68) $E(X^2) = \text{var}(X) + (E(X))^2 = \frac{(\beta-\alpha)^2}{12} + \frac{(\alpha+\beta)^2}{4}$

$= \frac{\alpha^2 - 2\alpha\beta + \beta^2}{12} + \frac{3\alpha^2 + 6\alpha\beta + 3\beta^2}{12} = \frac{4\alpha^2 + 4\alpha\beta + 4\beta^2}{12} = 43.8$

$E(X) = \frac{\alpha+\beta}{2} = 5.8 \Rightarrow \alpha = 11.6 - \beta$

$\boxed{\begin{matrix} \beta = 0.28 \,\&\, 11.32 \\ \alpha = 11.32 \,\&\, 0.28 \end{matrix}}$

69) $E(\hat\theta_1) = 0.9\theta \Rightarrow E\left(\frac{10}{9}\hat\theta_1\right) = \theta$

$\text{Var}\left(\frac{10}{9}\hat\theta_1\right) = \left(\frac{10}{9}\right)^2 \text{Var}(\hat\theta_1) = \frac{100}{81} \cdot 3 = \frac{100}{27}$

$E\left(\frac{1.0}{1.2}\hat\theta_2\right) = \theta \qquad \text{Var}\left(\frac{10}{12}\hat\theta_2\right) = \left(\frac{10}{12}\right)^2 \cdot 2$

$\therefore \hat\theta_2$ is more efficient

72) $E(f(x)) = \theta \qquad E\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{2}(E(X_1) + E(X_2)) = \frac{1}{2}(2\theta) = \theta$

$E(X_1) = \theta \qquad \text{Var}\left(\frac{X_1+X_2}{2}\right) = \left(\frac{1}{2}\right)^2 (\text{Var}(X_1) + \text{Var}(X_2)) = \frac{2}{4}\theta^2$

$\text{Var}(X_1) = \theta^2$

$E\left(\frac{X_1 + 2X_2}{3}\right) = \frac{3\theta}{3} = \theta$

$\text{Var}\left(\frac{X_1 + 2X_2}{3}\right) = (1/9)(\text{Var}(X_1) + 4\text{Var}(X_2)) = 5/9\,\theta^2$

$\therefore \hat\theta_2$ is more efficient based on the variance

67 $L(p) = \prod_{i=1}^{n} \binom{n}{x_i} p^{\sum_{i=1}^{n} x} (1-p)^{n - \sum_{i=1}^{n} x_i} = \prod_{i=1}^{n} \binom{n}{x_i} p^x (1-p)^{n-x}$

$\log L(p) = \underbrace{\sum_{i=1}^{n} \log\binom{n}{x_i}}_{c} + \log(p^x) + \log((1-p)^{n-x}) = c + x\log(p) + (n-x)\log(1-p)$

$\log L'(p) = \frac{x}{p} - \frac{n+x}{1-p} = 0 \Rightarrow \frac{x}{p} = \frac{n-x}{1-p} \Rightarrow x(1-p) = p(n-x)$

$= x - xp = np - xp \Rightarrow x = np \Rightarrow p = \frac{x}{n}$