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Math 611 Stochastic Simulation
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Homework 3

Exercise 1

Since $Area_{square} = 1$, $Area_{circle} = \frac{\pi}{2^2} = \frac{\pi}{4}$ as stated in the exercise. Therefore the number of points that are expected in the circle is $\frac{\pi/4}{1} = \pi/4$ Given that the orgin of the circle is at $(1/2, 1/2)$ with a radius of $r = 1/2$, the circle equation is $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$, so we should look for points with coordinates such that $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 < \frac{1}{4}$. I assume that the circumference doesn't count as in the circle.

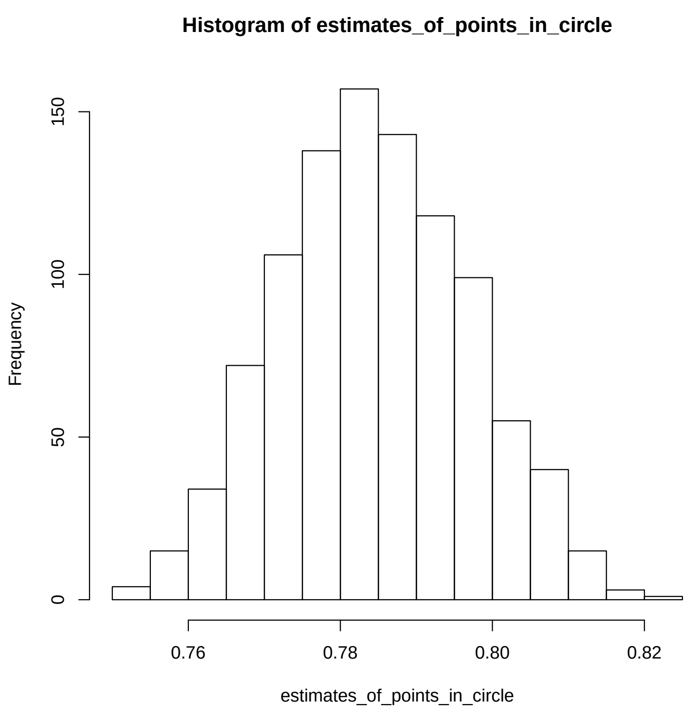
```
points_in_circle = function(z){
  points= data.frame(x = runif(1e3), y = runif(1e3))
  mean((points$x-1/2)^2+(points$y-1/2)^2<1/4)
}
estimates_of_points_in_circle = sapply(1:1e3, points_in_circle)
mean(estimates_of_points_in_circle)

## [1] 0.785135

var(estimates_of_points_in_circle)

## [1] 0.0001555804

hist(estimates_of_points_in_circle)
```



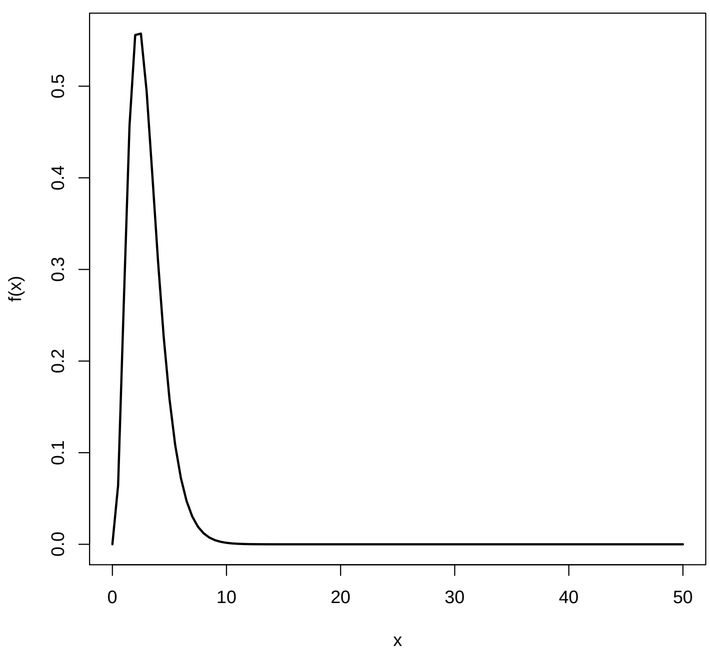
Based on the estimated mean and the variance, the fixed range is very close to the expected fraction.

Exercise 2

Please note: I used the methods from "http://math.stackexchange.com/questions/1200443/evaluating-difficult-monte-carlo-integration-in-r" to do monte carlo integration in R

Part A

```
f = function(x){
  return(exp(-4*x/3)*x^3)
}
curve(f, lwd=2,to = 50)
```



Based on the graph, it is clear that $f(x)$ converges to 0 close to 10, so using the method described in class, we will do the following

```
n = 1e4
a = 0
b = 10 #as any value past 10 may be significantly greater than zero, but not x = 20, f(20) is approxima
x = runif(n, a, b)
y = f(x)
(b-a)/n*sum(y)

## [1] 1.887044
```

Part B

$$\begin{aligned}\int_0^\infty e^{\frac{-4x}{3}} x^3 \delta x \implies \\ \int_0^\infty x^3 * e^{\frac{-4x}{3}} \delta x \implies \\ \int_0^\infty x^{4-1} * e^{\frac{x}{3/4}} \delta x \implies \\ \Gamma(4) * (\frac{3}{4})^4 \int_0^\infty \frac{1}{\Gamma(4) * (\frac{3}{4})^4} * x^{4-1} * e^{\frac{x}{3/4}} \delta x\end{aligned} \tag{1}$$

please recognize that $\frac{1}{\Gamma(4)*(\frac{3}{4})^4} * x^{4-1} * e^{\frac{x}{3/4}} \sim gamma(4, 3/4)$

$\therefore \Gamma(4) * (\frac{3}{4})^4 \int_0^\infty \frac{1}{\Gamma(4)*(\frac{3}{4})^4} * x^{4-1} * e^{\frac{x}{3/4}} \delta x = \Gamma(4) * (\frac{3}{4})^4 = 1.898$

Exercise 3

Exercise 4

Exercise 5