UNSTEADY MHD CASSON FLUID FLOW IN A VERTICAL CYLINDER WITH POROSITY AND SLIP VELOCITY EFFECTS

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Introduction

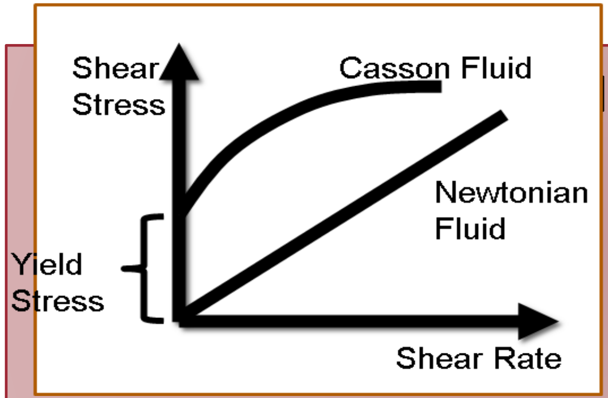

Figure 1. Rheology behaviour of Casson fluid

Casson Fluid

MHD

Magnetohydrodynamics

i. **study concerned with exploring the effects of crossing magnetic field within a moving electrically conducting fluid.**

Slip Velocity

i. **Finite velocity of a fluid at a boundary or surrounded within in that area.**
ii. **Applications: blood flow in arteries, drilling process**

Porous Medium

i. **Solid material with pores to enable a fluid to pass through or around them.**
ii. **Examples: fatty plaques and blood clots are formed in the artery.**

Objectives

1. To investigate the unsteady pulsatile flow of Casson fluid past through a cylinder in porous medium with MHD and slip velocity effects

2. To obtain analytical solution for velocity profile by using the Laplace and finite Hankel Transform methods.

3. To analyze the obtained analytical solutions for velocity profile with the involvement of physical parameters graphically.

Literature Review/Justifications

MHD fluid flow through a porous medium in a cylinder

Porous effects on the fractional modeling of magnetohydrodynamic pulsatile flow: an analytic study via strong kernels
Abro, K. et al. (2020)

Caputo–Fabrizio fractional order model on MHD blood flow with heat and mass transfer through a porous vessel in the presence of thermal radiation
Maiti et al. (2020)

Slip velocity at boundary of the cylinder

Transient electro-osmotic slip flow of an Oldroyd-B fluid with time fractional Caputo-Fabrizio derivative
Shah et al. (2019)

Effects of slip and magnetic field on the pulsatile flow of a Jeffrey fluid with magnetic nanoparticles in a stenosed artery
Padma et al. (2019)

