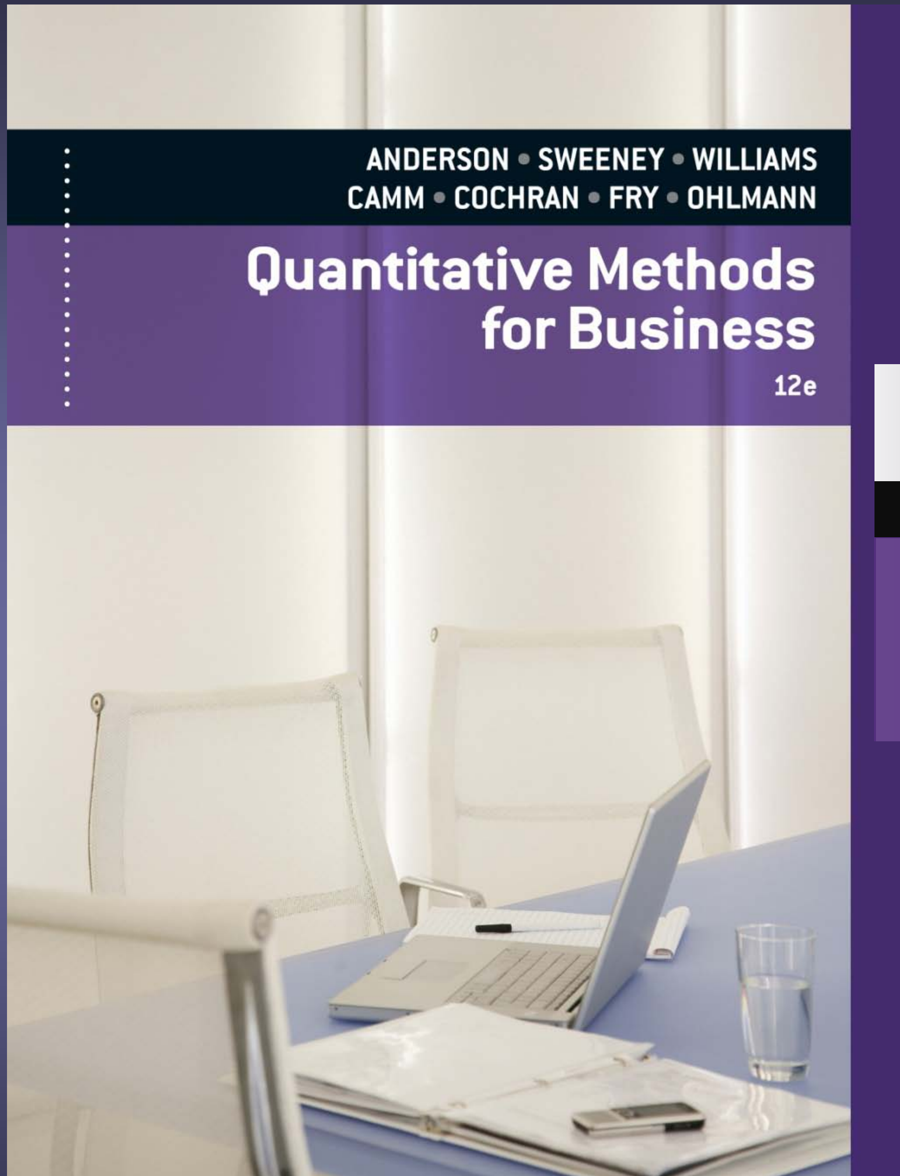

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Chapter 14, Part A
Inventory Models with Deterministic Demand

- Economic Order Quantity (EOQ) Model
- Economic Production Lot Size Model
- Inventory Model with Planned Shortages
- Quantity Discounts for the EOQ Model

Inventory Models

- The study of inventory models is concerned with two basic questions:
  - How much should be ordered each time
  - When should the reordering occur
- The objective is to minimize total variable cost over a specified time period (assumed to be annual in the following review).

