ON THE TRANSFORMATION OF NONNEWTONIAN BOUNDARY VALUE PROBLEM TO INITIAL VALUE PROBLEM

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Abstract: **A generalized dimensional analysis method is applied to develop the similarity solution of the steady two dimensional boundary layer flow of non Newtonian fluid Sisko fluid model with appropriate boundary condition. The non-linear boundary value problem (BVP) is transformed into initial value problem (IVP). Taylor series method is used to solve IVP .It is observed that velocity of set increases with similarity independent variable.**

Keywords: **Dimensional analysis, Group theory, boundary layer, non-Newtonian, Sisko fluid, laminar flow, Stress, Strain, Similarity solutions**

Nomenclature:
u, v - Velocity components in *X* and *Y* directions, respectively u
***U* - Main stream velocities in *X* direction**
ψ **- Stream function**
***a* ,*b* - Sisko fluid parameters**
***n*- Flow behaviour indices**
η **- Similarity variable**
f- Similarity functions
A, A_1, A_2, A_3, A_4, α_1, α_2, β_1, β_2- Real constants

I. INTRODUCTION

The term fluid includes both gases and liquids. Fluid mechanics deals with the motion of fluids and the forces exerted on solid bodies in contact with the fluids. From last two decades, the study of boundary layer flows of non-Newtonian fluids has attracted researcher, Scientists, Mathematicians and Engineers because of its real world applications. A fluid is characterized by the behavior of its viscosity. Non-Newtonian behavior is due to very long chains, coils or a platelet type of molecular structure of the fluids. Examples of non-Newtonian fluids are the mud encountered in drilling oil wells, the paper pulp suspensions of paper industry, the molten rayon, molten plastics, high polymers, pastes, thick suspensions, emulsions, oils and lubricating greases, substances of food industries, slurries, blood etc. Because of different properties of non-Newtonian fluids one cannot propose single model so in literature there are various model of non-Newtonian fluids available proposed by scientists working in this area. Observation of literature survey is that most of flow analysis is done for Newtonian fluids and limited analysis done for non-Newtonian fluids almost up to power law fluid.Power law fluid model for non-Newtonian fluid have been analyzed by Pakedemirli (1994), Djukic (1974), Kapur (1963), Naet al. (1967), Nagler (2014), Patel et al (2011,2012). Different fluid models are discussed by Manisha et al. (2013)
In this paper we have analyzed the Sisko fluid model proposed by Sisko (1958) which is combination of viscous and power-law fluid model. Several fluids exist in nature which satisfies the property of shear thinning and thickening. Lubricating greases, cement slurries, polymeric suspensions are industrial Sisko fluids. In spite of wide industrial applications of Sisko fluid limited work has done in this area. Lie group

analysis of Sisko fluid has been done by Gozde and Pakdemirli (2012). Olanrewaju and his co-worker (2013) have obtained numerical solution for unsteady free convective flow of Sisko fluid with radioactive heat transfer. Laminar boundary layer flow of Sisko fluid is recently analyzed using one parameter scaling group of transformations by Manisha Patel (2015).

So, in this paper our aim is to demonstrate that Generalized dimensional analysis method is used in finding similarity solution of non-Newtonian fluids for Sisko fluid model. Helmholtz (1973) first discovered the concept of 'Mathematical Similarity' through the dimensional analysis approach. In the same time, the Russian mathematician S. Lie (1975) has presented famous Lie algorithm based on the continuous group transformations for the same kind of simplification of partial differential equation. Moran (1971) has worked on the generalized Dimensional Analysis method.

Further numerical solution is obtained by transferring BVP to IVP and then applying Taylor's series method. This method of transformation was introduced by Topfer (1957). Further the method was extended to solve many similar type of equation by Klamkin (1962).by applying linear group of transformation, the method was seen to be applicable to Blasius solution with suction or blowing by T.Y. Na (1970).Hema et al has applied a linear group of transformation to convert BVP to IVP in two dimensional jet flow.

