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Math 611 Stochastic Simulation
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Homework 3

Exercise 1

Since $Area_{square} = 1$, $Area_{circle} = \frac{\pi}{2^2} = \frac{\pi}{4}$ as stated in the exercise. Therefore the number of points that are expected in the circle is $\frac{\pi/4}{1} = \pi/4$ Given that the orgin of the circle is at $(1/2, 1/2)$ with a radius of $r = 1/2$, the circle equation is $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$, so we should look for points with coordinates such that $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 < \frac{1}{4}$. I assume that the circumference doesn't count as in the circle.

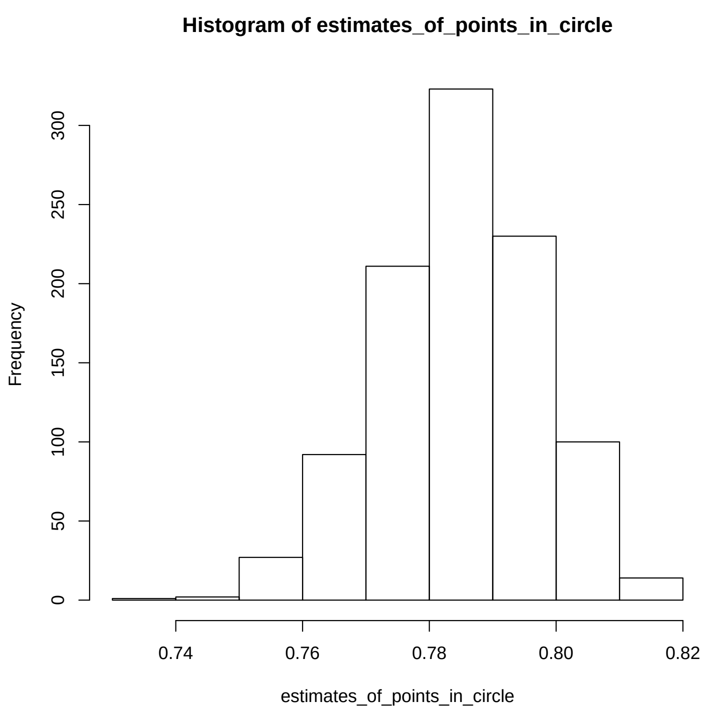
```
points_in_circle = function(z){
  points= data.frame(x = runif(1e3), y = runif(1e3))
  mean((points$x-1/2)^2+(points$y-1/2)^2<1/4)
}
estimates_of_points_in_circle = sapply(1:1e3, points_in_circle)
mean(estimates_of_points_in_circle)

## [1] 0.785235

var(estimates_of_points_in_circle)

## [1] 0.0001556374

hist(estimates_of_points_in_circle)
```



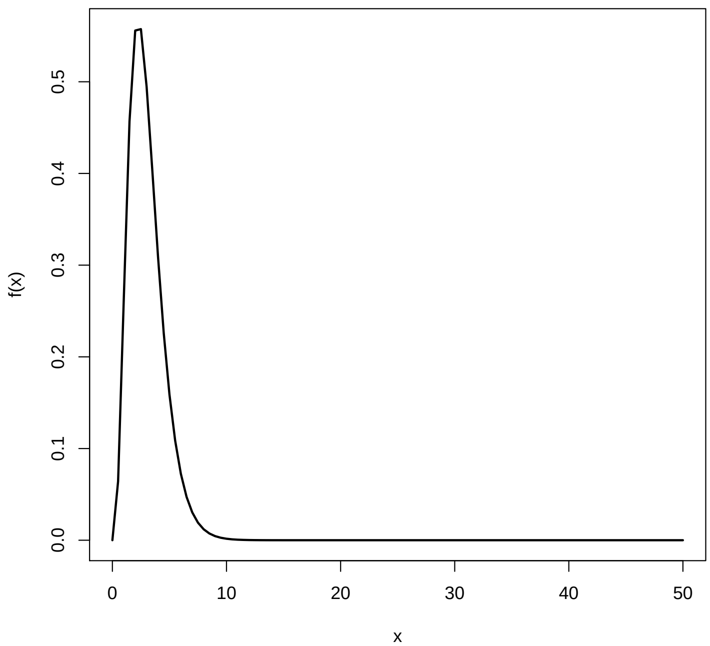
Based on the estimated mean and the variance, the fixed range is very close to the expected fraction.

