Cawangan Kedah
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INTERNATIONAL CONFERENCE ON COMPUTING, MATHEMATICS AND STATISTICS

ROTATING FLOW OF A NANOFLUID PAST A NONLINEARLY SHRINKING SURFACE WITH FLUID SUCTION

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PRESENTATION OUTLINE

1. Introduction
2. Scopes, Problem Statement, Objective
3. Literature review
4. Research Methodology & Basic Equations
5. Results and Discussion
6. Conclusion
7. References

INTRODUCTION

Motivation of heat exchanger

- Heat exchanger is a device used to transfer heat from one medium to another. Heat exchangers are used in both cooling and heating processes.
- The mechanical design of heat exchanger depends on the operating pressure and temperature.
- Since, the conventional heat transfer fluids (oil, ethylene glycol and water) have their limitation in heat transfer performance, hence, a new fluid is introduced to overcome this situation.
- In 1995, Choi found an alternative way by introducing the term nanofluid in industries. Some of applications of heat exchanger are:

Aircraft engine

Air conditioning

Microelectronic device

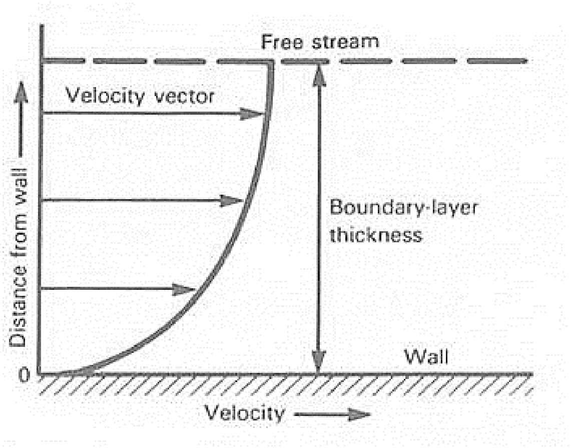
Refrigerator

To prevent the system from overheating & increase the efficiency

INTRODUCTION

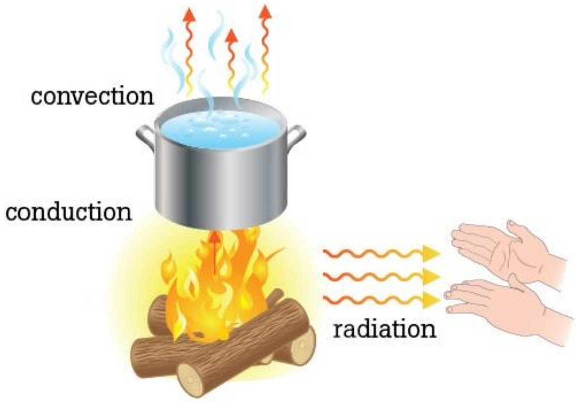
Boundary layer

The layer of fluid closest to the surface of an object where the fluid flow through its



Heat transfer

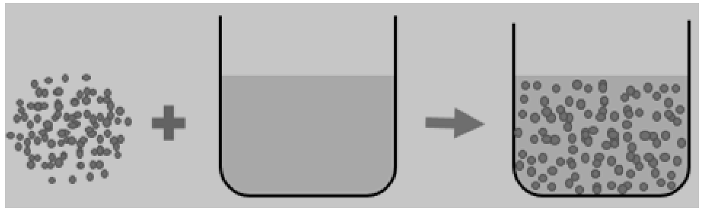
The movement of heat energy from one region to another regions of different temperature



INTRODUCTION

Nanofluid

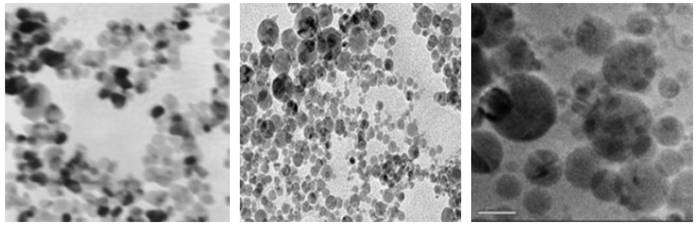
- Combination between nanoparticles and base fluids
- To enhance effective thermal conductivity and heat transfer coefficient