II. GOVERNING EQUATIONS

The Problem of two dimensional laminar boundary layer flow of Sisko fluid past a semi infinite flat plate is examined. The governing continuity and momentum equations of laminar boundary layer flow of Sisko fluid are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\frac{\partial}{\partial y}\left\{\left[a+b\left(\frac{\partial u}{\partial y}\right)^{n-1}\right]\frac{\partial u}{\partial y}\right\} \tag{2}$$

Subject to the boundary conditions:

$$y=0: u=0, v=0$$
$$y=\infty, u=U(x) \tag{3}$$

Where a and b are the Sisko fluid parameter

We define stream function ψ(x, y), to reduce one dependent variable which satisfies equation (1).

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \tag{4}$$

Equation (2) is transformed in following equation with associated boundary conditions.

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U\frac{dU}{dx}+\frac{\partial}{\partial y}\left\{\left[a+b\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\right]\frac{\partial^2\psi}{\partial y^2}\right\} \tag{5}$$

Subject to the boundary conditions;

$$y=0 \Rightarrow \frac{\partial\psi}{\partial y}=-\frac{\partial\psi}{\partial x}=0$$

$$y=\infty \Rightarrow \frac{\partial\psi}{\partial y}=U(x) \tag{6}$$

III. GENERLIZED DIMENSIONAL ANALYSIS

Consider the following group of transformation:

$$G:\begin{bmatrix} \bar{y} = A_2 y; \bar{x} = A_3 x \ \ (Independent\ variables) \\ \bar{\psi} = A_1 \psi \qquad (Dependent\ variable) \\ \bar{U} = A_4 U \qquad (Physical\ variable) \end{bmatrix} \tag{7}$$

Where A_i's i=1, 2, 3, 4 are group parameters.

Equations (5) and (6) remain invariant under group of transformations defined by G in equation (7)

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} = \bar{U}\frac{d\bar{U}}{d\bar{x}} + \frac{\partial}{\partial\bar{y}}\left\{\left[a + b\left(\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right)^{n-1}\right]\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right\} \tag{8}$$

$$A_1{A_2}^{-1}A_1{A_2}^{-1}{A_3}^{-1}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - A_1{A_3}^{-1}A_1{A_2}^{-2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}$$

$$= A_4A_4{A_3}^{-1}U\frac{dU}{dx} + \quad \frac{\partial}{\partial y}\left\{\left[a + b\left(A_1{A_2}^{-2}\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\right]A_1{A_2}^{-2}\frac{\partial^2\psi}{\partial y^2}\right\}{A_2}^{-1} \tag{9}$$

Equations (5) and (6) remains invariant under group of transformations defined by G in equation (7) provided $A_i^{'}s$ are interrelated in following form.

$${A_1}^2{A_2}^{-2}{A_3}^{-1} = {A_4}^2{A_3}^{-1} \text{ and } \quad {A_1}^2{A_2}^{-2}{A_3}^{-1} = A_1{A_2}^{-3} \tag{10}$$

$${A_1}^1{A_2}^{-1} = A_4 \quad \text{ and } {A_1}^1{A_2}^1 = A_3 \tag{11}$$

Two parameter group of transformations

$$\Gamma:\begin{bmatrix} \bar{y} = {A_1}^0{A_2}^1 y; \bar{x} = {A_1}^1{A_2}^1 x \ (Independent\ variables) \\ \bar{\psi} = {A_1}^1{A_2}^0\psi \qquad (Dependent\ variable) \\ \bar{U} = {A_1}^1{A_2}^{-1}U \qquad (Physical\ variable) \end{bmatrix} \tag{12}$$

Now according to pi theorem stated by Moran et el (1972), (given insection-2) the rank of dimensional matrix associated with independent and physical variables *BC* and physical variables *C* is required to be determined.

The associated dimensional matrix will be

BC: $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$ where matrix C corresponds last row.

