THE EFFECT OF COMPETITION BETWEEN TWO SPECIES ON A LOTKA-VOLTERRA FISHERY MODEL WITH THE PRESENCE OF TOXICITY

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Introduction

- Interaction between organisms in a population is now typical in today's ecosystem; they collaborate to keep the ecosystem running.
- Indirect competition - two or more organismic units compete for the same resources or when those resources are limited.
- Consumptive competition - exploitation competition - a process that leads to resource depletion or depression.
- Interspecific competition - competition for limited resources between two or more species and competition among individuals of the same species, usually within the same population.
- When one species is a stronger competitor, interspecific rivalry harms the other species by reducing population sizes and growth rates, which harms the competitor's population dynamics.

Objectives

1. To employ the stability analysis of competition model with the presence of toxicity.
2. To investigate the effect of competition coefficient between two species by using one-parameter bifurcation analysis.
3. To analyze the dynamical behavior of the two species as bifurcation parameters increased.

Literature Review

- Nearly a century ago, Lotka and Volterra's competitive equations had a significant impact on the development of modern ecological theory.

- Swain and Chatterjee (2017) employed the Lotka-Volterra population dynamics model; they summarized that determining the competitive coefficients is a critical component of competition.

Literature Review

- Gavina et al. (2018) studied the multi-species coexistence of competing systems with crowding efficiencies; the findings indicate that certain animals prefer to coexist.

- Haque and Sarwardi (2016) investigated the effect of toxicity on the harvested fisheries model of two competing fish species that released harmful toxic compounds; their findings indicate that the strength of the toxins emitted by both species may alter the qualitative nature of the proposed model.

Methodology

The competition fishery model, which is an extension of the Lotka-Volterra model, is represented as follows:

$$\frac{dX}{dt} = r_1 X\left(1 - \frac{X}{K}\right) - \alpha XY - \gamma_1 X^3 Y$$

$$\frac{dY}{dt} = r_2 Y\left(1 - \frac{Y}{L}\right) - \beta XY - \gamma_2 XY^2.$$

In the competition fishery model, populations of two species are forced into the competition, with the outcome being either a win or a loss.

Methodology

- In this research, the competition model are employed, and stability analysis is investigated using MAPLE software.
- For one-parameter bifurcation analysis, the parameter variation technique is used by choosing the competition coefficient (α) as a bifurcation parameter.
- The transcritical bifurcation point is captured by plotting the bifurcation diagram using the numerical software XPPAUT.
- The transcritical bifurcation point demonstrates how the system's stability changes for both species; these stability changes are then investigated.
- Finally, four different values of the bifurcation parameter are chosen to observe the population system's dynamics as the bifurcation parameter increases.

RESULTS AND DISCUSSION

1. Stability Analysis
2. Numerical Bifurcation Analysis
3. Dynamical Behavior of *X* And *Y* Species as Bifurcation Parameter is Varied