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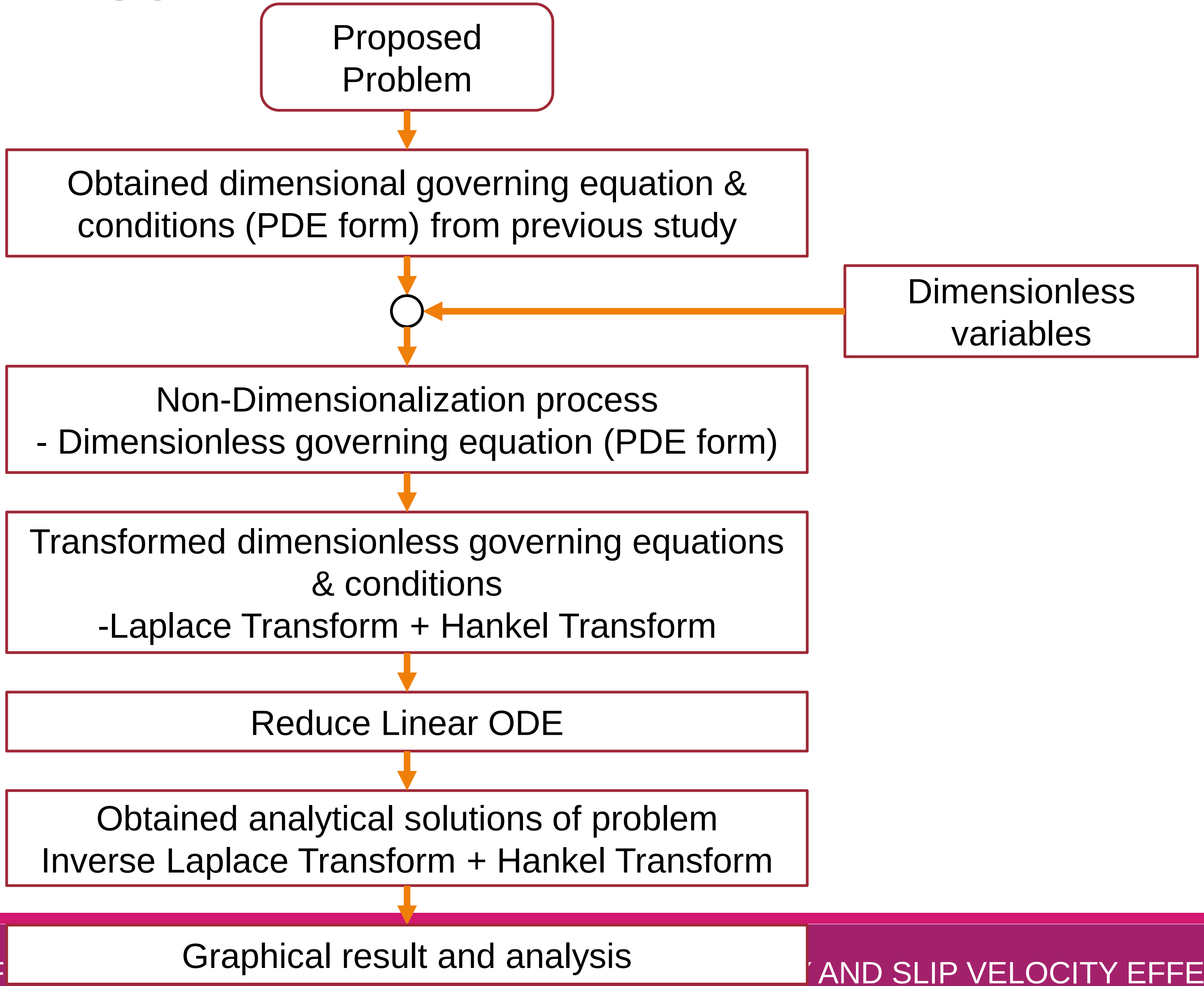
Methodology



Governing Equation

$$\rho\frac{\partial u^*}{\partial t^*}=-\frac{\partial p^*}{\partial z^*}+\mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)-\frac{\mu}{k_p}u^*-\sigma B_0^2 u^* \qquad (1)$$

Initial and Boundary Conditions

$$\left.\begin{aligned}
&u^*(r^*,0)=0 && ;\ r\in[0,r_0],\\
&\frac{\partial u^*(0,t^*)}{\partial r^*}=0 && ;\ t^*>0,\\
&u^*(r_0^*,t^*)=u_s^* && ;\ t^*>0
\end{aligned}\right\} \qquad (2)$$

Dimensionless Variables

$$t=\frac{t^{*}\nu}{r_0^{2}},\quad r=\frac{r^{*}}{r_0},\quad u=\frac{u^{*}}{u_0},\quad u_s=\frac{u_s^{*}}{u_0},\quad z^{*}=\frac{z}{r_0},\quad p^{*}=\frac{pr_0}{\mu u_0}. \tag{3}$$

Dimensionless Governing Equations

$$\frac{\partial u}{\partial t}=\left(A_0+A_1\cos\left(\omega t\right)\right)+\beta_1\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\frac{1}{Da}u-Mu. \qquad (4)$$