Inventory Costs

- <u>Ordering cost</u> -- salaries and expenses of processing an order, regardless of the order quantity
- <u>Holding cost</u> -- usually a percentage of the value of the item assessed for keeping an item in inventory (including cost of capital, insurance, security costs, taxes, warehouse overhead, and other related variable expenses)
- <u>Backorder cost</u> -- costs associated with being out of stock when an item is demanded (including lost goodwill)
- <u>Purchase cost</u> -- the actual price of the items
- Other costs

Deterministic Models

- The simplest inventory models assume demand and the other parameters of the problem to be <u>deterministic</u> and constant.
- The deterministic models covered in this chapter are:
  - Economic order quantity (EOQ)
  - Economic production lot size
  - EOQ with planned shortages
  - EOQ with quantity discounts

Economic Order Quantity (EOQ)

- The most basic of the deterministic inventory models is the economic order quantity (EOQ).
- The variable costs in this model are annual holding cost and annual ordering cost.
- For the EOQ, annual holding and ordering costs are equal.

Economic Order Quantity

- Assumptions
  - Demand *D* is known and occurs at a constant rate.
  - The order quantity *Q* is the same for each order.
  - The cost per order, $\$C_o$, is constant and does not depend on the order quantity.
  - The purchase cost per unit, *C*, is constant and does not depend on the quantity ordered.
  - The inventory holding cost per unit per time period, $\$C_h$, is constant.
  - Shortages such as stock-outs or backorders are not permitted.
  - The lead time for an order is constant.
  - The inventory position is reviewed continuously.

Economic Order Quantity

- Formulas
  - Optimal order quantity: $Q^* = \sqrt{2DC_o/C_h}$
  - Number of orders per year: D/Q^*
  - Time between orders (cycle time): Q^*/D years
  - Total annual cost: $[C_h(Q^*/2)] + [C_o(D/Q^*)]$
    (holding + ordering)

Example: Bart's Barometer Business

- Economic Order Quantity Model

Bart's Barometer Business is a retail outlet that deals exclusively with weather equipment. Bart is trying to decide on an inventory and reorder policy for home barometers.

Barometers cost Bart $50 each and demand is about 500 per year distributed fairly evenly throughout the year.

Example: Bart’s Barometer Business

- Economic Order Quantity Model

  Reordering costs are $80 per order and holding costs are figured at 20% of the cost of the item. Bart's Barometer Business is open 300 days a year (6 days a week and closed two weeks in August). Lead time is 60 working days.

Example: Bart's Barometer Business

- Total Variable Cost Model

Total Costs = (Holding Cost) + (Ordering Cost)

$$
\begin{aligned}
TC &= [C_h(Q/2)] + [C_o(D/Q)] \\
&= [.2(50)(Q/2)] + [80(500/Q)] \\
&= 5Q + (40{,}000/Q)
\end{aligned}
$$

Example: Bart's Barometer Business

- Optimal Reorder Quantity

$$Q^* = \sqrt{2DC_o/C_h} = \sqrt{2(500)(80)/10} = 89.44 \approx \boxed{90}$$

- Optimal Reorder Point

Lead time is m = 60 days and daily demand is d = 500/300 or 1.667.

Thus the reorder point $r = dm = (1.667)(60) = \boxed{100}$

Bart should reorder 90 barometers when his inventory position reaches 100 (that is 10 on hand and one outstanding order).

Example: Bart's Barometer Business

- Number of Orders Per Year

  Number of reorder times per year = (500/90) = 5.56 or once every (300/5.56) = 54 working days (about every 9 weeks).

- Total Annual Variable Cost

  TC = 5(90) + (40,000/90) = 450 + 444 = $894

Sensitivity Analysis for the EOQ Model

- Optimal Order Quantities for Several Costs

Possible Inventory Holding Cost	Possible Cost Per Order	Optimal Order Qnty. (*Q**)	Projected Total Annual Cost Using *Q**	Projected Total Annual Cost Using *Q* = 90
18%	$75	91 units	$822	$822
18	85	97	875	877
22	75	83	908	912
22	85	88	967	967

Example: Bart's Barometer Business

We'll now use a spreadsheet to implement the Economic Order Quantity model. We'll confirm our earlier calculations for Bart's problem and perform some sensitivity analysis.

