Safety and Reliability of Embedded Systems - SRES (WS 19/20)
Problem Set 6

Problem 1: Weibull distribution

The so-called Weibull distribution is often used to describe the wear-out phase of components (caused by e.g. fatigue failures).

Assume that the lifetime T of a component can be described by such a Weibull distribution with parameter β = 2.

In order to determine the parameter λ, you perform the following experiment:

The experiment is commenced with a large number of components all being initially intact. After 250 hours, the number of components that survived so far is recorded. After another 50 hours, it is observed that 25% of these components now have failed.

Please calculate the parameter λ of the corresponding Weibull distribution.

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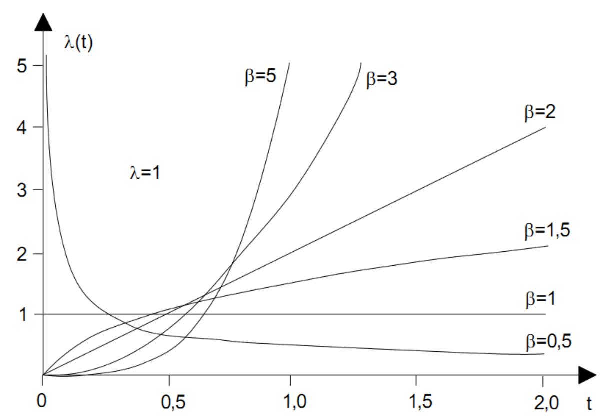
Failure rate of the Weibull distribution depending on the form parameter β

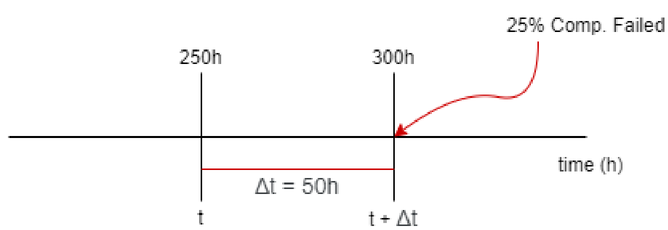
- Life Distribution : $F(t) = 1 - e^{-(\lambda t)^\beta}$; $\lambda, \beta > 0$
- or:
  $F(t) = 1 - e^{-\frac{1}{\alpha} t^\beta}$; $\alpha, \beta > 0$, d. h. $\frac{1}{\alpha} = \lambda^\beta$
- Density:
  $f(t) = \frac{dF(t)}{dt} = \lambda \beta (\lambda t)^{\beta - 1} e^{-(\lambda t)^\beta}$
- Reliability: $R(t) = e^{-(\lambda t)^\beta}$
- Failure rate:
  $\lambda(t) = \frac{f(t)}{R(t)} = \lambda \beta (\lambda t)^{\beta - 1}$



- $F(t) = P\{T \leq t\}$ -> probability that lifetime T is less or equal to t, meaning that a system has already failed by t
- In our case: P (T ≤ 300) give that P (T > 250) -> Conditional probability
- The conditional probability that a system has survived until t and fails within Δt is: $1 - \frac{R(t+\Delta t)}{R(t)}$

$$\frac{F(t+\Delta t) - F(t)}{1 - F(t)} => \frac{F(250+50) - F(250)}{1 - F(250)} = 0.25$$

$$\frac{1 - e^{-(\lambda .\, 300h)^2} - (1 - e^{-(\lambda .\, 250h)^2})}{e^{-(\lambda .\, 250h)^2}} = 0.25$$



Because lifetime is exponentially distributed

$$\frac{e^{-(\lambda.250h)^2} - e^{-(\lambda.300h)^2}}{e^{-(\lambda.250h)^2}} = 0.25$$

$$e^{-(\lambda .\, 250h)^2} - e^{-(\lambda .\, 300h)^2} = 0.25 * e^{-(\lambda .\, 250h)^2}$$

$$e^{-(\lambda .\, 250h)^2} - 0.25 * e^{-(\lambda .\, 250h)^2} = e^{-(\lambda .\, 300h)^2}$$

$$(1 - 0.25) * \mathrm{e}^{-(\lambda\,.\,250\mathrm{h})^2} = \mathrm{e}^{-(\lambda\,.\,300\mathrm{h})^2}$$

$$(0.75) * \mathrm{e}^{-(\lambda\,.\,250\mathrm{h})^2} = \mathrm{e}^{-(\lambda\,.\,300\mathrm{h})^2}$$

$$\ln(\mathrm{e}^{-(\lambda\,.\,250\mathrm{h})^2} * 0.75) = \ln(\mathrm{e}^{-(\lambda\,.\,300\mathrm{h})^2})$$

$$\ln(0.75) - (\lambda\,.\,250\mathrm{h})^2 = -(\lambda\,.\,300\mathrm{h})^2$$

$$(\lambda\,.\,300\mathrm{h})^2 - (\lambda\,.\,250\mathrm{h})^2 = -\ln(0.75)$$

$$27500\, h^2 \,.\, \lambda^2 = -\ln(0.75)$$

$$\lambda^2 = \frac{-\ln(0.75)}{27500\, h^2}$$

$$\lambda = \sqrt{\frac{-\ln(0.75)}{27500\, h^2}}$$

$$\lambda = 3.234 * 10^{-3} h^{-1}$$

Problem 2: Musa's execution time model

Based on your experience you expect a total number of 200 failures for a software system. At present you run the system test. The initial failure rate was 0.05 / CPU-second. Your goal is to reduce the failure rate to 0.005 / CPU-second. Use Musa's execution time model to answer the following questions:

a) How much total execution time will be necessary? How much additional execution time will be necessary to observe a failure rate of 0.005 / CPU-second after you have reached a failure rate of 0.01 / CPU-second?

