Arif Ali
Math 611 Stochastic Simulation
Sept 8, 2016

Homework 1

Exercise 1

It is given that $U \sim Unif(0,1)$ and that $c > 0$

Let $X = g(u) = -c * ln(u) \implies -x/c = ln(u) \implies e^{-x/c} = u$
$\therefore g^{-1}(u) = e^{-u/c}$
$X = \int_0^x e^{-x/c} = -\frac{1}{c}e^{-x/c} + \frac{1}{c}$
$dX/dx = 1/c * e^{-x/c} \sim exp(1/c)$

Exercise 2

Exercise 3

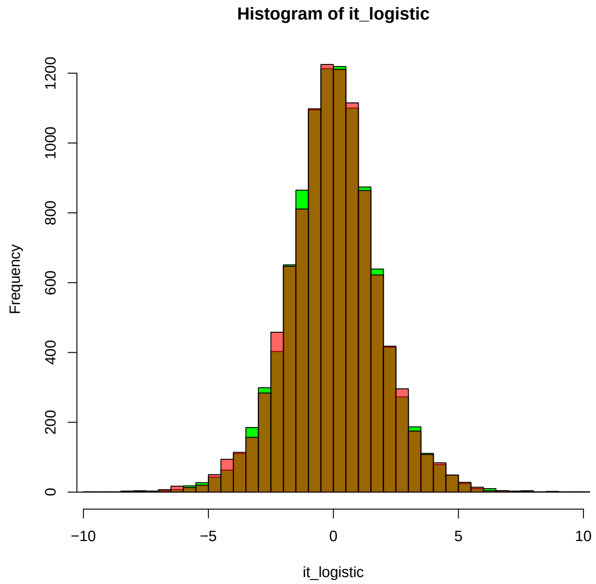
Part A

Let $z = 1 + e^{-(x-\mu)/\beta} \implies dz = -\frac{1}{\beta}e^{-(x-\mu)/\beta} * dx \implies dx = \frac{dz}{-\frac{1}{\beta}e^{-(x-\mu)/\beta}}$

$$F(x) = \int_{-\infty}^{x} f(x)dx = \int \frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{(1+e^{-(x-\mu)/\beta})^2}dx = \int \frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{z^2} * \frac{dz}{-\frac{1}{\beta}e^{-(x-\mu)/\beta}} = \int -\frac{1}{z^2}dz = \frac{1}{z} = |_{-\infty}^{x} \frac{1}{1+e^{-(x-\mu)/\beta}} = \frac{}{1 -} \tag{1}$$

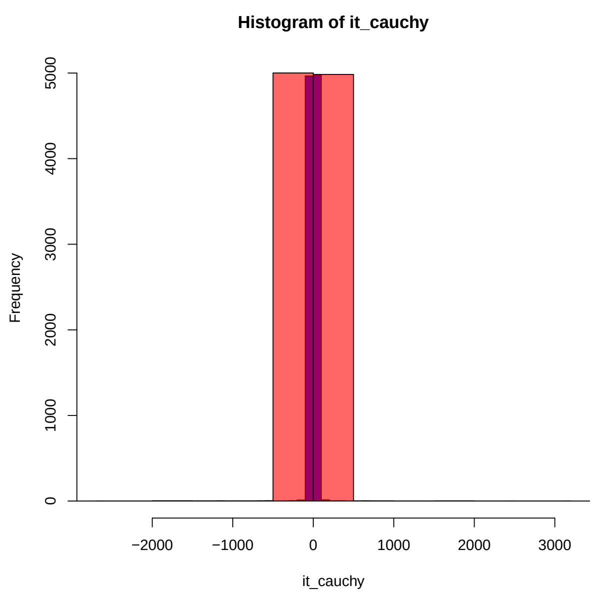
To find the inverse, we do the following
$Y = \frac{1}{1+e^{-(x-\mu)/\beta}} \implies$
$1 + e^{-(x-\mu)/\beta} = \frac{1}{Y} \implies$
$e^{-(x-\mu)/\beta} = \frac{1}{Y} - 1 \implies$
$-(x-\mu)/\beta = ln(\frac{1}{Y} - 1) \implies$
$-(x-\mu) = ln(\frac{1}{Y} - 1)\beta \implies$
$x = \mu - ln(\frac{1}{Y} - 1)\beta$

```
u = runif(1e4)

inverse_logistic_cdf = function(x, mu = 0 , B = 1){
  mu - log(1/x-1)*B
}

it_logistic = inverse_logistic_cdf(u)
r_logistic = rlogis(1e4)

hist(it_logistic, breaks = 50, col = "green")
hist(r_logistic, breaks = 50, col = "#FF000099", add = T)
```



