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Math 640 Bayesian Statistics
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Final Exam

Exercise 1

Part A

$$F(x) = \int_k^x \theta k^\theta * x^{-(\theta+1)} dx = \theta k^\theta \int_k^x x^{-(\theta+1)} dx = \theta k^\theta \Big|_k^x - \frac{1}{\theta} x^{-\theta} dx = \theta k^\theta \left(-\frac{1}{\theta} x^{-\theta} + \frac{1}{\theta} k^{-\theta} \right) = 1 - \left(\frac{k}{x} \right)^\theta \tag{1}$$

Let $u \sim unif(0,1)$

$$u = F(x) \implies u = 1 - \left(\frac{k}{x} \right)^\theta \implies (1-u) = \left(\frac{k}{x} \right)^\theta \implies (1-u)^{1/\theta} = \left(\frac{k}{x} \right) \implies x = \frac{k}{(1-u)^{1/\theta}}$$
$$\therefore F^{-1}(x) = \frac{k}{(1-x)^{1/\theta}} \tag{2}$$

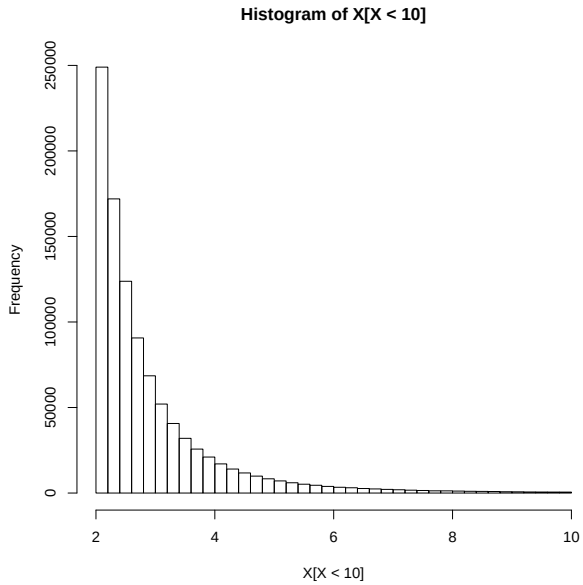
Obtain a random sample, u, from $U \sim unif(0,1)$. Obtain x by computing $F^{-1}(x)$ and ensure that the values of $x \leq k$ will be evaluated to zero (this will only occur when u = 0).

Part B

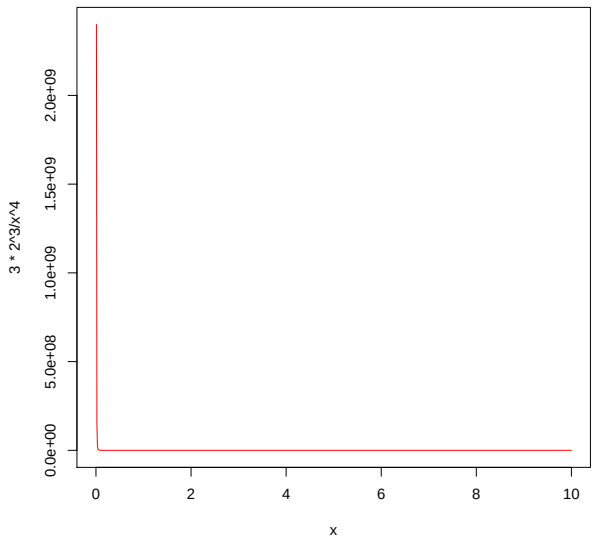
```
set.seed(1)
k=2
theta = 3
u = runif(1e6)
X = k/(1-u)^(1/theta)
X[u==0]=0
```

Part C

```
hist(X[X<10], breaks = 50)
```




```
x = seq(0,10, 0.01)
plot(x, 3*2^3/x^4, col = "red", type = "l")
```



Part D

$P(X < 3) = F(x = 3) = 1 - (\frac{2}{3})^3) = 0.7037037$

```
mean(X<3)
## [1] 0.703963
```

Exercise 2