

# Analytics 512 Homework 6

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## Exercise 2

### Part A

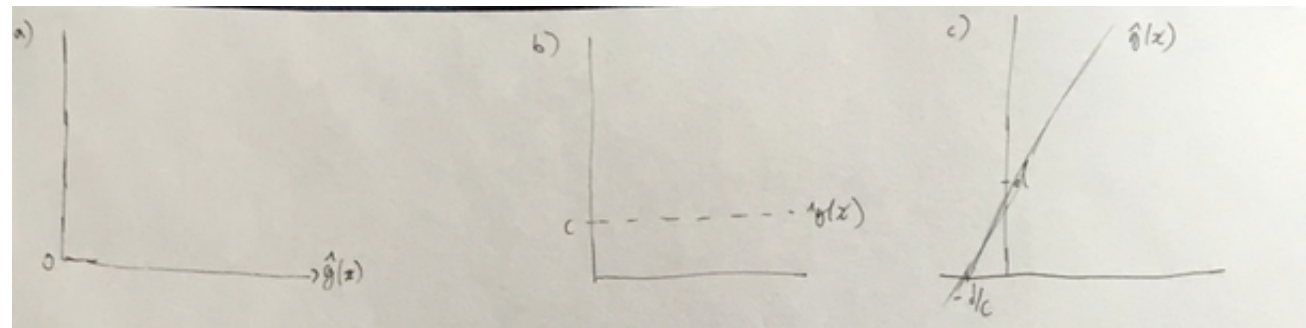
In order for the arg min to be achieved,  $\lambda$  would have to be minimized since it is  $\infty$ , therefore,  $g(x) = 0$

### Part B

by the logic in part A,  $g'(x) = 0$ , so  $g(x) = \int g'(x)dx = c$  where  $c$  is a constant

### Part C

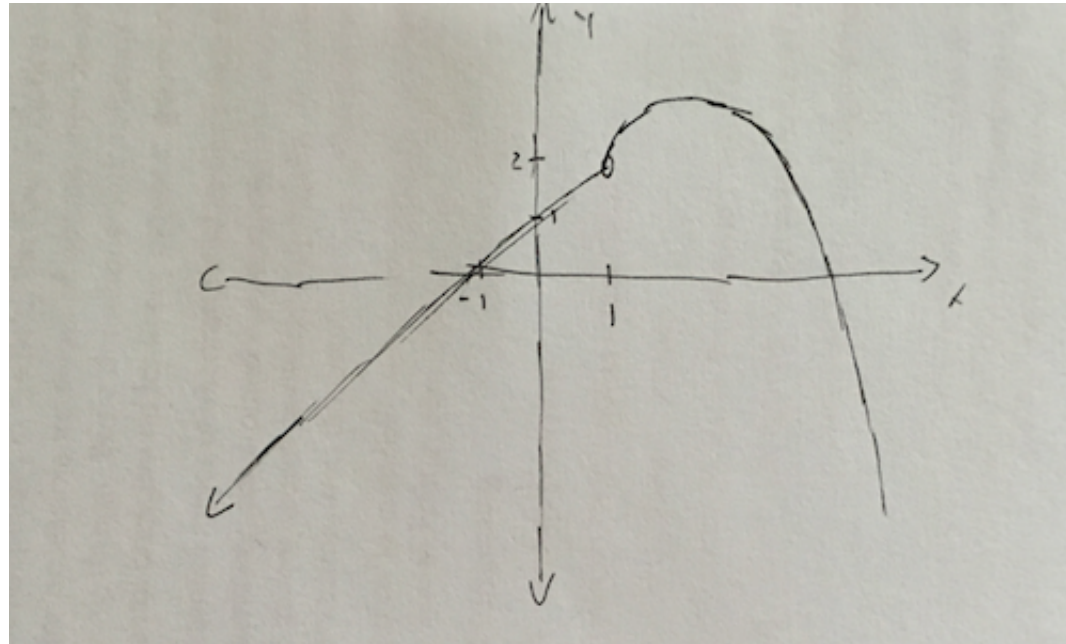
$g''(x) = 0 \implies g'(x) = c \implies g(x) = c * x + d$  where  $c$  and  $d$  are both constants



