Homework Assessment: Climate Change Analysis and Radiative Forcing

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CE 107 Assignment #2
Climate Change Mitigation
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Contents

1 Problem 1: Methane Control as a Climate–Change Mitigation Strategy

Introduction

This problem analyzes methane (CH_4) radiative forcing and its implications for climate mitigation using given data and formulas.

Given Data and Relationships

- Global CH_4 emission rate: 650 Tg/yr.
- CH_4 loss rate constant: $k = 0.083\,\text{yr}^{-1}$.
- Radiative efficiency of CH_4: $\Delta f_{CH_4} = 3.6 \times 10^{-4}\,\text{W/m}^2\text{/ppb}$ (at ~ 1800 ppb).
- Radiative efficiency of CO_2: $\Delta f_{CO_2} = 1.37 \times 10^{-5}\,\text{W/m}^2\text{/ppb}$.
- For CH_4,
$$\Delta F_{CH_4} = 0.036\left(\sqrt{M} - \sqrt{M_0}\right) - \left(0.036 + 6.38 \times 10^{-5}\,M\right) \quad (\text{W/m}^2),$$
- For CO_2,
$$\Delta F_{CO_2} = 5.35\,\ln\left(\frac{C}{C_0}\right) \quad (\text{W/m}^2),$$
- Pre–industrial values: $M_0 = 722$ ppb and $C_0 = 280$ ppm.
- Current (2023) values: $M = 1922$ ppb and $C = 424$ ppm.
- Conversion: $1\,\text{ppb}\,CH_4 \approx 2.84\,\text{Tg}$.

Solutions

(a) Radiative Forcing in 2023. For CH_4, substituting $M = 1922$ ppb and $M_0 = 722$ ppb:

$$\sqrt{1922} \approx 43.85, \quad \sqrt{722} \approx 26.87, \quad \sqrt{1922} - \sqrt{722} \approx 16.98.$$

Multiplying by 0.036 gives $0.036 \times 16.98 \approx 0.611$. The second term is

$$6.38 \times 10^{-5} \times 1922 \approx 0.123, \quad 0.036 + 0.123 \approx 0.159.$$

Thus,

$$\Delta F_{CH_4} \approx 0.611 - 0.159 \approx 0.45\,\text{W/m}^2.$$

For CO_2, with $C = 424$ ppm and $C_0 = 280$ ppm:

$$\frac{424}{280} \approx 1.514, \quad \ln(1.514) \approx 0.415, \quad \Delta F_{CO_2} \approx 5.35 \times 0.415 \approx 2.22\,\text{W/m}^2.$$

The total forcing is approximately

$$\Delta F_{tot} \approx 0.45 + 2.22 \approx 2.67\,\text{W/m}^2.$$

(b) Steady–State CH_4 Abundance at 650 Tg/yr. Using the relation

$$E = k\,(M_{\text{ss}} \times 2.84),$$

with $E = 650$ Tg/yr:

$$M_{\text{ss}} = \frac{650}{0.083 \times 2.84} \approx \frac{650}{0.2357} \approx 2758\,\text{ppb}.$$

(c) Emissions for a Steady–State CH_4 Abundance of 1000 ppb. For a target abundance $M = 1000$ ppb, the carbon burden is

$$B = 1000 \times 2.84 = 2840\,\text{Tg}.$$

Then,

$$E = k\,B = 0.083 \times 2840 \approx 236\,\text{Tg/yr}.$$

(d) CO_2 Increase to Offset a CH_4 Drop from 1922 to 722 ppb. At $M = 722$ ppb, the first term in ΔF_{CH_4} cancels, leaving

$$\Delta F_{CH_4}(722) \approx -\Big(0.036 + 6.38 \times 10^{-5} \times 722\Big) \approx -0.0821\,\text{W/m}^2.$$

Thus, the change in forcing when CH_4 drops from 1922 to 722 ppb is

$$0.45 - (-0.0821) \approx 0.5321\,\text{W/m}^2.$$

To compensate, the CO_2 forcing must increase from $2.22\,\text{W/m}^2$ to

$$2.22 + 0.5321 \approx 2.7521\,\text{W/m}^2.$$

Solving

$$5.35\,\ln\!\Big(\frac{C'}{280}\Big) = 2.7521,$$

yields

$$\ln\!\Big(\frac{C'}{280}\Big) \approx 0.514, \quad \frac{C'}{280} \approx e^{0.514} \approx 1.672, \quad C' \approx 280 \times 1.672 \approx 468\,\text{ppm}.$$

Thus, an increase of about $468 - 424 = 44$ ppm in CO_2 is required.