Where $Da=\frac{k_p}{r_0^2},\ M=\frac{\sigma B_0^2 r_0^2}{\rho\nu},\ \beta_1=\frac{1}{\beta_0},\ \beta_0=1+\frac{1}{\beta}$

Dimensionless Initial and Boundary Conditions

$$\left.\begin{aligned}
&u(r,0)=0, && ;\ r\in[0,1],\\
&\frac{\partial u(0,t)}{\partial r}=0, && ;\ t>0,\\
&u(1,t)=u_s, && ;\ t>0
\end{aligned}\right\} \qquad (5)$$

Laplace Transform

$$s\bar{u}(r,s)=\frac{A_0}{s}+\frac{A_1 s}{s^2+\omega^2}+\beta_1\left[\frac{\partial^2\bar{u}(r,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}(r,s)}{\partial r}\right]-\frac{1}{Da}\bar{u}(r,s)-M\bar{u}(r,s) \quad (6)$$

$$\frac{\partial\bar{u}(0,s)}{\partial r}=0, \qquad \bar{u}(1,s)=\frac{u_s}{s} \quad (7)$$

Finite Hankel Transform

$$\bar{u}_H(r_n,s)=\int_0^1 r\bar{u}(r,s)J_0(rr_n)\,dr \qquad (8)$$

$$\bar{u}_H(r_n,s)=\left[\left(\frac{A_0}{s}+\frac{A_1 s}{s^2+\omega^2}\right)\frac{J_1(r_n)}{r_n}+\frac{\beta_1 r_n J_1(r_n)u_s}{s}\right]\frac{1}{s+F_3} \qquad (9)$$

where $F_3=\dfrac{1+Da\left(M+\beta_1 r_n^2\right)}{Da}$

Inverse Laplace Transform

$$u_H(r_n,t)=\left(f_4(t)+f_5(t)\right)\frac{J_1(r_n)}{r_n}+\beta_1 r_n J_1(r_n)f_6(t) \qquad (10)$$

where

$$f_4(t)=\frac{A_0}{F_3}\left(1-\exp\left(-F_3 t\right)\right),$$

$$f_5(t)=\frac{A_1}{F_3^2+\omega^2}\left(F_3\cos(\omega t)+\omega\sin(\omega t)-F_3\exp\left(-F_3 t\right)\right),$$

$$f_6(t)=\frac{u_s}{F_3}\left(1-\exp\left(-F_3 t\right)\right).$$

Inverse Finite Hankel Transform

$$u(r,t)=2\sum_{n=1}^{\infty}u_H(r_n,t)\frac{J_0(rr_n)}{J_1^2(r_n)},$$ 11

$$u(r,t)=u_s-2u_s\sum_{n=1}^{\infty}\left[\exp(-F_3t)+\frac{c\left(1-\exp(-F_3t)\right)}{c+\beta_1r_n^2}\right]\frac{J_0(rr_n)}{r_nJ_1(r_n)}$$

$$+2\sum_{n=1}^{\infty}\left[\frac{A_0}{F_3}\left(1-\exp(-F_3t)\right)\right]\frac{J_0(rr_n)}{r_nJ_1(r_n)}$$

$$+2\sum_{n=1}^{\infty}\left[\frac{A_1}{F_3^2+\omega^2}\left(F_3\left(\cos(\omega t)-\exp(-F_3t)\right)+\omega\sin(\omega t)\right)\right]\frac{J_0(rr_n)}{r_nJ_1(r_n)}$$ 12

