Problem 1: MLE for Bernoulli Distribution A coin is flipped n = 10 times, and we observe x = 7 heads. Assume the coin follows a Bernoulli distribution with parameter p. Tasks:

1. Write down the likelihood function L(p).

2. Derive the log-likelihood function.

3. Find the Maximum Likelihood Estimator (MLE) for p

Ans)

1) $L(p)=P(\text{Data}|pp7(1-p)3$

2) $\ell(p)=\ln L(p)=7\ln p+3\ln(1-p)$

3) 0.7

Problem 2: MLE for Binomial Distribution A factory produces light bulbs, and in a sample of n = 20 bulbs, k = 15 are found to be defective. Assume defects follow a Binomial distribution with parameters n and p. Tasks:

1. Define the likelihood function L(p).

2. Derive the log-likelihood function.

3. Compute the MLE for p

Ans)

1) $L(p)=p15(1-p)5$

2) $\ell(p)=\ln L(p)=15\ln p+5\ln(1-p)$

3) $p=0.75$

Problem 3: MLE for Poisson Distribution The number of customers arriving at a store per hour follows a Poisson distribution with an unknown rate λ. Suppose in a given hour, 5 customers arrive. Tasks:

1. Write the likelihood function L(λ).

2. Compute the log-likelihood function.

3. Determine the MLE for λ.

Ans)

1) $L(\lambda)=e-\lambda\lambda5$
2) $\ell(\lambda)=\ln L(\lambda)=-\lambda+5\ln\lambda$
3) $\lambda\hat{}=5$

Problem 4: MLE for Normal Distribution Suppose we have a sample of size n from a normal distribution Xi ~ N(μ, σ2), where both μ and σ 2 are unknown. Tasks:

1. Write the likelihood function for μ and σ 2 .

2. Find the MLE for μ assuming σ 2 is known.

3. Derive the MLE for σ 2

Ans)

Problem 1: MOM for Bernoulli Distribution Suppose a coin is flipped n times, and the probability of getting heads is p. The observed mean number of heads is X¯ = 0.6. Tasks:

1. Write the first moment equation based on the expected value.

2. Solve for the estimator of p using the method of moment

Ans)

1) $E[X]=X$
2) $p\hat{}MOM=X=0.6$

3)

Problem 2: MOM for Binomial Distribution A die is rolled $n = 30$ times, and the number of times a six appears follows a Binomial distribution $X \sim Bin(n, p)$. The sample mean is observed to be $\bar{X} = 5$. Tasks:

1. Set up the moment equation using the expected value.

2. Find the estimator for p.

Ans)

1) $30p=5$

2) $p=nX=305=61\approx0.1667$

Problem 3: MOM for Poisson Distribution Suppose the number of emails received per hour follows a Poisson distribution with parameter λ. Given a sample with observed mean $\bar{X} = 4.5$. Tasks:

1. Write the moment equation using the expected value of a Poisson distribution.

2. Solve for the estimator of λ.

Ans)

1) $X=4.5$

2)$MOM=X=4.5$

Problem 4: MOM for Normal Distribution Let X1, X2, . . . , Xn be a random sample from a normal distribution N(μ, σ2). The sample mean and variance are given by X¯ = 10 and S 2 = 4. Tasks:

1. Write the moment equations based on the first and second moments.
2. Solve for the estimators of μ and σ 2 .

Ans)

1) ^$MOM=X=10$

2) σ^$MOM2=S2=4$