

Final Exam

Exercise 1

Part A

$$F(x) = \int_k^x \theta k^\theta * x^{-(\theta+1)} dx = \theta k^\theta \int_k^x x^{-(\theta+1)} dx = \theta k^\theta \left[-\frac{1}{\theta} x^{-\theta} \right]_k^x = \theta k^\theta \left(-\frac{1}{\theta} x^{-\theta} + \frac{1}{\theta} k^{-\theta} \right) = 1 - \left(\frac{k}{x} \right)^\theta \quad (1)$$

Let $u \sim \text{unif}(0, 1)$

$$u = F(x) \implies u = 1 - \left(\frac{k}{x} \right)^\theta \implies (1 - u) = \left(\frac{k}{x} \right)^\theta \implies (1 - u)^{1/\theta} = \left(\frac{k}{x} \right) \implies x = \frac{k}{(1 - u)^{1/\theta}} \implies \therefore F^{-1}(x) = \frac{k}{(1 - x)^{1/\theta}} \quad (2)$$