where $c=\frac{1}{Da}+M$

Results and Discussions

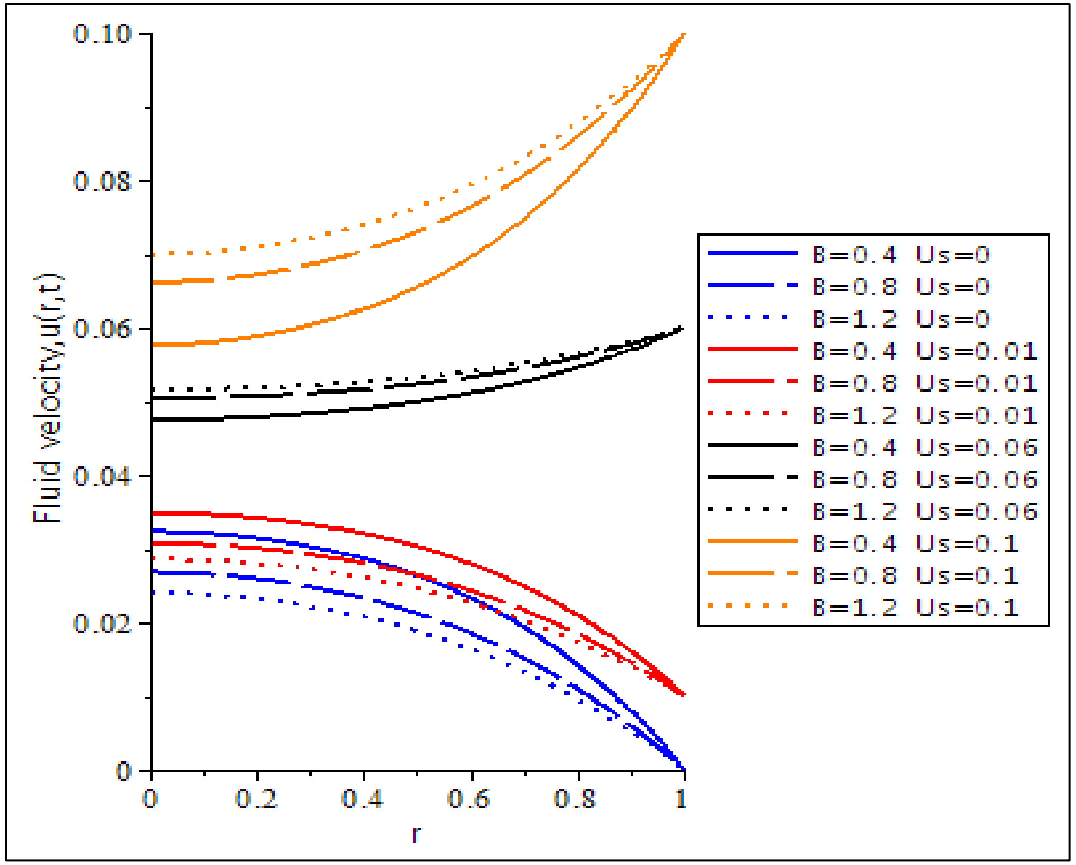


Figure 1: Velocity profiles $u(r,t)$ at different values of Casson parameter, β and slip velocity, u_s