Nano-particles | Base fluid | Nanofluid

Tiwari and Das model

- One phase model
- No-slip condition
- Thermal equilibrium
- Effect: nanoparticle volume fraction φ



Copper | Titania | Alumina

INTRODUCTION

Shrinking Surface

Surface that have a shrunk surfaces in its own plane

Rotating Flow

Flow in which the fluid particles rotate about their own axes while flowing

Suction

The process of removing the air or water from a space in order to pull something into that space

SCOPES, PROBLEM STATEMENT, OBJECTIVE

Scopes

Nanofluid, Rotating flow, Shrinking surface, Suction

Problem Statement

How the nanoparticle volume fraction, rotation, suction and nonlinear parameters affect the fluid flow and heat transfer characteristics

Objective

To solve the rotating flow and heat transfer over a nonlinearly shrinking surface in a nanofluid with suction numerically using a shooting method

LITERATURE REVIEW

Rotating Flow in Nanofluids

Nadeem et al. (2014)
Boundary layer flow of rotating nanofluid over a stretching surface

Nasir et al. (2018)
3D rotating flow of MHD SWCNT over a stretching sheet in presence of thermal radiation

Salleh et al. (2016)
Rotating flow over a permeable shrinking surface in a nanofluid

Muhammad et al. (2018)
Rotating flow of MHD carbon nanotubes over a stretching sheet with radiation & heat generation or absorption

Hayat et al. (2018)
Rotating flow of hybrid nanofluid with radiation and slip effects

Anuar et al. (2021)
Radiative hybrid nanofluid flow past a rotating permeable stretching or shrinking sheet

LITERATURE REVIEW

Nonlinear Surface in Nanofluids

Rana & Bhargava (2012)

Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet

Hayat et al. (2018)

Flow of nanofluid by nonlinear stretching velocity

Das (2015)

Nanofluid flow over a nonlinear permeable stretching sheet with slip effect

Eid et al. (2020)

Nanofluid flow over a convectively heated nonlinear stretching surface with chemically reactive species

Hayat et al. (2016)

On magnetohydrodynamic flow of nanofluid over a nonlinear stretching sheet

Abbas et al. (2021)

MHD hybrid nanofluid flow over nonlinear stretching cylinder

RESEARCH METHODOLGY

1. Prepare mathematical model
2. Transform PDEs to ODEs using similarity transformations
3. Obtain numerical solutions by using Shooting method

Shooting Method

- The numerical procedure used to solve ODEs - form a two-point boundary value problem (BVP). By using a shooting method, the BVP is converted into an initial value problem (IVP).
- The main reason of using the shooting method is that this method attempts to diagnose the applicable initial conditions for a related IVP which bring the solution to the BVP.
- This method is applied in MAPLE programming language based on dsolve command and shoot implementation.

BASIC EQUATIONS

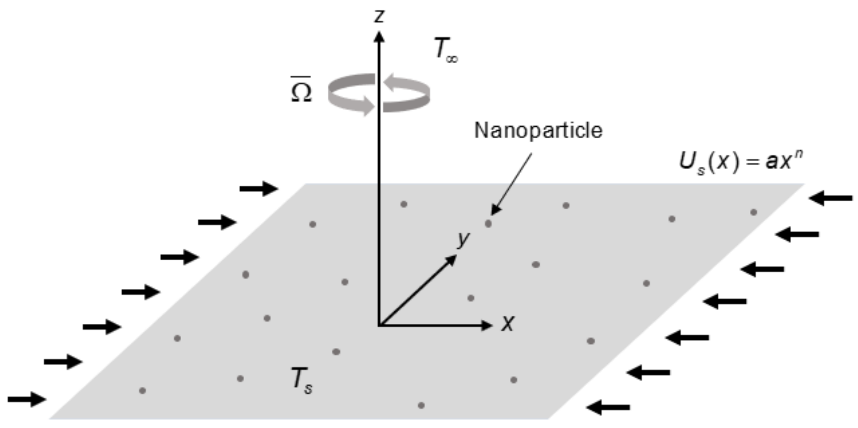


Figure 1: Geometric of the flow.

