

Exercise 21

$$P(X_1 = 1) = P(X_1 = 1|X_0 = 1) = 0.6$$

$$P(X_1 = 0) = P(X_1 = 0|X_0 = 1) = 0.4$$

$$P(X_2 = 1) = P(X_2 = 1|X_1 = 1) * P(X_1 = 1) + P(X_2 = 1|X_1 = 0) * P(X_1 = 0) = 0.6 * 0.6 + 0.2 * 0.4 = 0.44$$

$$P(X_2 = 0) = P(X_2 = 0|X_1 = 1) * P(X_1 = 1) + P(X_2 = 0|X_1 = 0) * P(X_1 = 0) = 0.4 * 0.6 + 0.8 * 0.4 = 0.56$$

$$P(X_3 = 1) = P(X_3 = 1|X_2 = 1) * P(X_2 = 1) + P(X_3 = 1|X_2 = 0) * P(X_2 = 0) = 0.6 * 0.44 + 0.2 * 0.6 = 0.376$$

$$P(X_3 = 0) = P(X_3 = 0|X_2 = 1) * P(X_2 = 1) + P(X_3 = 0|X_2 = 0) * P(X_2 = 0) = 0.4 * 0.44 + 0.8 * 0.56 = 0.624$$

Exercise 23

Part A

$$CDF(x) = 1 - e^{-\lambda x}$$

Part B

$$P(X > A) = 1 - P(X \leq A) = 1 - (1 - e^{-\lambda A}) - (1 - 1) = e^{-\lambda A}$$

Part C

if $A < z$

$$P(X > z \& X > A) = P(X > z) = 1 - P(X \leq z) = e^{-\lambda z}$$

if $z \leq A$

$$P(X > z \& X > A) = P(X > A) = 1 - P(X \leq A) = e^{-\lambda A}$$

Part D

$$P(X > z|X > A) = \frac{P(X > z \text{ and } X > A)}{P(X > A)}$$

if $A < z$

$$P(X > z|X > A) = \frac{e^{-\lambda z}}{e^{-\lambda A}} = e^{-\lambda(z-A)}$$

if $z \leq A$

$$P(X > z|X > A) = \frac{e^{-\lambda A}}{e^{-\lambda A}} = 1$$

Part E

let $z = y + A$

if $A < z \implies A < y + A$

$$P(X > z|X > A) = P(X > y + A|X > A) = \frac{e^{-\lambda(y+A)}}{e^{-\lambda A}} = e^{-\lambda(y+A-A)} = e^{-\lambda y} = 1 - P(Y \leq y) = P(Y > y)$$

if $y + A \leq A$

$$P(X > z|X > A) = \frac{e^{-\lambda A}}{e^{-\lambda A}} = 1$$

Part F

From Part E we know that $e^{-\lambda y} = 1 - P(Y \leq y) \implies P(Y \leq y) = 1 - e^{-\lambda y}$

This is the same as the CDF of X

Exercise 24

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