Rank of matrix BC is two and of matrix C is 1. (r>s) So by theorem 3 stated by Moran and Murshek n+m+p-r =2 absolute invariant exists in the form of

$$\Pi = \psi[x]^{\alpha_1}[U]^{\alpha_2} \text{ and } \pi = y[x]^{\beta_1}[U]^{\beta_2} \tag{13}$$

Solving following system of equations.

$$\begin{bmatrix}1\\0\end{bmatrix} + \alpha_1\begin{bmatrix}1\\1\end{bmatrix} + \alpha_2\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \tag{14}$$

$$\begin{bmatrix}0\\1\end{bmatrix} + \beta_1\begin{bmatrix}1\\1\end{bmatrix} + \beta_2\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \tag{15}$$

We obtained exponents $\alpha_1 = -\frac{1}{2}, \alpha_2 = -\frac{1}{2}, \beta_1 = -\frac{1}{2}, \beta_2 = \frac{1}{2}$.

So we have similarity transformations

$$\eta = yx^{-\frac{1}{2}}U^{-\frac{1}{2}} \tag{16}$$

$$f(\eta) = \psi x^{-\frac{1}{2}}U^{\frac{1}{2}} \tag{17}$$

It is to be observed that similarity solutions for all such Non-Newtonian fluids exist only for the flow past 90° wedge only.

Take $U = x^{\frac{1}{3}}$ (18)

Using chain rule of differentiation for transformations given in equations (16), (17), (18) equations (5) -(6) are transformed into ODE in the following form

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3af'''(\eta) - 3bn(f''(\eta))^{n-1}f'''(\eta) - 1 = 0 \quad (19)$$

With boundary conditions $f(0) = 0, f'(0) = 0$ and $f'(\infty) = 1$ (20)

IV. REDUCTION TO AN INITIAL VALUE PROBLEM

A. CASE(I): NEWTONIAN FLUID MODEL

Put $a = 1, b = 0, n = \frac{1}{3}$

Equation (19) is transformed as

$$f'^2(\eta) - 2f(\eta)f''(\eta) = 1 + 3f'''(\eta) \quad (21)$$

To transform equation (21) with boundary condition into initial value problem a one parameter linear group of transformation is defined as under

$$\eta = A^{\alpha_1}\bar{\eta}, \;\; f = A^{\alpha_2}\bar{f} \quad (22)$$

Where A is parameter of transformation and α_1, α_2 are constants which we have to determined

Substituting the transformation in equation (21)

$$A^{2\alpha_2-2\alpha_1}\bar{f}'^2(\eta) - 2A^{2\alpha_2-2\alpha_1}\bar{f}(\eta)\bar{f}''(\eta) - 1 - 3A^{\alpha_2-3\alpha_1}\bar{f}'''(\eta) =$$
$$f'^2(\eta) - 2f(\eta)f''(\eta) - 1 - 3f'''(\eta)$$

Now under this transformation equation (21) must remain invariant it is possible only if

$$2\alpha_2 - 2\alpha_1 = \alpha_2 - 3\alpha_1 = 0$$
$$\alpha_1 + \alpha_2 = 0 \quad (23)$$

To get missing initial condition, we put

$\eta = 0, f''(\eta) = A$

so $\bar{\eta} = 0, A^{\alpha_2-2\alpha_1}\bar{f}''(\eta) = A$

The boundary condition is independent of α_1, α_2 and A if

$$\alpha_2 - 2\alpha_1 = 1 \quad (24)$$

Which then gives $\bar{\eta} = 0, \bar{f}''(\eta) = 1$

From (23) and (24) $\alpha_1 = -\frac{1}{3}, \; \alpha_2 = \frac{1}{3}$

Finally, the value of A can be computed from the boundary condition at the second point, i.e., $f'(\infty) = 1$

$$A^{\alpha_2-\alpha_1}\bar{f}'(\infty) = 1$$
$$A^{\frac{2}{3}}\bar{f}'(\infty) = 1$$
$$A = \left(\frac{1}{\bar{f}'(\infty)}\right)^{\frac{3}{2}} \quad (25)$$

Thus boundary value problem is converted into initial value problem

$$\bar{f}'^2(\bar{\eta}) - 2\bar{f}(\bar{\eta})\bar{f}''(\eta) = 1 + 3\bar{f}'''(\bar{\eta}) \quad (26)$$

