Calculus Cheat Sheet

@kouariga

March 2019

1 Derivatives of (univariate) functions

	(Scalar-valued) functions	Vector-valued functions
First-order	$rac{df}{dx}$	$\begin{bmatrix} \frac{df_1}{dx} \\ \vdots \end{bmatrix}$
		$\left\lfloor rac{df_M}{dx} ight floor$
Second-order	$\frac{d^2f}{dx^2}$	$\begin{bmatrix} \frac{d^2 f_1}{dx^2} \\ \cdot \end{bmatrix}$
	$\overline{dx^2}$	$\left \lfloor rac{d^2 f_M}{dx^2} ight floor$

2 Partial derivatives of multivariate functions

	(Scalar-valued) functions	Vector-valued functions
	Gradient, ∇	Jacobian, J
First-order	$\left\lceil \frac{\partial f}{\partial x_1} \right\rceil$	$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial x_1} & \frac{\partial f_M}{\partial x_2} & \cdots & \frac{\partial f_M}{\partial x_N} \end{bmatrix}$
	$\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_N} \end{bmatrix}$	$ \frac{\partial f_2}{\partial x_2} \frac{\partial f_2}{\partial x_2} \dots \frac{\partial f_2}{\partial x_N} $
	:	1 : : : : :
	$\left\lfloor rac{\partial f}{\partial x_N} ight floor$	$\begin{bmatrix} \frac{\partial f_M}{\partial x_1} & \frac{\partial f_M}{\partial x_2} & \dots & \frac{\partial f_M}{\partial x_N} \end{bmatrix}$
	Hessian, H	
Second-order	$\begin{bmatrix} \frac{\partial f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_N} \\ \frac{\partial f}{\partial x_2 \partial x_1} & \frac{\partial f}{\partial x_2^2} & \cdots & \frac{\partial f}{\partial x_2 \partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f}{\partial x_N \partial x_1} & \frac{\partial f}{\partial x_N \partial x_2} & \cdots & \frac{\partial f}{\partial x_N^2} \end{bmatrix}$	
	$ \frac{\partial f}{\partial x_2 \partial x_1} \qquad \frac{\partial f}{\partial x_2^2} \qquad \dots \qquad \frac{\partial f}{\partial x_2 \partial x_N} $	
	$\left[\begin{array}{cccc} \frac{\partial f}{\partial x_N \partial x_1} & \frac{\partial f}{\partial x_N \partial x_2} & \dots & \frac{\partial f}{\partial x_N^2} \end{array}\right]$	

3 Remark

Gradient is a vector-valued function, so Jacobian of Gradient is well-defined. It turns out,

$$H(f) = J(\nabla f).$$