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18-06 Linear Algebra

Homework 2

Exercises on elimination with matrices

Problem 2.1. In the two-by-two system of linear equations below, what multiple of the first equation should be subtracted from the second equation when using the method of elimination? Convert this system of equations to matrix form, apply elimination (what are the pivots?), and use back substitution to find a solution. Try to check your work before looking up the answer.

$$\begin{aligned}2x + 3y &= 5 \\6x + 15y &= 12\end{aligned}$$

Answer. Multiplying the first equation by 3 and subtracting it from the second equation will eliminate x from the second equation. Performing this operation in matrix form will result in an Upper Triangular matrix:

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 6 & 15 & 12 \end{array} \right] \xrightarrow{r_2 - 3r_1} \left[\begin{array}{cc|c} \boxed{2} & 3 & 5 \\ 0 & \boxed{6} & -3 \end{array} \right]$$

Back Substitution: Convert the matrix back to equations:

$$\begin{aligned}6y &= -3 \implies \boxed{y = \frac{-1}{2}} \\2x + 3y &= 5 \implies 2x + 3\left(\frac{-1}{2}\right) = 5 \implies 2x - \frac{3}{2} = 5 \implies 2x = 5 + \frac{3}{2} \implies 2x = \frac{13}{2} \implies \boxed{x = \frac{13}{4}}\end{aligned}$$

Problem 2.2. (2.3 #29. Introduction to Linear Algebra: Strang) Find the triangular matrix E that reduces Pascal's matrix to a smaller Pascal:

$$E \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Which matrix M (multiplying several E s) reduces Pascal all the way to I ?

Answer. Call "Pascal's Matrix" P and the smaller Pascal Q . We want to find E such that $EP = Q$. Note that the first row of Q is identical to the first row of P and each

subsequent row of Q is equal to the corresponding row of P minus the previous row of P . Thus, E is a difference matrix:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

To find M , we note that to convert P to I , $I_1 = P_1$, $I_2 = P_2 - P_1$, $I_3 = P_3 - 2P_2 + P_1$ and $I_4 = P_4 - 3P_3 + 3P_2 - P_1$. Hence, we can determine the inverse matrix M :

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$