PARTIAL DIFFERENTIAL EQUATIONS (PDEs)

Continuity	$u_x + v_y + w_z = 0$
Momentum x-axis	$\rho_{nf}\left(uu_x + vu_y + wu_z - 2\bar{\Omega}v\right) = \mu_{nf}u_{zz}$
Momentum y-axis	$\rho_{nf}\left(uv_x + vv_y + wv_z + 2\bar{\Omega}u\right) = \mu_{nf}v_{zz}$
Energy	$uT_x + vT_y + wT_z = \alpha_{nf}T_{zz}$

Boundary conditions	$u = U_s(x),\ v = 0,\ w = -\sqrt{\frac{a\nu_f(n+1)}{2}}x^{\frac{n-1}{2}}s,\ T = T_s \text{ at } z = 0,$ $u \to 0,\ v \to 0,\ T \to T_\infty \text{ as } z \to \infty.$

BASIC EQUATIONS

NANOFLUID TERMS

$$\frac{\rho_{nf}}{\rho_f} = 1 - \varphi + \varphi\left(\frac{\rho_s}{\rho_f}\right), \quad \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = 1 - \varphi + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}.$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}},$$

SIMILARITY TRANSFORMATIONS

$$u = ax^n f'(\eta), \;\; v = ax^n h(\eta), \;\; w = -\sqrt{\frac{a\nu_f(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right],$$

$$\eta = \sqrt{\frac{a(n+1)}{2\nu_f}}x^{\frac{n-1}{2}}z, \;\; \theta(\eta) = \frac{T - T_\infty}{T_s - T_\infty},$$

ORDINARY DIFFERENTIAL EQUATIONS (ODEs)

Momentum x-axis	$$\frac{f'''}{(1-\varphi)^{2.5}\left[(1-\varphi)+\varphi\left(\rho_s/\rho_f\right)\right]}+ff''-\frac{2n}{n+1}f'^2+\frac{4\Omega}{(n+1)}h=0$$
Momentum y-axis	$$\frac{h''}{(1-\varphi)^{2.5}\left[(1-\varphi)+\varphi\left(\rho_s/\rho_f\right)\right]}+fh'-\frac{2n}{n+1}f'h-\frac{4\Omega}{(n+1)}f'=0$$
Energy	$$\frac{(k_{nf}/k_f)}{\Pr\left[(1-\varphi)+\varphi\left(\rho C_p\right)_s/\left(\rho C_p\right)_f\right]}\theta''+f\theta'=0$$

Boundary conditions	$f(0)=s,\;\; f'(0)=-1,\;\; h(0)=0,\;\; \theta(0)=1,$ $f'(\infty)\to 0,\;\; h(\infty)\to 0,\;\; \theta(\infty)\to 0.$

PHYSICAL QUANTITIES OF INTEREST

Skin friction coefficient of x-axis	$(Re_x)^{\frac{1}{2}} Cf_x = \dfrac{f''(0)}{(1-\varphi)^{2.5}}\sqrt{\dfrac{n+1}{2}}$
Skin friction coefficient of y-axis	$(Re_x)^{\frac{1}{2}} Cf_y = \dfrac{h'(0)}{(1-\varphi)^{2.5}}\sqrt{\dfrac{n+1}{2}}$
Local Nusselt number	$(Re_x)^{-\frac{1}{2}} Nu_x = -\dfrac{k_{nf}}{k_f}\theta'(0)\sqrt{\dfrac{n+1}{2}}$

RESULTS AND DISCUSSION

Table 1: Comparison values of the local heat flux $|-\theta'(0)|$ when the boundary conditions (11); $f(0) = 0$ and $f'(0) = 1$ and $\Omega = \varphi = 0$.

