Stat 155 Homework # 1 Due February 3

Problems:

Q 1 Let N be uniform on $\{1, 2, \ldots, 10\}$ and let X be a binomial $Bin(N, 1/2)$. Find the mean and variance of N.

Q 2 Let X_1, X_2, X_3 be independent $Exp(1)$ random variables. Calculate the density and mean of the second largest of the three random variables.

Q 3 A seller can produce a product for \$1. In each case, what is 1) the optimal price for selling the item and 2) the expected profit per customer if the distribution of the value of the product is (in dollars),

- Distribution Exp(1).
- Has CDF, $F(x) = x/3$ when $0 \leq x \leq 2$ and $F(x) = \frac{2}{3} + (x-2)/9$ for $2 \leq x \leq 5$.
- Has CDF, $F(x) = x/9$ when $0 \leq x \leq 3$ and $F(x) = \frac{1}{3} + (x-3)/3$ for $3 \leq x \leq 5$.

Q 4 Suppose that there are three agents in a sealed first price auction with independent values uniform on $[0, 1]$.

- Verify from the definition that bidding $\beta(v) = \frac{2}{3}v$ is a Bayes-Nash equilibrium.
- Calculate the expected revenue in this auction.