With initial conditions

$\bar{f}(0) = 0, \bar{f}'(0) = 0$ and $\bar{f}''(0) = 1$ again

$$3f'''(\bar{\eta}) = f'^2(\bar{\eta}) - 2f(\bar{\eta})f''(\bar{\eta}) - 1$$
$$f'''(0) = -\frac{1}{3}$$

Differentiating (26) with respect to $\bar{\eta}$

$$3f^{(iv)}(\bar{\eta}) = -2f(\bar{\eta})f'''(\bar{\eta}) \quad (27)$$

$f^{(iv)}(0)=0$

Differentiating (27) with respect to $\bar{\eta}$

$$3f^{(v)}(\bar{\eta}) = -2f'(\bar{\eta})f''{}'(\bar{\eta}) - 2f(\bar{\eta})f^{(iv)}(\bar{\eta}) \tag{28}$$

$$f^{(v)}(0) = 0$$

Differentiating (28) with respect to $\bar{\eta}$

$$3f^{(vi)}(\bar{\eta}) = -2f''(\bar{\eta})f''(\bar{\eta})) - 4f'(\bar{\eta})f^{(iv)}(\bar{\eta}) - 2f(\bar{\eta})f^{(v)}(\bar{\eta})$$

$$f^{(vi)}(0) = \frac{2}{9}$$

Using Taylor's series expansion and ignoring higher order terms we get,

$$\bar{f}(\eta) = \frac{\bar{\eta}^2}{2} - \frac{\bar{\eta}^3}{18} \tag{29}$$

$$\bar{f}'^{(\eta)} = \bar{\eta} - \frac{\bar{\eta}^2}{6} \tag{30}$$

Table 1

$\bar{\eta}$	$\bar{f}(\eta)$	$\bar{f}'(\eta)$
0	0	0
0.5	0.1180	0.4583
1	0.444	0.8333
1.5	0.9375	1.125
2	1.555	1.3333
2.5	2.257	1.4583
3	3	1.5

Thus $A = \left(\frac{1}{\bar{f}'(3)}\right)^{\frac{3}{2}} = 0.544$

$\eta = A^{\alpha_1}\bar{\eta} \Rightarrow \eta = 1.2249\,\bar{\eta}$ and $f = A^{\alpha_2}\bar{f} \Rightarrow f = 0.816\,\bar{f}$

and $f' = 0.6664\,\bar{f}'$

The calculated values of η, $f(\eta)$ and $f'(\eta)$ are given in Table 2.

Table 2

η	$f(\eta)$	$f'(\eta)$
0	0	0
0.6124	0.09629	0.3054
1.2249	0.3623	0.55531
1.8373	0.765	0.7947
2.4498	1.2688	0.8883
3.06225	1.8417	0.97183
3.6747	2.448	0.9996

B. CASE(II): POWER LAW FLUID MODEL

Put $a = 0, b = 1, n = 1/3$

Equation (19) is transformed as

$$f'^2(\eta) - 2f(\eta)f''(\eta) = 1 + 3(f''(\eta))^{\frac{-2}{3}} f'''(\eta) \qquad (29)$$

With boundary conditions

$f(0) = 0, f'(0) = 0$ *and* $f'(\infty) = 1$

Using the same method as above discussed in case (i) to convert boundary value problem into initial value problem we get

$$\bar{f}'^2(\bar{\eta}) - 2\bar{f}(\bar{\eta})\bar{f}''(\bar{\eta}) = 1 + 3\left(\bar{f}''(\bar{\eta})\right)^{\frac{-2}{3}} \bar{f}'''(\bar{\eta}) \qquad (30)$$