(e) Radiative Efficiency of CH_4 at 722 ppb. Differentiating the forcing function with respect to M,

$$\frac{dF}{dM} = \frac{0.036}{2\sqrt{M}} - 6.38 \times 10^{-5}.$$

At $M = 722$ ppb ($\sqrt{722} \approx 26.87$):

$$\frac{0.036}{2 \times 26.87} \approx \frac{0.036}{53.74} \approx 6.69{\times}10^{-4}, \quad \frac{dF}{dM} \approx 6.69{\times}10^{-4} - 6.38{\times}10^{-5} \approx 6.05{\times}10^{-4}\,\text{W/m}^2/\text{ppb}.$$

Final Answers for Problem 1

(a) $\Delta F_{CH_4} \approx 0.45\,\mathrm{W/m^2}$ and $\Delta F_{CO_2} \approx 2.22\,\mathrm{W/m^2}$; total forcing $\approx 2.67\,\mathrm{W/m^2}$.

(b) Steady–state CH_4 abundance for 650 Tg/yr emissions is approximately 2758 ppb.

(c) A target steady–state abundance of 1000 ppb corresponds to an emission rate of roughly 236 Tg/yr.

(d) Offsetting a CH_4 drop from 1922 to 722 ppb (a forcing change of about $0.53\,\mathrm{W/m^2}$) requires CO_2 to rise from 424 ppm to about 468 ppm (an increase of approximately 44 ppm).

(e) The local radiative efficiency at 722 ppb is about $6.05 \times 10^{-4}\,\mathrm{W/m^2/ppb}$.

2 Problem 2: Modeling Future CO_2 Levels Using Atmospheric Data and Mass Balance

Problem Statement

Historical data show that the atmospheric CO_2 concentration increased from 388.6 ppm in January 2010 to 413.4 ppm in January 2020 (a net increase of 24.8 ppm). During this period, the average anthropogenic emission rate was about 11.0 GtC/year. Assuming that in the absence of anthropogenic emissions the natural system would be in balance, the net increase represents the fraction of anthropogenic emissions that remained in the atmosphere.

A mass–balance model is given by

$$\frac{dS}{dt} = E(t) - L(t),$$

where $E(t)$ is the emission rate (GtC/year), $S(t)$ the atmospheric CO_2 stock (GtC), and $L(t)$ the loss rate (GtC/year). Assuming $L(t) = L$ (a constant), consider a future scenario (January 2020–December 2050) with constant emissions of 11.0 GtC/year and constant L.

Solutions

(a) Fraction of Emissions Remaining. The observed increase of 24.8 ppm corresponds to

$$\Delta S \approx 24.8\,\text{ppm} \times 2.12\,\text{GtC/ppm} \approx 52.6\,\text{GtC}.$$

Total emissions over 10 years are

$$11.0\,\text{GtC/yr} \times 10\,\text{yr} = 110\,\text{GtC}.$$

Thus, the fraction remaining is

$$\frac{52.6}{110} \approx 0.478 \quad (48\%).$$

(b) Determining the Loss Rate L. The net accumulation per year is

$$E - L = \frac{52.6\,\text{GtC}}{10\,\text{yr}} = 5.26\,\text{GtC/yr}.$$

Hence,

$$L = 11.0 - 5.26 \approx 5.74\,\text{GtC/yr}.$$

(c) Future Atmospheric CO_2 Level (2020–2050). Over 31 years (2020–2050), the net accumulation is

$$5.26\,\text{GtC/yr} \times 31\,\text{yr} \approx 163.1\,\text{GtC}.$$

This corresponds to an increase in ppm of

$$\frac{163.1}{2.12} \approx 77\,\text{ppm}.$$

Adding this to the 2020 level of 413.4 ppm gives

$$413.4 + 77 \approx 490.4\,\text{ppm}.$$

(d) Discussion. A level of approximately 490 ppm by 2050 exceeds the target of 450 ppm for limiting warming below 2 °C. Therefore, stabilizing emissions at 11.0 GtC/year is insufficient; additional emission reductions or carbon removal measures are necessary.

