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Math 611 Stochastic Simulation
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Homework 6

Exercise 1

```
x = c(169.14353, 135.73850, 102.46566,  80.91151, 148.45425, 144.68948, 106.56257, 104.83559,
94.81216, 109.47048,  95.94150, 123.84673,  87.18401, 104.73420, 111.94364, 119.69467,
151.77627,  81.80692, 116.58660,  98.28933)

mu_1 <- c(100)
mu_2 <- c(120)
## as well as the latent variable parameters
#tau_1 <- c(1-0.7) # 1-W
W <- c(0.7) # W
for( i in 1:12) {
  ## Given the observed data, as well as the distribution parameters,
  ## what are the latent variables?
  T_1 <- (1-W[i]) * dnorm(x, mu_1[i], sd = sqrt(20))
  T_2 <- W[i] * dnorm(x, mu_2[i], sd = sqrt(25))
  P_1 <- T_1 / (T_1 + T_2)
  P_2 <- T_2 / (T_1 + T_2) ## note: P_2 = 1 - P_1
  #tau_1[i+1] <- mean(P_1)
  W[i+1] <- mean(P_2)
  ## Given the observed data, as well as the latent variables,
  ## what are the population parameters
  mu_1[i+1] <- sum( P_1 * x) / sum(P_1)
  mu_2[i+1] <- sum( P_2 * x) / sum(P_2)
}
data.frame(mu_1, mu_2, W)[-1,]
```

```
##         mu_1     mu_2         W
## 2   96.04482 133.4018 0.4925329
## 3   98.20615 138.6084 0.4019140
## 4   99.49604 141.4485 0.3563159
## 5  100.38478 143.5734 0.3255391
## 6  101.02974 145.4095 0.3022692
## 7  101.17853 145.8059 0.2972582
## 8  101.24208 145.9554 0.2952656
## 9  101.27532 146.0340 0.2942234
## 10 101.29500 146.0809 0.2936057
## 11 101.30746 146.1106 0.2932139
## 12 101.31569 146.1304 0.2929552
## 13 101.32127 146.1437 0.2927797
```

Exercise 2

Part A

Since $X \sim N(0,1) \implies X^2 \sim Chi-squared(1)$ then $Z_1^2 \sim Chi-squared(1)$.

Exercise 3

Exercise 4