This spreadsheet can be modified to accommodate other inventory models presented in this chapter.

Example: Bart's Barometer Business

- Partial Spreadsheet with Input Data

	A	B
1	BART'S ECONOMIC ORDER QUANTITY	
2		
3	Annual Demand	500
4	Ordering Cost	$80.00
5	Annual Holding Rate %	20
6	Cost Per Unit	$50.00
7	Working Days Per Year	300
8	Lead Time (Days)	60

Example: Bart's Barometer Business

- Partial Spreadsheet Showing Formulas

	A	B	C
10	Econ. Order Qnty.	=SQRT(2*B3*B4/(B5*B6/100))	
11	Request. Order Qnty		
12	% Change from EOQ		=(C11/B10-1)*100
13			
14	Annual Holding Cost	=B5/100*B6*B10/2	=B5/100*B6*C11/2
15	Annual Order. Cost	=B4*B3/B10	=B4*B3/C11
16	Tot. Ann. Cost (TAC)	=B14+B15	=C14+C15
17	% Over Minimum TAC		=(C16/B16-1)*100
18			
19	Max. Inventory Level	=B10	=C11
20	Avg. Inventory Level	=B10/2	=C11/2
21	Reorder Point	=B3/B7*B8	=B3/B7*B8
22			
23	No. of Orders/Year	=B3/B10	=B3/C11
24	Cycle Time (Days)	=B10/B3*B7	=C11/B3*B7

Example: Bart's Barometer Business

- Partial Spreadsheet Showing Output

	A	B	C
10	Econ. Order Qnty.	89.44	
11	Request. Order Qnty.		75.00
12	% Change from EOQ		-16.15
13			
14	Annual Holding Cost	$447.21	$375.00
15	Annual Order. Cost	$447.21	$533.33
16	Tot. Ann. Cost (TAC)	$894.43	$908.33
17	% Over Minimum TAC		1.55
18			
19	Max. Inventory Level	89.44	75
20	Avg. Inventory Level	44.72	37.5
21	Reorder Point	100	100
22			
23	No. of Orders/Year	5.59	6.67
24	Cycle Time (Days)	53.67	45.00

Example: Bart's Barometer Business

- Summary of Spreadsheet Results
  - A 16.15% negative deviation from the EOQ resulted in only a 1.55% increase in the Total Annual Cost.
  - Annual Holding Cost and Annual Ordering Cost are no longer equal.
  - The Reorder Point is not affected, in this model, by a change in the Order Quantity.

Economic Production Lot Size

- The economic production lot size model is a variation of the basic EOQ model.
- A replenishment order is not received in one lump sum as it is in the basic EOQ model.
- Inventory is replenished gradually as the order is produced (which requires the production rate to be greater than the demand rate).
- This model's variable costs are annual holding cost and annual set-up cost (equivalent to ordering cost).
- For the optimal lot size, annual holding and set-up costs are equal.

Economic Production Lot Size

- Assumptions
  - Demand occurs at a constant rate of D items per year or d items per day.
  - Production rate is P items per year or p items per day (and $P > D$, $p > d$).
  - Set-up cost: $\$C_o$ per run.
  - Holding cost: $\$C_h$ per item in inventory per year.
  - Purchase cost per unit is constant (no quantity discount).
  - Set-up time (lead time) is constant.
  - Planned shortages are not permitted.

Economic Production Lot Size

- Formulas
  - Optimal production lot-size:

$$Q^* = \sqrt{2DC_o / [(1-D/P)C_h]}$$

  - Number of production runs per year: D/Q^*
  - Time between set-ups (cycle time): Q^*/D years
  - Total annual cost: $[C_h(Q^*/2)(1-D/P)] + [C_o/(D/Q^*)]$
    (holding + ordering)
  - Length of the production run: $t = Q^*/p$

Example: Beauty Bar Soap

- Economic Production Lot Size Model

Beauty Bar Soap is produced on a production line that has an annual capacity of 60,000 cases. The annual demand is estimated at 26,000 cases, with the demand rate essentially constant throughout the year.