Final Answers for Problem 2

(a) Approximately 48% of the decade's emissions remained in the atmosphere.

(b) The estimated constant loss rate is about 5.74 GtC/year.

(c) Under constant emissions, the atmospheric CO_2 level is projected to reach roughly 490 ppm by December 2050.

(d) Since 490 ppm exceeds the 450 ppm target, further emission reductions or carbon removal measures are needed.

3 Problem 3: Climate-Change Dynamics: Heat Capacity of the Earth and Its Oceans

Problem Statement

The Earth's large thermal capacity delays the response of temperature to changes in climate forcing. On intermediate time scales, the primary heat reservoirs are:

(a) The entire atmosphere.

(b) The surface oceans (assumed to be 100 m deep).

(c) The surface soils (assumed to be 10 m deep).

Estimate the total heat capacity (in J/K) of these three compartments. Then, given a steady–state temperature sensitivity of 0.3 K per (W/m^2) and a sudden extra forcing of 1 W/m^2 (with no increase in heat loss), determine the characteristic time for the Earth's average surface temperature to approach its new equilibrium. Finally, if the deep ocean (volume $\approx 1.4 \times 10^{18}$ m^3) absorbed a forcing of 1.6 W/m^2, compute its annual rate of temperature increase in °C/year.

Solutions

(a) Total Heat Capacity.

- *Atmosphere:* Mass $\approx 5.1 \times 10^{18}\,\mathrm{kg}$, specific heat $\approx 1000\,\mathrm{J/(kg\,K)}$, so

$$C_{\text{atm}} \approx 5.1 \times 10^{21}\ \mathrm{J/K}.$$

- *Surface Oceans:* Ocean area $\approx 0.70 \times 5.1 \times 10^{14}\,\mathrm{m}^2 \approx 3.57 \times 10^{14}\,\mathrm{m}^2$; for 100 m depth, volume $\approx 3.57 \times 10^{16}\,\mathrm{m}^3$; mass $\approx 3.57 \times 10^{19}\,\mathrm{kg}$; specific heat $\approx 4186\,\mathrm{J/(kg\,K)}$; thus,

$$C_{\text{ocean}} \approx 1.49 \times 10^{23}\ \mathrm{J/K}.$$

- *Surface Soils:* Land area $\approx 0.30 \times 5.1 \times 10^{14}\,\mathrm{m}^2 \approx 1.53 \times 10^{14}\,\mathrm{m}^2$; for 10 m depth, volume $\approx 1.53 \times 10^{15}\,\mathrm{m}^3$; assuming density $\approx 1500\,\mathrm{kg/m}^3$, mass $\approx 2.295 \times 10^{18}\,\mathrm{kg}$; specific heat $\approx 800\,\mathrm{J/(kg\,K)}$; hence,

$$C_{\text{soil}} \approx 1.84 \times 10^{21}\ \mathrm{J/K}.$$

The total heat capacity is

$$C_{\text{total}} \approx 5.1 \times 10^{21} + 1.49 \times 10^{23} + 1.84 \times 10^{21} \approx 1.56 \times 10^{23}\ \mathrm{J/K}.$$

(b) Characteristic Time Scale. A forcing of 1 W/m^2 yields a temperature increase of 0.3 K, so the energy needed is

$$Q = C_{\text{total}} \times 0.3 \approx 1.56 \times 10^{23} \times 0.3 \approx 4.68 \times 10^{22}\,\text{J}.$$

With Earth's surface area $A_{\text{Earth}} \approx 5.1 \times 10^{14}\,\text{m}^2$, the total power is

$$P = 1\,\text{W/m}^2 \times 5.1 \times 10^{14} \approx 5.1 \times 10^{14}\,\text{W}.$$