With initial conditions

$$\bar{f}(0) = 0, \bar{f}(0) = 0 \text{ and } \bar{f}''(0) = 1 \qquad (31)$$

And $A = \left(\frac{1}{\bar{f}'(\infty)}\right)^{\frac{11}{6}}$

Differentiating (30) successively and neglecting higher order derivatives we get following equation using Taylor's series expansion

$$\bar{f}(\bar{\eta}) = \frac{\bar{\eta}^2}{2} - \frac{\bar{\eta}^3}{6} \qquad (32)$$

$$\bar{f}'(\bar{\eta}) = \bar{\eta} - \frac{\bar{\eta}^2}{2} \qquad (33)$$

Table 3

$\bar{\eta}$	$\bar{f}(\eta)$	$\bar{f}'(\eta)$
0	0	0
0.1	0.0048	0.095
0.2	0.0186	0.18
0.3	0.0405	0.255
0.4	0.0693	0.32
0.5	0.1042	0.375
0.6	0.144	0.42
0.7	0.1878	0.455
0.8	0.2346	0.48
0.9	0.2835	0.495
1	0.333	0.5

Thus $A = \left(\frac{1}{\bar{f}'(1)}\right)^{\frac{3}{2}} = 3.563$

$\eta = A^{\alpha_1}\bar{\eta} \Rightarrow \eta = 0.5612\,\bar{\eta}$ and $f = A^{\alpha_2}\bar{f} \Rightarrow f = 1.1224\,\bar{f}$

and $f' = 1.9998\,\bar{f}'$

The calculated values of η, f (η) and f' (η) are given in Table 4.

Table 4

η	$f(\eta)$	$f'(\eta)$
0	0	0
0.0561	0.005382	0.1899
0.1122	0.02087	0.3599
0.1683	0.0454	0.5099

0.2245	0.07778	0.6399
0.2806	0.1169	0.7499
0.3367	0.1616	0.8399
0.3928	0.21078	0.9099
0.44896	0.2633	0.9599
0.50508	0.3182	0.9899
0.5612	0.3737	0.999

C. CASE(III): SISKO FLUID MODEL

Put $a = \frac{1}{2}, b = \frac{1}{2}, n = \frac{2}{3}$

Equation (20) is transformed as

$f'^2(\eta) - 2f(\eta)f''(\eta) = 1 + \frac{3}{2}f'''(\eta) + 3(f''(\eta))^{\frac{-1}{3}}f'''(\eta)$(34)

With boundary conditions $f(0) = 0, f'(0) = 0$ *and* $f'(\infty) = 1$ (35)

Using the same method to convert boundary value problem into initial value problem we get

$\bar{f}'^2(\bar{\eta}) - 2\bar{f}(\bar{\eta})\bar{f}''(\bar{\eta}) = 1 + \frac{3}{2}\bar{f}'''(\bar{\eta}) + 3\left(\bar{f}''(\bar{\eta})\right)^{\frac{-1}{3}}\bar{f}'''(\bar{\eta}))$ (36)

With initial conditions

$\bar{f}(0) = 0, \bar{f}'(0) = 0$ *and* $\bar{f}''(0) = 1$(37)

And $A = \left(\frac{1}{\bar{f}'(\infty)}\right)^{\frac{3}{2}}$

Differentiating (36) successively and neglecting higher order derivatives we get following Taylor's series expansion

$\bar{f}(\bar{\eta}) = \frac{\bar{\eta}^2}{2} - \frac{\bar{\eta}^3}{15}$ (37)

$\bar{f}'(\bar{\eta}) = \bar{\eta} - \frac{\bar{\eta}^2}{5}$ (38)

Table 5

$\bar{\eta}$	$\bar{f}(\eta)$	$\bar{f}'(\eta)$
0	0	0
0.1	0.005	0.09802
0.2	0.01947	0.19208
0.3	0.0432	0.2818
0.4	0.0757	0.36382
0.5	0.11675	0.4505
0.6	0.1657	0.52872
0.7	0.222	0.60298
0.8	0.2862	0.67328
0.9	0.35688	0.73962
1	0.434	0.802

Thus $A = \left(\frac{1}{\bar{f}'(2.5)}\right)^{\frac{3}{2}} = 0.7155$

$\eta = A^{\alpha_1}\bar{\eta} \Rightarrow \eta = 1.11805\,\bar{\eta}$ and $f = A^{\alpha_2}\bar{f} \Rightarrow f = 0.8944\,\bar{f}$
and $f' = 0.7999\,\bar{f}'$

The calculated values of η, $f(\eta)$ and $f'(\eta)$ are given in Table 6.