### Exercise 3

$$Y = 1 + X - 2(X - 1)^2 = 1 + X - 2(X^2 - 2X + 1) = -2X^2 + 5X - 1 \text{ when } X \geq 1$$
$$Y = 1 + X \text{ o. w.}$$

Intercepts at  $(0,1)$ ,  $(-1,0)$ ,  $(\sqrt{17}/4 + 5/4, 0)$



### Exercise 6

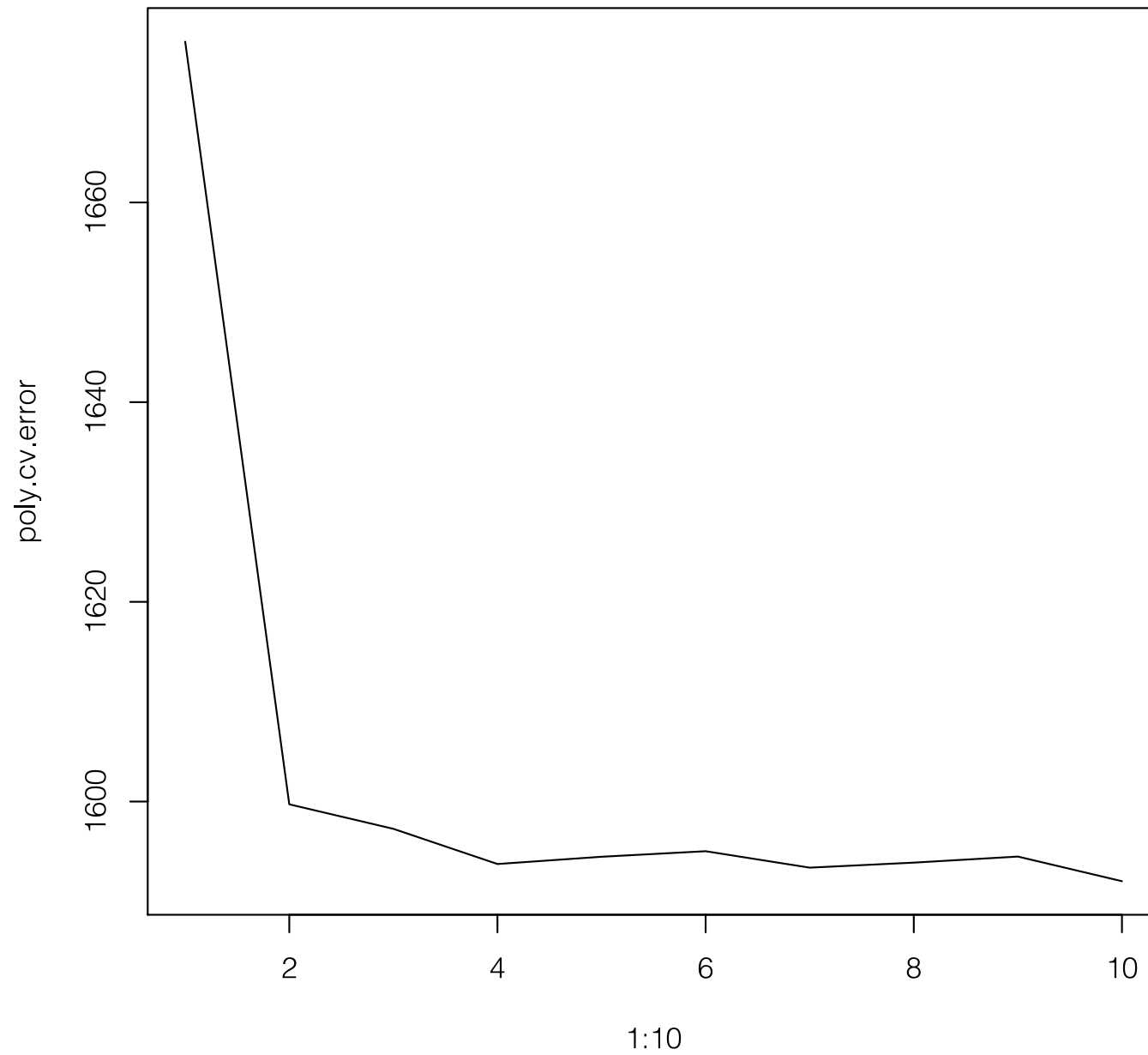
#### Part A

```
In [1]: d = 10
library(ISLR)
library(boot)
poly.cv.error = c()
different.d = as.list(1:10)
for(i in 1:d){
  wage.pm = glm(wage~poly(age, i), data = Wage)
  poly.cv.error[i] = cv.glm(Wage, wage.pm, K = 10)$delta[2]
  different.d[[i]] = lm(wage~poly(age, i), data = Wage)
}
poly.cv.error
which.min(poly.cv.error)
```

```
Out[1]:      1676.10420087314  1599.72101804354  1597.26734459586  1593.74316802434  1594.46944707439
          1595.02556440882  1593.37772701369  1593.88216375312  1594.4884242901  1592.02258842504
```

```
Out[1]: 10
```

```
In [2]: plot(1:10,poly.cv.error, type = "l")
```



```
In [3]: print(anova(different.d[[1]], different.d[[2]],different.d[[3]],different.d[[4]],different.d[[5]],
different.d[[6]], different.d[[7]],different.d[[8]],different.d[[9]],different.d[[10]]))
```