Results and Discussions

i) Stability Analysis

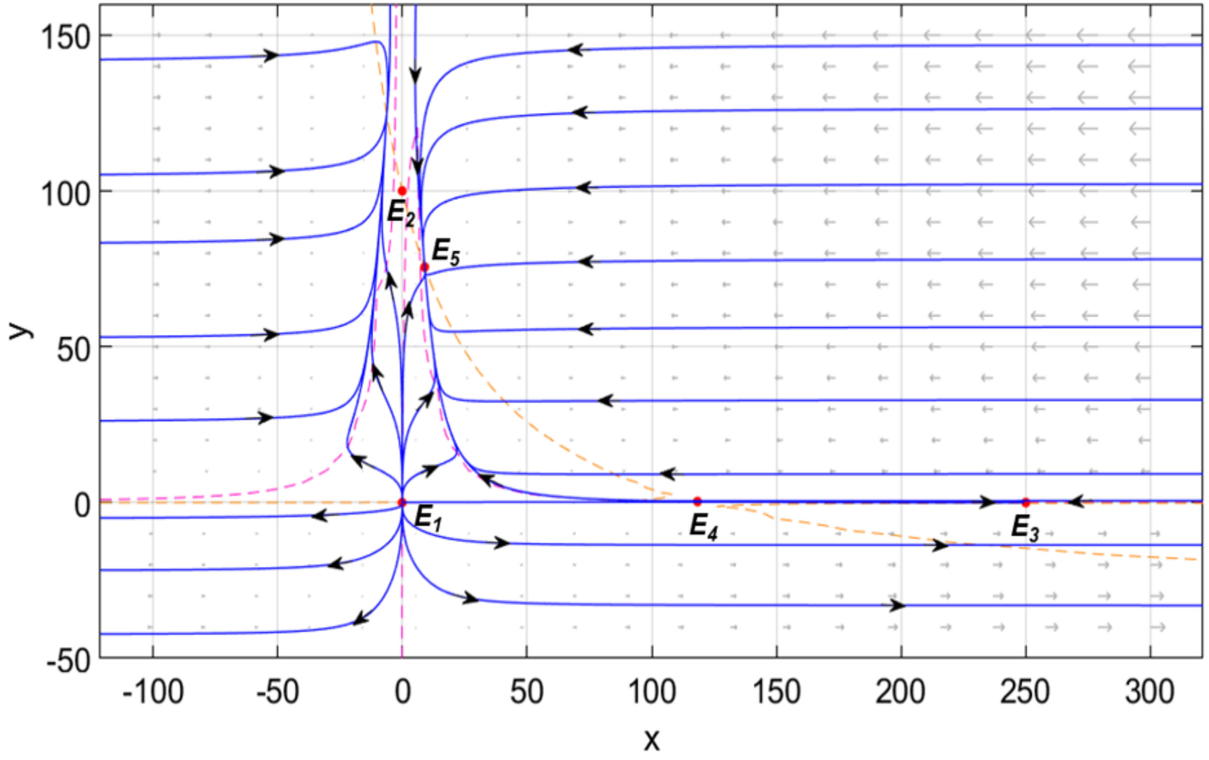

Figure 1: The phase-plane diagram of species X and Y of system (3) with parameter values $\alpha = 0.015, \beta = 0.01, r_1 = 3.7, r_2 = 1.2, \gamma_1 = 0.0004, \gamma_2 = 0.003, K = 250$ and $L = 100$.

Table 1: Summary of stability analysis of competition fishery model system (3).

Critical Points	Eigenvalues	Results
$E_1 = (0,0)$	$\lambda_1 = 3.7, \lambda_2 = 1.2$	Unstable node
$E_2 = (0,100)$	$\lambda_1 = 2.2, \lambda_2 = -1.2$	Unstable saddle node
$E_3 = (250,0)$	$\lambda_1 = -1.3, \lambda_2 = -3.7$	Asymptotically stable node
$E_4 = (118.352,0.347)$	$\lambda_1 = 0.3712, \lambda_2 = -6.0257$	Unstable saddle node
$E_5 = (8.973,75.571)$	$\lambda_1 = -0.8578, \lambda_2 = -5.2525$	Asymptotically stable node

