ANLY-511 Homework assigned on 09/24/15
Email a pdf file with solutions (preferred)
or hand it in at my office door by Wednesday, 9/30 11:59PM.
Five "short" and three "long" problems.

Explain your work and give concise reasoning. Attach `R` *code with comments if applicable. Do not print out any data or any detailed results of simulations. Your solutions for this homework set should fit on no more than six pages including graphs. In problems requiring graphs, you are allowed to just give the code to make the graphs and to describe them, without actually including them.*

17. (2 points) Let X be a random variable with a beta(-.5, -.8) distribution. Use a suitable simulation to approximate $\mathcal{E}(X)$, $s(X)$, $\mathcal{E}(X^{-1/3})$.

18. (2 points) Let X be a random variable with a beta(-.5, -.8) distribution. Use a suitable simulation to approximate $\mathcal{E}(\sqrt{X})$ with an error of less than 10^{-3} and explain why you think you have achieved this accuracy. *1 point will be subtracted if you use more than 100 times as many simulations as is necessary .*

19. (2 points) Suppose X and Y are independent random variables that both have a uniform $U(0,1)$ distribution. Define the new random variable

$$Z = X|Y \leq \sin^2(2\pi X)\,.$$

Use `R` to generate a random sample of size at least 10000 for Z, make a histogram, and describe what you see.

20. (2 points) A breathalyzer that is used by the police to detect drunk drivers correctly identifies a drunk driver with probability 0.99 and falsely identifies a sober driver as drunk with probability 0.02. Then answer the following questions:
a) What is the sensitivity of this breathalyzer? What is its specificity? *Look up the terms "specificity" and "sensitivity".*
b) About one in 500 drivers are drunk. Somebody gets pulled over and fails the breathalyzer test. What is the probability that she is drunk?

21. (2 points) Consider a Bayesian network of the simple form

$$X_0 \to X_1 \to X_2 \to X_3$$

where the X_i are random variables that can only attain the values 0 and 1. Here are the conditional probabilities for the network: $\mathcal{P}(X_{i+1} = 1|X_i = 0) = .2$, $\mathcal{P}(X_{i+1} = 1|X_i = 1) = .6$, for $i = 0, 1, 2$. Find the distribution of X_3 if $X_0 = 1$.

22. (5 points) The Cauchy distribution has the property that the mean $\frac{1}{n}\sum_{i=1}^{n} X_i$ of n independent copies $X_1, \ldots, X_n$ has the same distribution as each individual X_i. Demonstrate this as best as you can with simulations for different values of n, using suitable plots. Include one such plot in your solution and explain it.

23. (5 points) Given an exponentially distributed random variable X with intensity λ and some number $A > 0$, defined the new random variable

$$Y = (X|X > A) - A\,.$$

Intuitively, think of X as the unknown time when a bomb goes off after you push a button. You push the button and watch a timer. If the bomb hasn't gone off by the time A, you reset the timer to 0 and keep it running. Then Y is displayed on the

timer when the bomb goes off.
Show that Y has the same cumulative distribution function as X. If you're unsure how to do this, I suggest you use the following steps and explain them.
a) Write down the distribution function of X.
b) Find a formula for $\mathcal{P}(X > A)$.
c) Given z, find a formula for $\mathcal{P}(X > z \text{ and } X > A)$.
d) Find a formula for $\mathcal{P}(X > z | X > A)$.
e) Explain why $\mathcal{P}(Y > y) = \mathcal{P}(X > y + A | X > A)$.
f) Find the formula for $\mathcal{P}(Y \leq y)$.

24. (5 points) Consider the Bayesian network given in the figure below, with probabilities given in the tables underneath. *Note that these probabilities are different from those of the class below. Also, only half of the probabilities are given. You can compute the other half for yourself.*

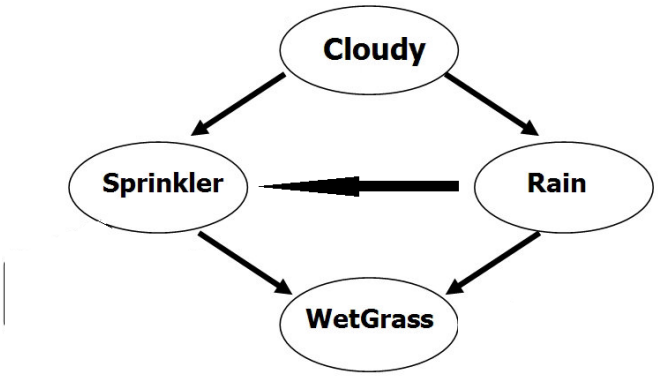

FIGURE 1. Bayesian network for problem 24

Probabilities for cloudiness

Cloudy = T
..6

Conditional probabilities for rain

Cloudy	Rain = T
T	.7
F	.1

Conditional probabilities for sprinkler

Cloudy	Rain	Sprinker = T
T	T	.05
T	F	.4
F	T	.1
F	F	.8

Conditional probabilities for wet lawn

Rain	Sprinker	Wet lawn = T
T	T	.95
T	F	.9
F	T	.9
F	F	.1

Answer the following questions, with explanation:

a) What is the probability that the lawn is wet?
b) The lawn is wet. What is the probability that it rained?
c) The sprinkler is on. What is the probability that it rained?