Analysis of Variance Table

```
Model 1: wage ~ poly(age, i)
Model 2: wage ~ poly(age, i)
Model 3: wage ~ poly(age, i)
Model 4: wage ~ poly(age, i)
Model 5: wage ~ poly(age, i)
Model 6: wage ~ poly(age, i)
Model 7: wage ~ poly(age, i)
Model 8: wage ~ poly(age, i)
Model 9: wage ~ poly(age, i)
Model 10: wage ~ poly(age, i)
```

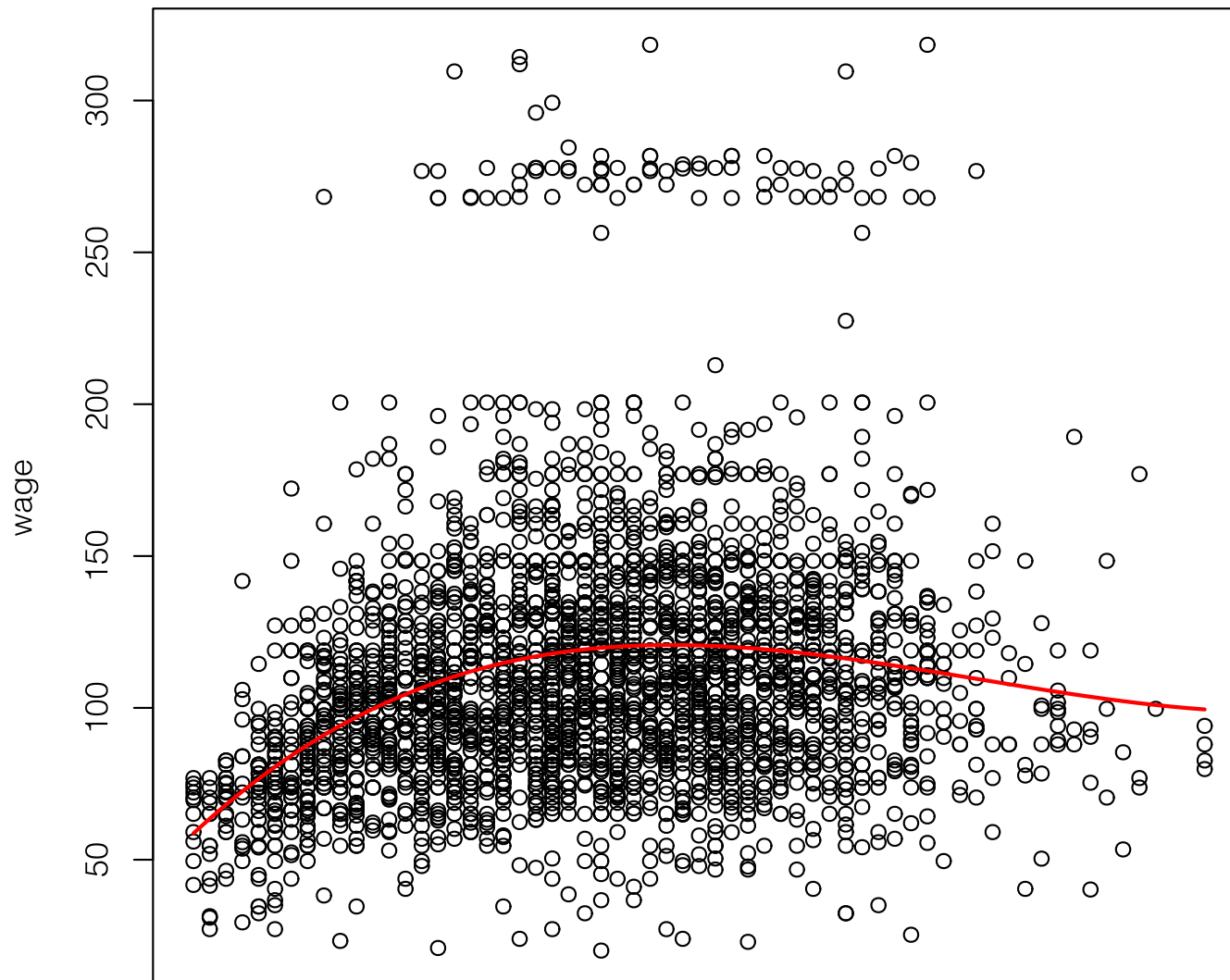
	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)	
1	2998	5022216					
2	2997	4793430	1	228786	143.7638	< 2.2e-16	***
3	2996	4777674	1	15756	9.9005	0.001669	**
4	2995	4771604	1	6070	3.8143	0.050909	.
5	2994	4770322	1	1283	0.8059	0.369398	
6	2993	4766389	1	3932	2.4709	0.116074	
7	2992	4763834	1	2555	1.6057	0.205199	
8	2991	4763707	1	127	0.0796	0.777865	
9	2990	4756703	1	7004	4.4014	0.035994	*