Results and Discussions

ii) Numerical Bifurcation Analysis

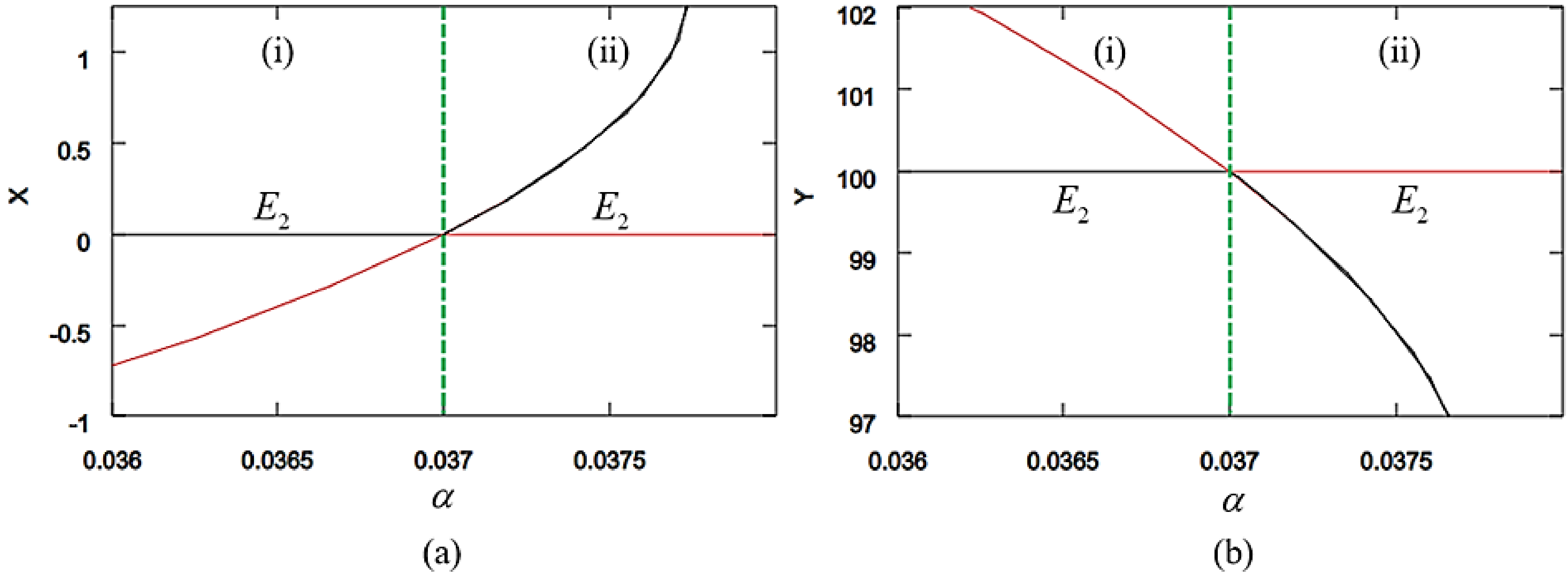

Figure 2: Bifurcation diagram of competition system with respect to competition coefficient α where $\beta = 0.01, \quad r_1 = 3.7, r_2 = 1.2, \gamma_1 = 0.0004, \gamma_2 = 0.003, K = 250$ and $L = 100$ for (a) X species and (b) Y species.

Results and Discussions

ii) Numerical Bifurcation Analysis

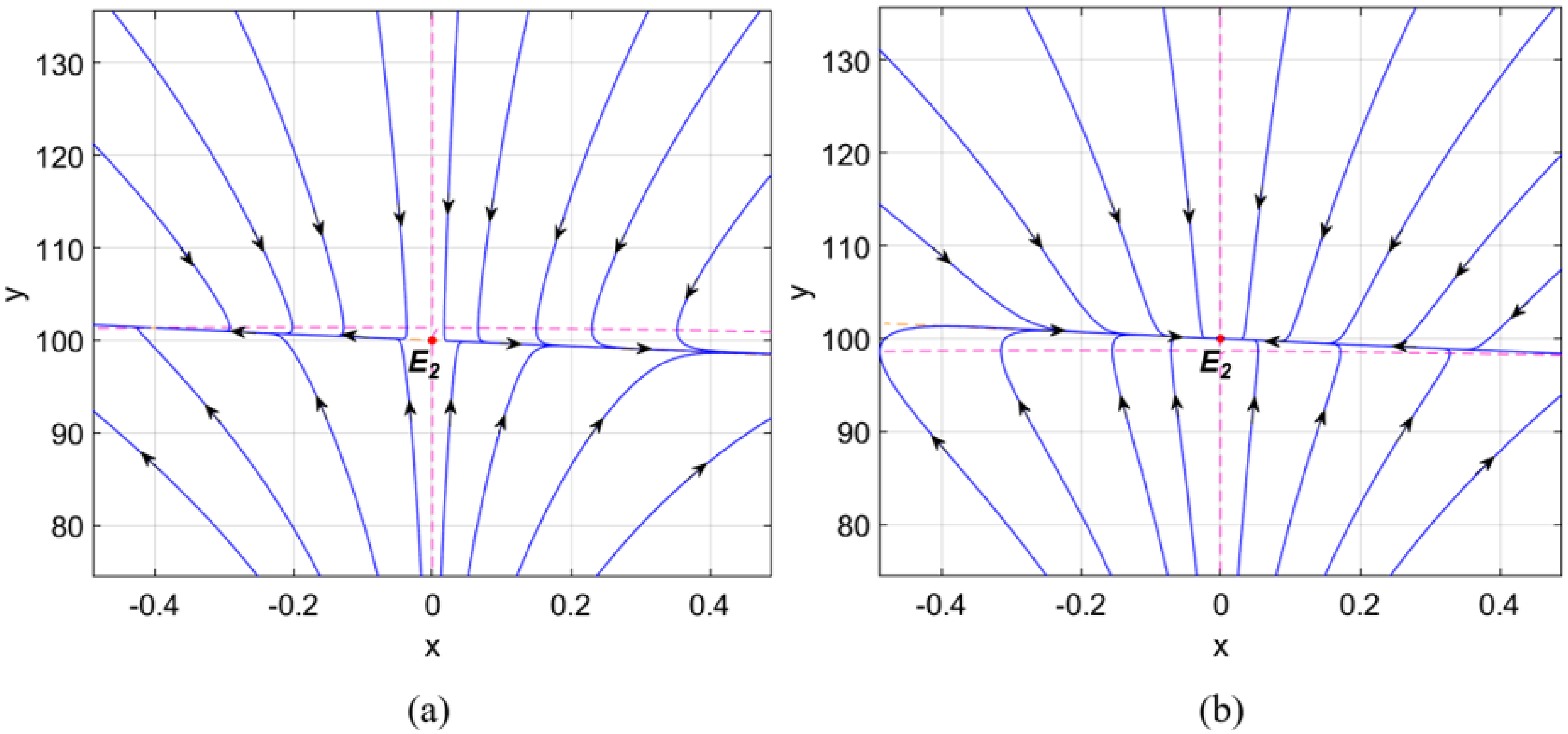

Figure 3: The phase-plane diagram of species X and Y of competition system with bifurcation parameter (a) $\alpha = 0.0365$ and (b) $\alpha = 0.0375$.