Exercise 2

Please note: I used the methods from "http://math.stackexchange.com/questions/1200443/evaluating-difficult-monte-carlo-integration-in-r" to do monte carlo integration in R

Part A

```
f = function(x){
  return(exp(-4*x/3)*x^3)
}
curve(f, lwd=2,to = 50)
```



Based on the graph, it is clear that $f(x)$ converges to 0 close to 10, so using the method described in class, we will do the following:

```
n = 1e4
a = 0
b = 10
x = runif(n, a, b)
y = f(x)
(b-a)/n*sum(y)

## [1] 1.882511
```

Part B

$$\begin{aligned}\int_0^\infty e^{\frac{-4x}{3}} x^3 \delta x \implies \\ \int_0^\infty x^3 * e^{\frac{-4x}{3}} \delta x \implies \\ \int_0^\infty x^{4-1} * e^{-\frac{x}{3/4}} \delta x \implies \\ \Gamma(4) * (\frac{3}{4})^4 \int_0^\infty \frac{1}{\Gamma(4) * (\frac{3}{4})^4} * x^{4-1} * e^{-\frac{x}{3/4}} \delta x\end{aligned} \tag{1}$$

please recognize that $\frac{1}{\Gamma(4)*(\frac{3}{4})^4} * x^{4-1} * e^{\frac{x}{3/4}} \sim gamma(4, 3/4)$

$\therefore \Gamma(4) * (\frac{3}{4})^4 \int_0^\infty \frac{1}{\Gamma(4)*(\frac{3}{4})^4} * x^{4-1} * e^{\frac{x}{3/4}} \delta x = \Gamma(4) * (\frac{3}{4})^4 = 1.898$

Exercise 3

```
n = 1e4
a = -10
b = 10
x = runif(n, a, b)
y = exp(-(dnorm(x, 0, 2))^2)
(b-a)/n*sum(y)

## [1] 19.86204
```

$$\begin{aligned}E_f(h(x)) = \int_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2} * e^{-x^2} \delta x = \\ \int_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2 - \frac{2\sigma^2}{2\sigma^2}x^2} \delta x = \\ \int_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2(1+2\sigma^2)} \delta x = \\ \int_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{\frac{2\sigma^2}{(1+2\sigma^2)}}x^2} \delta x = \\ 1/\sqrt{(1+2\sigma^2)} \int_x \frac{1}{\sqrt{2\pi\sigma^2/(1+2\sigma^2)}} e^{-\frac{1}{\frac{2\sigma^2}{(1+2\sigma^2)}}x^2} \delta x = 1/\sqrt{(1+2\sigma^2)}\end{aligned} \tag{2}$$

This is true because $\frac{1}{\sqrt{2\pi\sigma^2/(1+2\sigma^2)}} e^{-\frac{1}{\frac{2\sigma^2}{(1+2\sigma^2)}}x^2} \sim N(0, \sigma^2/(1+2\sigma^2))$

Exercise 4

The min probably of transiting to state 3 is $min 0.1, 0.2, 0.4 = 0.1 \therefore P_3(T_3 > n) \leq (0.9)^n$ $(0.9)^n \to 0$ as $n \to \infty$, so state 3 is recurrent.

The min probably of transiting to state 2 is $min 0.2, 0.5, 0.4 = 0.2 \therefore P_2(T_2 > n) \leq (0.8)^n$ $(0.2)^n \to 0$ as $n \to \infty$, so state 2 is recurrent.

The min probably of transiting to state 1 is $min 0.7, 0.3, 0.2 = 0.2 \therefore P_1(T_1 > n) \leq (0.8)^n$ $(0.2)^n \to 0$ as $n \to \infty$, so state 1 is recurrent.

Exercise 5