Assignment 3

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Feb 11, 2023

import numpy as np

def householder(A):

    m, n = A.shape

    R = np.copy(A)

    Q = np.identity(m)

    for i in range(n):

        x = R[i:, i]

        e = np.zeros\_like(x)

        e[0] = np.linalg.norm(x)

        u = x - e

        v = u / np.linalg.norm(u)

        H = np.identity(m)

        H[i:, i:] -= 2 \* np.outer(v, v)

        R = np.dot(H, R)

        Q = np.dot(Q, np.transpose(H))

        print(f"Step ======= {i} =========")

        print(f"Q: {Q} \n R: {R}")

    return Q, R

A = np.array([[1, -1, 4], [1, 4, -2], [1, 4, 2], [1, -1, 0]])

Q, R = householder(A)

print(f"A:\n {A}")

print(f"Q Result :\n {Q}")

print(f"R Result :\n {R}")

print(f"Varify Result (multiply Q \* R) -> \n got: \n {np.dot(Q, R)} \n given: \n {A}")

OUTPUT:

Step ======= 0 =========

Q: [[ 0.5 0.5 0.5 0.5]

[ 0.5 0.5 -0.5 -0.5]

[ 0.5 -0.5 0.5 -0.5]

[ 0.5 -0.5 -0.5 0.5]]

R: [[ 2. 3. 2.]

[ 0. 0. 0.]

[ 0. 0. 4.]

[ 0. -5. 2.]]

Step ======= 1 =========

Q: [[ 0.5 -0.5 0.5 -0.5]

[ 0.5 0.5 -0.5 -0.5]

[ 0.5 0.5 0.5 0.5]

[ 0.5 -0.5 -0.5 0.5]]

R: [[ 2.00000000e+00 3.00000000e+00 2.00000000e+00]

[ 0.00000000e+00 5.00000000e+00 -2.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 4.00000000e+00]

[ 0.00000000e+00 -1.11022302e-15 4.44089210e-16]]

Step ======= 2 =========

Q: [[ 0.5 -0.5 0.5 0.5]

[ 0.5 0.5 -0.5 0.5]

[ 0.5 0.5 0.5 -0.5]

[ 0.5 -0.5 -0.5 -0.5]]

R: [[ 2.00000000e+00 3.00000000e+00 2.00000000e+00]

[ 0.00000000e+00 5.00000000e+00 -2.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 4.00000000e+00]

[ 0.00000000e+00 1.11022302e-15 -4.44089210e-16]]

A:

[[ 1 -1 4]

[ 1 4 -2]

[ 1 4 2]

[ 1 -1 0]]

Q Result :

[[ 0.5 -0.5 0.5 0.5]

[ 0.5 0.5 -0.5 0.5]

[ 0.5 0.5 0.5 -0.5]

[ 0.5 -0.5 -0.5 -0.5]]

R Result :

[[ 2.00000000e+00 3.00000000e+00 2.00000000e+00]

[ 0.00000000e+00 5.00000000e+00 -2.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 4.00000000e+00]

[ 0.00000000e+00 1.11022302e-15 -4.44089210e-16]]

Varify Result (multiply Q \* R) ->

|  |  |
| --- | --- |
| got:  [[ 1. -1. 4.]  [ 1. 4. -2.]  [ 1. 4. 2.]  [ 1. -1. 0.]] | given:  [[ 1 -1 4]  [ 1 4 -2]  [ 1 4 2]  [ 1 -1 0]] |