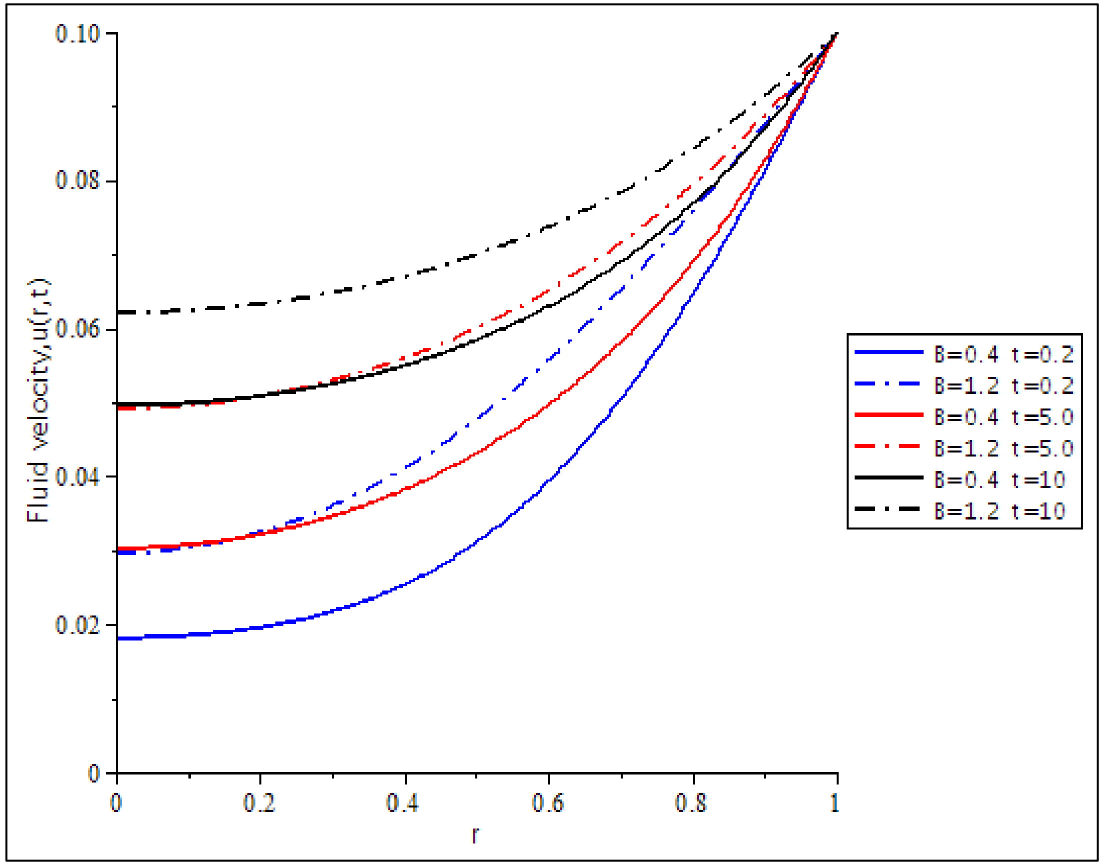


Figure 2: Velocity profiles $u(r,t)$ at different values of Casson parameter, β and time, t

