Safety and Reliability of Embedded Systems - SRES (WS 19/20)

Problem Set 4

Problem 1: Quantitative Markov Modeling

Consider the two given Markov models below. They depict a pump system with two pumps, P1 and P2, where P2 runs in a cold standby (P1 = ON with a workload of 100%, P2 = OFF with a workload of 0%). If P1 fails, P2 will be switched ON and overtake the complete workload immediately. The Pumps are identical and have a failure rate of 0 at a workload of 0% and a failure rate of 2 per year (λ = 2/365 d) at a workload of 100%. If both pumps fail, the whole pump system fails. The two Markov models depict different repair strategies:

1. Both pumps have to be *repaired together*
2. Each pump has to be *repaired separately* after a failure

Derive the differential equations for both Markov models and ***determine a steady state*** analysis for both models. Assume λ=2/365 d and μ=1/d. Give a statement ***which repair strategy is the best*** w.r.t. a lower probability of the system's fail state.

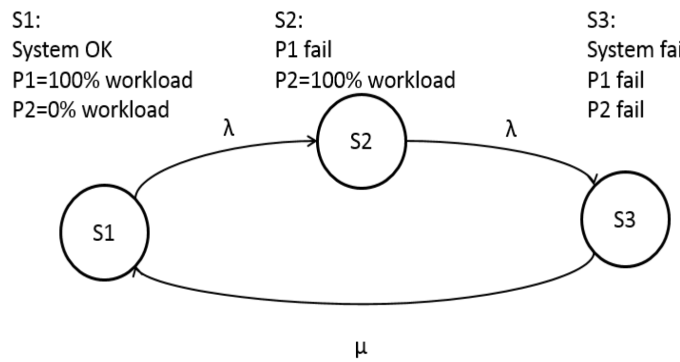
Hint: For calculating the stationary availability t is approximated to infinity. In this case we can set the differential equations ***to 0 ($dP_{Sx}(t)/dt=0$)*** *and we get constant probability values ($P_{Sx}(t) = P_{Sx}$). So we get a homogeneous linear equation system.*

Solution:

1. Both pumps have to be repaired together

-

Derive the differential equations: -



$$\frac{dS_1(t)}{dt} = -\lambda_{11} * S_1(t) + \mu * S_3(t) \ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{dS_2(t)}{dt} = \lambda_{11} * S_1(t) - \lambda_{22} * S_2(t) \ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{dS_3(t)}{dt} = \lambda_{22} * S_2(t) - \mu * S_3(t) \ldots\ldots\ldots\ldots\ldots (3)$$

From (1)

$$-\lambda_{11} * S_1(t) + \mu * S_3(t) = 0$$

$$S_1(t) = \frac{\mu * S_3(t)}{\lambda_{11}} \ldots\ldots\ldots\ldots\ldots\ldots. (4)$$

From (2)

$$\lambda_{11} * S_1(t) - \lambda_{22} * S_2(t) = 0 \ldots\ldots\ldots\ldots (5)$$

From (3)

$$\lambda_{22} * S_2(t) - \mu * S_3(t) = 0$$

$$S_2(t) = \frac{\mu * S_3(t)}{\lambda_{22}} \ldots\ldots\ldots\ldots\ldots\ldots. (6)$$

Nazrul.arif93@gmail.com

Strategy-1 Both pumps have to be repaired together

$S_1(t) + S_2(t) + S_3(t) = 1$

Steady state analysis,

From (4) & (6)

$$\frac{\mu * S_3(t)}{\lambda_{11}} + \frac{\mu * S_3(t)}{\lambda_{22}} + S_3(t) = 1$$

$$S_3(t) = \frac{\lambda}{2\mu + \lambda}$$

$$S_3(t) = \frac{\frac{2}{365d}}{2\frac{1}{d} + \frac{2}{365d}}$$

$$S_3(t) = 0.0027$$

From (4) & (6)

$$S_1(t) = \frac{\mu * S_3(t)}{\lambda_{11}}, \qquad S_1(t) = \frac{\frac{1}{d} * 0.0027}{\frac{2}{365d}} = 0.49275$$

$$S_2(t) = \frac{\mu * S_3(t)}{\lambda_{22}}, \qquad S_2(t) = \frac{\frac{1}{d} * 0.0027}{\frac{2}{365d}} = 0.49275$$

$\lambda_{11} = \lambda_{22}$

So,

$S_1(t) = 0.49275$

$S_2(t) = 0.49275$

$S_3(t) = 2.7 * 10^{-1}$

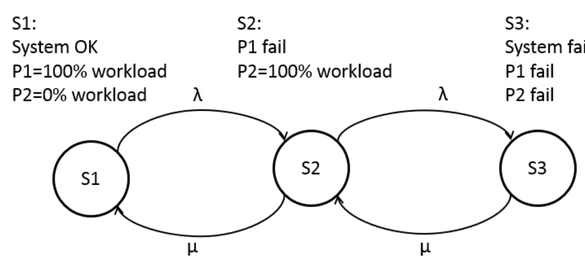
2. Each pump has to be repaired separately after a failure

-

$$\frac{dS_1(t)}{dt} = -\lambda_{11} * S_1(t) + \mu * S_2(t) \;\ldots\ldots(1)$$

$$\frac{dS_2(t)}{dt} = \lambda_{11} * S_1(t) - (\lambda_{22} + \mu) * S_2(t) + \mu * S_3(t) \ldots\ldots(2)$$

$$\frac{dS_3(t)}{dt} = \lambda_{22} * S_2(t) - \mu * S_3(t) \;\ldots\ldots(3)$$



So,

$S_1(t) = 0.9945$

$S_2(t) = 5.4 * 10^{-3}$

$S_3(t) = 2.9 * 10^{-5}$

Nazrul.arif93@gmail.com

Repaired together (Strategy 1)	Repaired separately (Strategy 2)
$S_1(t) = 0.49275$	$S_1(t) = 0.9945$
$S_2(t) = 0.49275$	$S_2(t) = 5.4 * 10^{-3}$
$S_3(t) = 2.7 * 10^{-1}$	$S_3(t) = 2.9 * 10^{-5}$

$$S_3(t) = 2.9 * 10^{-5} \ll S_3(t) = 2.7 * 10^{-1}$$

At state 3, Strategy 2 is better than Strategy 1