Pr	n	Rana and Bhargava (2012)	Das (2015)	Present results
1	0.2	0.6113	0.610571	0.610202
	0.5	0.5967	0.595719	0.595201
	1.5	0.5768	0.574525	0.574730
5	0.2	1.5910	1.607130	1.607787
	0.5	1.5839	1.586190	1.586782
	1.5	1.5496	1.557190	1.557695

RESULTS AND DISCUSSION

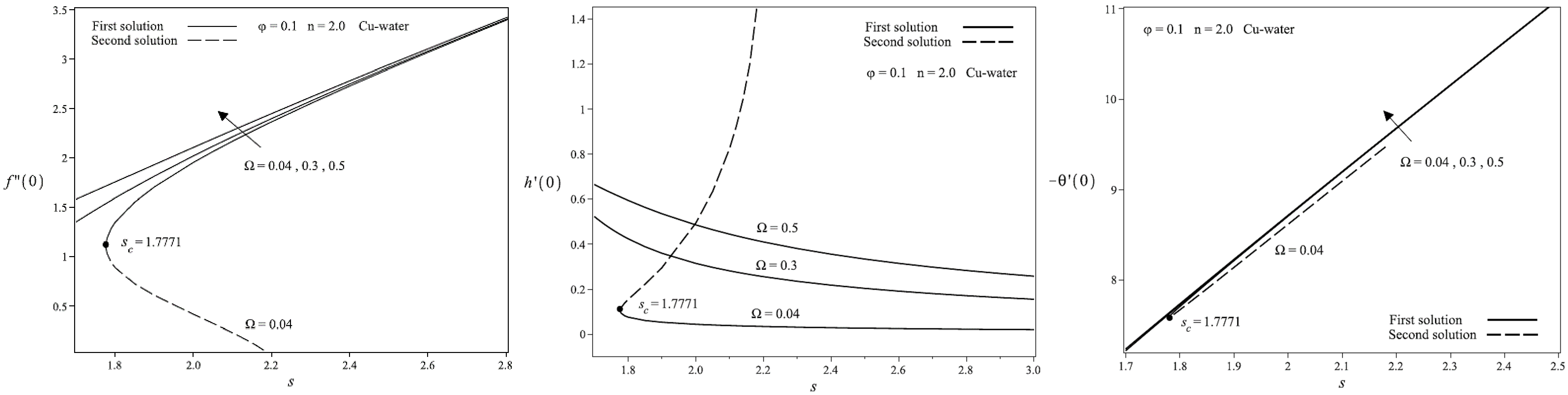


Figure 2: Influence of rotation on (a) $f''(0)$ (b) $h'(0)$ and (c) $-\theta'(0)$ versus s for Cu nanoparticle.

- The shear stress of both velocity components and the local heat flux increase as the rotation rate enhance.
- Dual solutions exist when Ω takes the lowest value that is $\Omega = 0.04$.
- Dual solutions exist in a certain region of $s_c < s \leq 2.18$ where s_c represents the turning point that connects first and second solutions.