Table 6

η	$f(\eta)$	$f'(\eta)$
0	0	0
0.1118	0.004	0.0784
0.2236	0.0174	0.1536
0.3354	0.0386	0.2257
0.4472	0.0677	0.2946
0.559	0.1044	0.36035
0.6078	0.1482	0.4229
0.7826	0.1985	0.48232
0.8944	0.2559	0.53855
1.006	0.3191	0.5916
1.11805	0.3875	0.64151
1.67707	0.80496	0.83989
2.2361	1.31655	0.96627
2.7951	1.8633	0.9998

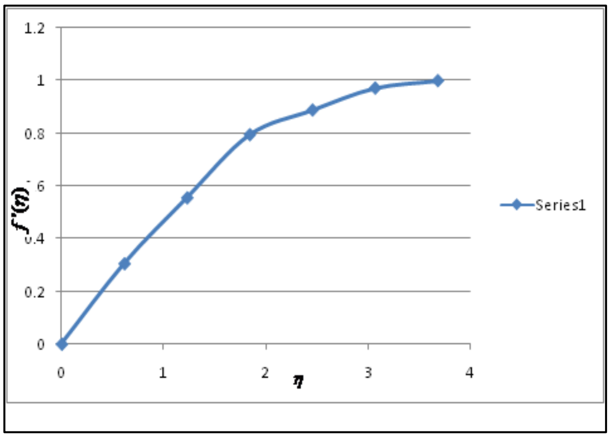

Figure 1: Velocity profile for Newtonian fluid

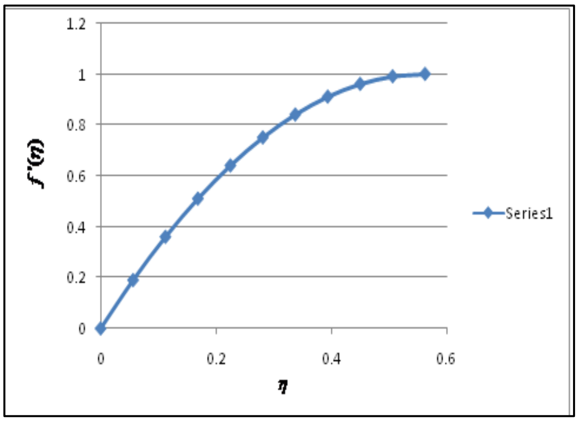

Figure 2: Velocity profile for Power law fluid

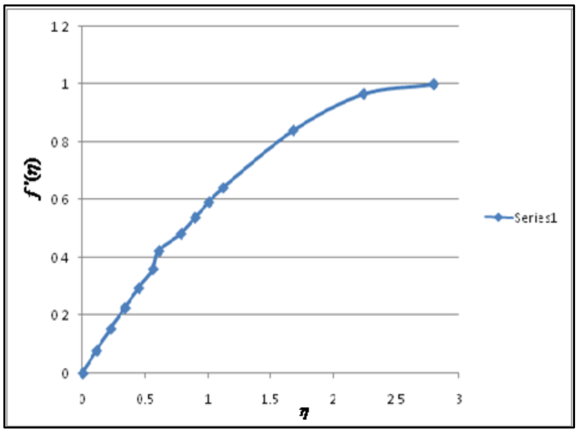

Figure 3: Velocity profile for Sisko fluid

V. CONCLUSION

Similarity solutions for steady two dimensional boundary layer equations for Non-Newtonian Sisko fluids are derived. The generalized Dimensional Analysis method with two parameter group transformation is used to derive similarity solutions. The similarity equations that are highly non-linear ordinary differential equations with boundary conditions are transferred to initial value problem(IVP). The numerical solution of resulting IVP is obtained by simple Taylor's series method. The advantage of present analysis is that highly non-linear BVP can be solved by simple numerical technique like Taylor's series method after converting it into IVP. This method can be applied to the solutions of equations where certain parameters appear either in differential equations or in boundary conditions.

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