Results and Discussions

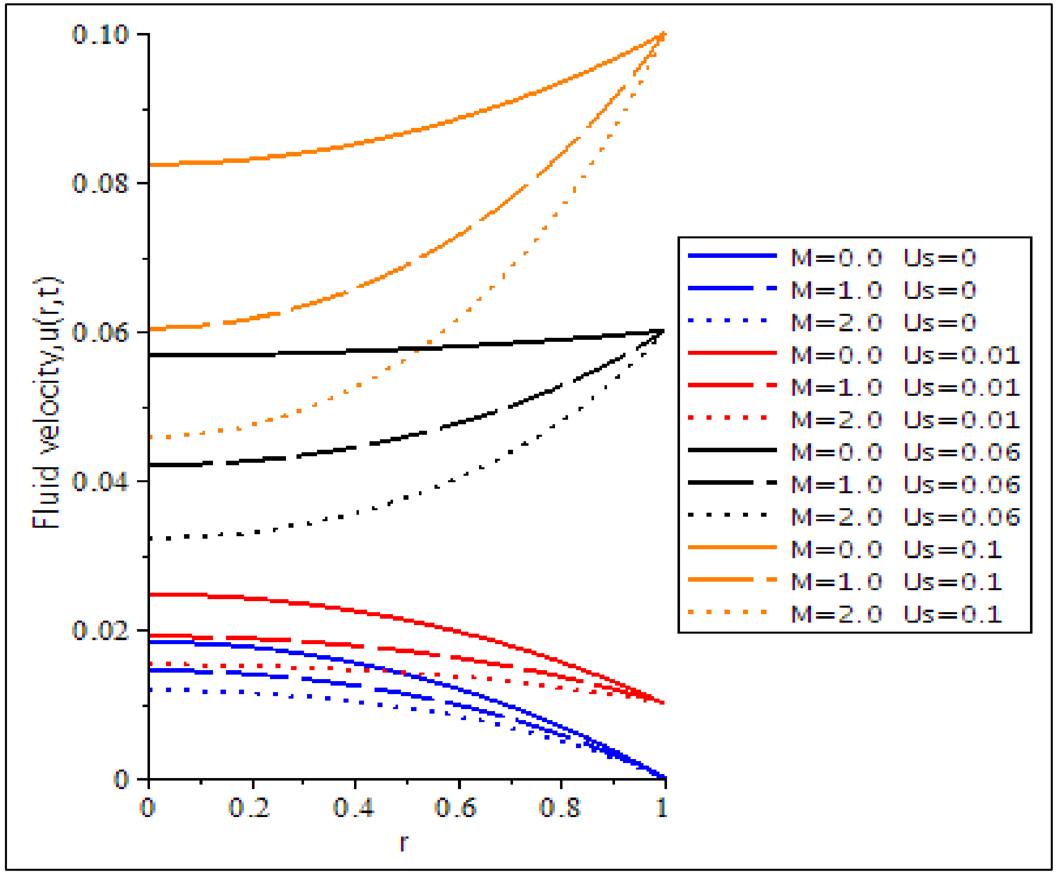


Figure 3: Velocity profiles $u(r,t)$ at different values of magnetic parameter, M and slip velocity, u_s