RESULTS AND DISCUSSION

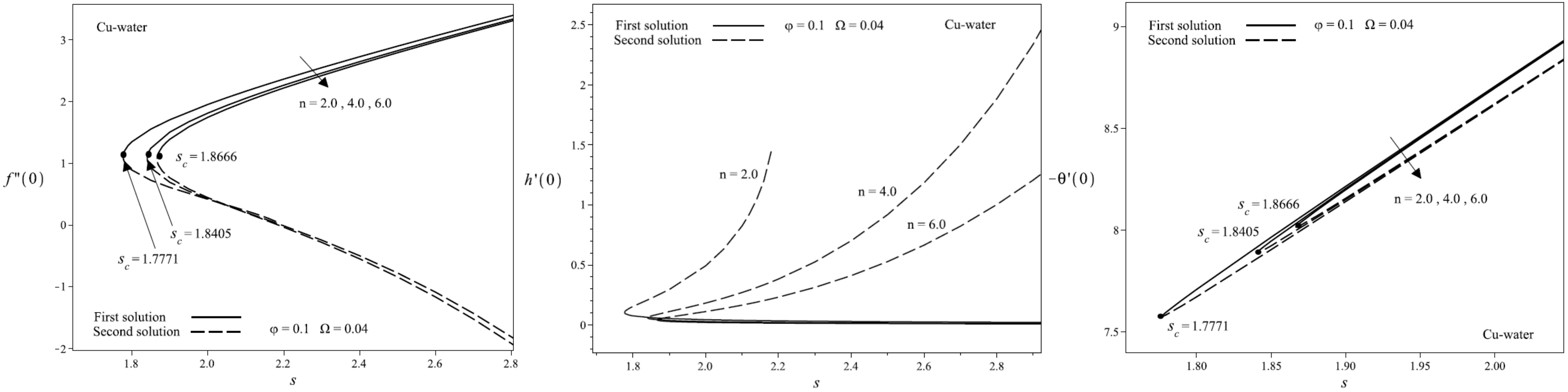


Figure 3: Influence of nonlinear rate on (a) $f''(0)$ (b) $h'(0)$ and (c) $-\theta'(0)$ versus s.

- The shear stress of both velocity components and the heat transfer reduce as the nonlinear parameter increase.
- An increase in the parameter n enhance the critical values of s from $s_c = 1.7771$ to $s_c = 1.8666$. This implies that the imposition of a higher value of n faster the boundary layer separation in the flow.

RESULTS AND DISCUSSION

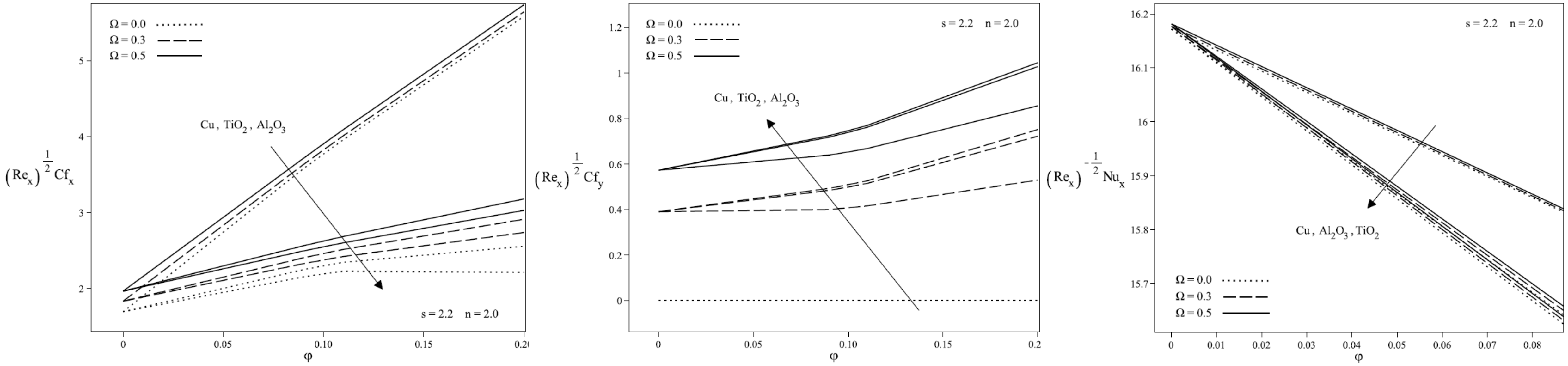


Figure 4: Influence of rotation and nanomaterials on (a) skin friction coefficient of x-component, (b) skin friction coefficient of y-component and (c) heat transmission rate versus φ.

- The presence of Ω and φ increase the coefficient of the skin friction for both velocity components.
- Cu has the highest values of skin friction coefficients followed by TiO_2 and Al_2O_3.
- Cu has the highest values of heat transfer rate than Al_2O_3 and TiO_2 - Cu has greater thermal conductivity.
- The heat transmission increase with the greater value of the rotation parameter.
- The heat transmission rate diminishes as the φ increase.

