Problem 15 Algebra Proof

$f(x) = \lambda e^{-\lambda x}$ $F(X) = 1 - e^{-\lambda x}$

$U(0,1) = t$

$P(F(x) \leq t) = P(1 - e^{-\lambda x} \leq t) = P(e^{-\lambda x} \geq 1 - t) = P(x \leq (-\lambda^{-1} * ln(1-t))$

$= \int_0^{-\lambda^{-1} * ln(1-t)} \lambda e^{-\lambda x} = \left(-e^{-\lambda x}\right)_0^{-\lambda^{-1} * ln(1-t)} = -e^{-\lambda - \lambda^{-1} * ln(1-t)} + 1 = -(1-t) + 1 = t$

$\therefore P(F(x) \leq t) = t = U(0,1)$

This can also be done without the integrals by using only CDF

$P(F(x) \leq t) = P(1 - e^{-\lambda x} \leq t) = P(e^{-\lambda x} \geq 1 - t) = P(x \leq (-\lambda^{-1} * ln(1-t))$

$= F(-\lambda^{-1} * ln(1-t)) - F(0) = 1 - e^{-\lambda * (-\lambda^{-1} * ln(1-t))} = 1 - e^{ln(1-t)} - (1 - e^0) = 1 - (1-t) - 0 = t$