10	2989	4756701	1	3	0.0017	0.967529	

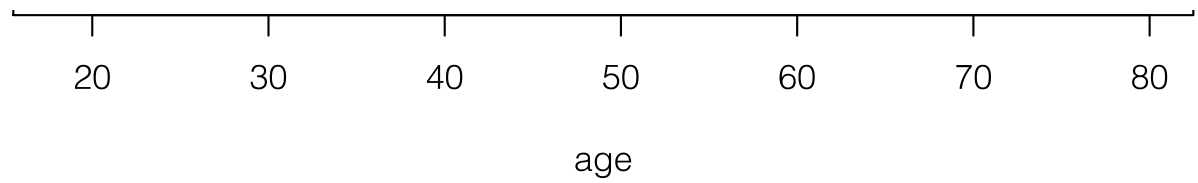
---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Via the cross validated method, a polynomial with the degree of 9 was chosen. Under the ANOVA test, the 9th degree polynomial model is considered significant; however, not as significant as the second or third degree polynomial, so the third degree is probably the most optimal

```
In [4]: plot(wage ~ age, data = Wage)
        agelims = range(Wage$age)
        age.grid = seq(from = agelims[1], to = agelims[2])
        wage.d3m = lm(wage ~ poly(age, 3), data = Wage)
        preds = predict(wage.d3m, newdata = list(age = age.grid))
        lines(age.grid, preds, col = "red", lwd = 2)
```