RESULTS AND DISCUSSION

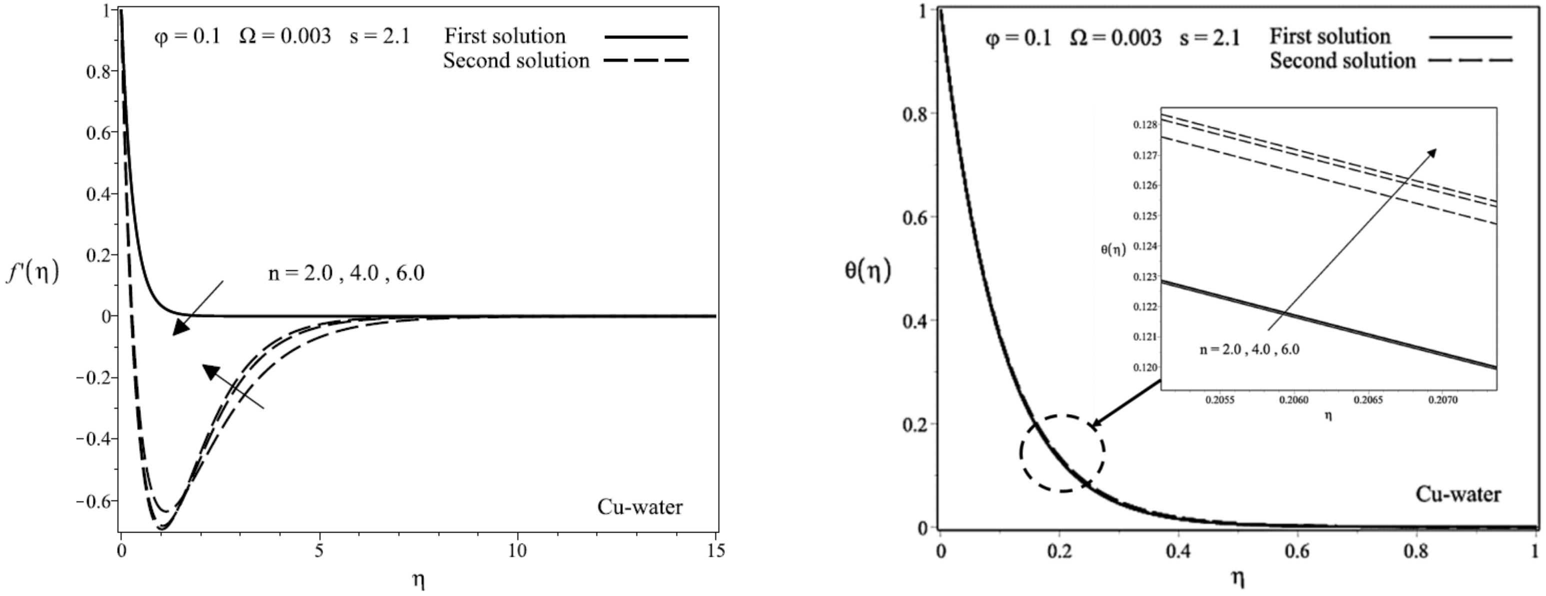


Figure 5: Influence of nonlinear rate on (a) velocity field of x-component and (b) temperature field for Cu-water.

- All profiles obtained satisfied the requirement of the endpoint boundary restrictions (11) asymptotically. Hence, it can be concluded with confidence that the computational outcomes obtained in this research are accurate.
- The thickness of the boundary layer for the first solution is thinner as opposed to the second solution.

CONCLUSION

- The presence of rotation boost the coefficient of skin friction and heat transmission rate.
- The enhancement of nonlinear rate accelerates the boundary layer separation where the dual solution meets.
- The imposition of nanoparticles in the flow rises the skin friction coefficients, whilst it reduces the heat transmission rate at the wall.
- The dual solutions show up when the rotation parameter takes the lowest value that is $\Omega = 0.04$ and when the value of suction exceeds a particular value; $s > 1.7771$.
- Copper has the highest coefficient of skin friction and heat transfer rate compared to aluminum oxide and titanium oxide.

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Any Questions...???

THANK YOU

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