Modeling Epidemic Dynamics Using Neural Ordinary Differential Equations

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1. Introduction and Background

Understanding and predicting the spread of infectious diseases is a fundamental task in epidemiology. One of the most widely used compartmental models is the SIR model, which divides the population into Susceptible (S), Infected (I), and Recovered (R) compartments. The dynamics are governed by the following system of ODEs:

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta \frac{SI}{N} \\
\frac{dI}{dt} &= \beta \frac{SI}{N} - \gamma I \\
\frac{dR}{dt} &= \gamma I
\end{aligned}
$$

Here, β is the transmission rate and γ is the recovery rate. These equations model the time evolution of disease spread in a population of size N.

In many real-world situations, the underlying dynamics may not be fully known or may vary with time. This motivates the use of **Neural Ordinary Differential Equations (Neural ODEs)**, which offer a data-driven approach to modeling such systems by learning the right-hand side of the differential equation from observed trajectories.

2. Methodology

The assignment was carried out in the following stages:

(a) Generate Synthetic Data

We used the `DifferentialEquations.jl` package to simulate the SIR model with:

- $N = 1000$, $\beta = 0.3$, $\gamma = 0.1$
- Initial conditions: $S_0 = 999$, $I_0 = 1$, $R_0 = 0$
- Time span: 0 to 160 days

(b) Define the Neural ODE

We constructed a neural network using the `Lux.jl` and `DiffEqFlux.jl` libraries to approximate the system's dynamics. The model was:

```
Lux.Chain(
  x -> x ./ N,
  Dense(3, 32, tanh),
  Dense(32, 32, tanh),
  Dense(32, 3)
)
```

(c) Train the NeuralODE

We trained the neural network using the Adam optimizer for 500 epochs followed by BFGS for refinement. The loss was calculated as the normalized MSE between the predicted and true time series. During training, we visualized the predicted vs. true trajectories and the decreasing loss.

3. Results and Discussion

Figure 1 shows the final static summary of the NeuralODE model's performance.

The top panel displays the predicted trajectories of the susceptible, infected, and recovered populations over time, overlaid with the ground truth from the classical SIR model. The bottom panel plots the training loss across epochs, showing steady convergence of the neural network to an accurate representation of the dynamics.

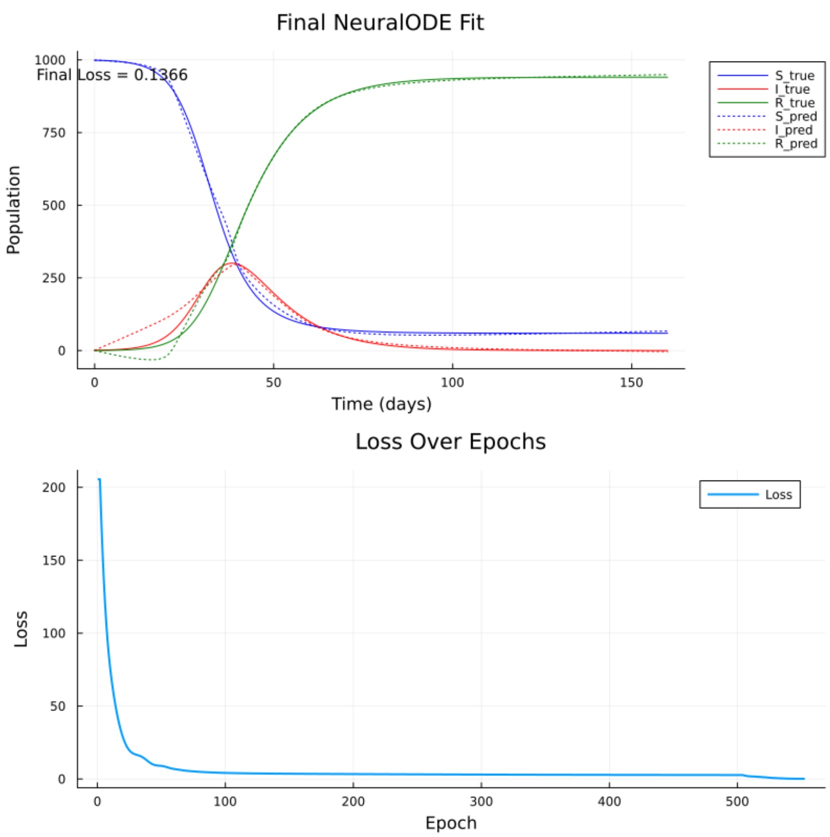

Figure 1: Top: Predicted vs. true SIR dynamics. Bottom: Loss curve during training.

The NeuralODE model learned to closely replicate the true SIR dynamics. The predicted curves aligned well with the ground truth for all three compartments. Minor discrepancies were visible early in training, particularly around the infection peak, but resolved with further optimization.

The loss curve showed a steady decrease, indicating successful convergence. The results demonstrate the feasibility of learning ODE dynamics purely from data, even for nonlinear systems.

4. Conclusion

Neural ODEs provide a powerful tool to learn continuous-time dynamics from data. In this assignment, we successfully applied a NeuralODE to approximate the classical SIR model. The learned model was able to reproduce the full trajectory of an epidemic, without requiring explicit access to the original equations.

This method is promising for real-world epidemic modeling, particularly when physical models are unknown, incomplete, or time-varying. In future work, it can be extended to more complex models like SEIR or spatial SIR PDEs.