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ANLY-511 Prob. Modeling & Stat. Computing
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Homework #6

Problem 41

```
Z = (51-48)/(9/sqrt(30))
pnorm(Z, lower.tail = F)
## [1] 0.03394458
```

Problem 42

For each Xi U(0,1) the mean is .5 so the mean of Z is 12*.5-6 = 0. For the variance of each Xi, it is 1/12*(1-0) = 1/12. The Variance of Z is 1/12*12 - 0 = 1. Z has the mean and variance of a standard normal distribution. By theorem 4.2, since all X are i.i.d witht the same variances and means, then any constant z will follow the normal distribution.

Problem 43

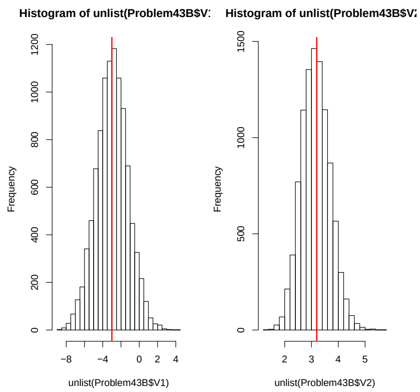
Part A

```
#Part A
meanY_X = 7-10
#[1] -3
sdY_X = sqrt(9/sqrt(9)+25/sqrt(12))
#[1] 3.196385
```

Part B

```
simX_Y = function(i){
  X = rnorm(9, mean = 7, sd = 3)
  Y = rnorm(12, mean = 10, sd = 5)
  meanY_X = mean(X) - mean(Y)
  sdY_X = sqrt((sd(X))^2/sqrt(length(X))+(sd(Y))^2/sqrt(length(Y)))
  return(list(meanY_X, sdY_X))
}
Problem43B = sapply(1:10000, simX_Y)

Problem43B = as.data.frame(t(Problem43B))
par(mfrow = c(1,2))
hist(unlist(Problem43B$V1), breaks = 25)
abline(v = meanY_X, col = "red", lwd = 2)
mean(unlist(Problem43B$V1))
## [1] -2.979591
hist(unlist(Problem43B$V2), breaks = 25)
abline(v = sdY_X, col = "red", lwd = 2)
```



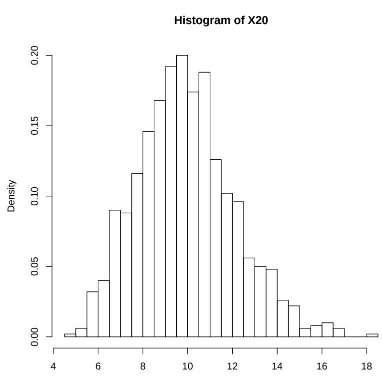

```
mean(unlist(Problem43B$V2))
## [1] 3.157558
```

Based on the histogram and the means of the simulationed SE and mean, both seem close to the theoretical ones.

Problem 44

Part A

```
X20 = replicate(1000, sum(rexp(20, rate = 2)))
hist(X20, probability = T, breaks = 25)
```



Part B

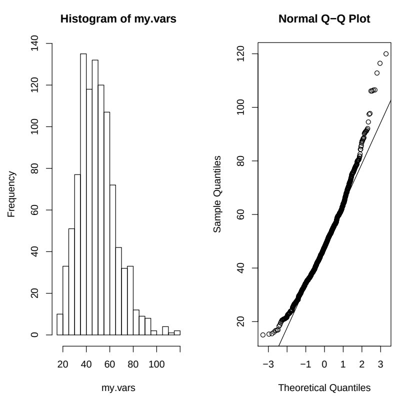
```
mean(X20)
## [1] 9.935507
var(X20)
## [1] 4.639166
```