Thus, the characteristic time is

$$\tau = \frac{Q}{P} \approx \frac{4.68 \times 10^{22}}{5.1 \times 10^{14}} \approx 9.18 \times 10^{7}\,\text{s} \approx 2.9\,\text{yr}.$$

(c) Deep Ocean Warming Rate. The deep ocean has volume $1.4 \times 10^{18}\,\text{m}^3$; with density $1000\,\text{kg/m}^3$, mass is

$$M_{\text{deep}} \approx 1.4 \times 10^{21}\,\text{kg}.$$

Its heat capacity is then

$$C_{\text{deep}} = M_{\text{deep}} \times 4186 \approx 5.86 \times 10^{24}\,\text{J/K}.$$

A forcing of 1.6 W/m^2 over the entire Earth gives

$$P_{\text{deep}} = 1.6 \times 5.1 \times 10^{14} \approx 8.16 \times 10^{14}\,\text{W}.$$

Thus, the temperature increase rate is

$$\frac{dT}{dt} = \frac{P_{\text{deep}}}{C_{\text{deep}}} \approx \frac{8.16 \times 10^{14}}{5.86 \times 10^{24}} \approx 1.39 \times 10^{-10}\,\text{K/s}.$$

Converting to °C/yr:

$$1.39 \times 10^{-10} \times 3.16 \times 10^{7} \approx 4.4 \times 10^{-3}\,\text{K/yr} \approx 0.0044\,^{\circ}\text{C/yr}.$$

Final Answers for Problem 3

(a) Total heat capacity: $1.56 \times 10^{23}\,\text{J/K}$.

(b) Characteristic time scale: Approximately 3 years.

(c) Deep ocean warming rate: Approximately $0.0044\,^{\circ}\text{C/yr}$.

4 Problem 4: Atmospheric Persistence of CO_2

Problem Statement

The persistence function for CO_2 is given by

$$I(t) = 0.2173 + 0.224\, e^{-t/394.4} + 0.2824\, e^{-t/36.54} + 0.2763\, e^{-t/4.304},$$

with t in years (valid for $t < 1000$ years). (Source: F. Joos et al., *Atmospheric Chemistry and Physics* 13, 2793, 2013.) Compute:

(a) The ultimate persistence, $\lim_{t\to\infty} I(t)$.

(b) The time t_{50} when $I(t) = 0.5$.

(c) The integral $\int_0^T I(t)\,dt$ for $T = 20$, 100, and 500 years.

Solutions

(a) Ultimate Persistence. As $t \to \infty$, the exponential terms vanish. Thus,

$$I(\infty) = 0.2173.$$

(b) Time t_{50}. We solve for t in

$$0.2173 + 0.224\, e^{-t/394.4} + 0.2824\, e^{-t/36.54} + 0.2763\, e^{-t/4.304} = 0.5.$$

Numerical evaluation yields $t_{50} \approx 44.5$ years.

(c) Time Integrals. Since

$$\int_0^T e^{-t/\tau}\, dt = \tau\left[1 - e^{-T/\tau}\right],$$

we have

$$\int_0^T I(t)\, dt = 0.2173\, T + 0.224\,(394.4)[1-e^{-T/394.4}] + 0.2824\,(36.54)[1-e^{-T/36.54}] + 0.2763\,(4.304)[1-e^{-T/4.304}].$$

For $T = 20$ years:

$$\int_0^{20} I(t)\, dt \approx 14.25\,\text{years}.$$

For $T = 100$ years:

$$\int_0^{100} I(t)\, dt \approx 52.32\,\text{years}.$$

For $T = 500$ years:

$$\int_0^{500} I(t)\, dt \approx 183.66\,\text{years}.$$

Final Answers for Problem 4

(a) Ultimate persistence: $I(\infty) \approx 0.2173$.

(b) $t_{50} \approx 44.5$ years.

(c) $\int_0^{20} I(t)\,dt \approx 14.25$ years, $\int_0^{100} I(t)\,dt \approx 52.32$ years, and $\int_0^{500} I(t)\,dt \approx 183.66$ years.