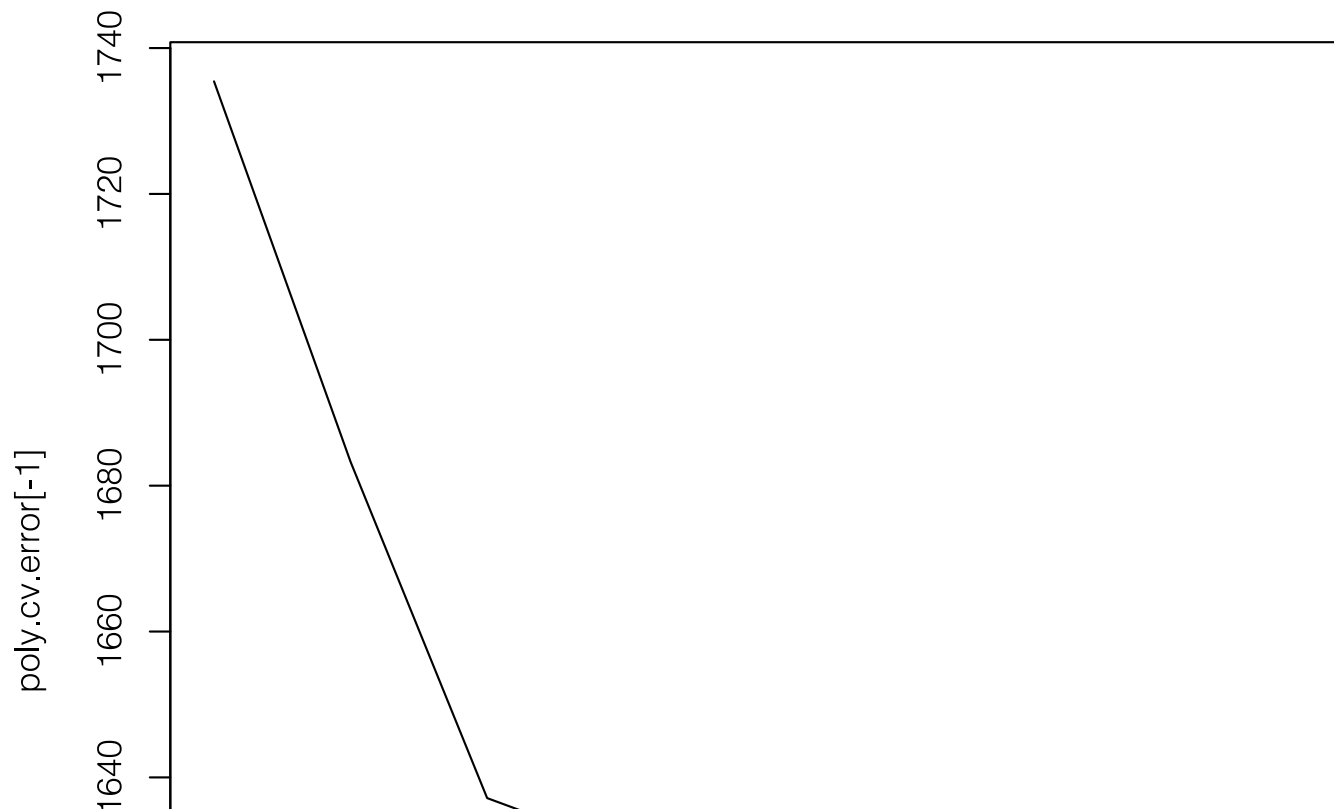


## Part B

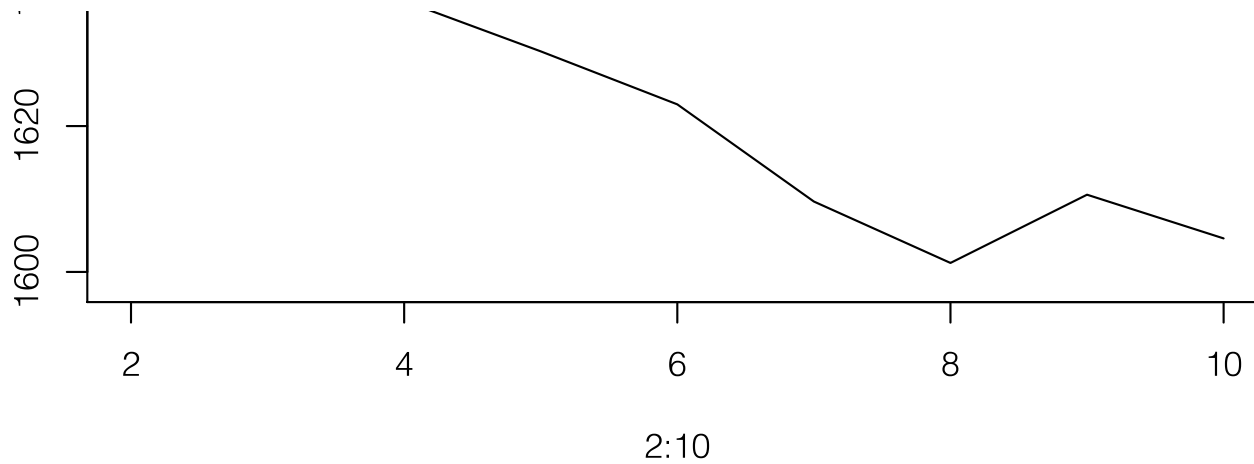
```
In [5]: d = 10
        for(i in 2:d){
          Wage$age_cuts = cut(Wage$age, i)
          wage.pm = glm(wage~age_cuts, data = Wage)
          poly.cv.error[i] = cv.glm(Wage, wage.pm, K = 10)$delta[2]
        }
        poly.cv.error[-1]
        which.min(poly.cv.error[-1])
        plot(2:10,poly.cv.error[-1], type = "l")
```

```
Out[5]:      1735.43659534234  1683.26493861114  1637.16324930901  1630.24366180349  1622.97659710992
          1609.65316531392  1601.23375699219  1610.59146799314  1604.59775913335
```

```
Out[5]: 7
```







Eight cuts results in the lowest MSE from a cross-validated model.

## Exercise 9

### Part A

In [6]: `library(MASS)`

```
In [7]: nox.nlm = lm(nox~poly(dis,3),data = Boston)
summary(nox.nlm)
```

Out[7]:

Call:

```
lm(formula = nox ~ poly(dis, 3), data = Boston)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.121130	-0.040619	-0.009738	0.023385	0.194904

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	0.554695	0.002759	201.021	< 2e-16	***
poly(dis, 3)1	-2.003096	0.062071	-32.271	< 2e-16	***
poly(dis, 3)2	0.856330	0.062071	13.796	< 2e-16	***
poly(dis, 3)3	-0.318049	0.062071	-5.124	4.27e-07	***

---

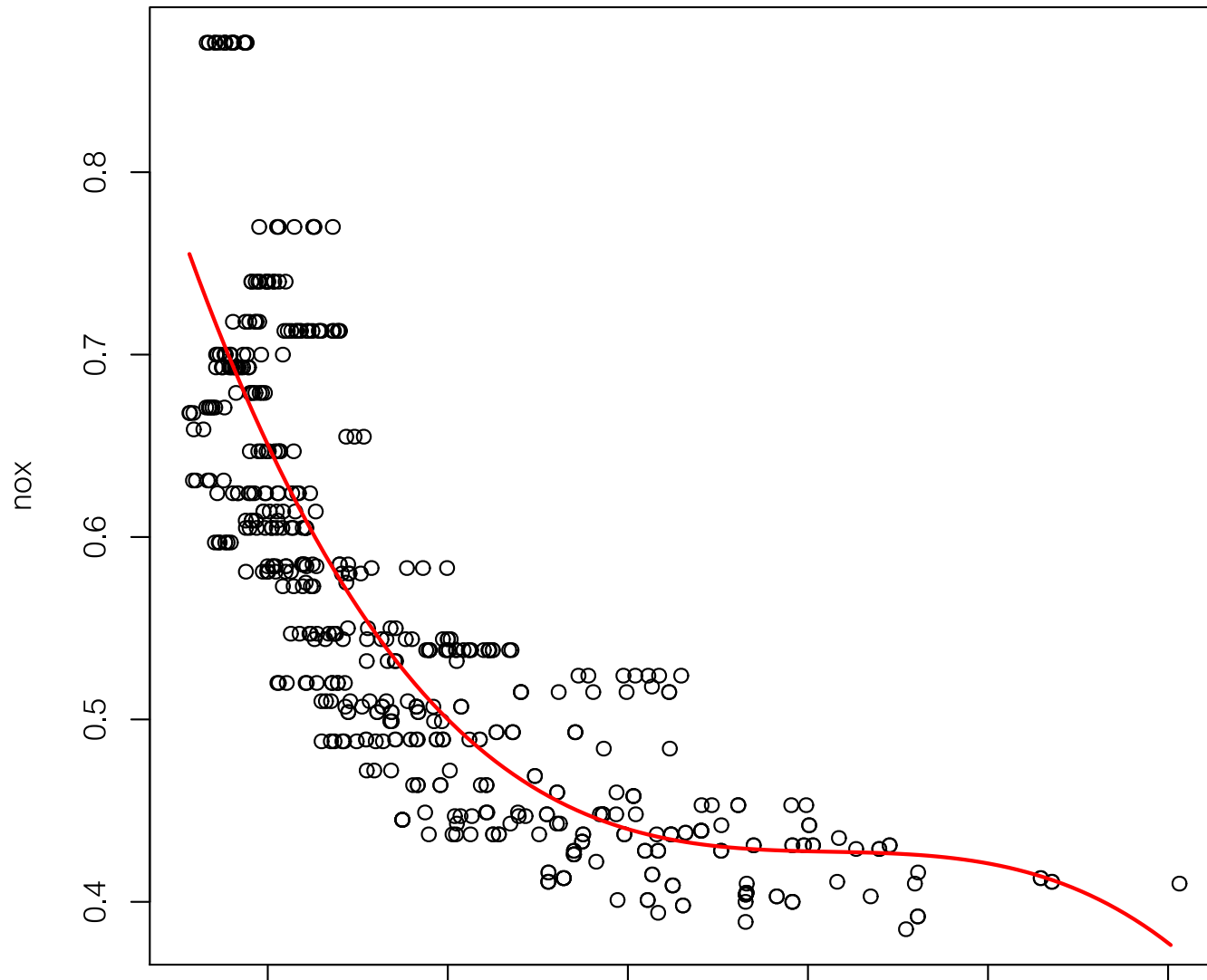
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.06207 on 502 degrees of freedom

Multiple R-squared: 0.7148, Adjusted R-squared: 0.7131

F-statistic: 419.3 on 3 and 502 DF, p-value: < 2.2e-16

```
In [8]: dislims = range(Boston$dis)
dis.grid = seq(from = dislims[1], to = dislims[2], by = 0.1)
preds = predict(nox.nlm, list(dis = dis.grid))
plot(nox ~ dis, data = Boston)
lines(dis.grid, preds, col = "red", lwd = 2)
```





## Part B