Part C

```
mean(X20<=10)
## [1] 0.54
```

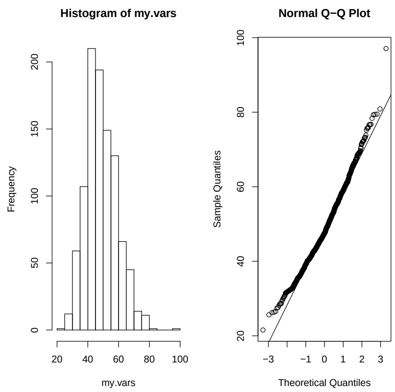
Problem 45

```
par(mfrow = c(1,2))
my.vars = sapply(rep(20, times = 1000), function(x){var(rnorm(x, 25, 7))})
mean(my.vars)
## [1] 49.25078
var(my.vars)
## [1] 253.393
hist(my.vars, breaks = 25)
#dev.new()
qqnorm(my.vars)
qqline(my.vars)
```



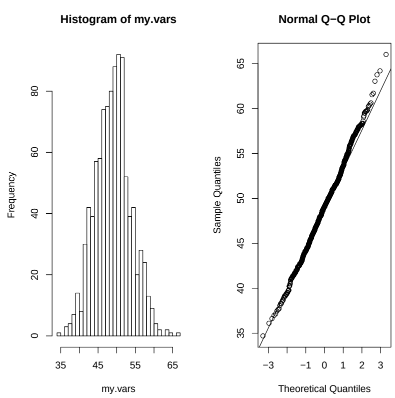
At n = 20, the qqnorm plot does not follow the the qqline; therefore, it does not appear to be normally distributed. The Histogram is skrewed

```
par(mfrow = c(1,2))
my.vars = sapply(rep(50, times = 1000), function(x){var(rnorm(x, 25, 7))})
mean(my.vars)
## [1] 48.85999
var(my.vars)
## [1] 102.7943
hist(my.vars, breaks = 25)
#dev.new()
qqnorm(my.vars)
qqline(my.vars)
```



At n = 50, the qqnorm plot follows the qqline close than at n = 20, but it still deviates a significant amount of the time so, it does not appear to be normally distributed. The histogram has two peaks.

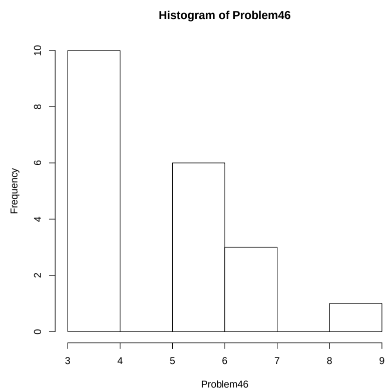
```
par(mfrow = c(1,2))
my.vars = sapply(rep(200, times = 1000), function(x){var(rnorm(x, 25, 7))})
mean(my.vars)
## [1] 49.00263
var(my.vars)
## [1] 22.51143
hist(my.vars, breaks = 25)
#dev.new()
qqnorm(my.vars)
qqline(my.vars)
```



At n = 200, it closely follows the line so it is normally distributed. The histogram is a rough, but looks somewaht bell shaped.

Problem 46

```
pop = c(3,6,7,9,11,14)
Problem46 = combn(pop, m = 3, FUN = min)
mean(Problem46)
## [1] 4.8
hist(Problem46)
```



It looks like we are trying to estimate the min of the population. That seems to be the parameter we are trying to find.

Problem 47

Part A

$$E(X) = 10$$

Part B

```
my.means = replicate(1000, mean(rexp(30, rate = 1/10)))
mean(my.means>=12)
## [1] 0.15
```

Part C

The proportion doesn't seem too small, so it doesn't seem to be unusual.

Problem 48

Part A

$$f_{min}(x) = n(1-(1-e^{-\lambda x}))^{n-1} * \lambda e^{-\lambda x} = n * e^{(n-1)-\lambda x} * \lambda^{-\lambda x} = n\lambda e^{-n\lambda}$$

$$\therefore X_{min} \sim Exp(n\lambda)$$

Part B

```
Problem48B = replicate(1000, min(rexp(25, rate = 7)))
1/(25*7) - mean(Problem48B)
## [1] -0.000425932
```