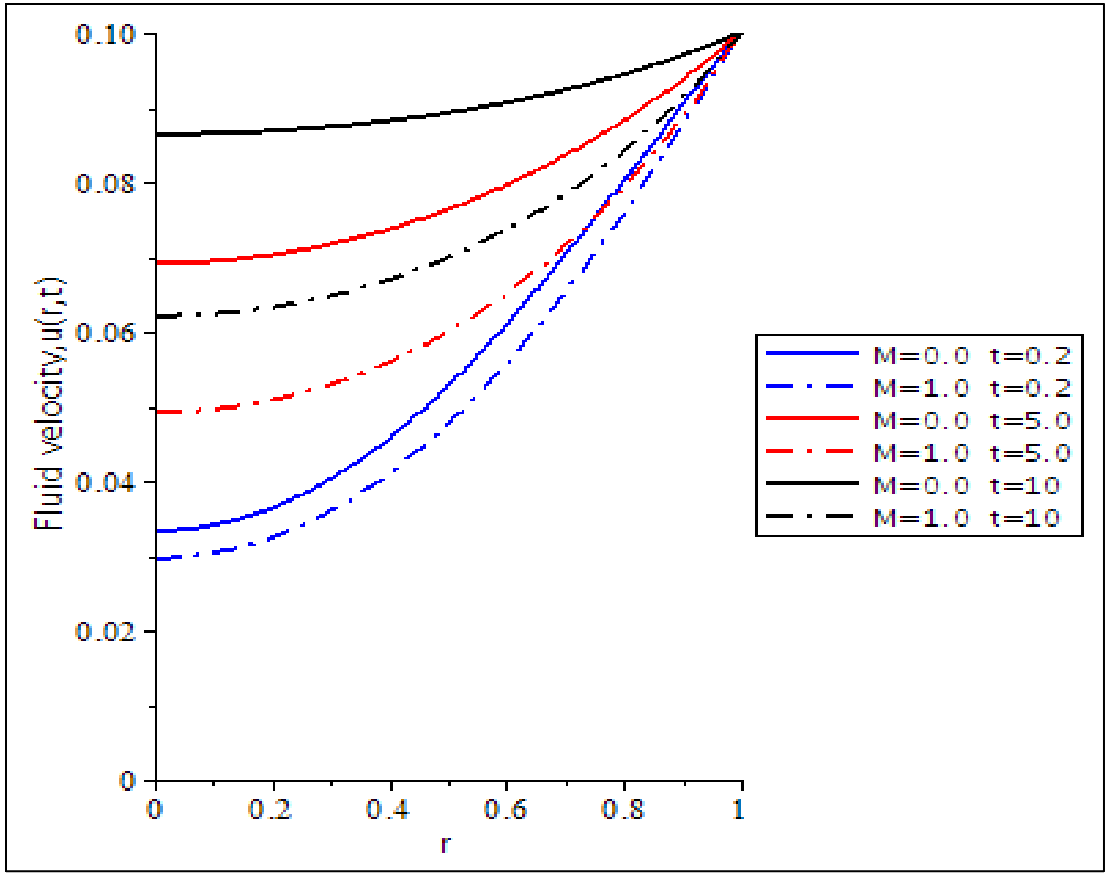


Figure 4: Velocity profiles $u(r,t)$ at different values of magnetic parameter, M and time, t

Results and Discussions

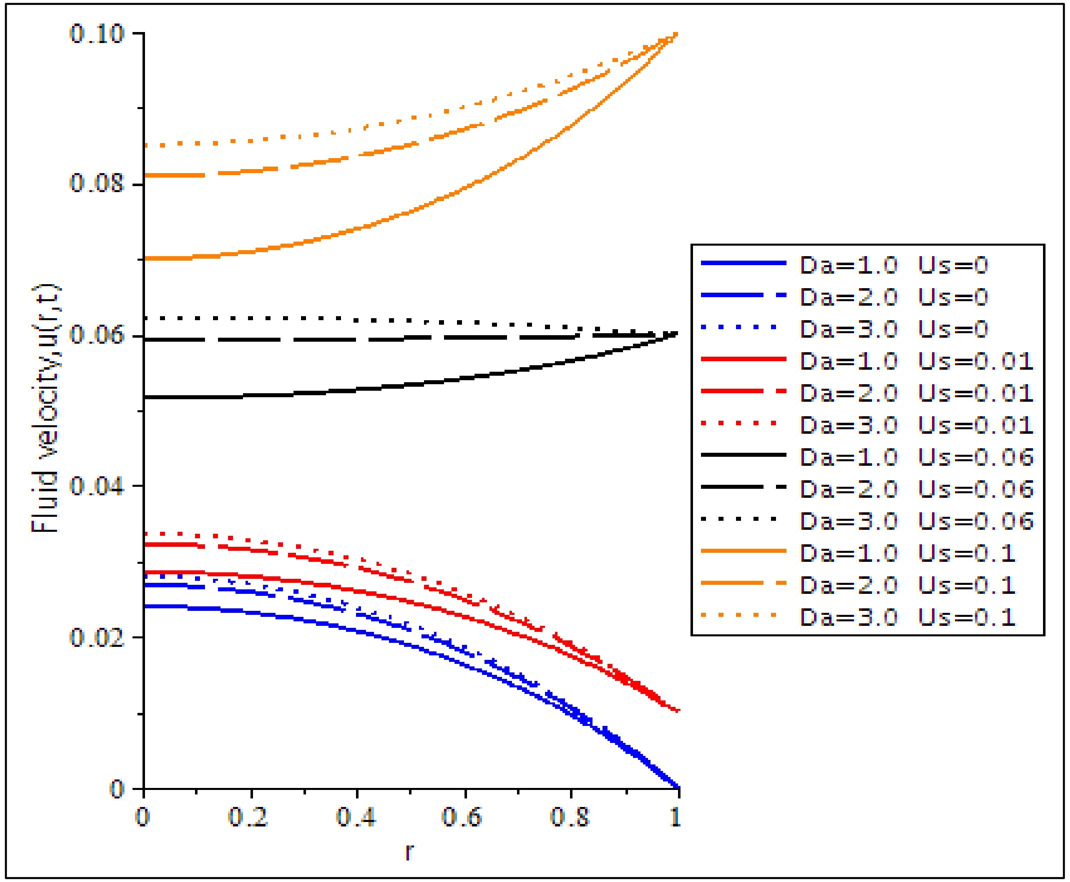


Figure 5: Velocity profiles $u(r,t)$ at different values of Darcy number, Da and slip velocity, u_s