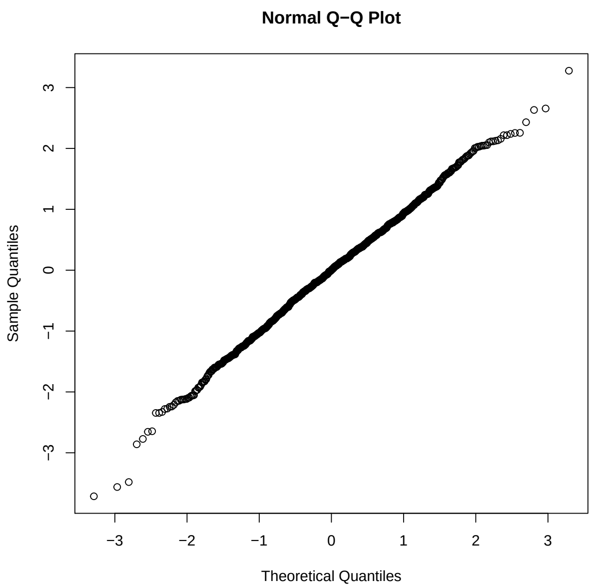
Part B

```
inverse_cauchy_cdf = function(x, mu = 0, sigma = 1){
  sigma*tan(pi*(x-0.5)) + mu
}

it_cauchy = inverse_cauchy_cdf(u)
r_cauchy = rcauchy(1e4)

u = runif(1e4)
hist(it_cauchy, breaks = 50, col = "blue")
hist(r_cauchy, breaks = 50, col = "#FF000099", add = T)
```

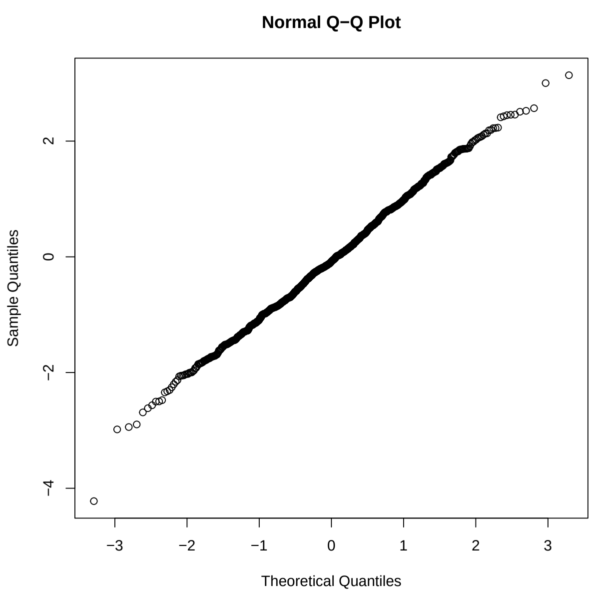


Exercise 4

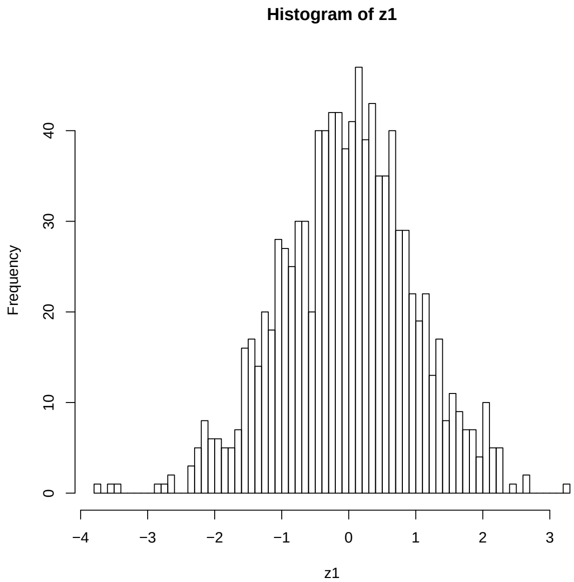
```
u1 = runif(1e3)
u2 = runif(1e3)

z1 = sqrt(-2*log(u1))*cos(2*pi*u2)
z2 = sqrt(-2*log(u1))*sin(2*pi*u2)
qqnorm(z1)
```



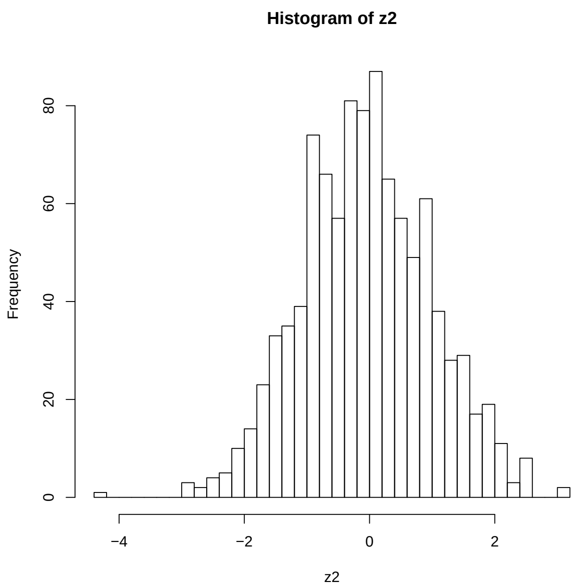

```
qqnorm(z2)
```



```r
hist(z1, breaks = 50)
```




```r
hist(z2, breaks = 50)
```



Exercise 5