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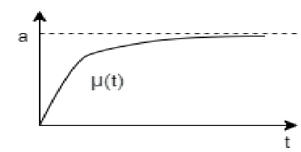
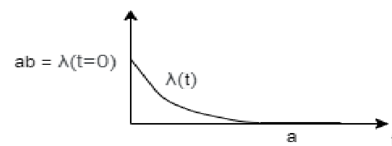
1. How much **total execution time** will be necessary?

The initial failure rate is proportional to the expected number of failure a, with the constant of proportionality b.

$$\lambda(t) = \lambda_0 . e^{-\frac{\lambda_0}{a}t} \qquad \mu(t) = a . (1 - e^{-\frac{\lambda_0}{a}t})$$

<u>Musa's execution model</u>





$$\ln(\lambda(t)) = ln(\lambda_0) - \frac{\lambda_0}{a} t$$

$$t = a . \frac{\ln(\lambda_0) - \ln(\lambda(t))}{\lambda_0}$$

$$t = 200 * \frac{\ln(0.05) - \ln(\lambda(0.005))}{0.05} \ CPU \ sec$$

Total execution time, $t = 9210.340372 \ CPU \ sec$

2. How much **additional execution time will be necessary** to observe a failure rate of 0.005 / CPU-second **after you have reached a failure rate of 0.01 / CPU-second**? *(see slide no 37 in Chapter 10: Reliability)*

The additional time Δt until this target is reached,

$$\Delta t = \frac{a}{\lambda_0} \ln \frac{\lambda_{current}}{\lambda_{target}}$$

$$\Delta t = \frac{200}{0.05} \ln \frac{0.01}{0.005}$$

$$\Delta t = 2772.6 \ CPU \ sec$$

b) How many failures will have occurred when you reach the failure rate of 0.005 / CPU-second? How many additional failures will occur after you have reached a failure rate of 0.01 / CPU-second until you observe a failure rate of 0.005 / CPU-second?

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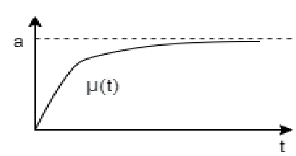
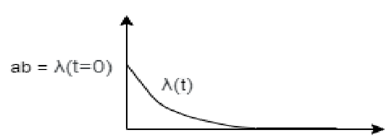
1. How many **failures will have occurred** when you reach the failure rate of 0.005 / CPU-second?





$$\mu(t) = a . (1 - e^{-\frac{\lambda_0}{a}t})$$

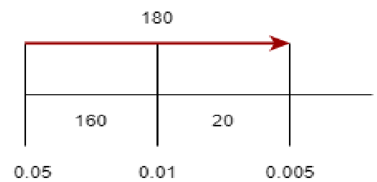
$$\mu(t) = 200 . (1 - e^{-\frac{0.05}{200} 9210.340372})$$

$$\mu(t) = 180 \ failures$$

2. How many **additional failures will occur** after you have reached a failure rate of 0.01 / CPU-second until **you observe a failure rate of 0.005 / CPU-second**?

If λ is the present failure rate and a target λ_z is defined, $\Delta\,\mu$ additional failures will occur until this target is reached.

$$\Delta\mu = a\,.\,(\frac{\lambda - \lambda_z}{\lambda_0})$$



$$\Delta\mu = 200\,.\,(\frac{0.01 - 0.005}{0.05})$$

$$\Delta\mu = 20\ failures$$

c) What will be the **failure rate** after you have observed 100 failures?

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$$\lambda(\mu) = \lambda_0\,.\,(1 - \frac{\mu}{a})$$

$$\lambda(\mu) = 0.05 * (1 - \frac{100}{200})$$

$$\lambda(\mu) = 0.025\ \frac{1}{CPU\ sec}$$

d) What will be the failure rate after 5000 CPU-seconds and how many failures have occurred until then?

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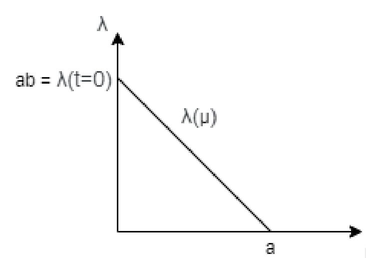
1. What will be the **failure rate after 5000 CPU-seconds**?

$$\lambda(t) = \lambda_0\,.\,e^{-\frac{\lambda_0}{a}t}$$

$$\lambda(t) = 0.05 * e^{-\frac{0.05}{200}5000}\ \frac{1}{CPU\ sec}$$



$$\lambda(t) = 0.014325\ \frac{1}{CPU\ sec}$$

2. **how many failures have occurred** until then?

$$\mu(t) = a\,.\,(1 - e^{-\frac{\lambda_0}{a}t})$$

$$\mu(t) = 200\,.\,(1 - e^{-\frac{0.05}{200}5000})$$

$$\mu(t) = 143\ failures$$

Nazrul.arif93@gmail.com