1D Wave Equation Simulation using Physics-Informed Neural Networks (PINNs)

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Introduction

In this assignment, we solve the one-dimensional wave equation using Physics-Informed Neural Networks (PINNs). The wave equation describes how disturbances (such as vibrations or sound waves) propagate in a medium. Mathematically, it is given by:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

subject to Dirichlet boundary conditions:

$$u(0,t) = u(1,t) = 0$$

and initial conditions:

$$u(x,0) = x(1-x), \quad \frac{\partial u(x,0)}{\partial t} = 0$$

Here, the wave speed c is assumed to be 1. The domain of the solution is $x \in [0,1]$ and $t \in [0,1]$. PINNs learn the solution by minimizing the residuals of the PDE and boundary conditions using a neural network.

Comparison of True Solution and PINN Approximation

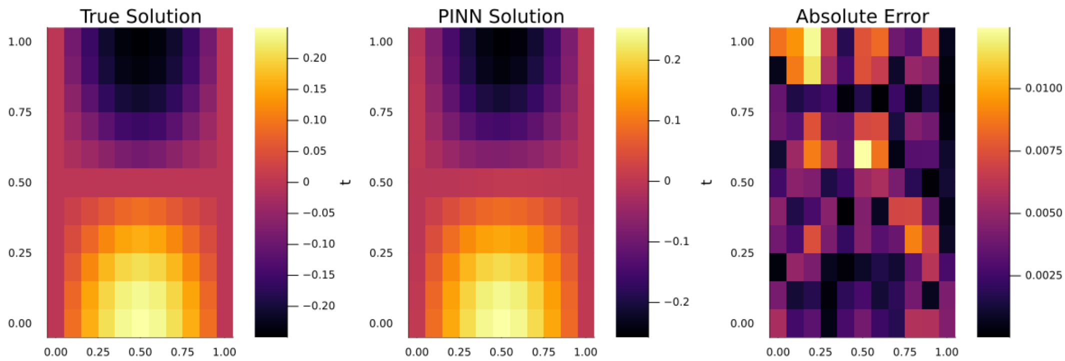

Figure 1: Comparison of the true solution, PINN solution, and absolute error across the domain.

Discussion

The neural network used in this simulation consists of two hidden layers with 16 neurons each and uses the `tanh` activation function. The training process utilized the BFGS optimizer for 1000 iterations with quadrature-based sampling over the domain.

As seen in the visualizations:

- The first plot shows the analytical (true) solution derived from a truncated Fourier series.
- The second plot displays the solution learned by the PINN.
- The third plot presents the absolute error between the two.

The PINN solution effectively captures the main structure of the wave. The absolute error remains below 0.01 across the domain, with small variations that are likely due to training limitations or the approximation capacity of the network.

This demonstrates that even with a modest network size and limited training, PINNs can provide accurate approximations of PDE solutions when guided by physical laws.