Results and Discussions

iii) Dynamical behavior of X and Y species as bifurcation parameter α increases

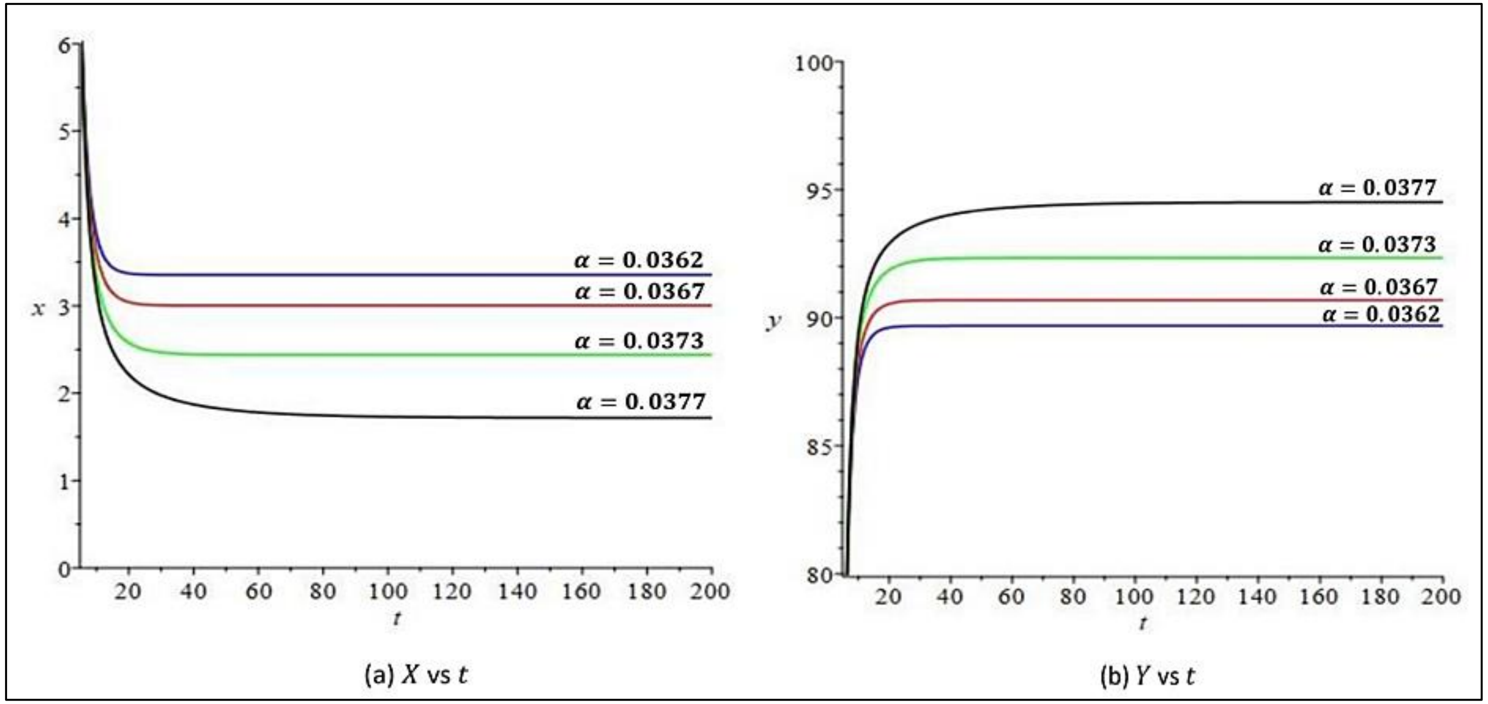

Figure 4: The time-series graph of species X and Y with varying bifurcation parameters α.

Table 2: The critical points for four different values of bifurcation parameter α

Bifurcation parameters	$E_5\ (x*, y*)$
$\alpha = 0.0362$	$(3.355, 89.681)$
$\alpha = 0.0367$	$(3.004, 90.686)$
$\alpha = 0.0373$	$(2.439, 92.337)$
$\alpha = 0.0377$	$(1.718, 94.509)$

Conclusions

- The competition coefficient between two species in a population with toxicity has been investigated intensively using one-parameter bifurcation analysis.
- The competition coefficient parameter was treated as a bifurcation parameter since it is a significant parameter that shapes the competition interaction model.
- Our findings suggest that different rates of competition coefficient can influence the dynamical behavior of both species.
- Environmental disturbances, for example, can disrupt the ecosystem and eliminate the competitor's advantage.

THANK YOU