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Math 611 Stochastic Simulation
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Homework 10

Exercise 1

Please note the the joint density can be easily seperated:

$$f(x,y) = \frac{x^{a+y-1} * e^{-(1+b)x} * b^a}{y! * \Gamma(a)} = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx} * \frac{x^y e^{-x}}{y!} \tag{1}$$

From the reorganization of the joint density

$$f(x|y) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx} \sim gamma(a,b) \tag{2}$$

Interesting to note that $f(x|y)$ does not depend on y, so in effect $f(x|y) = f(x)$.

Exercise 2

From the reorganization of the joint density

$$f(y|x) = \frac{x^y e^{-x}}{y!} \tag{3}$$

Which follows a poisson distribution where $\lambda = x$

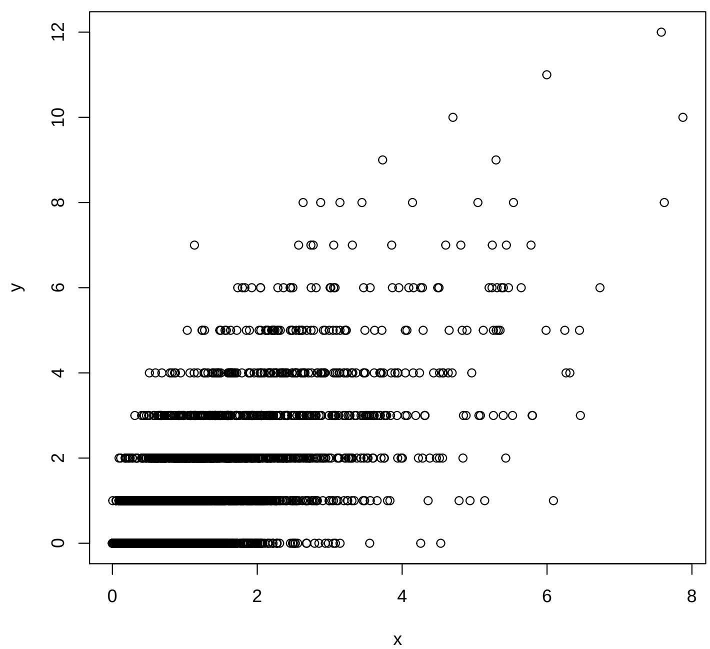
Exercise 3

```
joint = function(a,b){
  x = rgamma(1,a,b)
  y = rpois(1,x)
  return(data.frame(x,y,x*y))
}
```

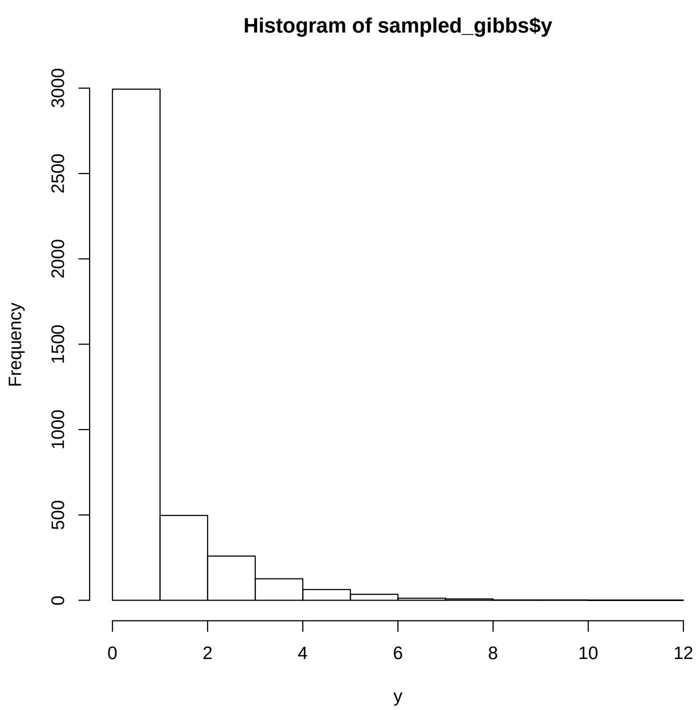
Exercise 4

```
N = 5000
prob_4 = data.frame("x", "y", "xy")[-1,]

for(t in 1:N){
  prob_4 = rbind(prob_4, joint(1,1))
}
names(prob_4) = c("x", "y", "xy")
burn = 1e3
sampled_gibbs = prob_4[-(1:burn),]
plot(sampled_gibbs$x, sampled_gibbs$y, xlab = "x", ylab = "y")
```




```r
hist(sampled_gibbs$y, xlab = "y")
```




```r
mean(sampled_gibbs$y)

## [1] 0.9925
```

