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Problem - 1 :

Decision Variables

c : # of scoops of chocolate chips
m: # of " " mint chocolate chips
r: # " " " Rocky road

objective functions,

$$\max \quad c + 2m + 3r$$

Constraints,

$$4c + 2m + r \le 35$$
$$3c + 5m + 4r \le 50$$
$$c + 2m + r \le 20$$
$$c, m, r \ge 0$$

Problem-2:

Decision variables

c : # of acres of cabbage to be planted

p : # " " " potatoes " "

objective function.

$$\max \quad 80c \times 5 + 40p \times 3$$

$$\text{or,} \quad \max \quad 400c + 120p$$

Constraints

$80c \le 250$ (cabbage production constraints)

$6c + 8p \le 48$ (labor constraints)

$c + p \le 40$ (Acres of firmland)

Problem-3:

(a) r is free,

Step-1

$$\max \quad 3q + 6(r^+ - r^-) - 7s$$

st,

$$q + 2(r^+ - r^-) \geq -4 \quad \text{--- (i)}$$

$$2.5q - (r^+ - r^-) \geq 7 \quad \text{--- (ii)}$$

$$q + (r^+ - r^-) = -4 \quad \text{--- (iii)}$$

$$q, s, r^+, r^- \geq 0 \quad \text{--- (iv)}$$

Step-2

Need to convert the above in a standard form

$$\min \quad -3q - 6(r^+ - r^-) + 7s$$

st,

$$q + 2(r^+ - r^-) \geq -4$$

$$2.5q - (r^+ - r^-) \geq 7$$

$$q + (r^+ - r^-) \geq -4$$

$$-q - (r^+ - r^-) \geq 4$$

$$q, s, r^+, r^- \geq 0$$

(b)
$$X = \begin{bmatrix} q \\ r^+ \\ r^- \\ s \end{bmatrix}, \quad b = \begin{bmatrix} -4 \\ 7 \\ -4 \\ 4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2.5 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}$$

$$C = [3 \;\; 6 \;\; -6 \;\; -7]$$

Problem - 4:

(a) In set notation,

$$\sum_{j=1}^{4} a_j x_j = b$$

(b) $x_j - y_j \geq 0 \quad \forall j \in \{1,2,3,4\}$

(c) $\sum_{j=1}^{3} a_{kj} x_j = b_k \quad \forall k \in \{1,2,3\}$

Using algebric notation,

(d) $c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 = 0$

(e)
$$X_1 + Y_1 + Z_1 = 1$$
$$X_2 + Y_2 + Z_2 = 1$$
$$X_3 + Y_3 + Z_3 = 1$$

(f)
$$b_1 W_1 + b_2 W_2 + b_3 W_3 = C_1$$
$$b_1 W_1 + b_2 W_2 + b_3 W_3 = C_2$$
$$b_1 W_1 + b_2 W_2 + b_3 W_3 = C_3$$

Problem - 5

(a) Sets

Several Musical pieces, M

(b) Parameters

Per minute value, v

minimum value, V_{min}

(c) Variables

s: stamina in minutes

m: a musical piece

(d) Objective function:

min (amount of stamina lost)

or, $\min \sum_{\forall s \in S, \forall m \in M} sm$

(e) Constraint

$$\sum_{\forall m \in M, \forall v \in V} mv \geq V_{min} \quad \text{(value constraint)}$$

$$m \geq 5 \quad \forall m \in M \quad \text{(Music Constraint)}$$

Scanned with CamScanner

Problem-6:

First constraint,

$$x_1 + 2x_2 \leq 12$$

Consider,

$$x_1 + 2x_2 = 12$$

$$\Rightarrow \frac{x_1}{12} + \frac{x_2}{6} = 1$$

The line will intercept at (12,0) & (0,6) point.

Second constraint,

$$2x_1 + 3x_2 = 12$$

$$\Rightarrow \frac{x_1}{6} + \frac{x_2}{4} = 1$$

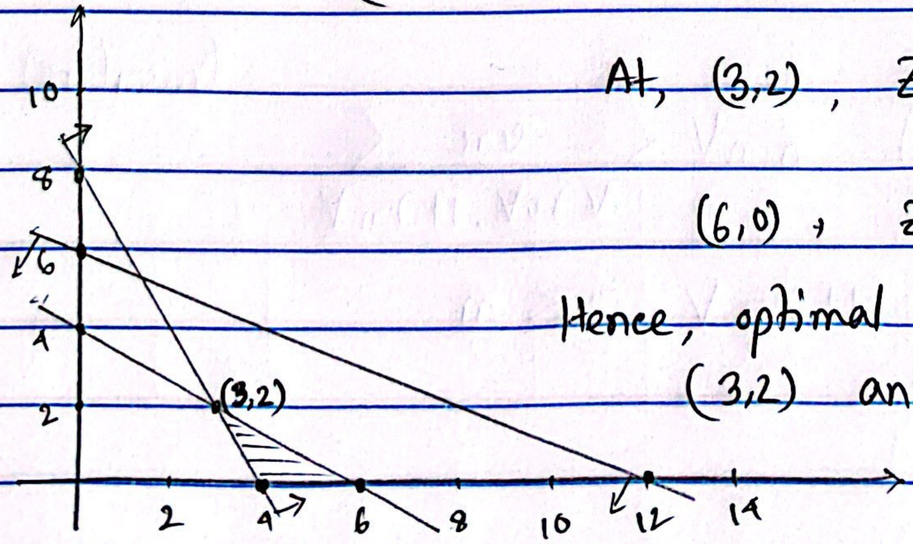
So, the point now becomes, (6,0) & (0,4)

Third constraint, consider,

$$2x_1 + x_2 = 8$$

$$\Rightarrow \frac{x_1}{4} + \frac{x_2}{8} = 1$$

So, now, (4,0) & (0,8).



At, (3,2), $Z = 3\times3 + 2\times2$
$= 13$

(6,0), $Z = 18$

Hence, optimal solution is at, (3,2) and $Z = 13$.