```
In [9]: for(i in 1:10){
        nox.nlm = lm(nox~poly(dis,i),data = Boston)
        statement = paste("For a polynomial of", i, "The RSS is",sum(nox.nlm$residuals^2), sep = " ")
        print(statement)
      }

[1] "For a polynomial of 1 The RSS is 2.76856285896928"
[1] "For a polynomial of 2 The RSS is 2.03526186893526"
[1] "For a polynomial of 3 The RSS is 1.93410670717907"
[1] "For a polynomial of 4 The RSS is 1.93298132729859"
[1] "For a polynomial of 5 The RSS is 1.9152899610843"
[1] "For a polynomial of 6 The RSS is 1.87825729850816"
[1] "For a polynomial of 7 The RSS is 1.84948361458298"
[1] "For a polynomial of 8 The RSS is 1.83562968906769"
[1] "For a polynomial of 9 The RSS is 1.8333308044916"
[1] "For a polynomial of 10 The RSS is 1.83217112393138"
```

## Exercise 11

### Part A

```
In [10]: esp = rnorm(100)
        x1 = rnorm(100)
        x2 = rnorm(100)
        y = x1 + 2*x2 + esp
```

### Part B

```
In [11]: beta1 = 14
```

## Part C

```
In [12]: a=y-beta1*x1  
beta2=lm(a~x2)$coef[2]
```

## Part D

```
In [13]: a=y-beta2*x2  
beta1=lm(a~x1)$coef[2]  
beta0=lm(a~x1)$coef[1]
```

## Part E

```
In [14]: betals = rep(beta1,1000)  
beta2s = rep(beta2,1000)  
beta0s = rep(beta0,1000)  
for(i in 2:1000){  
  a=y-beta1*x1  
  beta2=lm(a~x2)$coef[2]  
  beta2s[i] = beta2  
  a=y-beta2*x2  
  beta1=lm(a~x1)$coef[2]  
  betals[i] = beta1  
  beta0=lm(a~x1)$coef[1]  
  beta0s[i] = beta0  
}
```

```
In [15]: summary(betals)
unique(betals)
```

```
Out[15]:      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      0.9279  0.9279  0.9279  0.9279  0.9279  0.9289
```

```
Out[15]:      0.928854928102986  0.92789650362555  0.927896433350339  0.927896433345186  0.927896433345186
```

```
In [16]: summary(beta2s)
unique(beta2s)
```

```
Out[16]:      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      1.967   2.078   2.078   2.078   2.078   2.078
```

```
Out[16]:      1.96730839472973  2.07797035256653  2.07797846670868  2.07797846730364  2.07797846730368
```

```
In [17]: summary(beta0s)
unique(beta0s)
```

```
Out[17]:      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      0.05372 0.08331 0.08331 0.08328 0.08331 0.08331
```

```
Out[17]:      0.0537164601650667  0.08330699333951  0.0833091630263381  0.0833091631854278  0.0833091631854391
```

## Part F

```
In [18]: master.nlm = lm(y~x1+x2)
summary(master.nlm)
```

Out[18]:

Call:

lm(formula = y ~ x1 + x2)

Residuals:

	Min	1Q	Median	3Q	Max
	-3.0989	-0.7849	-0.0257	0.6801	2.7337

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.08331	0.11360	0.733	0.465
x1	0.92790	0.10638	8.723	7.64e-14 ***
x2	2.07798	0.10517	19.757	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.074 on 97 degrees of freedom

Multiple R-squared: 0.8288, Adjusted R-squared: 0.8252

F-statistic: 234.7 on 2 and 97 DF, p-value: < 2.2e-16

The values obtained from part E, are very close to the values obtained by E which the exception of the 0 percentile values for  $\beta_0$  and  $\beta_2$  and the 100 percentile with respect to  $\beta_1$