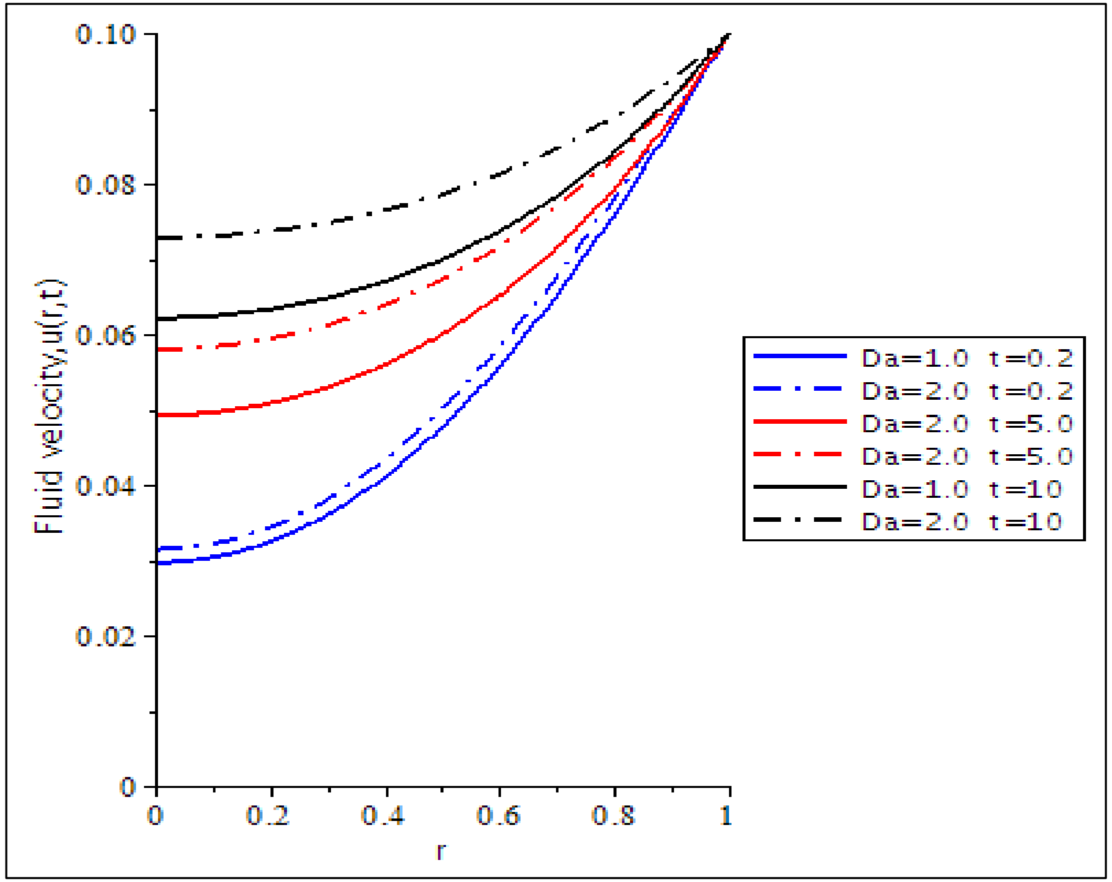


Figure 6: Velocity profiles $u(r,t)$ at different values of Darcy number, Da and time, t

Conclusions

The problem of unsteady pulsatile flow of Casson fluid past through a cylinder in porous medium with MHD and slip velocity effects has been investigated.

The Laplace and finite Hankel transform methods had been used to attain analytical solutions for this problem.

The obtained analytical solutions for velocity profile with the involvement of physical parameters have been analyzed graphically.

i. Fluid velocity increases with increases of *Da* and *t* while decreases as *M* increases.
ii. Fluid velocity increases with increases of β for larger slip velocity while decreases with increases of β for no slip velocity or small slip velocity. The viscosity and plasticity of Casson fluid are influenced by the slip velocity at the boundary conditions.

THANK YOU