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Math 611 Stochastic Simulation
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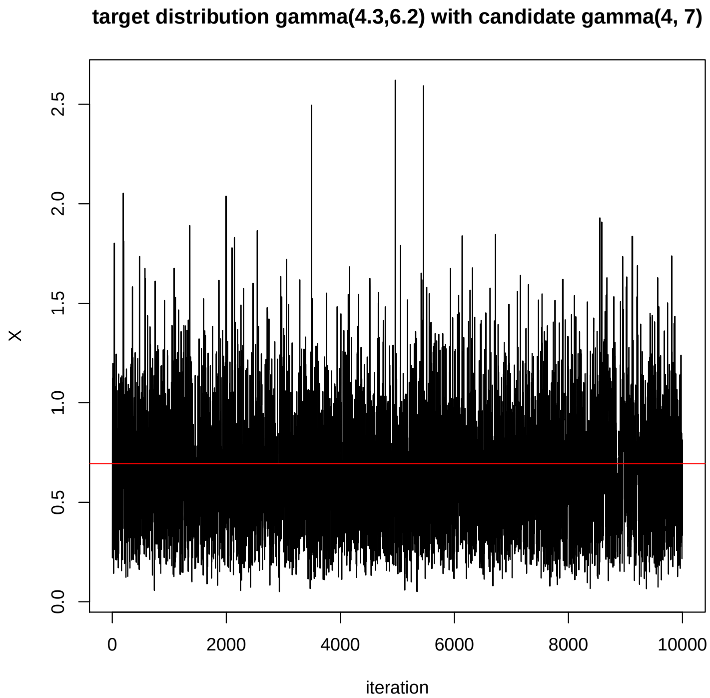
Homework 7

Exercise 1 & 2

Part A

```
f = function(x) dgamma(x, 4.3,6.2)
g = function(x) dgamma(x, 4, 7)


Nsim=10^4
X=rgamma(1, 4.3,6.2)    # initialize the chain from the stationary
accept = c(0)
for (t in 2:Nsim){
  Y=rgamma(1, 4,7)   # candidate normal
  rho=f(Y)*g(X[t-1])/(f(X[t-1])*g(Y))
  if(runif(1)<rho){
    X[t] = Y
    accept[t] = 1
  }
  else{
    X[t] =X[t-1]
    accept[t] = 0
  }
}
plot(X, type = "l",
     main = "target distribution gamma(4.3,6.2) with candidate gamma(4, 7)",
     xlab = "iteration")
abline(h = 4.3/6.2, col = "red")
```

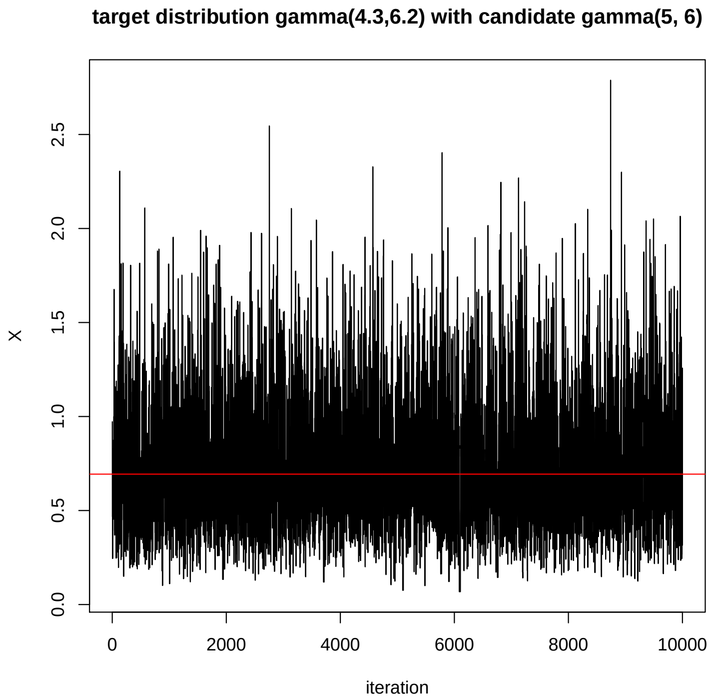


From the Metropolis-Hastings Algorithm the estimated mean of $gamma(4.3, 6.2)$ is 0.6950373 and the acceptance rate is 77.97%.

Part B

```
g = function(x) dgamma(x,5,6)

X=rgamma(1, 4.3,6.2)
accept = c(0)
for (t in 2:Nsim){
  Y=rgamma(1, 5,6)
  rho=f(Y)*g(X[t-1])/(f(X[t-1])*g(Y))
  if(runif(1)<rho){
    X[t] = Y
    accept[t] = 1
  }
  else{
    X[t] =X[t-1]
    accept[t] = 0
  }
}
plot(X, type = "l",
     main = "target distribution gamma(4.3,6.2) with candidate gamma(5, 6)",
     xlab = "iteration")
abline(h = 4.3/6.2, col = "red")
```



From the Metropolis-Hastings Algorithm the estimated mean of $gamma(4.3, 6.2)$ is 0.6846806 and the acceptance rate is 75.95%.

Exercise 3

```r
library('mixtools')

## mixtools package, version 1.0.4, Released 2016-01-11
## This package is based upon work supported by the National Science Foundation under Grant
No.  SES-0518772.

x = c(0.12,0.17,0.32,0.56,0.98,1.03,1.10,1.18, 1.23, 1.67, 1.68, 2.33)
out <- gammamixEM(x)

## number of iterations= 39

out[2:4]

## $lambda
## [1] 0.2464711 0.7535289
##
## $gamma.pars
##           comp.1    comp.2
## alpha 5.70365196 6.9064472
## beta  0.03578078 0.1884112
##
## $loglik
## [1] -9.071738
```

$Gamma(1, \beta) \equiv Exp(\beta)$ therefore the estimates for μ and λ are 0.0357808, 0.1884112 respectively. The one issue is that the gamma mixture gives outputs for the α in a gamma distribution. This is set at $\alpha = 1$ for an exponential distribution.

Exercise 4

Part A

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.019 & 0.98 & 0.001 \\ 0.02 & 0.03 & 0 & 0.95 \end{bmatrix} \end{array} \tag{1}$$

$$(I - R) = \begin{bmatrix} 1 - 0.98 & 0 - 0.001 \\ 0 - 0 & 1 - 0.95 \end{bmatrix} = \begin{bmatrix} 0.02 & -0.001 \\ 0 & 0.05 \end{bmatrix} \tag{2}$$

$$(I - R)^{-1} = \frac{1}{0.02 * 0.05 - (-0.001)(0)} \begin{bmatrix} 0.05 & 0.001 \\ 0 & 0.02 \end{bmatrix} = \begin{bmatrix} 50 & 1 \\ 0 & 20 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} 50 & 1 \\ 0 & 20 \end{bmatrix} * \begin{bmatrix} 0 & 0.019 \\ 0.02 & 0.03 \end{bmatrix} = \begin{bmatrix} 0.02 & 0.98 \\ 0.4 & 0.6 \end{bmatrix} \tag{4}$$

$$P(3, 1) = 0.02 \tag{5}$$

Part B

$$E(3, 3) = 50 \tag{6}$$