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

$Q \uparrow \quad f'(\eta) \uparrow \quad \theta(\eta) \downarrow$

Results and Discussions

Table 4: Skin friction coefficient, $f''(0)$ and local Nusselt number $-\theta'(0)$, for different values S and λ when $Rd = \Pr = Ec = A = \omega = Q = B = 1$ and $\varepsilon = 1$.

λ	S	$f''(0)$	$-\theta'(0)$
-1.0	-2.0	1.15464862	-0.04630545
	0.0	1.36294349	0.19533344
	2.0	1.51754415	0.41634897
1.0	-2.0	0.18002435	0.33130890
	0.0	0.17396785	0.46706254
	2.0	0.16114898	0.58940549

Results and Discussions

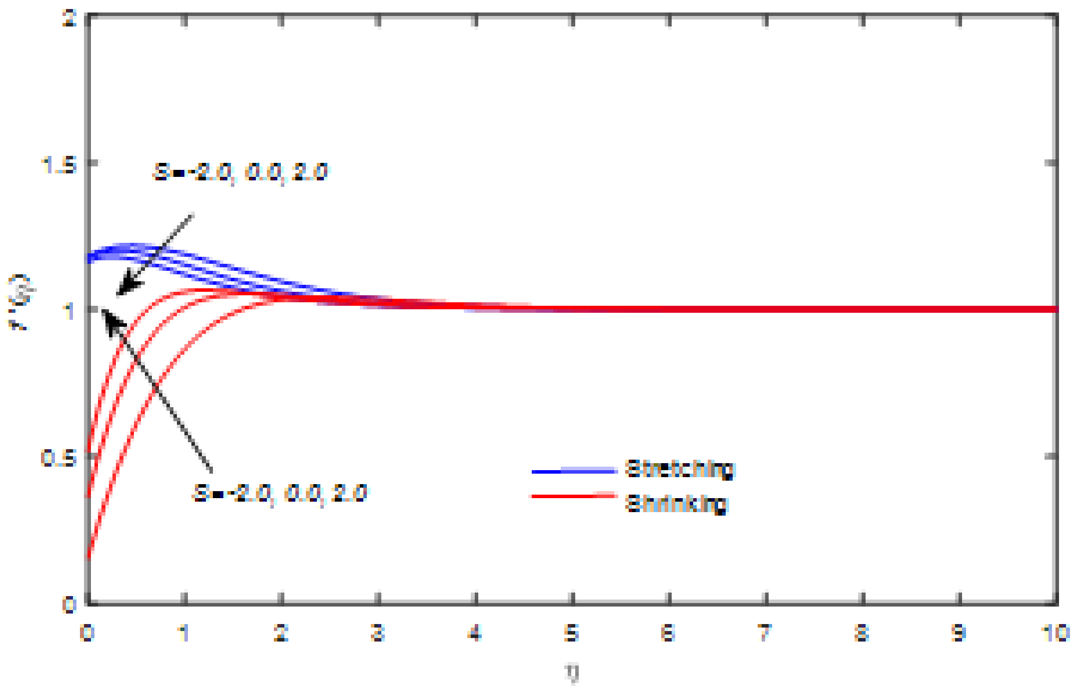


Figure 5: Velocity profile, $f'(\eta)$ for various values of S when $Rd = \Pr = Q = A = \omega = Ec = B = \varepsilon = 1$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

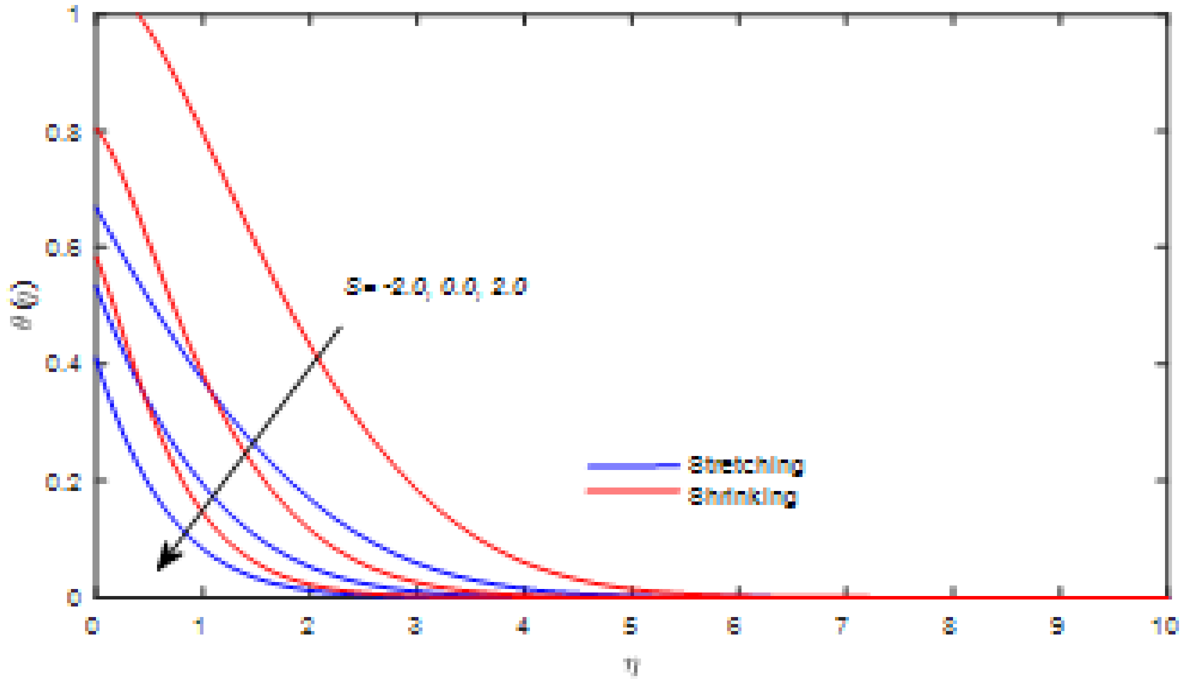


Figure 6: Temperature profile, $\theta(\eta)$ for various values of S when $Rd = \Pr = Q = A = \omega = Ec = B = \varepsilon = 1$ for $\lambda = 1$ (stretching) and $\lambda = -1$ (shrinking).

$S \uparrow$ $f'(\eta)$ $\uparrow$ **shrinking** $\downarrow$ **stretching** $\theta(\eta) \downarrow$

Conclusions

Summary Results of the Velocity and Temperature Profiles

Parameters	$f'(\eta)$	$\theta(\eta)$
Casson Parameter, *B*	**Increase**	**Decrease**
Modified Hartmann Number, *Q*	**Increase**	**Decrease**
Suction/ Injection, *S*	**Increase (Shrinking)** **Decrease (Stretching)**	**Decrease**

THANK YOU

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