Calculus Cheat Sheet

@kouariga

March 2019

1 Derivatives of (univariate) functions

	(Scalar-valued) functions	Vector-valued functions
First-order	$\frac{df}{dx}$	$\begin{bmatrix} \frac{df_1}{dx} \\ \vdots \\ \frac{df_M}{dx} \end{bmatrix}$
Second-order	$\frac{d^2 f}{dx^2}$	$\begin{bmatrix} \frac{d^2 f_1}{dx^2} \\ \vdots \\ \frac{d^2 f_M}{dx^2} \end{bmatrix}$

2 Partial derivatives of multivariate functions

	(Scalar-valued) functions	Vector-valued functions
First-order	Gradient, ∇ $\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_N} \end{bmatrix}$	Jacobian, J $\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial x_1} & \frac{\partial f_M}{\partial x_2} & \cdots & \frac{\partial f_M}{\partial x_N} \end{bmatrix}$
Second-order	Hessian, H $\begin{bmatrix} \frac{\partial f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_N} \\ \frac{\partial f}{\partial x_2 \partial x_1} & \frac{\partial f}{\partial x_2^2} & \cdots & \frac{\partial f}{\partial x_2 \partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f}{\partial x_N \partial x_1} & \frac{\partial f}{\partial x_N \partial x_2} & \cdots & \frac{\partial f}{\partial x_N^2} \end{bmatrix}$	

3 Remark

Gradient is a vector-valued function, so Jacobian of Gradient is well-defined. It turns out,

$$H(f) = J(\nabla f).$$