$E(Y)$ =0.9925, which is very close to the value of the mean of $gamma(1, 1)$ Since the x values were derived from that distribution, it would make sense that the average lambda value supplied to y is close to one.

Exercise 5

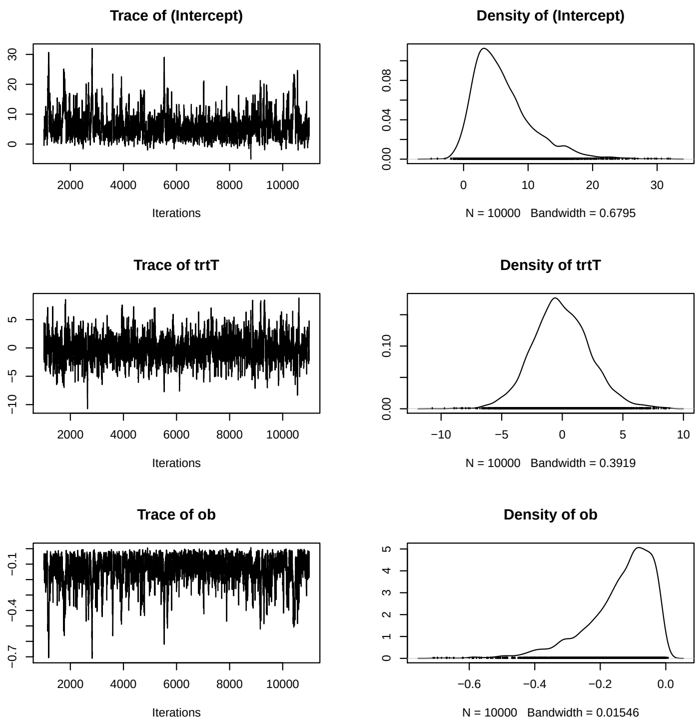
```
library(MCMCpack)

## Loading required package:  coda
## Loading required package:  MASS
## ##
## ## Markov Chain Monte Carlo Package (MCMCpack)
## ## Copyright (C) 2003-2016 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
## ##
## ## Support provided by the U.S. National Science Foundation
## ## (Grants SES-0350646 and SES-0350613)
## ##

beta_blockers <- read.csv("g.csv")
beta_blockers = beta_blockers[beta_blockers$center == 1,]
beta_blockers$death <- 1
beta_blockers$death[beta_blockers$value != "Death"] = 0
beta_blockers$trt = as.factor(beta_blockers$trt)
mc_posterior = MCMClogit(death~trt+ob, data = beta_blockers)
summary(mc_posterior)

##
## Iterations = 1001:11000
## Thinning interval = 1
## Number of chains = 1
## Sample size per chain = 10000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##                 Mean     SD Naive SE Time-series SE
## (Intercept)  6.06317 4.6184 0.046184       0.250883
## trtT        -0.06891 2.4006 0.024006       0.092080
## ob          -0.13856 0.1035 0.001035       0.005508
##
## 2. Quantiles for each variable:
##
##                 2.5%     25%     50%      75%   97.5%
## (Intercept) -0.04411  2.7965  5.0422  8.21607 17.5936
## trtT        -4.62908 -1.6543 -0.1683  1.47173  4.9107
## ob          -0.40264 -0.1868 -0.1122 -0.06347 -0.0144

plot(mc_posterior)
```



Trace of (Intercept)
Iterations
Density of (Intercept)
N = 10000 Bandwidth = 0.6795
Trace of trtT
Iterations
Density of trtT
N = 10000 Bandwidth = 0.3919
Trace of ob
Iterations
Density of ob
N = 10000 Bandwidth = 0.01546