The cleaning, preparation, and setup of the production line cost approximately \$135. The manufacturing cost per case is \$4.50, and the annual holding cost is figured at a 24% rate.

Other relevant data include a five-day lead time to schedule and set up a production run and 250 working days per year.

Example: Beauty Bar Soap

- Total Annual Variable Cost Model

This is an economic production lot size problem with
$D = 26{,}000$, $P = 60{,}000$, $C_h = 1.08$, $C_o = 135$

$$TC = \text{(Holding Costs)} + \text{(Set-Up Costs)}$$
$$= [C_h(Q/2)(1 - D/P)] + [C_o(D/Q)]$$
$$= [1.08(Q/2)(1 - 26{,}000/60{,}000)] + [135(26{,}000/Q)]$$
$$= .306Q + 3{,}510{,}000/Q$$

Example: Beauty Bar Soap

- Optimal Production Lot Size

$$Q^* = \sqrt{2DC_o/[(1-D/P)C_h]}$$

$$= \sqrt{2(26{,}000)(135)\ /[(1 - 26{,}000/60{,}000))1.08)]}$$

$$= \boxed{3{,}387}$$

- Number of Production Runs (Cycles) Per Year

$$D/Q^* = 26{,}000/3{,}387$$

$$= \boxed{7.6764} \text{ times per year}$$

Example: Beauty Bar Soap

- Total Annual Variable Cost

$$\begin{aligned} \text{Optimal } TC &= .306(3{,}387) + 3{,}510{,}000/3{,}387 \\ &= 1{,}036.42 + 1{,}036.32 \\ &= \boxed{\$2{,}073} \end{aligned}$$

Example: Beauty Bar Soap

- Idle Time Between Production Runs

There are 7.6764 cycles per year.

Thus, each cycle lasts (250/7.6764) = 32.567 days.

The time to produce 3,387 per run:

3387/240 = 14.1125 days.

Thus, the production line is idle for:

32.567 – 14.1125 = 18.4545 days between runs.

The production line is utilized:

14.1125/32.567(100) = 43.33%

Example: Beauty Bar Soap

- Maximum Inventory

  Maximum inventory = $(1-D/P)Q^*$

  $= (1-26{,}000/60{,}000)3{,}387 \approx \boxed{1{,}919.3}$

- Machine Utilization

  Machine is producing $D/P = 26{,}000/60{,}000$

  $= \boxed{.4333}$ of the time.

EOQ with Planned Shortages

- With the EOQ with planned shortages model, a replenishment order does not arrive at or before the inventory position drops to zero.
- Shortages occur until a predetermined backorder quantity is reached, at which time the replenishment order arrives.
- The variable costs in this model are annual holding, backorder, and ordering.
- For the optimal order and backorder quantity combination, the sum of the annual holding and backordering costs equals the annual ordering cost.

EOQ with Planned Shortages

- Assumptions
  - Demand occurs at a constant rate of D items/year.
  - Ordering cost: $\$C_o$ per order.
  - Holding cost: $\$C_h$ per item in inventory per year.
  - Backorder cost: $\$C_b$ per item backordered per year.
  - Purchase cost per unit is constant (no qnty. discount).
  - Set-up time (lead time) is constant.
  - Planned shortages are permitted (backordered demand units are withdrawn from a replenishment order when it is delivered).

EOQ with Planned Shortages

- Formulas
  - Optimal order quantity:

$$Q^* = \sqrt{2DC_o/C_h}\sqrt{(C_h+C_b)/C_b}$$

  - Maximum number of backorders:

$$S^* = Q^*(C_h/(C_h+C_b))$$

  - Number of orders per year: D/Q^*
  - Time between orders (cycle time): Q^*/D years
  - Total annual cost:

$$[C_h(Q^*-S^*)^2/2Q^*] + [C_o(D/Q^*)] + [S^{*2}C_b/2Q^*]$$

(holding + ordering + backordering)

Example: Higley Radio Components Co.

