ANLY-511 Homework assigned on 10/3/15
Email a pdf file with solutions (preferred)
or hand it in at my office door by Friday,10/9 11:59PM.
Five "short" and three "long" problems.

Explain your work and give concise reasoning. Attach `R` *code with comments if applicable. Do not print out any data or any detailed results of simulations. Your solutions for this homework set should fit on no more than six pages including graphs. In problems requiring graphs, you are allowed to just give the code to make the graphs and to describe them, without actually including them.*

25. (2 points) Consider the symmetric random walk that was discussed in class on 9/28, where the states are integers $x \in \mathbb{Z}$ and the transition probabilities are

$$\mathcal{P}(X_{i+1} = x + 1 | X_i = x) = \mathcal{P}(X_{i+1} = x - 1 | X_i = x) = \frac{1}{2}$$

for all i and all x and $X_0 = 0$. Let T be the random time when $X_T = 20$ for the first time. Use a simulation to generate a few hundred values of T and then make a box plot of T. *Your answer should consist of commented simulation code and either the box plot or a description (max, min, quartiles, median).*

26. (2 points) In American roulette it is possible to bet on a "line", consisting e.g. of the numbers 3, 6, 9, 12, ..., 36. You win twice your bet if one of these numbers comes up, and you lose your bet otherwise. Propose a modification of the St. Petersburg system for somebody who only uses this bet, and explain. *It is not necessary to triple your bet each time you lose!*

27. (2 points) Joe's preferred bet in American roulette consists in betting \$1 on red numbers and simultaneously \$1 on odd numbers (see the roulette board in the course slides). Find the probability distribution of the outcome of a single bit, and compute its expected value.

28. (2 points) Consider a room that is paved with $n \times n$ square tiles. A frog performs a random walk by hopping from one tile to a randomly chosen adjacent tile in each time step. All adjacent tiles are chosen with the same probability. The frog can never hops into a wall of the room.
True or not true: the transition matrix for this random walk is symmetric, that is, it satisfies $\mathcal{P}(X_{i+1} = k | X_i = j) = \mathcal{P}(X_{i+1} = j | X_i = k)$ for all i and all $1 \leq j, k \leq n$. Explain your answer.

29. (2 points) Consider the following game: you get to roll an n-sided fair die once. If the outcome is the number k, then you get to throw k darts at a target. The probability of hitting the target is p. Dart throws are independent of one another. Let X be the result of the roll of the die, and let Y be the number of hits when you throw the darts. Set up the joint probability mass function (pmf) for X and Y.

30. (5 points) Consider the random walk performed by the caveman in the class slides of 9/29.
a) Find all transition probabilities from room # 6.
b) Set up the entire transition matrix in `R` and compute $\mathcal{P}(X_2 = 0 | X_0 = 0)$ using the transition matrix. Then do the same computation directly. *Do not print out the entire transition matrix.*
c) Find the probability that the caveman is still alive after 30 steps, using `R` .

31. (5 points) Let N be a random variable with a Poisson distribution with parameter $\lambda > 0$. Given that $N = n$, let X be a binomial $B(n, p)$ distribution where $0 < p < 1$.
a) Set up the joint probability mass function for N and X, in terms of the parameters λ and p.
b) Write an `R` function with input λ, p, k that simulates k values of X.
c) Pick some values of λ and p and simulate sufficiently many instances in each case to obtain an estimate of $\mathcal{E}(X)$. Use `sapply` or `replicate`, do not use `for` loops. Then guess a formula for $\mathcal{E}(X)$ and explain why it makes sense to you. *To document this, only state your choice of λ and p, the number of simulations, and your estimate for the expected value in each case.*

32. (5 points) Given are two random variables X and Y with joint pmf

$$\mathcal{P}(X = x, Y = y) = c(x + 3y)$$

for $x = 1, 2, \ldots, 10$, $y = 1, \ldots, 8$, where $c > 0$ is some constant.
a) Find the constant c that makes this a pmf, using `R` .
b) Find the marginal distributions of X and Y, also using `R` .
c) Find the conditional distribution of $Z = Y|(X = 5)$, using `R` .