- EOQ with Planned Shortages Model

  Higley has a product for which the assumptions of the inventory model with shortages are valid. Demand for the product is 2,000 units per year. The inventory holding cost rate is 20% per year. The product costs Higley $50 to purchase. The ordering cost is $25 per order. The annual shortage cost is estimated to be $30 per unit per year. Higley operates 250 days per year.

Example: Higley Radio Components Co.

- Optimal Order Policy

D = 2,000; C_o = \$25; C_h = .20(50) = \$10; C_b = \$30

$$Q^* = \sqrt{2DC_o/C_h}\sqrt{(C_h + C_b)/C_b}$$

$$= \sqrt{2(2000)(25)/10} \times \sqrt{(10+30)/30}$$

$$= \boxed{115.47}$$

$$S^* = Q^*(C_h/(C_h+C_b))$$

$$= 115.47(10/(10+30)) = \boxed{28.87}$$

Example: Higley Radio Components Co.

- Maximum Inventory

  $Q - S = 115.47 - 28.87 = \boxed{86.6}$ units

- Cycle Time

  $T = Q/D(250) = 115.47/2000(250) = \boxed{14.43}$ working days

Example: Higley Radio Components Co.

- Total Annual Cost

Holding Cost:

$$C_h(Q - S)^2/(2Q) = 10(115.47 - 28.87)^2/(2(115.47))$$
$$= \$324.74$$

Ordering Cost:

$$C_o(D/Q) = 25(2000/115.47) = \$433.01$$

Backorder Cost:

$$C_b(S^2/(2Q) = 30(28.87)^2/(2(115.47)) = \$108.27$$

Total Cost:

$$324.74 + 433.01 + 108.27 = \boxed{\$866.02}$$

Example: Higley Radio Components Co.

- Stockout: When and How Long

Question:

How many days after receiving an order does Higley run out of inventory? How long is Higley without inventory per cycle?

Example: Higley Radio Components Co.

- Stockout: When and How Long

Answer:

Inventory exists for $C_b/(C_b+C_h) = 30/(30+10) = .75$ of the order cycle. (Note, $(Q^*-S^*)/Q^* = .75$ also, before Q^* and S^* are rounded.)

An order cycle is $Q^*/D = .057735$ years = 14.434 days. Thus, Higley runs out of inventory .75(14.434) = $\boxed{\text{10.823 days after receiving an order}}$

Higley is out of stock for approximately 14.434 – 10.823 = $\boxed{\text{3.611 days per order cycle}}$

Example: Higley Radio Components Co.

- Reorder Point

Question:

At what inventory or backorder level should Higley place an inventory replenishment order?

Example: Higley Radio Components Co.

- Reorder Point

Answer:

Higley is out of stock for approximately 3.611 working days per order cycle. Reorder lead time is 7 working days. Hence, Higley should reorder when it has inventory on hand to cover just 7 – 3.611 = 3.389 days of demand.

Demand per day is 2000/250 = 8 units. Hence, 3.389 days of inventory is 3.389 x 8 = 27.112 units.

Higley should place an order for 115 units when its inventory drops to 27.112 units

Example: Higley Radio Components Co.

- EOQs with and with<u>out</u> Planned Shortages

	With Shortages	Without Shortages
EOQ (units)	115.47	100
Max. Inventory (units)	86.60	100
Max. Shortages (units)	28.87	0
Reorder Point (units)	27.112	56
Cycle Time (days)	14.43	12.50
Shortage Time/Cycle (days)	3.611	0
Shortage Time/Cycle (%)	25	0

Example: Higley Radio Components Co.

- EOQs with and without Planned Shortages

Annual Costs	With Shortages	Without Shortages
Holding	$324.74	$500.00
Ordering	433.01	500.00
Backordering	108.27	0
Total	**$866.02**	**$1000.00**

EOQ with Quantity Discounts

- The EOQ with quantity discounts model is applicable where a supplier offers a lower purchase cost when an item is ordered in larger quantities.
- This model's variable costs are annual holding, ordering and purchase costs.
- For the optimal order quantity, the annual holding and ordering costs are **not** necessarily equal.

EOQ with Quantity Discounts

- Assumptions
  - Demand occurs at a constant rate of D items/year.
  - Ordering Cost is \$$C_o$ per order.
  - Holding Cost is \$$C_h$ = \$$C_iI$ per item in inventory per year (note holding cost is based on the cost of the item, C_i).
  - Purchase Cost is \$$C_1$ per item if the quantity ordered is between 0 and x_1, \$$C_2$ if the order quantity is between x_1 and x_2, etc.
  - Delivery time (lead time) is constant.
  - Planned shortages are not permitted.

EOQ with Quantity Discounts

- Formulas
  - Optimal order quantity: the procedure for determining Q^* will be demonstrated
  - Number of orders per year: D/Q^*
  - Time between orders (cycle time): Q^*/D years
  - Total annual cost: $[C_h(Q^*/2)] + [C_o(D/Q^*)] + DC$ (holding + ordering + purchase)

Example: Nick's Camera Shop

- EOQ with Quantity Discounts Model

  Nick's Camera Shop carries Zodiac instant print film. The film normally costs Nick $3.20 per roll, and he sells it for $5.25. Zodiac film has a shelf life of 18 months. Nick's average sales are 21 rolls per week. His annual inventory holding cost rate is 25% and it costs Nick $20 to place an order with Zodiac.

Example: Nick's Camera Shop

- EOQ with Quantity Discounts Model

If Zodiac offers a 7% discount on orders of 400 rolls or more, a 10% discount for 900 rolls or more, and a 15% discount for 2000 rolls or more, determine Nick's optimal order quantity.

D = 21(52) = 1092; C_h = .25(C_i); C_o = 20

Example: Nick's Camera Shop

- Unit-Prices' Economical Order Quantities
  - For C_4 = .85(3.20) = \$2.72

    To receive a 15% discount Nick must order at least 2,000 rolls. Unfortunately, the film's shelf life is 18 months. The demand in 18 months (78 weeks) is 78 x 21 = 1638 rolls of film.

    If he ordered 2,000 rolls he would have to scrap 372 of them. This would cost more than the 15% discount would save.

Example: Nick's Camera Shop

- Unit-Prices' Economical Order Quantities
  - For $C_3 = .90(3.20) = \$2.88$

$$Q_3^* = \sqrt{2DC_o/C_h} = \sqrt{2(1092)(20)/[.25(2.88)]} = 246.31$$

(not feasible)

The most economical, feasible quantity for C_3 is 900.

  - For $C_2 = .93(3.20) = \$2.976$

$$Q_2^* = \sqrt{2DC_o/C_h} = \sqrt{2(1092)(20)/[.25(2.976)]} = 242.30$$

(not feasible)

The most economical, feasible quantity for C_2 is 400.

Example: Nick's Camera Shop

- Unit-Prices' Economical Order Quantities
  - For $C_1 = 1.00(3.20) = \$3.20$

$$Q_1^* = \sqrt{2DC_o/C_h} = \sqrt{2(1092)(20)/.25(3.20)} = 233.67$$

(feasible)

When we reach a <u>computed</u> Q that is feasible we stop computing Q's. (In this problem we have no more to compute anyway.)

Example: Nick's Camera Shop

- Total Cost Comparison

Compute the total cost for the most economical, feasible order quantity in each price category for which a Q * was computed.

$$TC_i = (C_h)(Q_i^*/2) + (C_o)(D/Q_i^*) + DC_i$$

$$TC_3 = (.72)(900/2) + (20)(1092/900) + (1092)(2.88) = 3{,}493$$

$$TC_2 = (.744)(400/2) + (20)(1092/400) + (1092)(2.976) = 3{,}453$$

$$TC_1 = (.80)(234/2) + (20)(1092/234) + (1092)(3.20) = 3{,}681$$

Comparing the total costs for order quantities of 234, 400 and 900, the lowest total annual cost is \$3,453. Nick should order 400 rolls at a time.

End of Chapter 14, Part A